CS3319 Foundations of Data Science

4.Locality Sensitive Hashing

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Text Similarity

- Similarity check for a paper with all the published papers.


Running title • August 2012 • Vol. XXI, No. 1

Article Title

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Abstract

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I. Introduction

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II. Methods

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*A thank you or further information

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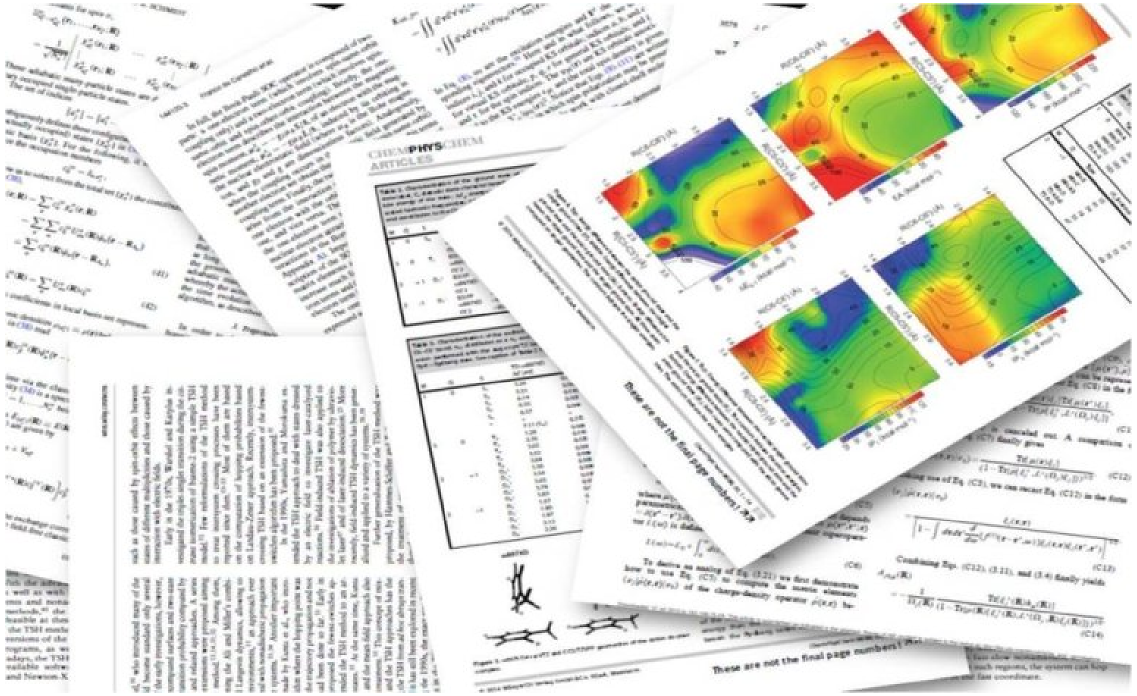


Task: Finding Similar Documents

- **Goal**: Given a large number ($\boldsymbol{N}$ in the millions or billions) of documents, find "near duplicate" pairs

- Challenges:
  - How to define the **similarity**?
    - Many small pieces of one document can appear **out of order** in another.
  - How to compute **efficiently**?
    - Documents are so **large** or so many that they cannot fit in main memory
    - **Too many** documents to compare **all pairs**. E.g. 1 million documents, we have 10^{12} pairs, if we compare 10^6 per second, it takes about 10 days.

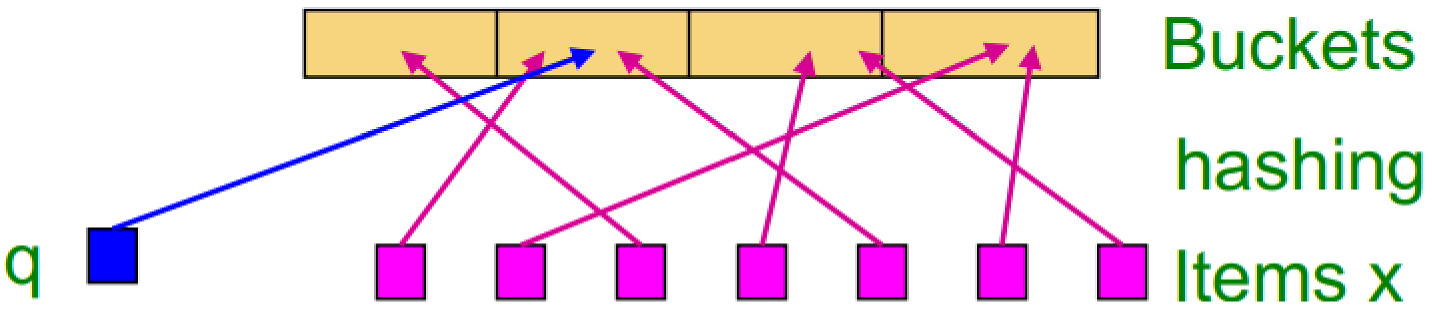
Problem Definition

- **Given**: High dimensional data points (e.g. Bag of Words) $x_1, x_2, \ldots$
- A **distance function** $d(x_1, x_2)$

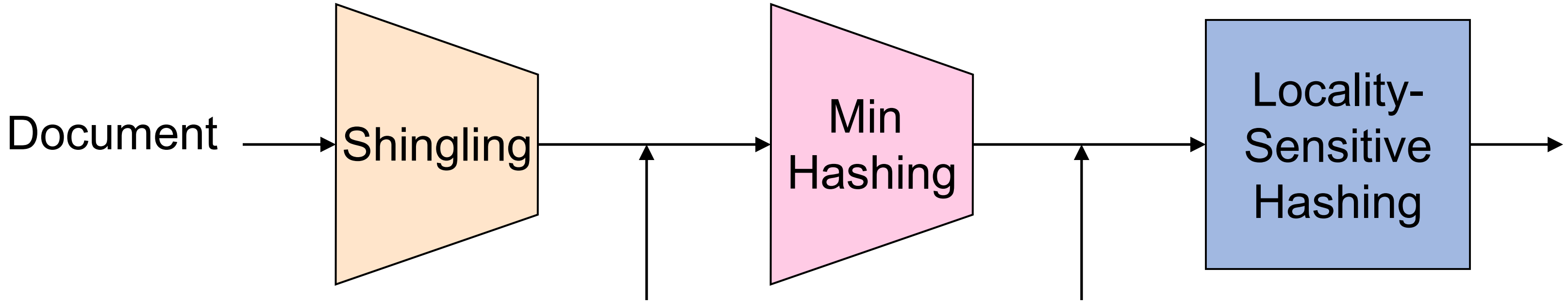
- **Goal:** Find all pairs of data points (x_i, x_j) that are within some distance threshold $d(x_i, x_j) \leq s$
- Naïve solution would take $O(N^2)$
  - where N is the number of data points
- This can be done in $O(N)$!

Key Idea

- **Hashing**
  - Throw items into buckets using several different **hash functions**.
  - Examine only those pairs of items that **share a bucket** for at least one of these hashings.



The Big Picture: 3 Steps for Similar Documents



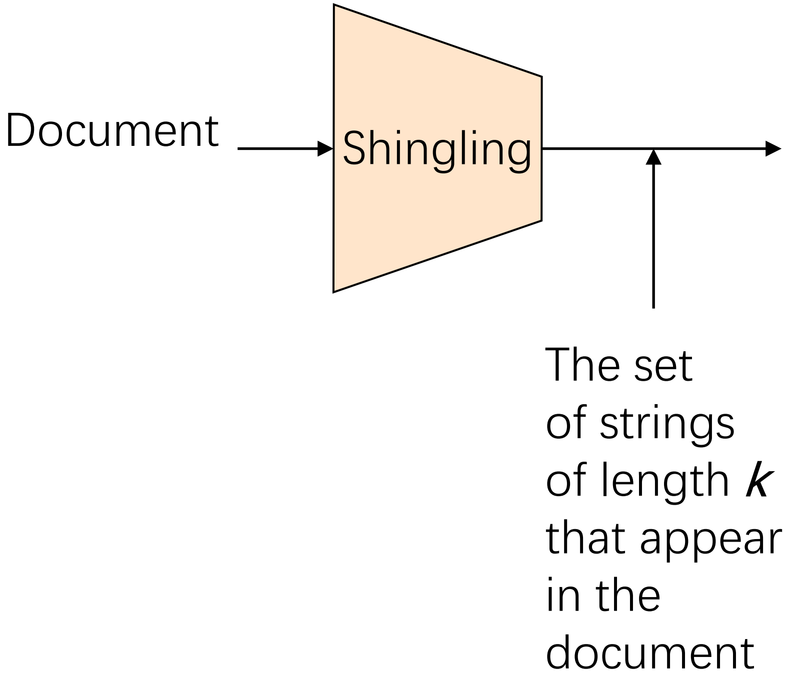
Candidate pairs: those pairs of signatures that we need to test for similarity

The set of strings of length ***k*** that appear in the document

Signatures: short integer vectors that represent the sets, and reflect their similarity

Shingling

- Step 1: Shingling: Convert documents to sets



Documents as High Dimensional Data

- Step 1: **Shingling**: Convert documents to sets

- Simple approaches:
  - Document = set of words appearing in document
  - Document = set of "important" words

- Need to account for **ordering** of words!
- A different way: Shingles!

Define: Shingles

- A **k-shingle** (or k-gram) for a document is a sequence of k tokens that appears in the document
  - Tokens can be characters, or words, depending on the application
- Represent a document by the set of its k-shingles

- Example: $k = 2$; document $D_1 = abcab$
  Set of 2-shingles: $S(D_1) = \{ab, bc, ca\}$

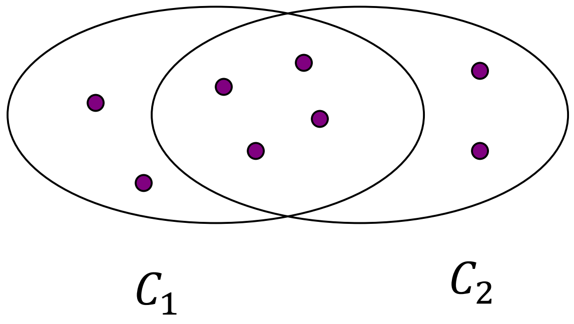
Similarity Metric for Shingles

- Document D_1 is a set of its k-shingles $C_1 = S(D_1)$
- A natural similarity measure is the **Jaccard similarity**:

$$sim(D_1, D_2) \; = \; |C_1 \cap C_2|/|C_1 \cup C_2|$$

- **Jaccard distance**:

$$\mathrm{d}(D_1, D_2) = 1 - sim(D_1, D_2)$$



$E.g.\ |C_1 \cup C_2| = 8$

$|C_1 \cap C_2| = 4$

$sim(D_1, D_2) = 0.5$

Set Representation

- Encode sets with 0/1 vectors
- Rows = elements (shingles)
- Columns = sets (documents)
  - 1 in row e and column s if and only if e is a member of s
  - Column similarity is the Jaccard similarity
  - Typical matrix is sparse!

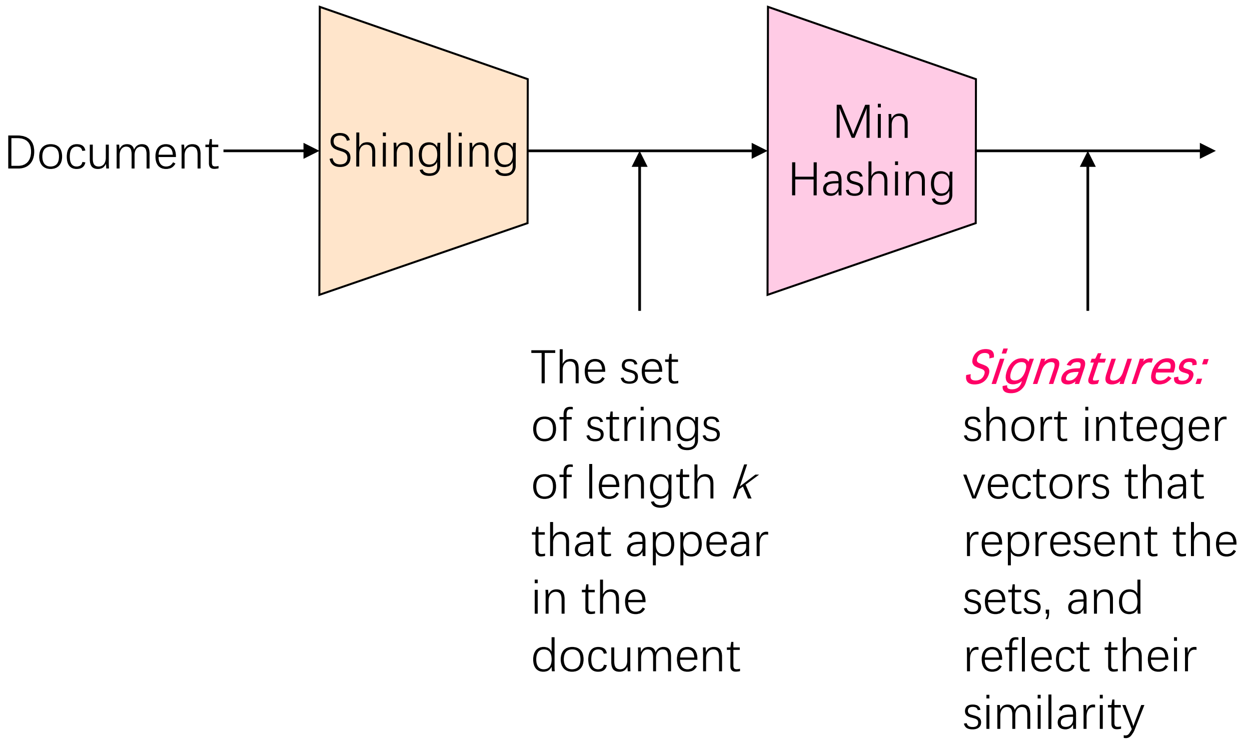
Documents

Shingles

	text1	text2		
ab	1	1	1	0
ac	1	1	0	1
	0	1	0	1
	0	0	0	1
	1	0	0	1
	1	1	1	0
	1	0	1	0

MinHashing

- Step 2: Min-hashing: Convert large sets to short signatures, while preserving similarity



Signatures

- Key idea: "hash" each column C to a small **signature** $h(C)$, such that:
  (1) $h(C)$ is small enough
  (2) $sim(C1, C2)$ is the same as the "similarity" of signatures $h(C1)$ and $h(C2)$

- **Goal**: Find a hash function h(·) such that:
  - If $sim(C1, C2)$ is high, then with high prob. $h(C1) = h(C2)$
  - If $sim(C1, C2)$ is low, then with high prob. $h(C1) \neq h(C2)$

- Hash function for the Jaccard similarity: **Min-Hashing**

Min-Hashing

2	1	0	1	0
3	1	0	0	1
7	0	1	0	1
6	0	1	0	1
1	0	1	0	1
5	1	0	1	0
4	1	0	1	0

- Imagine the rows of the boolean matrix permuted under **random permutation** π
- Define minhash function $h_{\pi}(C)$ = the index of the first (in the permuted order π) row in which column C has value 1:

$$h_{\pi}(C) = \min \pi(C)$$

- Use independent hash functions to create a signature of a column

Min-Hashing Example

2nd element of the permutation
is the first to map to a 1

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

4th element of the permutation
is the first to map to a 1

The Min-Hash Property

- Choose a random permutation π
- Claim: $\boldsymbol{Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)}$
- Why?
  - Let X be a doc (set of shingles), $y \in X$ is a shingle
  - Then: $\Pr[\pi(y) = \min(\pi(X))] = 1/|X|$
    - It is equally likely that any $y \in X$ is mapped to the min element
  - Let y satisfy $\pi(y) = \min(\pi(C_1 \cup C_2))$
  - Then either: $\pi(y) = \min(\pi(C_1))$ if $y \in C_1$, or
    $\pi(y) = \min(\pi(C_2))$ if $y \in C_2$
  - So the prob. that both are true is the prob. $y \in C_1 \cap C_2$
  - $\Pr[\pi(y) = \min(\pi(C_1)) = \min(\pi(C_2))] = |C_1 \cap C_2| / |C_1 \cup C_2| = sim(C_1, C_2)$

0	0
0	0
1	**1**
0	0
0	1
1	0

One of the two cols had to have 1 at position ***y***

Similarity for Signatures

- $We\ know$: $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- Now generalize to multiple hash functions

- The similarity of two signatures is the fraction of the hash functions in which they agree

- **Note**: Because of the Min-Hash property, the similarity of columns is the same as the expected similarity of their signatures, with **expected error** of $O\left(\frac{1}{\sqrt{k}}\right)$, k is the number of hash functions.

Min-Hashing Example

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	2	3	4
1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Similarity

	1-3	2-4	1-2	3-4
Jaccard	0.75	0.75	0	0
Sig.	0.67	1.00	0	0

How to pick a random hash function h(x)?

Universal hashing:

$h_{a,b}(x)=((a \cdot x+b) \bmod p) \bmod N$

where:

a,b … random integers

p … prime number (p > N)

Implementation Trick

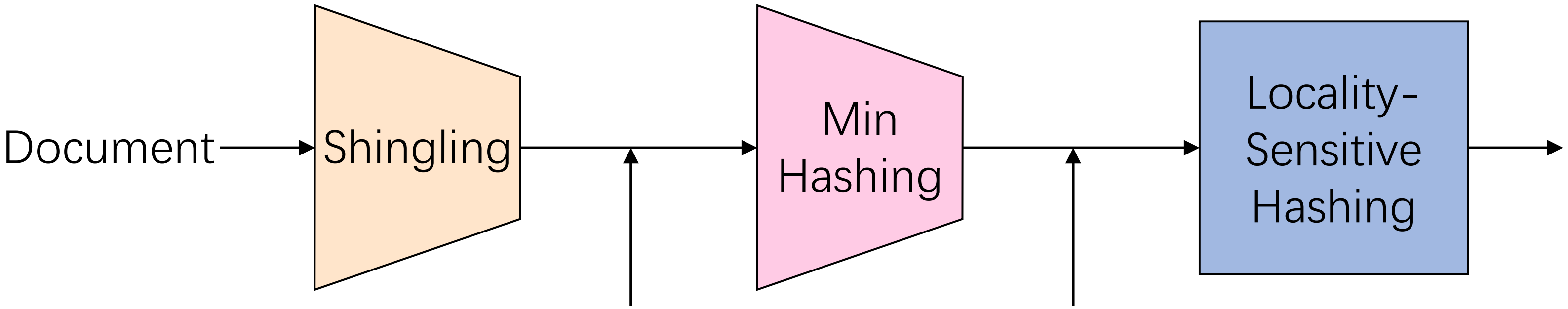
- Permuting rows is complicated, we only need the minimum hashing
- Row hashing
  - Pick $K = 100$ hash functions $\mathbf{h}_i$
  - Ordering under $\mathbf{h}_i$ gives a random row permutation
- One-pass implementation
  - For each column C and hash func. $\mathbf{h}_i$
  - Initialize all $M(i, C) = \infty$, to store the **smallest** hashing value of a document under $\mathbf{h}_i$
  - Scan rows looking for 1s
    - Suppose row j has 1 in column C
    - Then for each $\mathrm{h_i}$:
      - If $\mathrm{h_i}(j) < M(i, C)$,then $M(i, C) = \mathrm{h_i}(j)$.

Signature matrix M

$$\begin{array}{cc} & C \\ i & \begin{bmatrix} 2 & 1 & 2 & 1 \\ 2 & 1 & 4 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix} \end{array}$$

Locality Sensitive Hashing

- Step 3: Locality-Sensitive Hashing:
  Focus on pairs of signatures likely to be from similar documents



The set of strings of length *k* that appear in the document

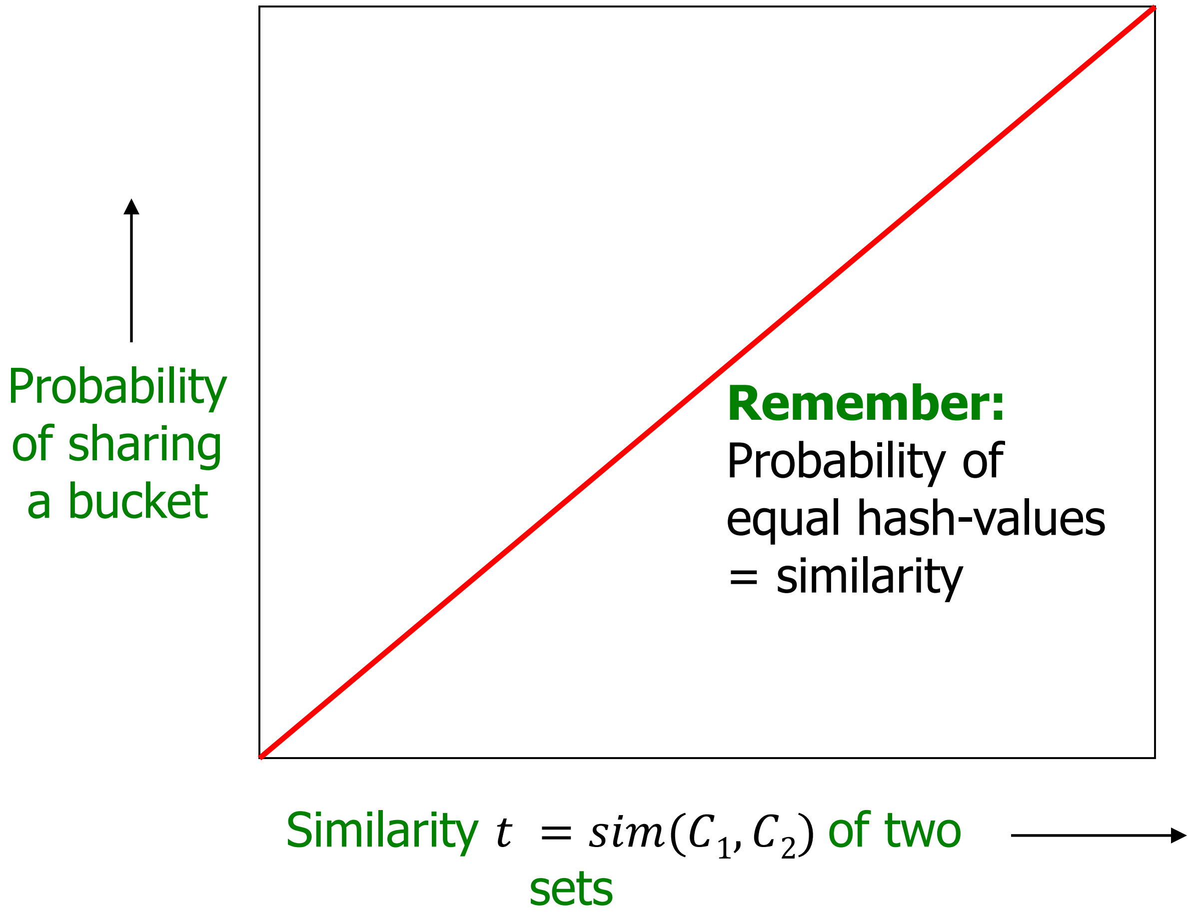
Signatures: short integer vectors that represent the sets, and reflect their similarity

Candidate pairs: those pairs of signatures that we need to test for similarity

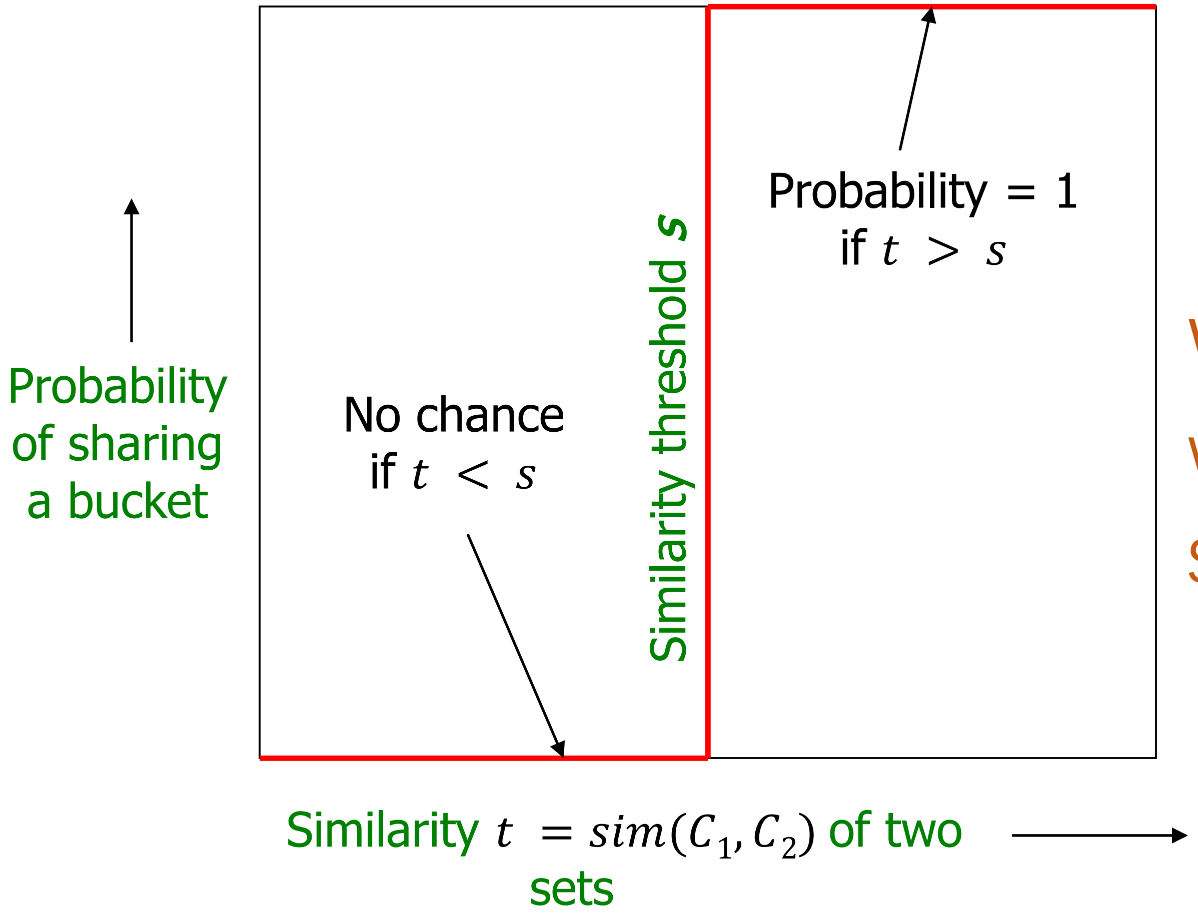
Locality Sensitive Hashing

- **Goal**: Find documents with Jaccard similarity at least s (for some similarity threshold, e.g., $s = 0.8$)
- **LSH – General idea**: Use a **function** $f(x, y)$ that tells whether x and y is a **candidate pair**
- **For Min-Hash matrices:**
  - **Hash** columns of signature matrix M to many buckets
  - Each pair of documents that hashes into the same bucket is a candidate pair

Jaccard Similarity Hashing (1 signature)

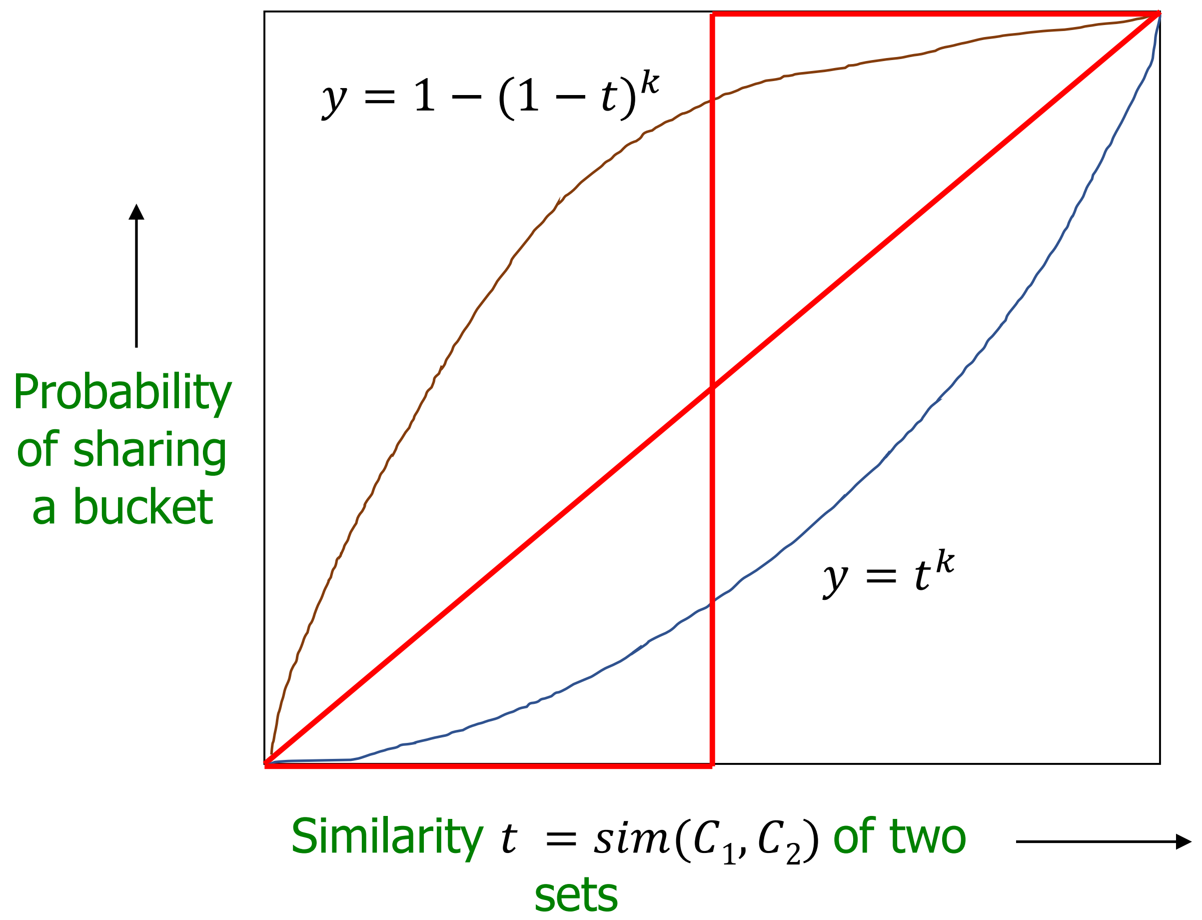


What We Want



What can we do with multiple minhash signatures?

Jaccard Similarity Hashing



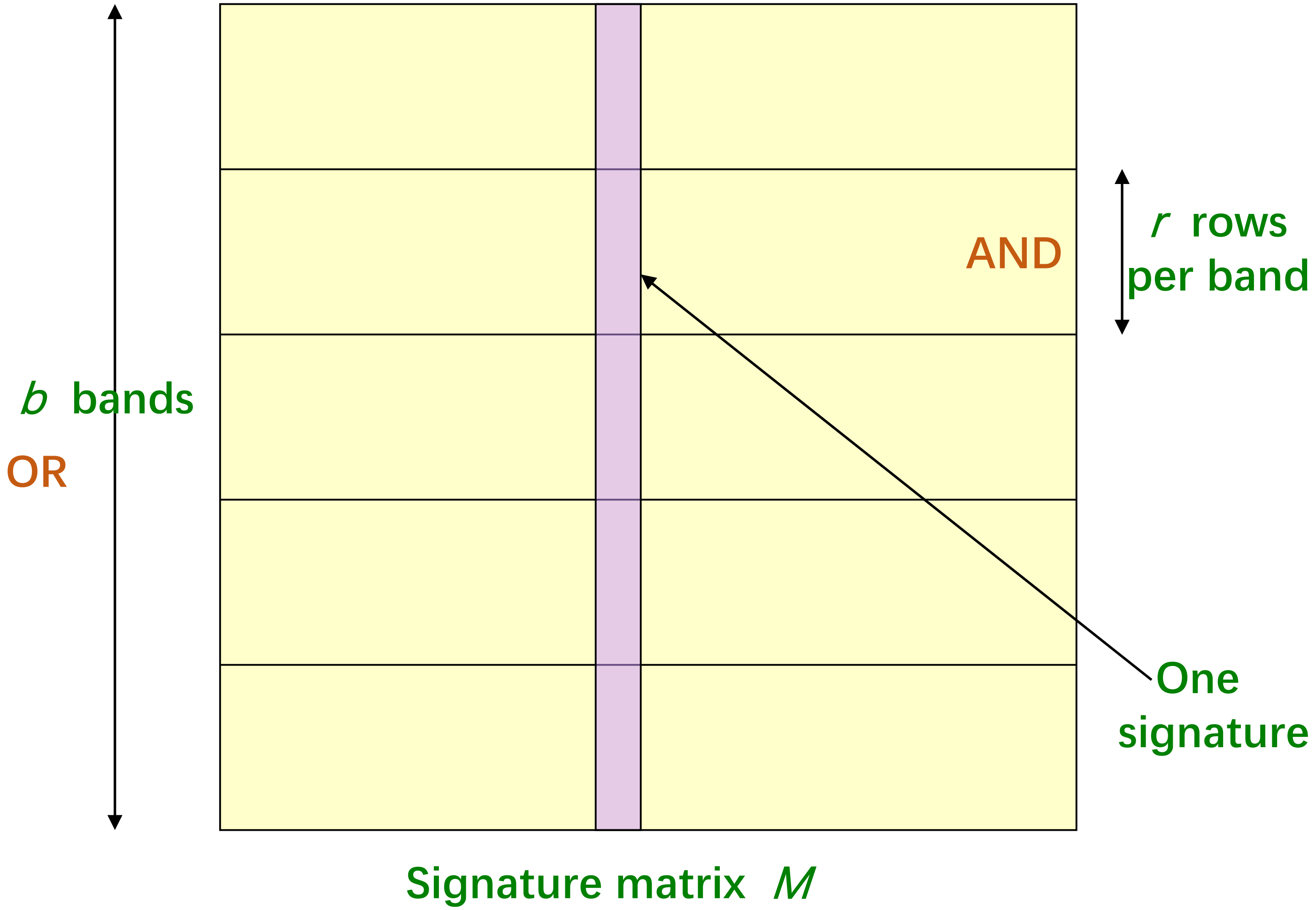
We have k hash functions.

- We consider C_1, C_2 to be a candidate pair, only if they share **all** the k Minhash values (**AND**)
- We consider C_1, C_2 to be a candidate pair, if they share at least **one** Minhash value (**OR**)

What can we do?

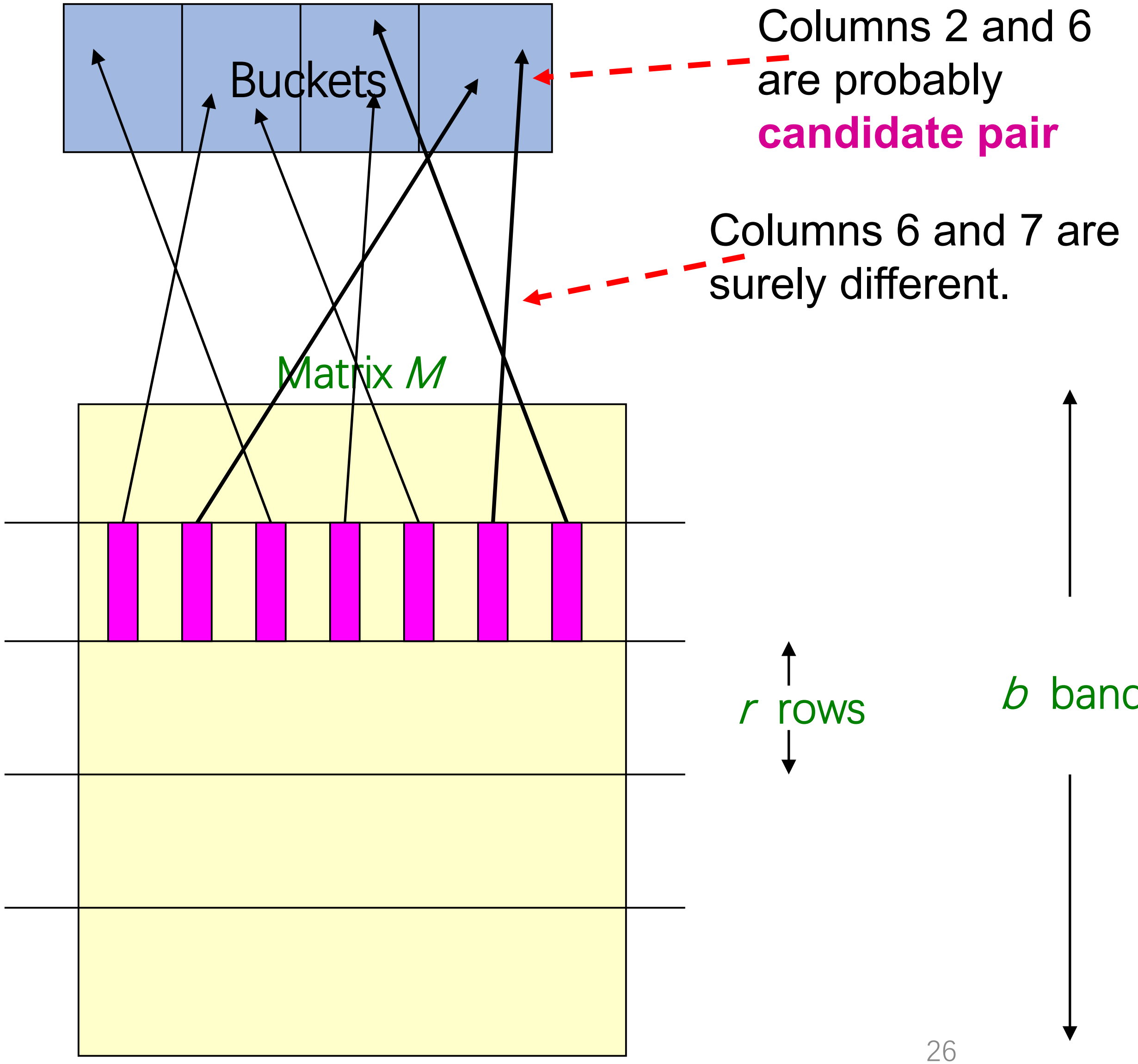
LSH for Min-Hash

- **Key Idea:**
  - Hash columns of signature matrix M several times
  - **Combine OR and AND**



Partition M into Bands

- Divide matrix M into b bands of r rows
- For each band, hash its portion of each column to a hash table (Buckets: as many as possible)
- Candidate column pairs are those that hash to the same bucket for $\geqslant 1$ band
- Tune b and r to catch most similar pairs, but few non-similar pairs



Example of Bands

- Assume the following case:
- Suppose 100,000 columns of M (100k docs)
- Signatures of 100 integers (rows)

- Choose b = 20 bands of r = 5 integers/band

- **Goal**: Find pairs of documents that are at least s = 0.8 similar

C_1, C_2 are 80% Similar, false negative rate?

- Find pairs, similarity $\geq s = 0.8$, set $b = 20, r = 5$
- Assume: $sim(C_1, C_2) = 0.8$
  - Since $sim(C_1, C_2) \geq s$, we want C_1, C_2 to be a candidate pair: we want them to hash to at least 1 common bucket (at least one band is identical)
- Probability C_1, C_2 identical in one particular band: $(0.8)^5 = 0.328$
- Probability C_1, C_2 are not similar in all of the 20 bands:

$$(1 - 0.328)^{20} = 0.00035$$

  - i.e., about 1/3000 of the 80%-similar column pairs are false negatives (we miss them)
  - We would find 99.965% pairs of truly similar documents

C_1, C_2 are 30% Similar, false positive rate?

- Find pairs of similarity ≥ s=0.8, set $b = 20, r = 5$
- Assume: $sim(C_1, C_2) = 0.3$
  - Since $sim(C_1, C_2) < s$ we want C_1, C_2 to hash to NO common buckets (all bands should be different)
- Probability C_1, C_2 identical in one particular band: $(0.3)^5 = 0.00243$
- Probability C_1, C_2 identical in at least 1 of 20 bands:

$$1 - (1 - 0.00243)^{20} = 0.0474$$

  - In other words, approximately 4.74% pairs of docs with 30% similarity end up becoming candidate pairs (false positive)
    - They are false positives since we will have to examine them (they are candidate pairs) but then it will turn out their similarity is below threshold s

b bands, r rows/band

Pick:

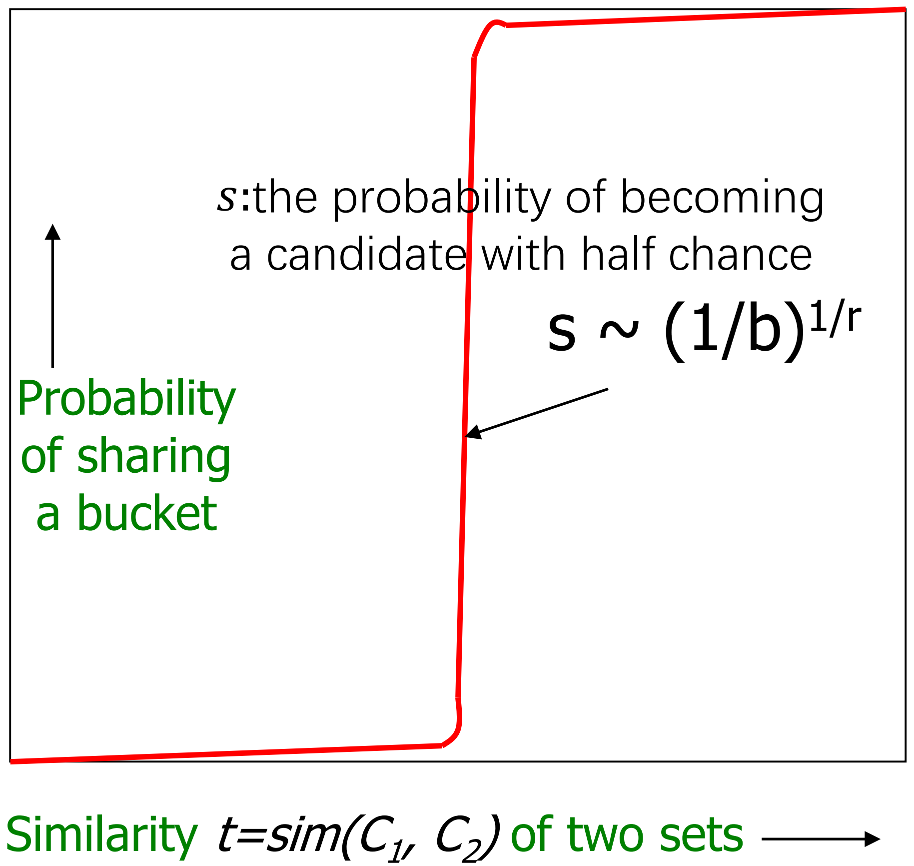
The number of Min-Hashes (rows of M)

The number of bands b, and

The number of rows r per band

to balance false positives/negatives

- Columns C_1 and C_2 have similarity t
- Pick any band (r rows)
  - Prob. that all rows in band equal $= t^r$
  - Prob. that some row in band unequal $= 1 - t^r$
- Prob. that no band identical $= (1 - t^r)^b$
- Prob. that at least 1 band identical $= 1 - (1 - t^r)^b$



At least one band identical

No bands identical

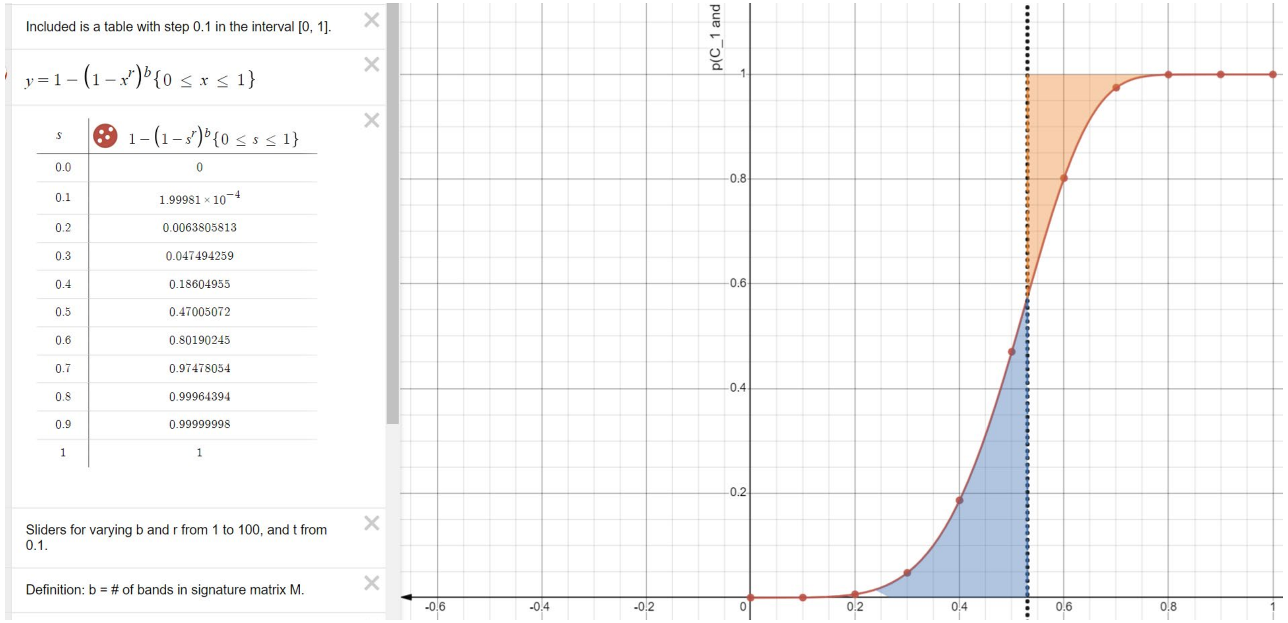
$$\mathbf{1 - (1 - t^r)^b}$$

Some row of a band unequal

All rows of a band are equal

Picking r and b: The S-curve

- Picking r and b to get the best S-curve
  - https://www.desmos.com/calculator/lzzvfjiujn?lang=zh-CN
  - r: hashed into the same bucket, b: identified as similar.



LSH Summary

- Tune M, b, r to get almost all pairs with similar signatures, and eliminate most pairs that do not have similar signatures

- Check in main memory that candidate pairs really do have similar signatures

- Optional: In another pass through data, check that the remaining candidate pairs really represent similar documents