CS3319 Foundations of Data Science

2.Data Fundamentals

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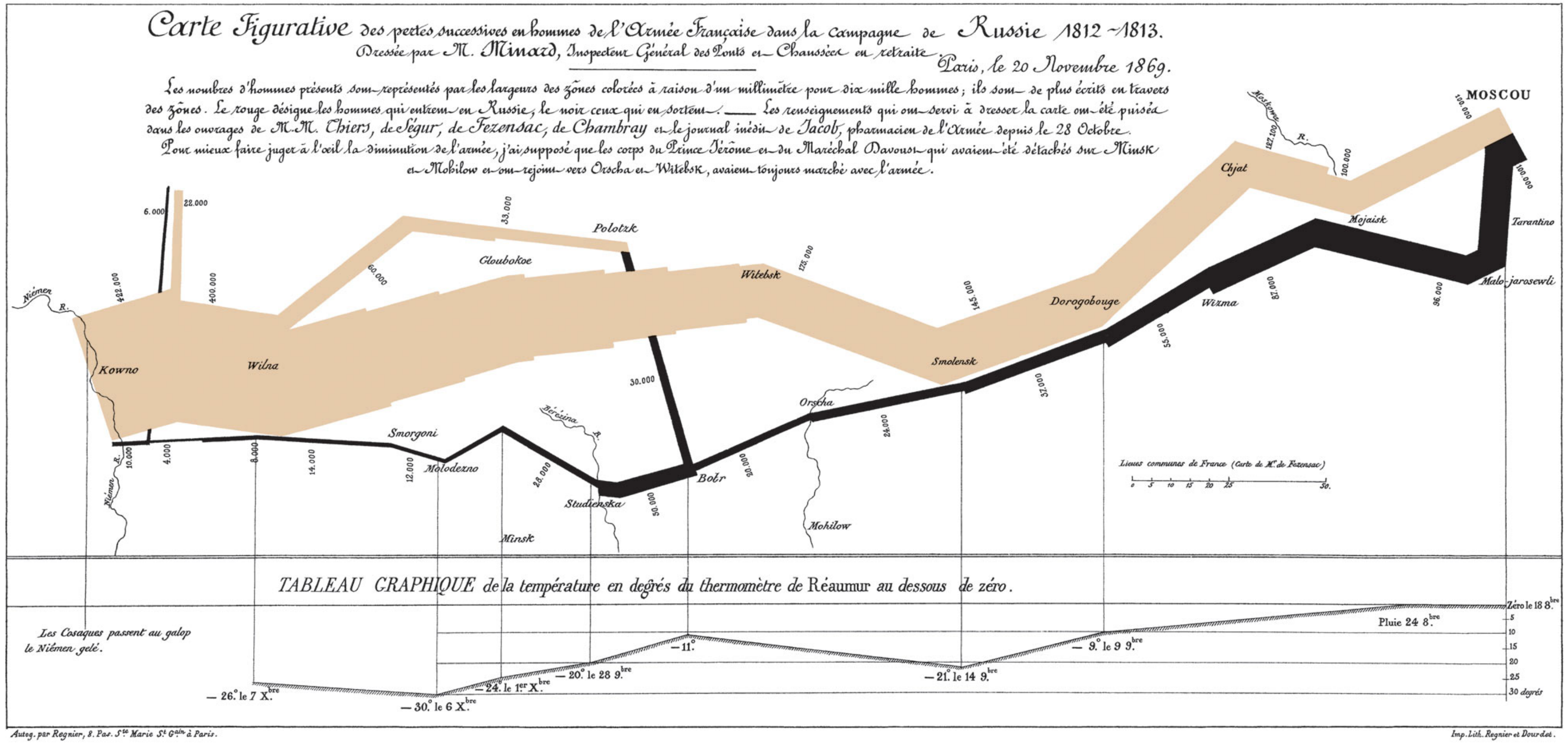
Understanding Data



Charles Minard's map of Napoleon's Russian campaign of 1812

Content

- Data Attributes
- Basic Statistical Descriptions of Data
- Measuring Data Similarity and Dissimilarity
- Probability Inequalities

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Data Attributes

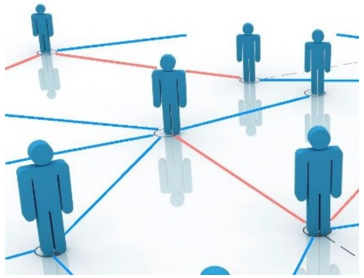
- **Data object**: an entity in the dataset
- A **data attribute** is a particular data field, representing a characteristic or feature of a data object (Feature)

学号	姓名	入学年份
1001	张三	2018
1003	李四	2019
1099	王二	2020

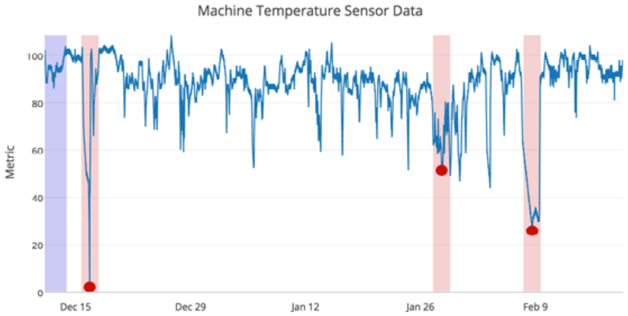
Name in the database

RGB value of a pixel

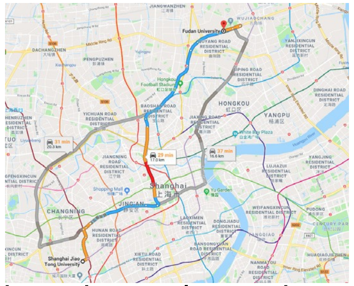
The frequency of a word



The friends of a user



The reading at time t



The time-location of a trajectory point

Record Data

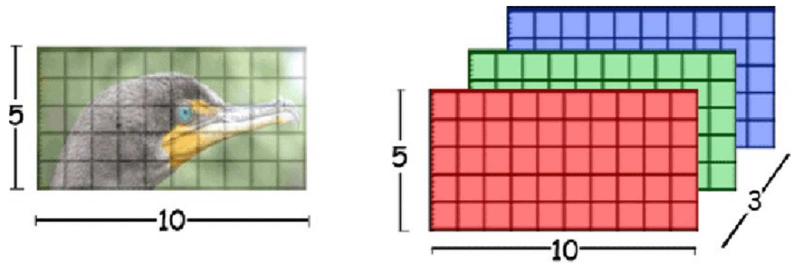
- Relational databases
  - Each row represents a data object
  - Each column represents a data attribute

WEEKDAY	GENDER	AGE	CITY
Tuesday	Male	28	London
Monday	Female	24	New York
Tuesday	Female	36	Hong Kong
Thursday	Male	17	Tokyo

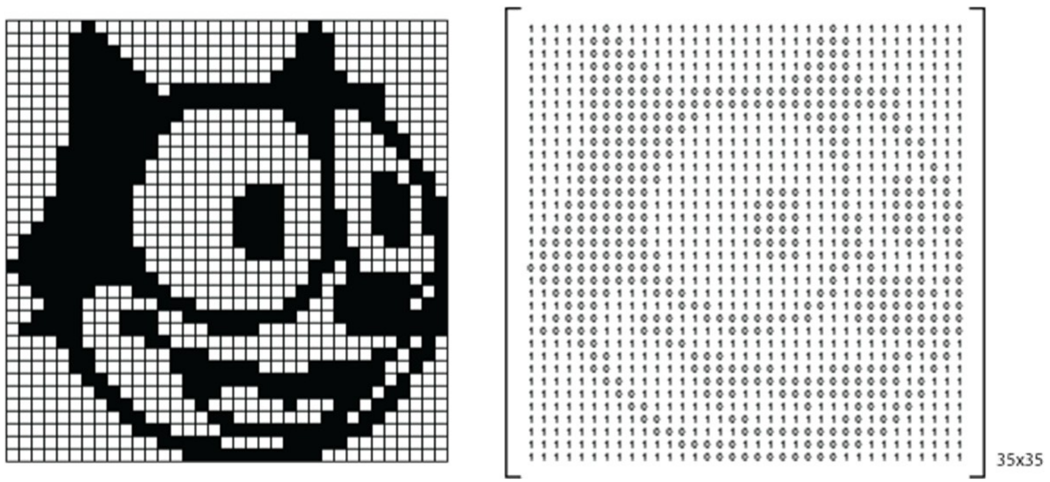
JSON Format:
{
WEEKDAY: Monday;
GENDER: Female;
AGE: 24;
CITY: New York;
}

Image Data

- A 3-layer matrix (3*height*width) of [0,255] real value



Original Color Image   Matlab RGB Matrix



Text Data

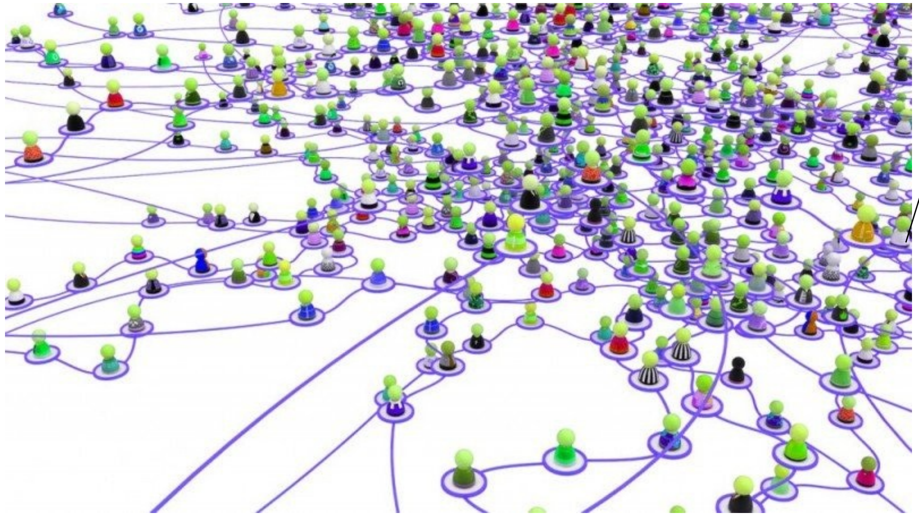
- A sequence of words/tokens that represents semantic meanings.

Text mining, also referred to as text data mining, roughly equivalent to text analytics, is the process of deriving high-quality information from text.

Bag-of-Words Format:
{
text: 4;
mining: 2;
also: 1;
referred: 1;
to: 2;
as: 1;
data: 1;
roughly: 1;
equivalent: 1;
analytics: 1;
is: 1;
the: 1;
process: 1;
of: 1;
deriving: 1;
high-quality: 1;
information: 1;
from: 1;
}

Graph Data

- A directed/undirected graph
  - Possibly with additional information for nodes and edges



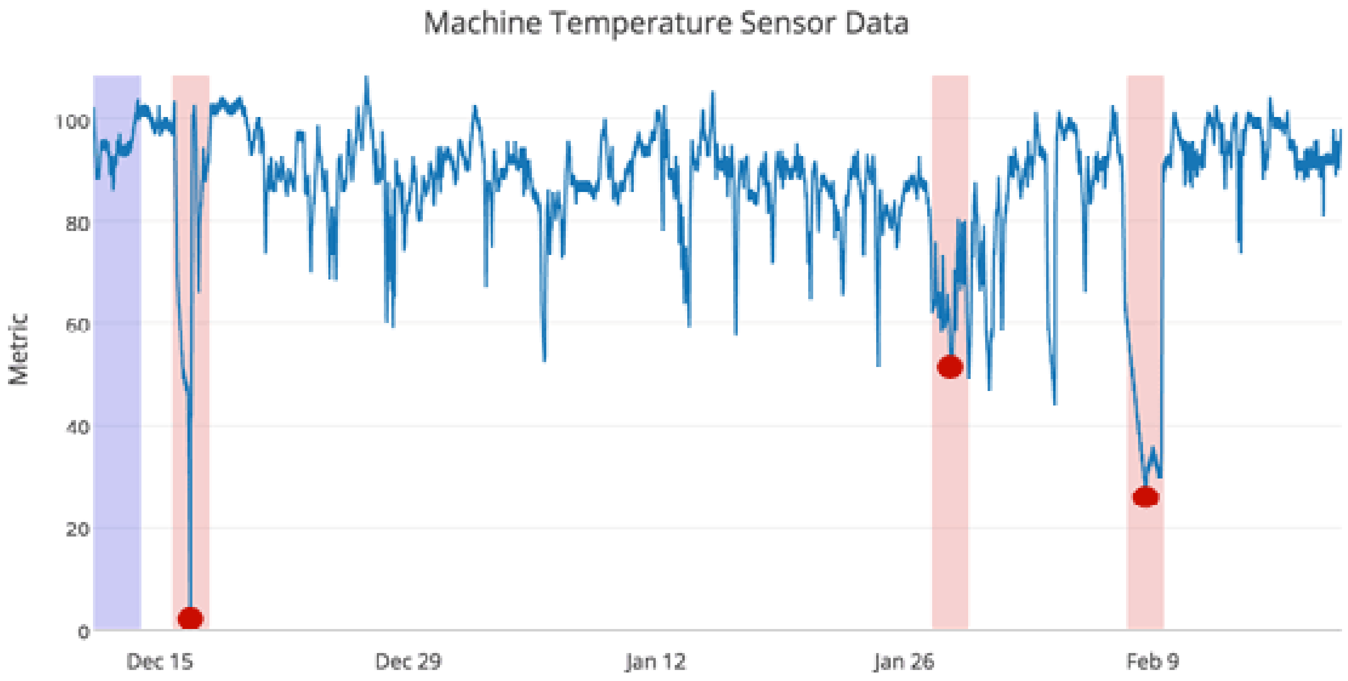
Friendship Format:

Alice	Bob
Bob	Carl
Carl	Victor
Bob	Victor
Alice	Victor
…	

Stanford network dataset collection: https://snap.stanford.edu/data/

Streaming Data

- A sequence of readings



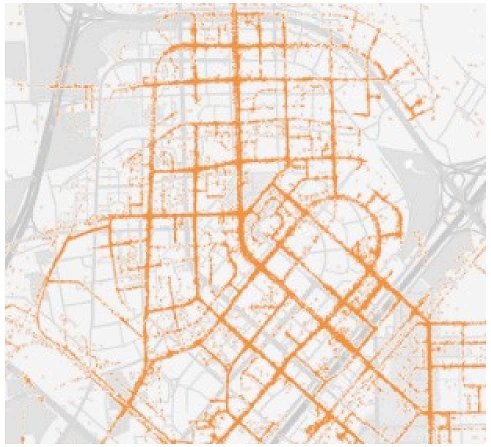
. . . 1, 5, 2, 7, 0, 9, 3

. . . a, r, v, t, y, h, b

. . . 0, 0, 1, 0, 1, 1, 0

Spatio-Temporal Data

- A sequence of (time, location, info) tuples



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- Data Attributes
- Basic Statistical Descriptions of Data
- Measuring Data Similarity and Dissimilarity
- Probability Inequalities

Basic Statistical Descriptions of Data

- **How to capture the properties of a given data set?**
  - **Central tendency:** describes the center around the data is distributed
  - **Dispersion:** describes the data spread

Measuring the Central Tendency

- **Mean** (algebraic measure)

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

  - Weighted arithmetic mean:

$$\mu = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

  - Geometric mean： $\mu = \sqrt[n]{\Pi x_i}$

    - The geometric mean is always <= arithmetic mean, and more sensitive to values near zero.
    - Geometric means make sense with ratios: 1/2 and 2/1 *should* average to 1.

Measuring the Central Tendency

- **Median**
  - Middle value if odd number of values, or average of the middle two values otherwise.
- Example:
  - Five data points **{1.2, 1.4, 1.5, 1.8, 10.2}**
  - Mean: 3.22 Median: **1.5**

- Mean is meaningful for symmetric distributions without outliers: e.g. height and weight.
- Median is better for skewed distributions or data with outliers: e.g. wealth and income.

Measuring the Central Tendency

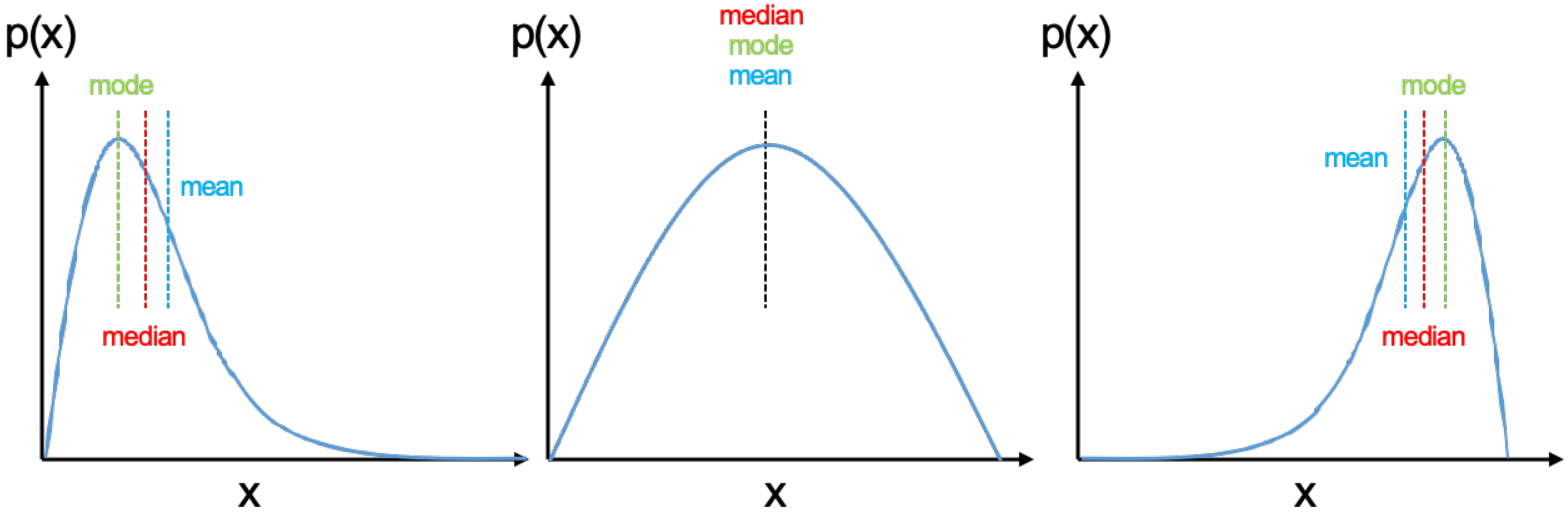
- **Mode**
  - Value that occurs most frequently in the data
  - Unimodal, bimodal, trimodal
  - Empirical formula (moderately skewed distribution):

$$\text{mean} - \text{mode} \simeq 3 \times (\text{mean} - \text{median})$$

- Example:
  - Five data points **{1, 1, 1, 1, 1, 2, 2, 2, 3, 3}**
  - Mean: 1.7 Median: 1.5 Mode: **1**

Symmetric vs. Skewed Data

- Median, mean and mode of symmetric, positively and negatively skewed data



Positively skewed data

mode < median

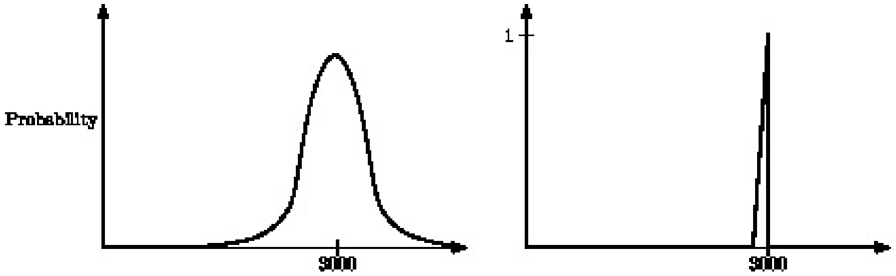
Symmetric data

mode = median

Negatively skewed data

mode > median

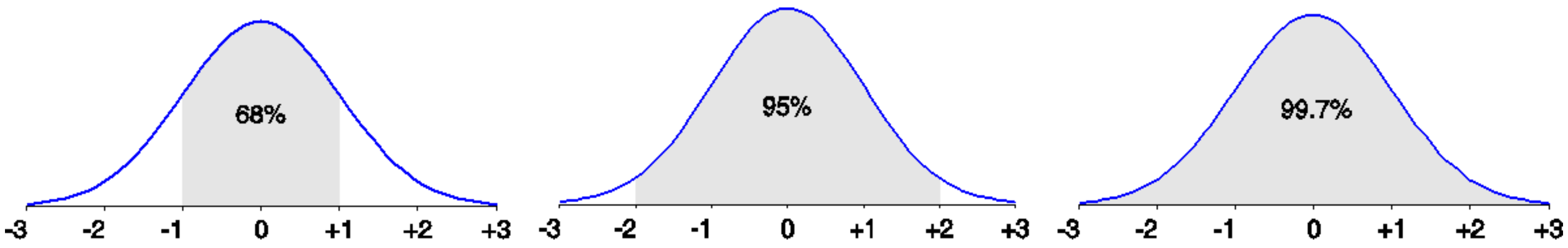
Measuring the Dispersion of Data



- Variance and standard deviation
  - Variance

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i = \mathbb{E}[x] \quad \sigma^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i - \mu)^2 = \mathbb{E}[x^2] - \mathbb{E}[x]^2$$

  - Standard deviation σ is the square root of variance σ^2
- **The normal distribution curve**
  - From $\mu-\sigma$ to $\mu+\sigma$: contains about 68% of the measurements
  - From $\mu-2\sigma$ to $\mu+2\sigma$: contains about 95% of it
  - From $\mu-3\sigma$ to $\mu+3\sigma$: contains about 99.7% of it

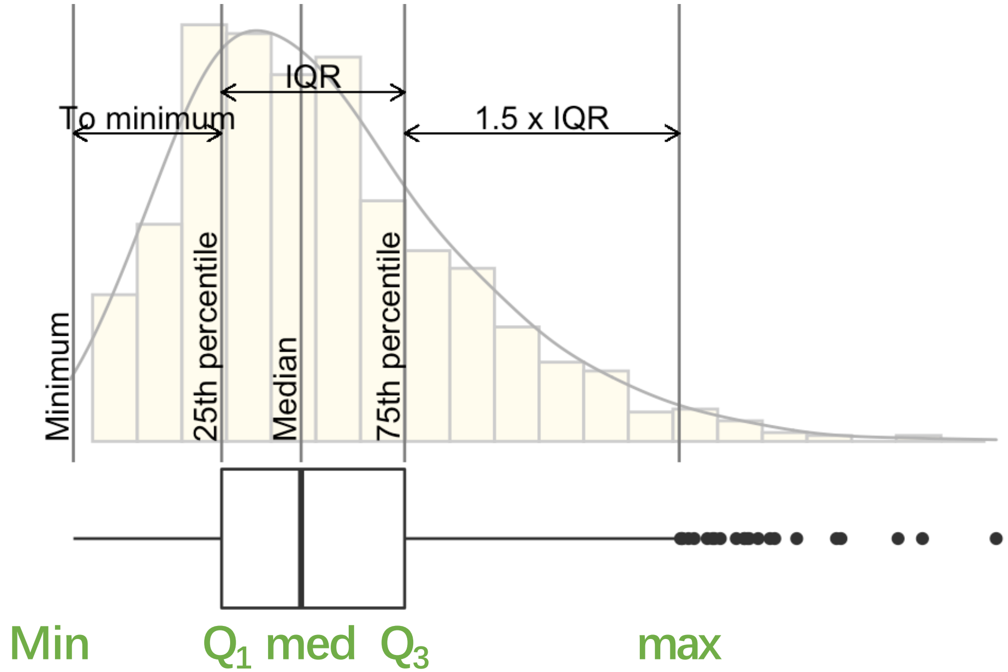


Measuring the Dispersion of Data

- Regardless of how data is distributed, at least $\left(1-\frac{1}{k^2}\right)$ of the points must lie within $k\sigma$ of the mean.
  - Thus at least 75% must lie within two sigma of the mean.
  - The normal distribution can achieve tighter bound.

Measuring the Dispersion of Data

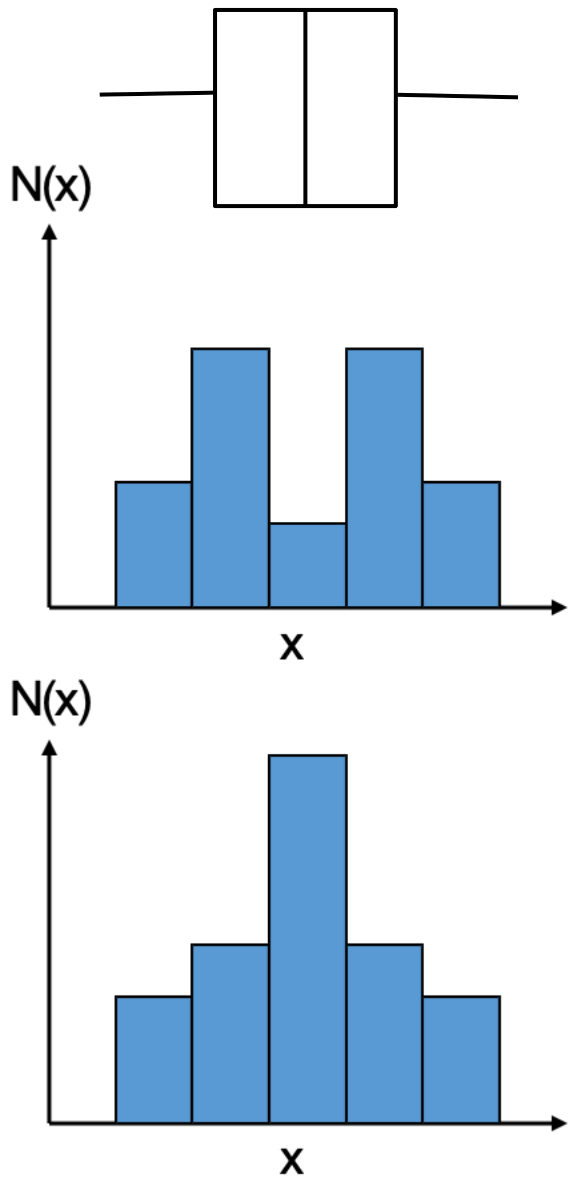
- Quartiles, outliers and boxplots
  - **Quartiles:** Q_1 (25th percentile), Q_3 (75th percentile)
  - **Inter-quartile range:** IQR = $Q_3 - Q_1$
  - **Five number summary:** min, Q_1, median, Q_3, max
  - **Outlier:** usually, a value higher(lower) than 1.5 x IQR than Q_3 (Q_1)



Histograms

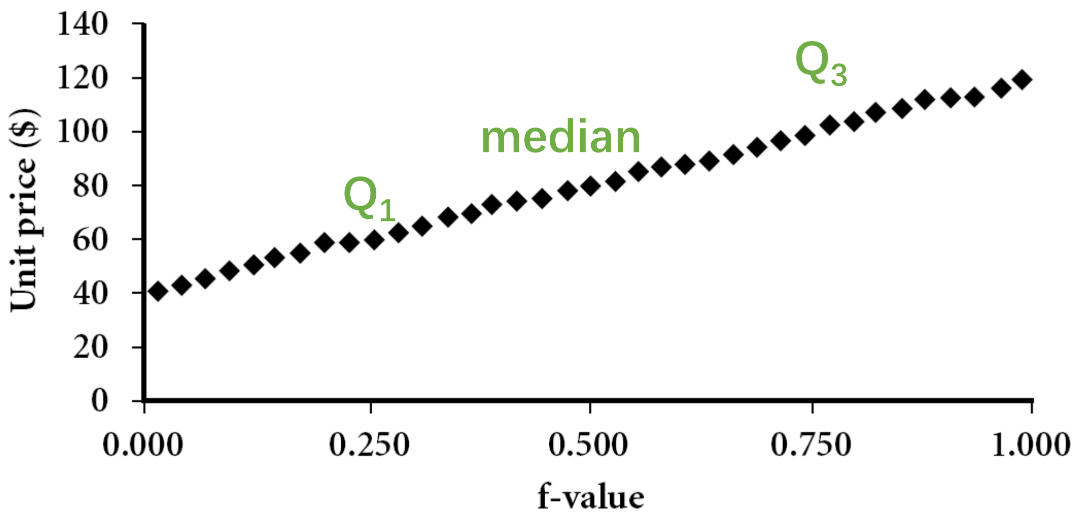
- **Histogram:** Graph display of tabulated frequencies, shown as bars. It shows what proportion of cases fall into each of several categories

- The two histograms shown may have the same boxplot representation
  - The same values for: min, Q1, median, Q3, max
  - But they have rather different data distributions



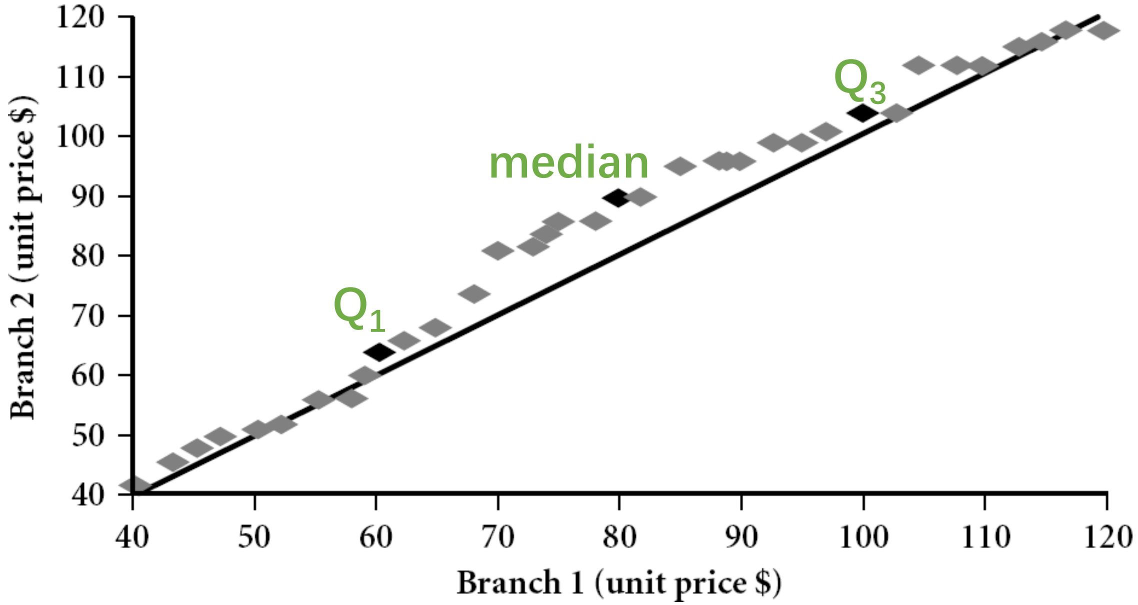
Quantile Plot

- **Quantile Plot**: Each value x_i is paired with f_i indicating that approximately $100 f_i\%$ of data $\leqslant x_i$



Quantile-Quantile (Q-Q) Plot

- **Quantile-Quantile (Q-Q) Plot:** graphs the quantiles of one univariate distribution against the corresponding quantiles of another
- Which branch has a lower price?
  - Example shows unit price of items sold at Branch 1 vs. Branch 2 for each quantile. Unit prices of items sold at Branch 1 tend to be lower than those at Branch 2.



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Proximity Measure

- Proximity refers to a similarity or dissimilarity of two data objects
- Similarity
  - Numerical measure of how alike two data objects are
  - Value is higher when objects are more alike
  - Often falls in the range [0,1]
- Dissimilarity (e.g., distance)
  - Numerical measure of how different two data objects are
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies
- Applications: clustering, anomaly detection, and nearest neighbor search

Proximity Measure for Binary Attributes

- A contingency table for binary data
  - E.g. (1,0,1,0,1,0,⋯)

		Object j		
		1	0	sum
Object i	1	q	r	$q+r$
	0	s	t	$s+t$
	sum	$q+s$	$r+t$	p

- Distance measure for symmetric binary variables:

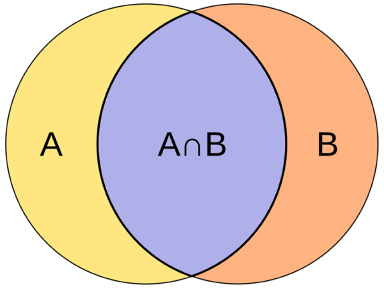
$$d(i,j) = \frac{r+s}{q+r+s+t}$$

- Distance measure for asymmetric binary variables(if t is too large):

$$d(i,j) = \frac{r+s}{q+r+s}$$

- Jaccard coefficient:

$$sim_{Jaccard}(i,j) = \frac{q}{q+r+s}$$



  - Note: Jaccard coefficient is the ratio of intersection over union of two sets.

Minkowski Distance

- Minkowski distance:

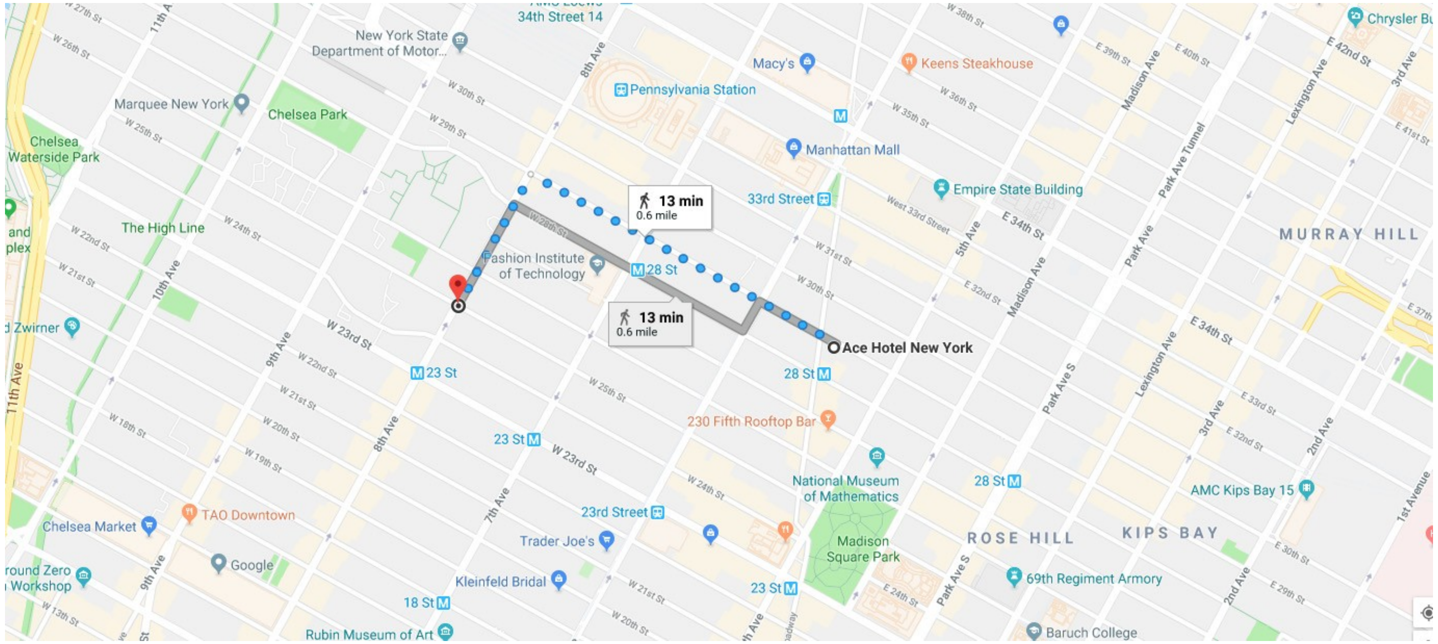
$$x_i = (x_{i1}, x_{i2}, \ldots, x_{ip})$$
$$x_j = (x_{j1}, x_{j2}, \ldots, x_{jp})$$
$$d(i,j) = \left(|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \cdots + |x_{ip} - x_{jp}|^h\right)^{\frac{1}{h}}$$

  - *h* is the order (the distance so defined is also called L_{h} norm)
- Properties
  - Positive definiteness: $d(i,j) > 0 \; if \; i \neq j, and \; d(i,i) = 0$
  - Symmetry: $d(i,j) = d(j,i)$
  - Triangle Inequality: $d(i,j) \leq d(i,k) + d(k,j)$
- A distance that satisfies these properties is a metric

Minkowski Distance

- $h = 1$: Manhattan (city block, L_1 norm) distance
  - $d(i,j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \cdots + |x_{ip} - x_{jp}|$

    E.g., the Hamming distance: the number of bits that are different between two binary vectors



Cosine Similarity

- A document can be represented by thousands of attributes, each recording the frequency of a particular word (such as keywords) or phrase in the document.

Document	Team	Coach	Hockey	Baseball	Soccer	Penalty	Score	Win	Loss	Season
d1	5	0	3	0	2	0	0	2	0	0
d2	3	0	2	0	1	1	0	1	0	1
d3	0	7	0	2	1	0	0	3	0	0
d4	0	1	0	0	1	2	2	0	3	0

- Cosine measure: If d_1 and d_2 are two vectors (e.g., term-frequency vectors), then

$$\cos(d_1, d_2) = (d_1 \cdot d_2)/(\| d_1 \| \cdot \| d_2 \|)$$

where • indicates vector dot product, $\| d \|$ is the length of vector d

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Markov's Inequality

- If $\boldsymbol{X}$ is a **non-negative** r.v. then for every $c > 0$:

$$\Pr\left[\boldsymbol{X} \geq c\mathbb{E}[\boldsymbol{X}]\right] \leq \frac{1}{c}$$

- **Proof**

$$\begin{aligned}
\mathbb{E}[\boldsymbol{X}] &= \textstyle\sum_i i \cdot \Pr[\boldsymbol{X} = i] && \text{(by definition)} \\
&\geq \textstyle\sum_{i=c\mathbb{E}[\boldsymbol{X}]}^{\infty} i \cdot \Pr[\boldsymbol{X} = i] && \text{(pick only some i's)} \\
&\geq \textstyle\sum_{i=c\mathbb{E}[\boldsymbol{X}]}^{\infty} c\mathbb{E}[\boldsymbol{X}] \cdot \Pr[\boldsymbol{X} = i] && (i \geq c\mathbb{E}[\boldsymbol{X}]) \\
&= c\mathbb{E}[\boldsymbol{X}] \textstyle\sum_{i=c\mathbb{E}[\boldsymbol{X}]}^{\infty} \Pr[\boldsymbol{X} = i] && \text{(by linearity)} \\
&= c\mathbb{E}[\boldsymbol{X}] \Pr\left[\boldsymbol{X} \geq c\ \mathbb{E}[\boldsymbol{X}]\right] && \text{(same as above)}
\end{aligned}$$

$$\Rightarrow \Pr\left[\boldsymbol{X} \geq c\ \mathbb{E}[\boldsymbol{X}]\right] \leq \frac{1}{c}$$

Pro: always works!

Cons:

Not very precise

Doesn't work for the lower tail: $\Pr\left[\boldsymbol{X} \leq c\ \mathbb{E}[\boldsymbol{X}]\right]$

Chebyshev's Inequality

Measuring the Dispersion of Data

- Regardless of how data is distributed, at least $\left(1 - \frac{1}{k^2}\right)$ of the points must lie within $k\sigma$ of the mean.
  - Thus at least 75% must lie within two sigma of the mean.

- For every $c > 0$:

$$\Pr\left[|\boldsymbol{X} - \mathbb{E}[\boldsymbol{X}]| \geq c\sqrt{Var[\boldsymbol{X}]}\right] \leq \frac{1}{c^2}$$

- Proof:

$$\Pr\left[|\boldsymbol{X} - \mathbb{E}[\boldsymbol{X}]| \geq c\sqrt{Var[\boldsymbol{X}]}\right]$$

$$= \Pr\left[|\boldsymbol{X} - \mathbb{E}[\boldsymbol{X}]|^2 \geq c^2 Var[\boldsymbol{X}]\right] \quad \text{(by squaring)}$$

$$= \Pr\left[|\boldsymbol{X} - \mathbb{E}[\boldsymbol{X}]|^2 \geq c^2 \mathbb{E}[|\boldsymbol{X} - \mathbb{E}[\boldsymbol{X}]|^2]\right] \quad \text{(def. of Var)}$$

$$\leq \frac{1}{c^2} \quad \text{(by Markov's inequality)}$$

Chernoff bound

- Let $X_1 \dots X_{\boldsymbol{t}}$ be **independent and identically distributed** random values with range [0,1] and expectation μ.
- Then if $X = \frac{1}{t}\sum_i X_i$ and $1 > \delta > 0$,

$$\Pr[|X - \mu| \geq \delta\mu] \leq 2\exp\left(-\frac{\mu \boldsymbol{t} \delta^2}{3}\right)$$

Chernoff v.s Chebyshev: Example

Let $X = \frac{1}{t}\sum_i X_i, \sigma = Var[X_i]$:

- Chebyshev: $\Pr[|\boldsymbol{X} - \mu| \geq c'] \leq \frac{Var[\boldsymbol{X}]}{c'^2} = \frac{\sigma}{\boldsymbol{t}\ \boldsymbol{c'^2}}$

$\Pr\left[|\boldsymbol{X} - \mathbb{E}[\boldsymbol{X}]| \geq c\sqrt{Var[\boldsymbol{X}]}\right] \leq \frac{1}{c^2}$

- Chernoff: $\Pr[|X - \mu| \geq \delta\mu] \leq 2\exp\left(-\frac{\mu \boldsymbol{t} \delta^2}{3c}\right)$

If $\boldsymbol{t}$ is very big:

- Values $\mu, \sigma, \delta, c, c'$ are all constants!
  - Chebyshev: $\Pr[|\boldsymbol{X} - \mu| \geq z] = O\left(\frac{1}{\boldsymbol{t}}\right)$
  - Chernoff: $\Pr[|\boldsymbol{X} - \mu| \geq z] = e^{-\Omega(\boldsymbol{t})}$

So is Chernoff always better for us?
Yes, if we have i.i.d. variables.

Summary

- Data Attributes
- Basic Statistical Descriptions of Data
  - Centrality/Dispersion
- Measuring Data Similarity and Dissimilarity
  - Distances for binary/numerical
- Probability Inequalities
  - Markov/Chebyshev/Chernoff