MA677 HW1.

7/ (a) $P(T=n)=\left(\frac{5}{6}\right)^{n-1}\cdot\frac{1}{6}$

(b). $P(T>3)=\left(\frac{5}{6}\right)^3$

$=\frac{125}{216}$

(c) $P(T>6\mid T>3)=\frac{P(T>6)}{P(T>3)}=\frac{\left(\frac{5}{6}\right)^6}{\left(\frac{5}{6}\right)^3}=\left(\frac{5}{6}\right)^3=\frac{125}{216}$

10. (a) $P(X=k)=\frac{\binom{n_1}{k}\cdot\binom{N-n_1}{n_2-k}}{\binom{N}{n_2}}$

(b). $P(X=n_{12})=$

16 $P(X\le 1)=P(X=1)+P(X=0)$

$=(1-\cancel{0.01})^{300}+(\cancel{3}\cdot e^{-3})$

$=0.99^{300}+3e^{-3}$

$=0.049+0.15$

$=0.199\approx 0.2$

18 (a) Use Possion Dist.

$P(X=0)=\left(1-\frac{\lambda}{n}\right)^n$

Since 500 cookies with 600 raisins.

$\lambda=n\cdot p=\frac{600}{500}=1.2.$

So $P(X=0)=e^{-1.2}=0.301.$

Scanned with CamScanner

(b) $P(X=2)$

$\lambda = n \cdot p = 400 \cdot \frac{1}{500} = \frac{400}{500} = 0.8$.

$P(X=2) = \frac{\lambda^2}{2!} e^{-0.8}$.

$= \frac{0.8^2}{2!} \cdot e^{-0.8}$

$= \frac{0.64}{2} \cdot e^{-0.8}$

$= 0.32 \cdot 0.45$

$= 14.4\%$.

(c) $\lambda = n \cdot p = \frac{400+600}{500} = 2$

$P(X \geq 2) = 1 - P(X=1) - P(X=0)$

$= 1 - \lambda e^{-\lambda} - e^{-\lambda}$

$= 1 - 3e^{-2}$

$= 0.594$

25. $E(y) = 2 \cdot \frac{5^2 \cdot e^{-5}}{2!} + (2+5) \frac{5^3 e^{-5}}{3!} + \cdots + (2+5 \cdot 98) \cdot \frac{5^{98} e^{-5}}{98!}$

Total payment $= 100 \cdot 0.1 = \$10$

27. $P(X \geq 1) = 1 - P(X=0)$

$= 1 - \binom{100}{0} \cdot 0.999$

$= 1 - 0.9047$

$= 0.0953$

28 $P(X \geq 2) = 1 - P(X=0) - P(X=1)$ $\lambda = 100 \cdot 0.04 = 4$

$= 1 - e^{-\lambda} - \lambda e^{-\lambda}$ -

$= 1 - 5e^{-4}$

Scanned with CamScanner

38. (a) $P(X=1) = \binom{5}{1} \cdot 0.25 \cdot 0.75^4 = 0.3955$

(3) $h(20,5,5,1) = \dfrac{\binom{5}{1}\cdot\binom{20-5}{5-1}}{\binom{20}{5}} = 0.4402$

1. (a) $Y = U + 2$

So $U \sim Unif[0,1] \Rightarrow Y \sim Unif[2,3]$

$f(Y) = \frac{1}{3-2} = 1$

$F(Y) = Y - 2$ on $[2,3]$

(b) $Y = U^3$

$U \sim Unif[0,1] \Rightarrow Y \sim Unif[0,1]$

$f(Y) = \frac{1}{3}X^{-\frac{2}{3}}$ on $[0,1]$

$F(x) = x^{1/3}$ on $[0,1]$

17. (a) $F(X) = \begin{cases} 0 & \text{if } X < 0 \\ \sin^2\left(\frac{\pi X}{2}\right) & \text{if } 0 \le X \le 1 \\ 1 & \text{if } 1 < X \end{cases}$

f_X ~~for~~ $X. = \begin{cases} \frac{\pi}{2}\sin(\pi X) & 0 \le X \le 1 \\ 0 & \text{otherwise} \end{cases}$

(b) $\sin^2\left(\frac{\pi}{8}\right) = .146$

21. $Y = F(X)$

$P(Y \le y) = P(F(X) \le y) = P(X \le F^{-1}(y)) = F(F^{-1}(y)) = y$ on $[0,1]$

Scanned with CamScanner

37. $X \sim N$

$Y = e^{X}$. $Y \sim 1$

$F_Y(y) = \frac{1}{\sqrt{2\pi}y} \cdot e^{-\frac{\log^2(y)}{2}}$ for $y > 0$

Scanned with CamScanner