Lecture 4

Quotient Rings

Recall that given a ring homomorphism $f : R \to S$, the kernel of f, $\operatorname{Ker} f$, is a subring of R.

Definition 4.1

Given a ring homomorphism $f : R \to S$, let $I = \operatorname{Ker} f$ and $r \in R$.
The **coset** of $r \in R$ with respect to f (or w.r.t I) is the set

$$r + I \coloneqq \{r + x | x \in I = \operatorname{Ker} f\}$$

The **quotient ring** of R by I is the set

$$R/I \coloneqq \{r + I | r \in R\}$$

Proposition 4.1

Given a ring homomorphism $f : R \to S$ with $I = \operatorname{Ker} f$, the quotient ring R/I is a ring with operations

$$(r + I) + (s + I) \coloneqq (r + s) + I$$
$$(r + I) \cdot (s + I) \coloneqq (r \cdot s) + I$$