HOMEWORK 7

QUESTION 1

(a.)

$$\nabla^2 f(\boldsymbol{w}) = \nabla\{-\sum_{i=1}^{m}\left[1-\sigma\left(y_i \boldsymbol{x}_i^T \boldsymbol{w}\right)\right] y_i \boldsymbol{x}_i\}$$

$$= \nabla\{\sum_{i=1}^{m}\left[\sigma\left(y_i \boldsymbol{x}_i^T \boldsymbol{w}\right)\right] y_i \boldsymbol{x}_i\} \tag{1}$$

$$= \sum_{i=1}^{m}\left[\frac{e^{-y_i \boldsymbol{x}_i^T \boldsymbol{w}}}{(1+e^{-y_i \boldsymbol{x}_i^T \boldsymbol{w}})^2}\right](y_i \boldsymbol{x}_i^T y_i \boldsymbol{x}_i)^T$$

And we know that $y_i^2 = 1$, therefore

$$\sum_{i=1}^{m}\left[\frac{e^{-y_i \boldsymbol{x}_i^T \boldsymbol{w}}}{(1+e^{-y_i \boldsymbol{x}_i^T \boldsymbol{w}})^2}\right]\boldsymbol{x}_i \boldsymbol{x}_i^T = \sum_{i=1}^{m}\sigma'\left(y_i \boldsymbol{x}_i^T \boldsymbol{w}\right)\boldsymbol{x}_i \boldsymbol{x}_i^T \tag{2}$$

(b.)

Case 1

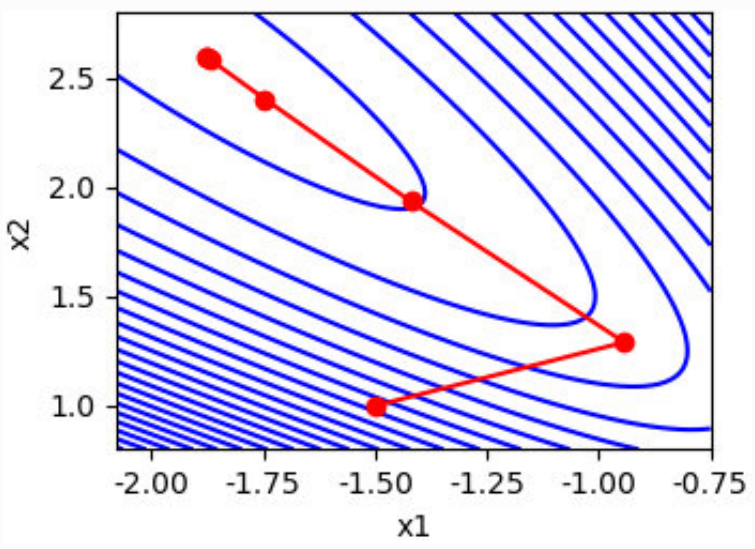
$w_0 = (-1.5, 1)^T$

Converges.

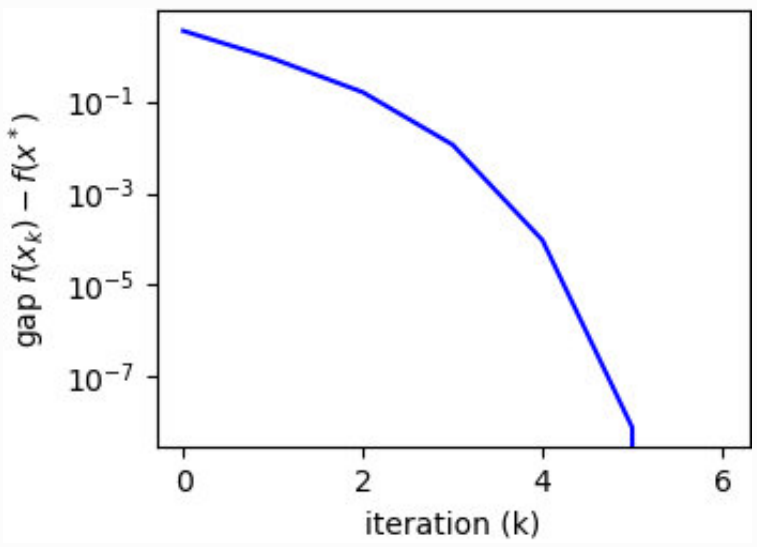
Log

number of iterations in outer loop	solution	value
6	(-1.87973941, 2.60188452)	3.3295135687527964

trajectory



gap



Case 2

$w_0 = (1, 1)^T$

The error message below indicates that in this case it **doesn't converge.**

```
  File "/Users/husky/opt/anaconda3/lib/python3.9/site-packages/numpy/linalg/linalg.py", line 545, in inv
    ainv = _umath_linalg.inv(a, signature=signature, extobj=extobj)
  File "/Users/husky/opt/anaconda3/lib/python3.9/site-packages/numpy/linalg/linalg.py", line 88, in _raise_li
nalgerror_singular
    raise LinAlgError("Singular matrix")
numpy.linalg.LinAlgError: Singular matrix
```

(c.)

Case 1

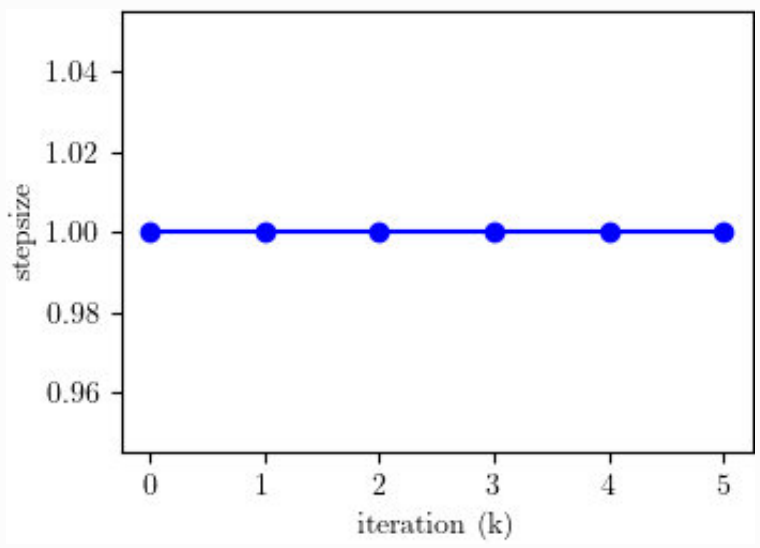
$w_0 = (-1.5, 1)^T$

Converges.

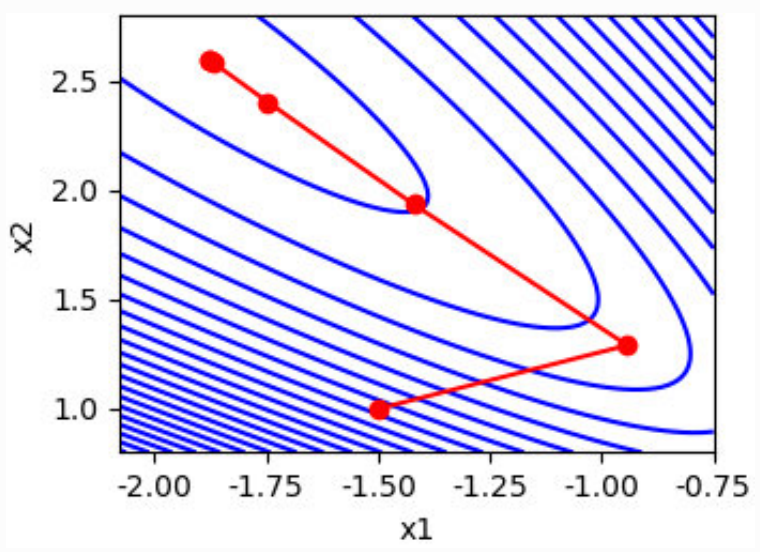
> *Log*

number of iterations in outer loop	total number of iterations in inner loop
6	0
solution	**value**
(-1.87973941 2.60188452)	3.3295135687527964

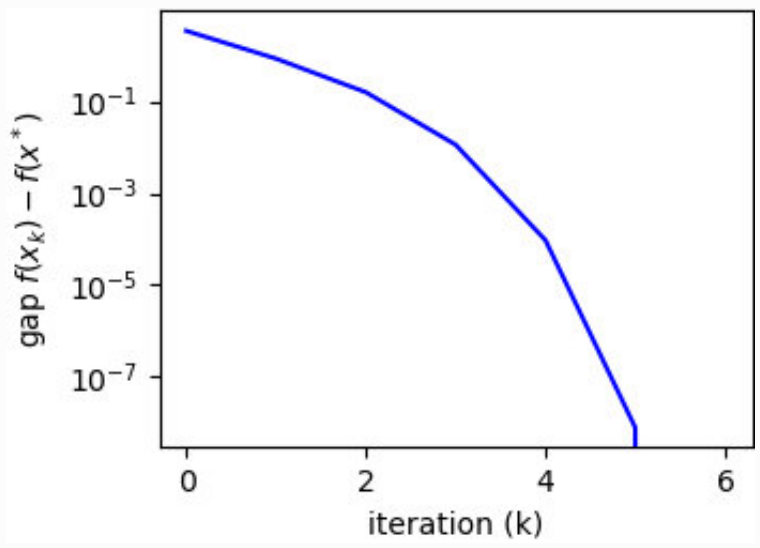
stepsize



trajectory



gap



Case 2

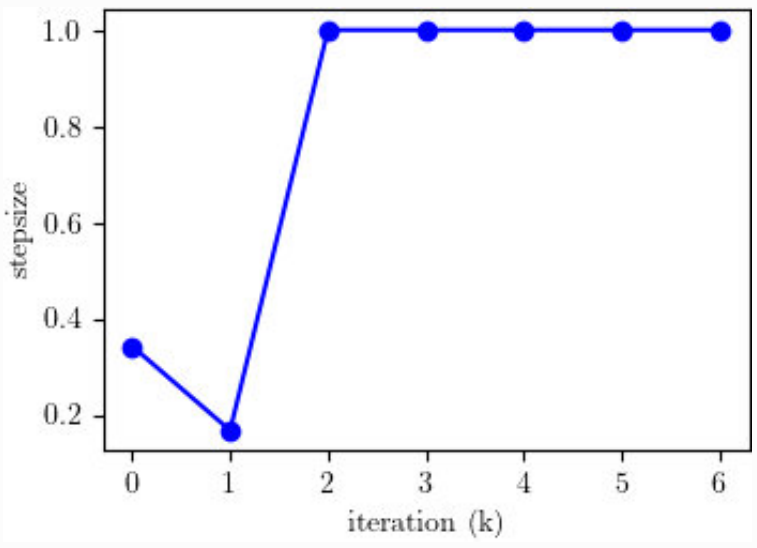
$w_0 = (1, 1)^T$

Converges.

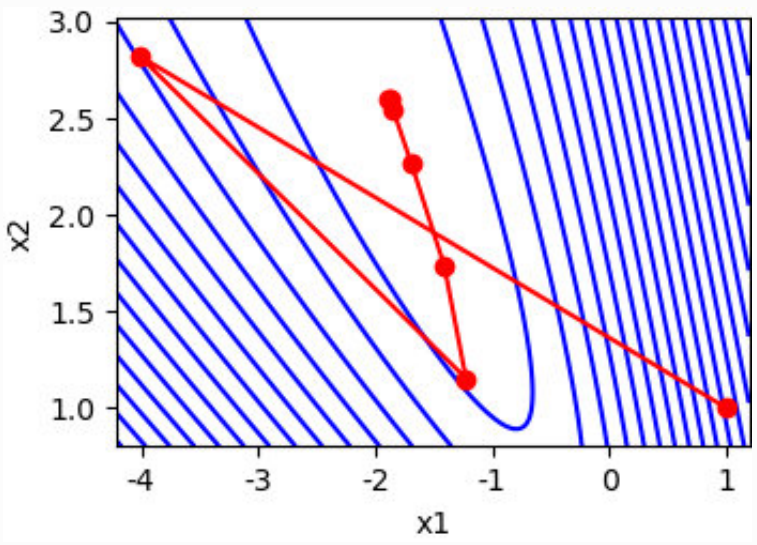
> *Log*

number of iterations in outer loop	total number of iterations in inner loop
7	8
solution	**value**
(-1.87973889 2.60188365)	3.329513568753013

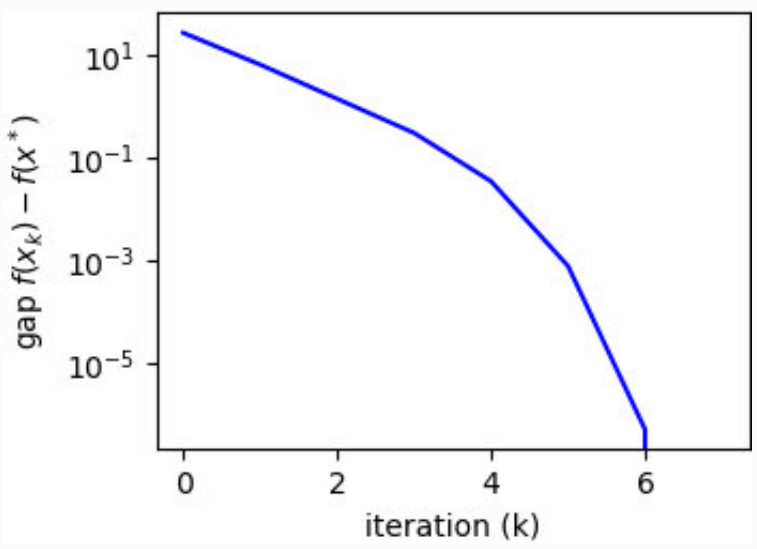
stepsize



trajectory



gap



By the experiments above, we could conclude that:

Pure Newton's method	***Damped Newton's method***
Not always converge, depending on **the initial point**	always converge
extremely fast	Slower

QUESTION 2

(a.)

Newton step:

$$\boldsymbol{x} \leftarrow \boldsymbol{x} - \left[\nabla^2 f(\boldsymbol{x})\right]^{-1} \nabla f(\boldsymbol{x}) \tag{3}$$

When $f(x) = (x-a)^6$, then its newton step is $\frac{a-x}{5}$.

(b.)

$$x_{k+1} - a = \frac{4(x_k - a)}{5} \leftarrow x_{k+1} = x_k + \frac{a - x_k}{5} \tag{4}$$

Therefore,

$$y_{k+1} = \frac{4}{5} y_k \tag{5}$$

(c.)

Since

$$|x_{k+1} - a| = y_{k+1} = \frac{4}{5} y_k = (\frac{4}{5})^2 y_{k-1} = \cdots = (\frac{4}{5})^k |x_1 - a| \tag{6}$$

$$\lim_{k \to +\infty} |x_k - a| = 0 \tag{7}$$

we conclude that x_k converges exponentially to a.

QUESTION 3

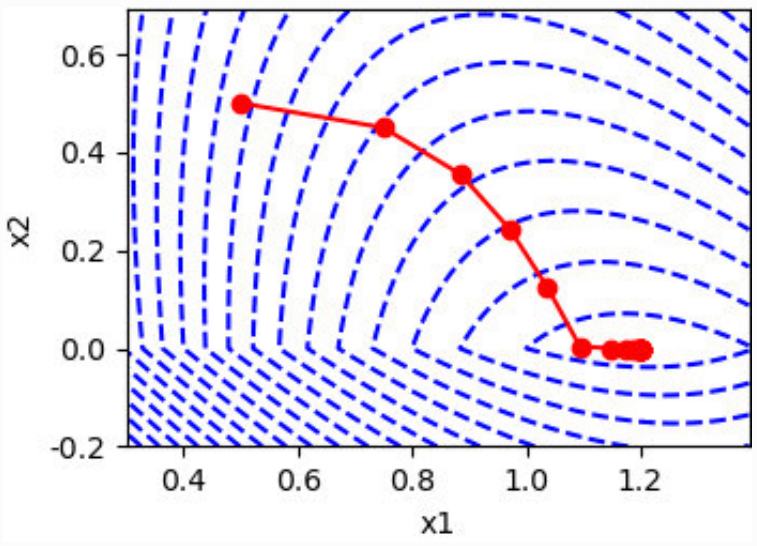
(a.)

Case 1

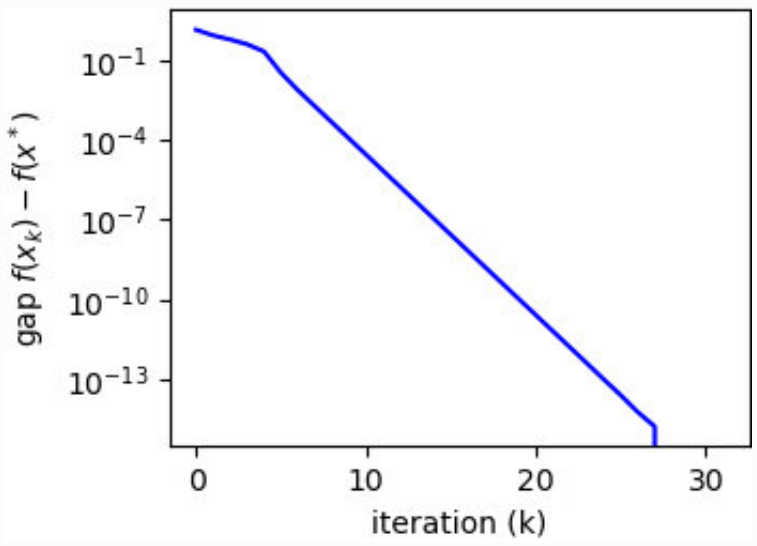
> *log*

lambda	number of iterations	solution	Value
2	31	(1.2, 0)	4.9

trajectory



gap



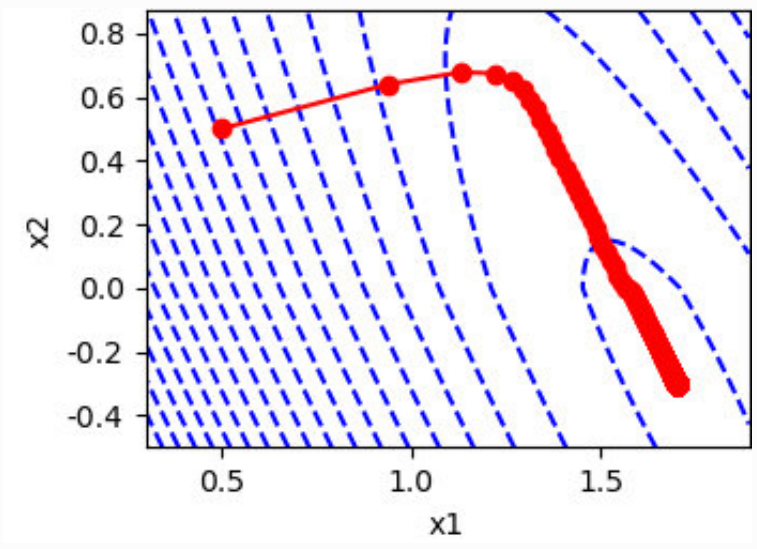
Case 2

log

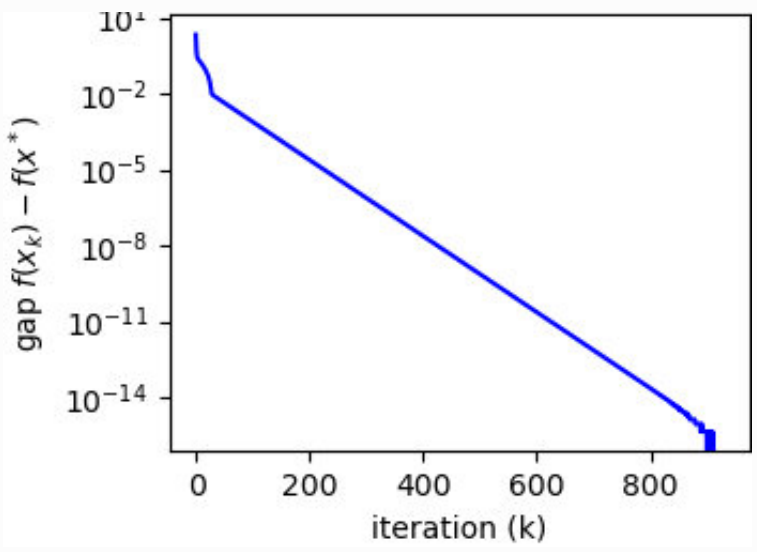
lambda	number of iterations	solution	Value
0.1	927	(1.69999998 , -0.29999995)	2.2500000000000004

No zero in the solution!

trajectory



gap



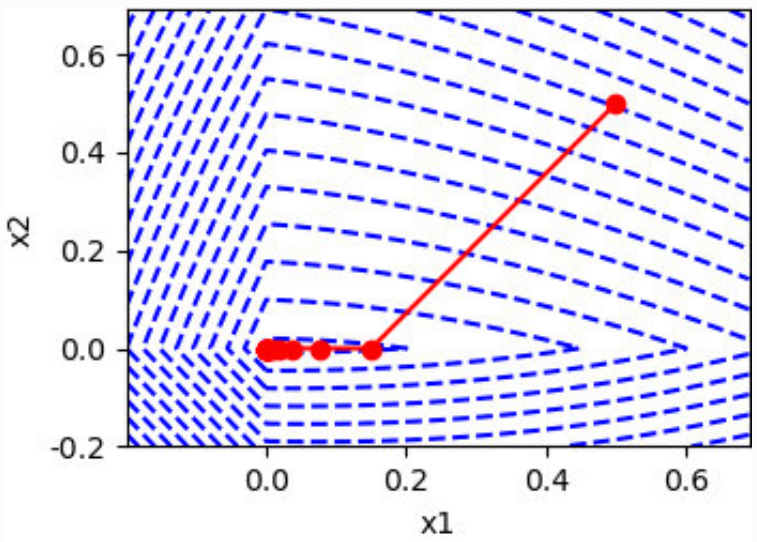
Case 3

log

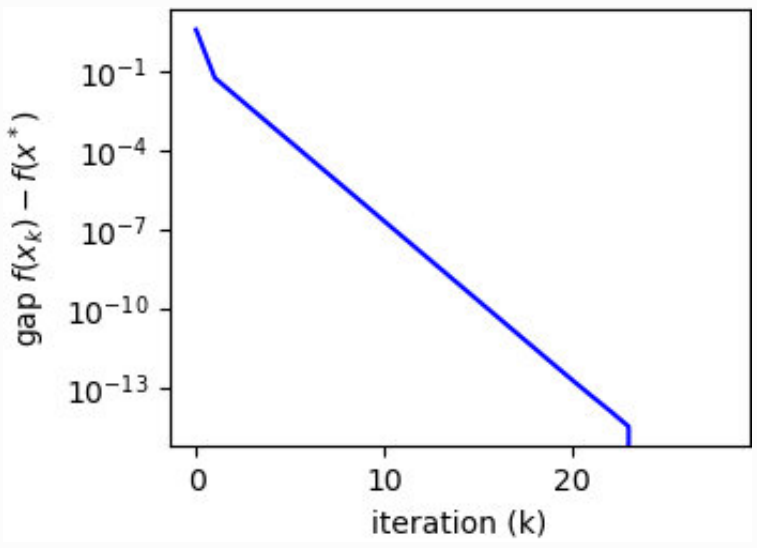
lambda	number of iterations	solution	Value
8	28	(1.11758702e-09, 0.00000000e+00)	8.5

Two zeros in the solution

trajectory


0.6
0.4
0.2
0.0
-0.2
x2
0.0
0.2
0.4
0.6
x1

gap


10^{-1}
10^{-4}
10^{-7}
10^{-10}
10^{-13}
gap $f(x_k) - f(x^*)$
0
10
20
iteration (k)