逻辑回归
Logistic Regression

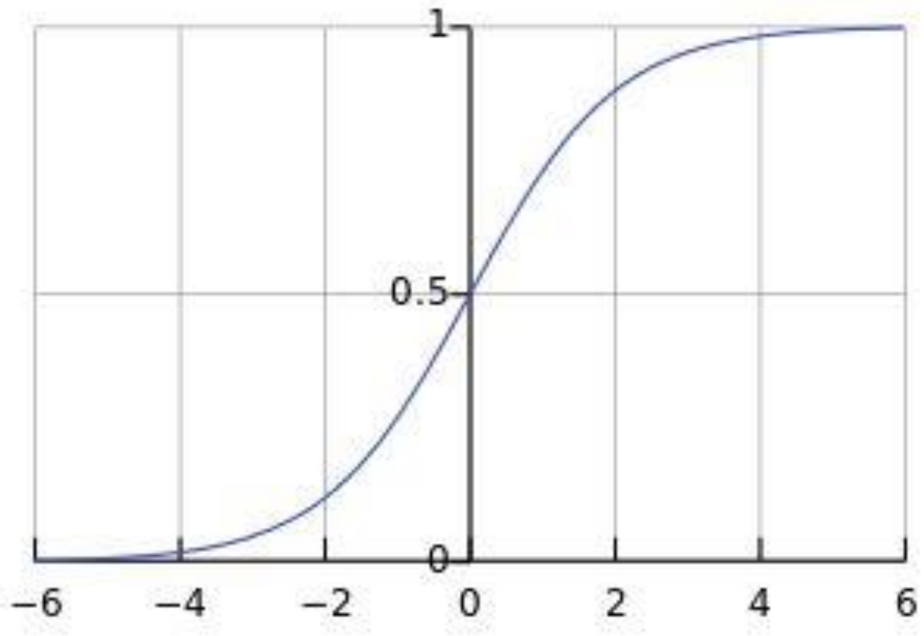
分类

- 垃圾邮件分类
- 预测肿瘤是良性还是恶性
- 预测某人的信用是否良好

Sigmoid/Logistic Function

我们定义逻辑回归的预测函数为$h_\theta(x) = g(\theta^T x)$，其中g(x)函数是sigmoid函数。

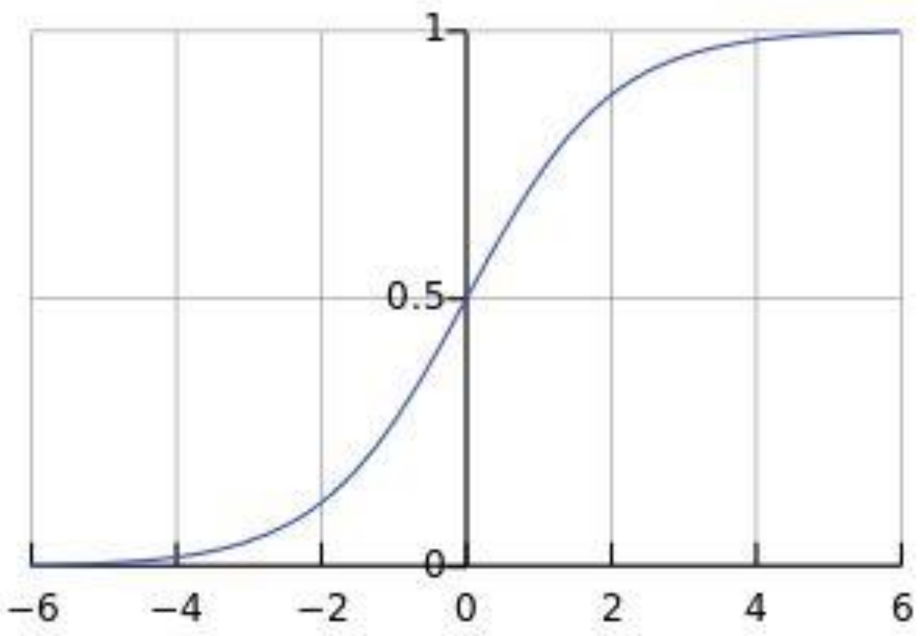
$$g_{(x)} = \frac{1}{1+\mathrm{e}^{-x}} \qquad h_\theta(x) = \frac{1}{1+\mathrm{e}^{-\theta^{\mathrm{T}}\mathrm{X}}}$$



Sigmoid/Logistic Function

$$g_{(x)} = \frac{1}{1 + e^{-x}}$$

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T X}}$$

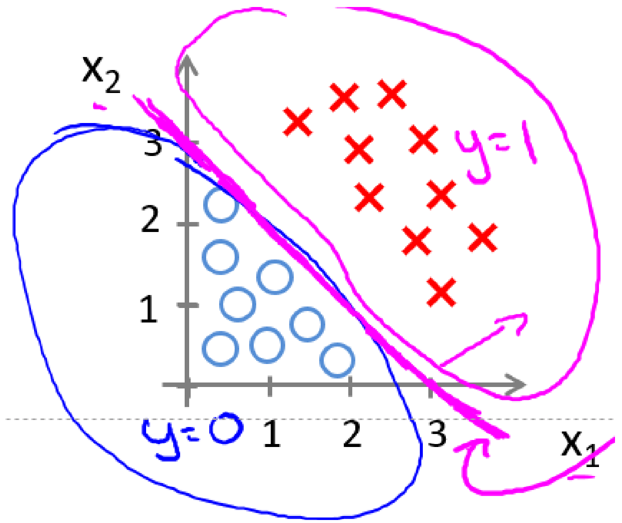


0.5可以作为分类的边界

当z≥0的时候g(z)≥0.5
当$\theta^T X$≥0的时候g($\theta^T X$)≥0.5

当z≤0的时候g(z)≤0.5
当$\theta^T X$≤0的时候g($\theta^T X$)≤0.5

决策边界

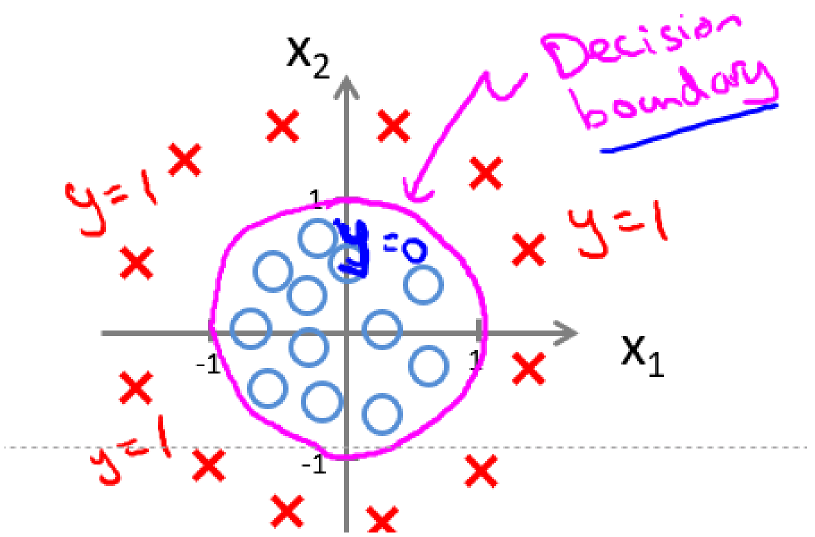


$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

-3 1 1

当 $-3 + x_1 + x_2 \geq 0$ 则 $y = 1$

决策边界



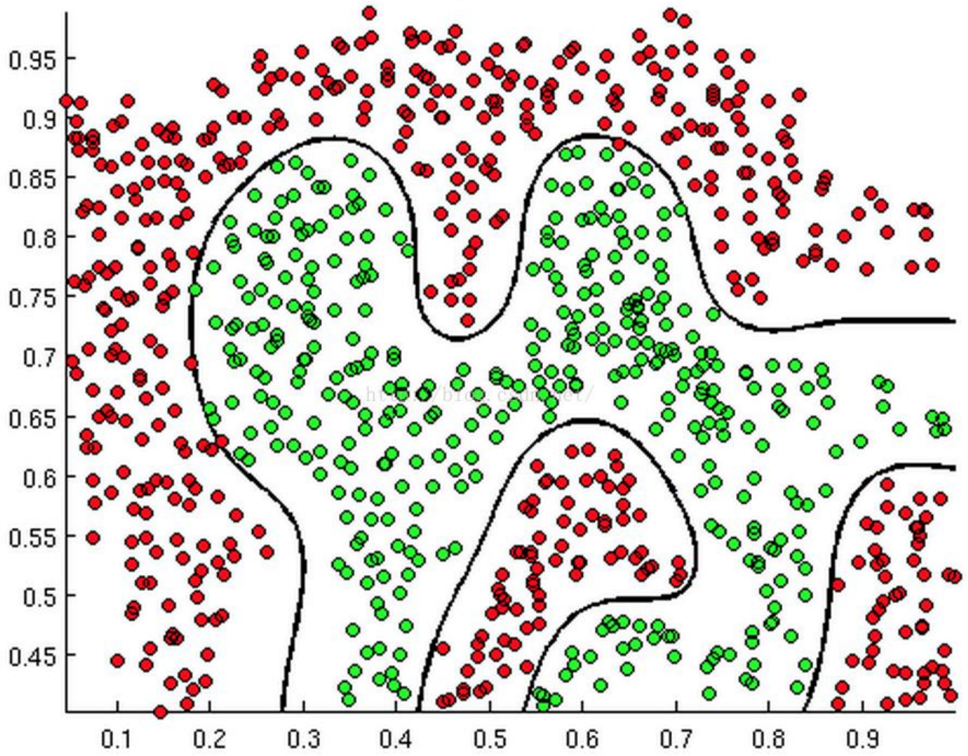
$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

-1 0 0 1 1

当 $-1 + x_1^2 + x_2^2 \geq 0$ 则 $y = 1$

决策边界

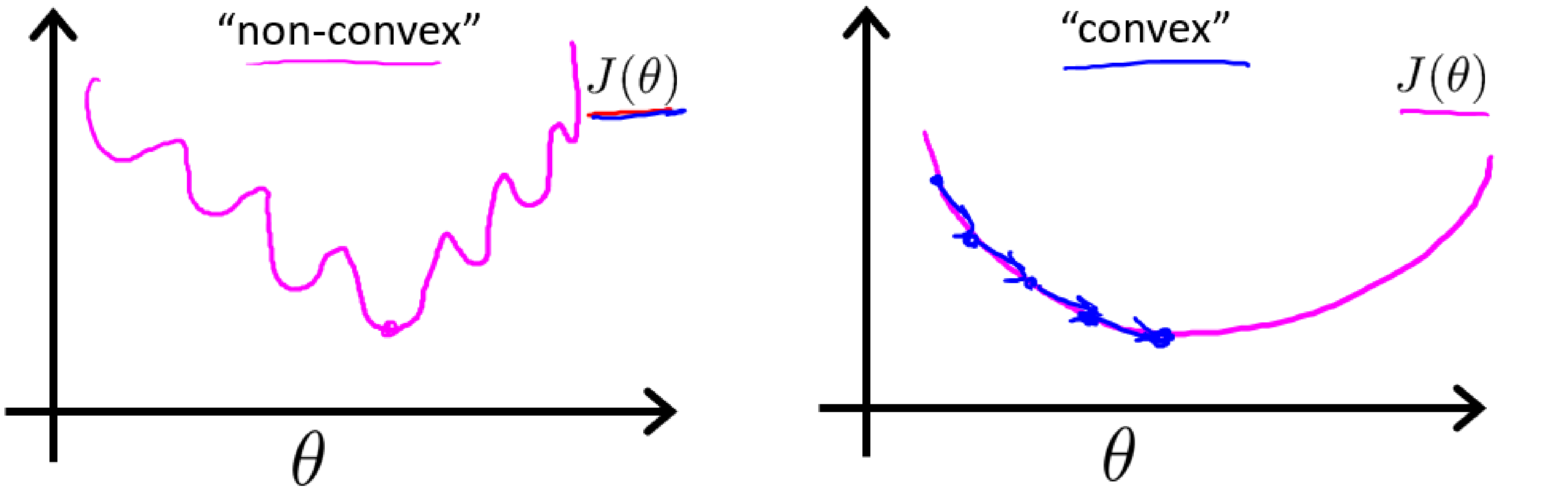
$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 \\ + \theta_4 x_1^2 x_2 + \theta_5 x_1^2 x_2^2 + \theta_6 x_1^3 x_2 + \ldots)$$



逻辑回归的代价函数

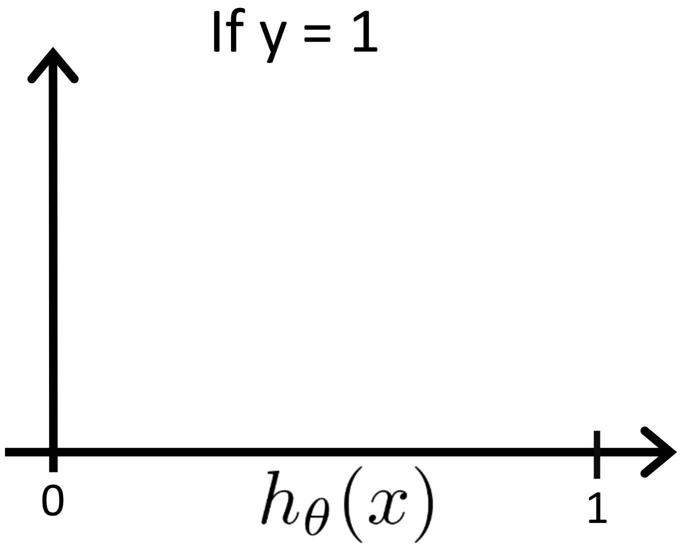
Linear regression:

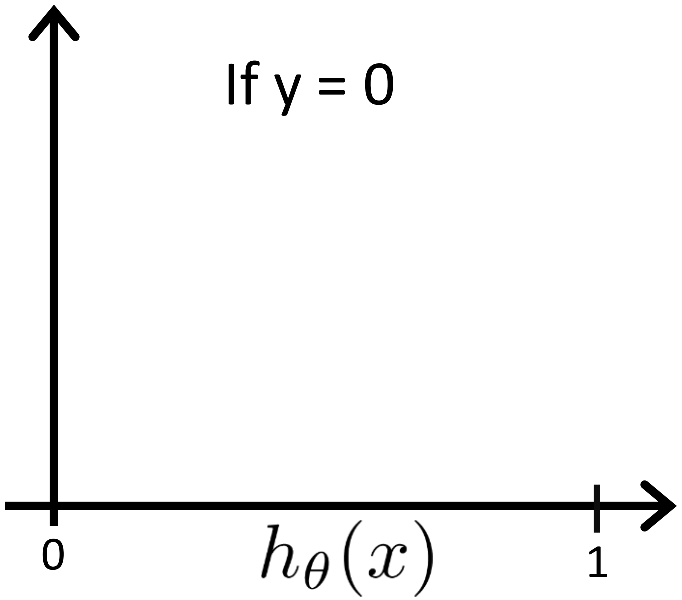
$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{2}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$



逻辑回归的代价函数

$$\mathrm{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$





当$y = 1$，$h_\theta(x) = 1$时，cost=0

当$y = 1$，$h_\theta(x) = 0$时，cost=∞

当$y = 0$，$h_\theta(x) = 1$时，cost=∞

当$y = 0$，$h_\theta(x) = 0$时，cost=0

逻辑回归的代价函数

$$\mathrm{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

$$Cost(h_\theta(x), y) = -ylog(h_\theta(x)) - (1 - y)log(1 - h_\theta(x))$$

当y=1时，$Cost(h_\theta(x), y) = -log(h_\theta(x))$
当y=0时，$Cost(h_\theta(x), y) = -log(1 - h_\theta(x))$

逻辑回归的代价函数

Gradient Descent

$$J(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)}))]$$

Want $\min_\theta J(\theta)$:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

(simultaneously update all θ_j)

$\}$

逻辑回归的代价函数

$$Cost(h_\theta(x),y) = -\frac{1}{m}\sum_{i=1}^{m} ylog(h_\theta(x)) + (1-y)log\left(1-h_\theta(x)\right)$$

$$\frac{\partial Cost(h_\theta(x),y)}{\partial\theta} = -\frac{1}{m}\sum_{i=1}^{m}\left(\frac{y}{h_\theta(x)} - \frac{(1-y)}{1-h_\theta(x)}\right)\frac{\partial h_\theta(x)}{\partial\theta}$$

$$= -\frac{1}{m}\sum_{i=1}^{m}\left(\frac{y}{h_\theta(x)} - \frac{(1-y)}{1-h_\theta(x)}\right)h_\theta{}'(x)\,x$$

$$= \frac{1}{m}\sum_{i=1}^{m}\frac{h_\theta{}'(x)x}{h_\theta(x)\left(1-h_\theta(x)\right)}(h_\theta(x)-y)$$

$$= \frac{1}{m}\sum_{i=1}^{m} x\,(h_\theta(x)-y)$$

$$h_\theta(x) = g(\theta^T x)$$

$$g(x) = \frac{1}{1+\mathrm{e}^{-x}}$$

$$h_\theta{}'(x) = h_\theta(x)\left(1-h_\theta(\mathrm{x})\right)$$

逻辑回归的代价函数

Gradient Descent

$$J(\theta) = -\frac{1}{m}[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1-y^{(i)}) \log(1-h_\theta(x^{(i)}))]$$

Want $\min_\theta J(\theta)$:

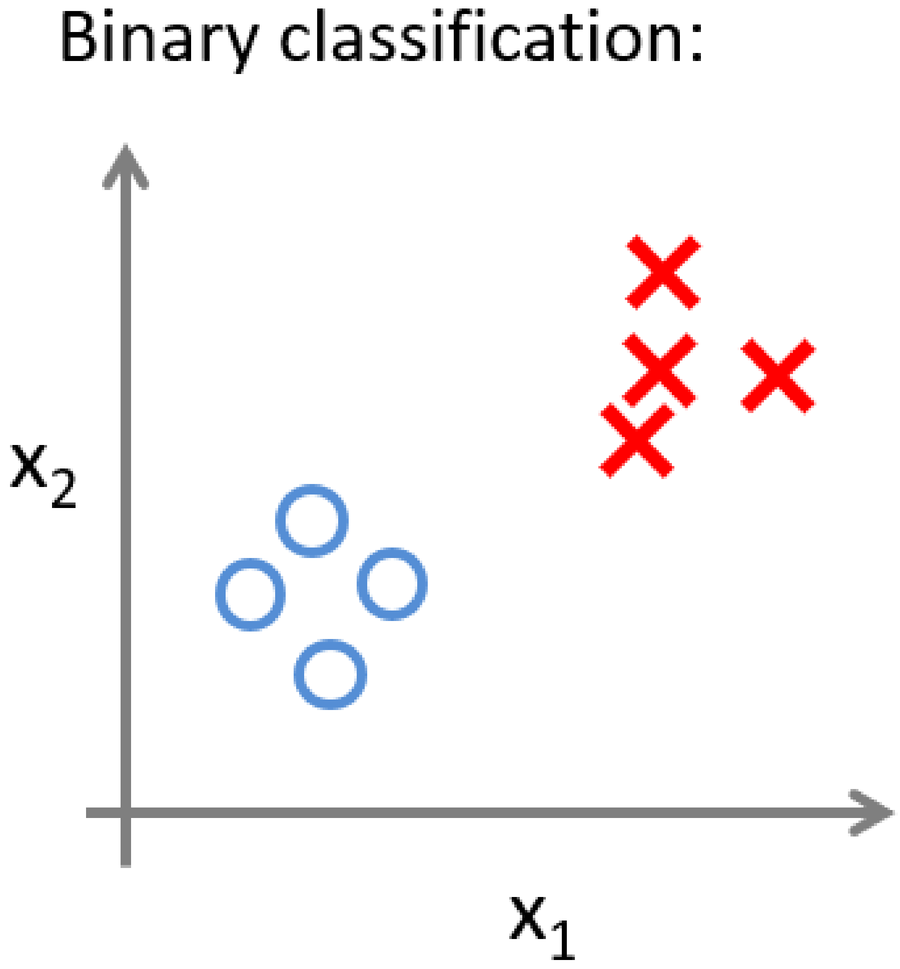
Repeat $\{$

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

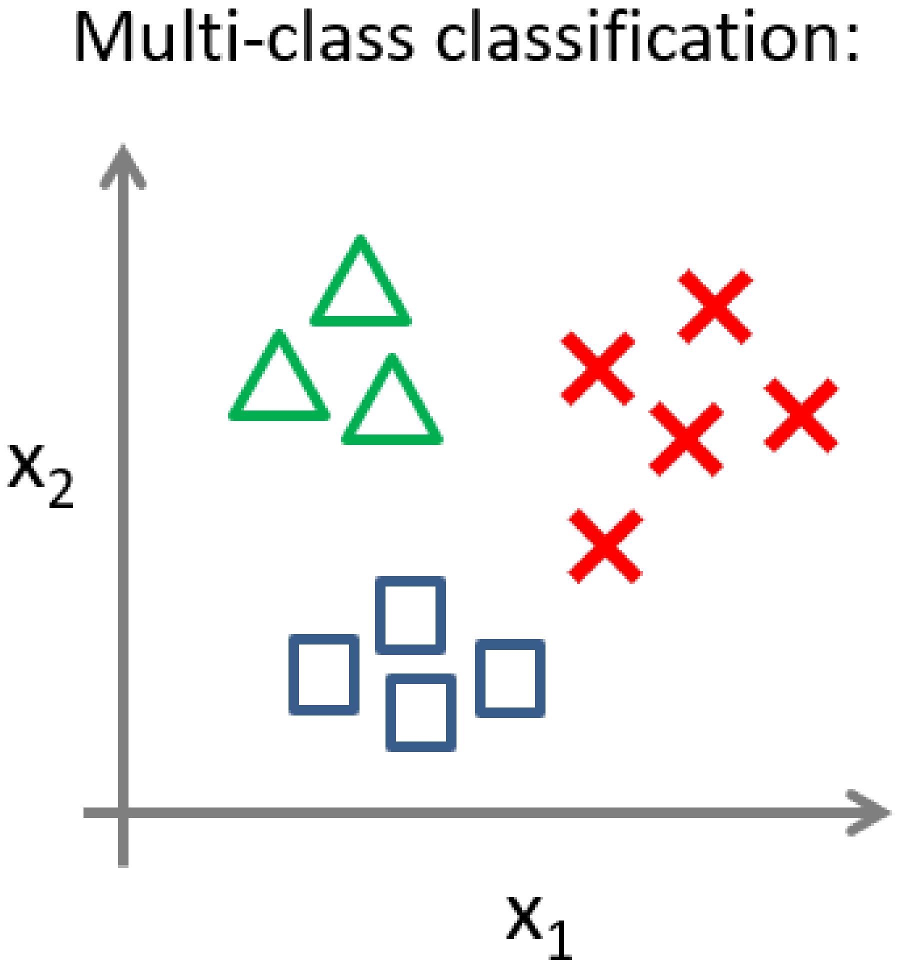
(simultaneously update all θ_j)

$\}$

多分类

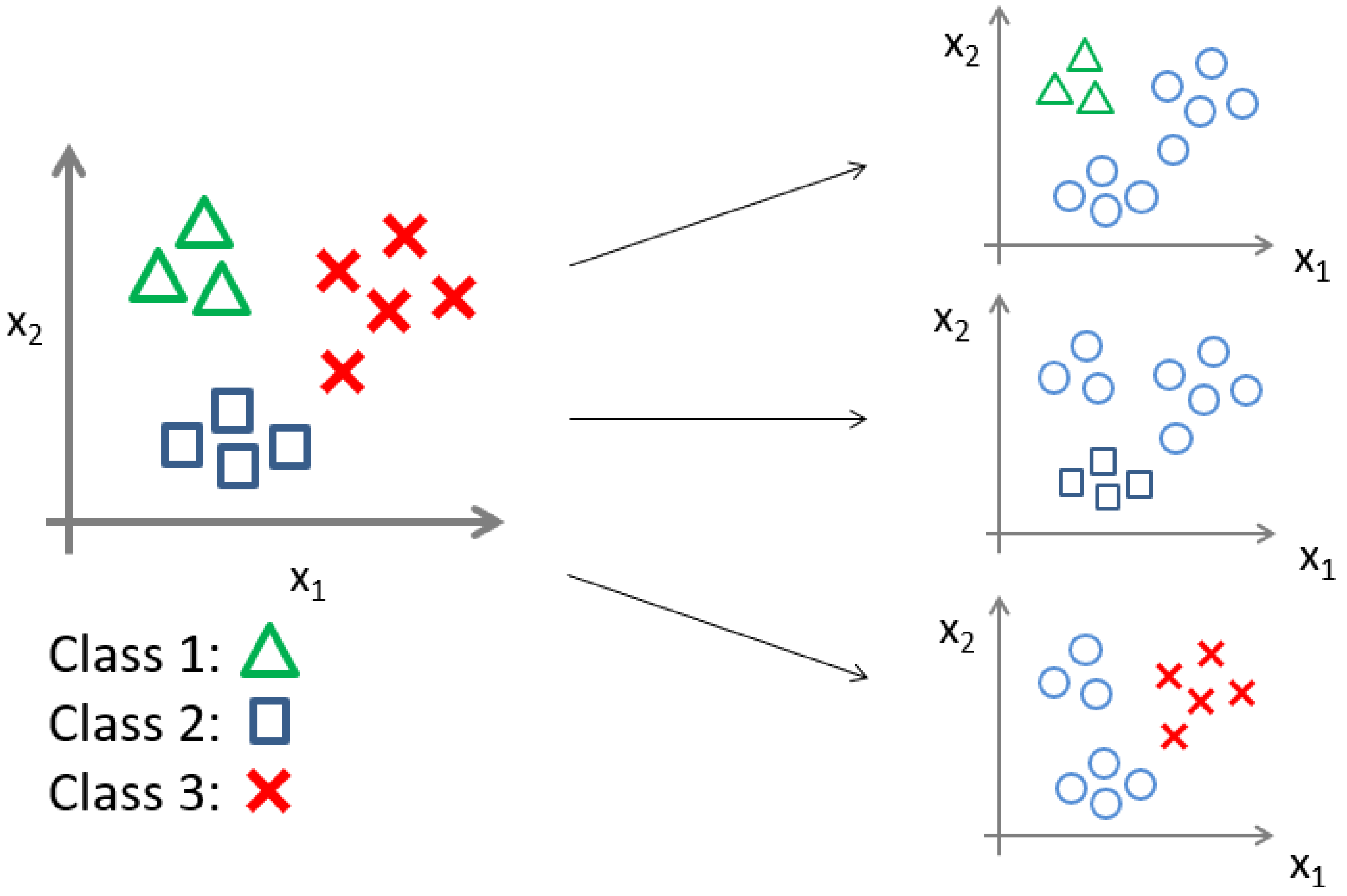


Binary classification:
x2
x1



Multi-class classification:
x2
x1

多分类



逻辑回归正则化

普通逻辑回归代价函数：

$$J(\theta) = -\left[\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1-y^{(i)}) \log(1-h_\theta(x^{(i)}))\right]$$

正则化逻辑回归代价函数：

$$J(\theta) = \left[-\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log\left(h_\theta(x^{(i)}) + (1-y^{(i)}) \log 1 - h_\theta(x^{(i)})\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$

求导：

$$\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_0^{(i)}$$

$$\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_1^{(i)} - \frac{\lambda}{m}\theta_1$$

$$\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_2^{(i)} - \frac{\lambda}{m}\theta_2$$

……

正确率/召回率/F1指标

正确率与召回率

正确率与召回率（Precision & Recall）是广泛应用于信息检索和统计学分类领域的两个度量值，用来评价结果的质量。

一般来说，正确率就是检索出来的条目有多少是正确的，召回率就是所有正确的条目有多少被检索出来了。

F1值$=2*\frac{正确率*召回率}{正确率+召回率}$。是综合上面二个指标的评估指标，用于综合反映整体的指标。

这几个指标的取值都在0-1之间，数值越接近于1，效果越好。

举例

某池塘有1400条鲤鱼，300只虾，300只鳖。现在以捕鲤鱼为目的。撒一大网，逮着了700条鲤鱼，200只虾，100只鳖。那么，这些指标分别如下：

正确率 = 700 / (700 + 200 + 100) = 70%
召回率 = 700 / 1400 = 50%
F值 = 70% * 50% * 2 / (70% + 50%) = 58.3%

举例

某池塘有1400条鲤鱼，300只虾，300只鳖。现在以捕鲤鱼为目的。
撒一大网，逮着了所有的鱼虾鳖：

正确率 = 1400 / (1400 + 300 + 300) = 70%
召回率 = 1400 / 1400 = 100%
F值 = 70% * 100% * 2 / (70% + 100%) = 82.35%

正确率与召回率

我们希望检索结果Precision越高越好，同时Recall也越高越好，但事实上这两者在某些情况下有矛盾的。比如极端情况下，我们只搜索出了一个结果，且是准确的，那么Precision就是100%，但是Recall就很低；而如果我们把所有结果都返回，那么比如Recall是100%，但是Precision就会很低。

因此在不同的场合中需要自己判断希望Precision比较高或是Recall比较高。

综合评价指标

正确率与召回率指标有时候会出现的矛盾的情况，这样就需要综合考虑他们，最常见的方法就是F-Measure（又称为F-Score）：

$$F_\beta = (1+\beta^2) \cdot \frac{\text{precision} \cdot \text{recall}}{(\beta^2 \cdot \text{precision}) + \text{recall}}$$

当β =1时，就是常见的F1指标：

$$F_1 = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

梯度下降法-逻辑回归

sklearn-逻辑回归

非线性逻辑回归

sklearn-非线性逻辑回归