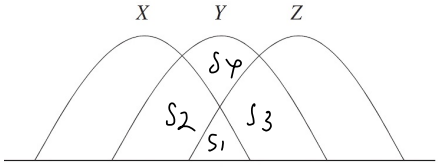
1.



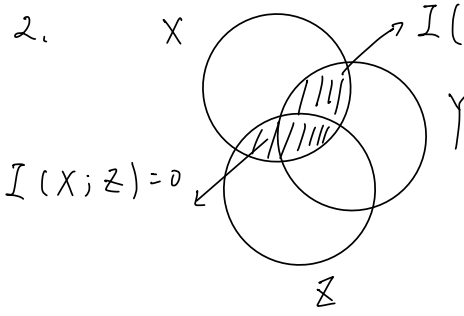
$X \perp Y \mid Z \Rightarrow S_2 = 0$

$X \perp Z \Rightarrow S_1 = 0$

$\therefore S_1 + S_2 = 0$

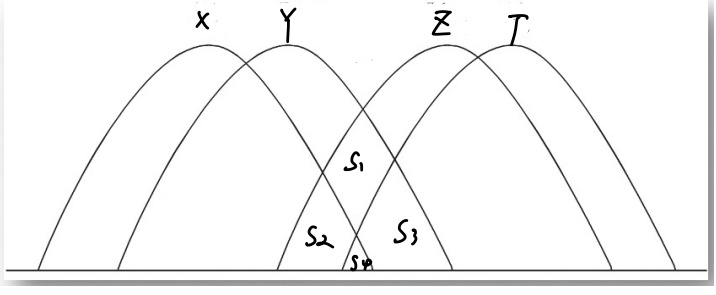
$\therefore X \perp Y$

2.



$\therefore X \perp Y$

3.



$Y \perp Z \mid T \Rightarrow$

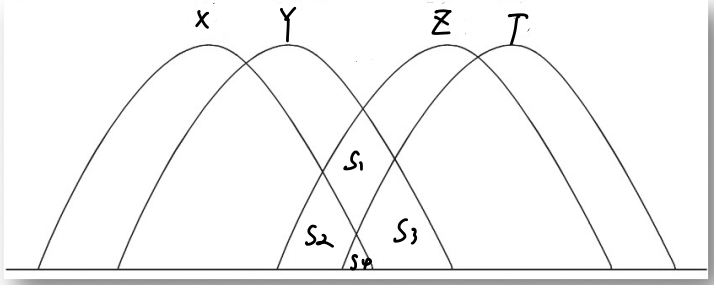
$S_1 + S_2 = 0, S_1 = S_2 = 0$

$I(Y; Z \mid (X,T))$

$= S_1 = 0$

$\therefore Y \perp Z \mid (X,T)$

4. ①



$I(X;T) + I(Y;Z)$

$= 2S_\varphi + S_1 + S_2 + S_3$

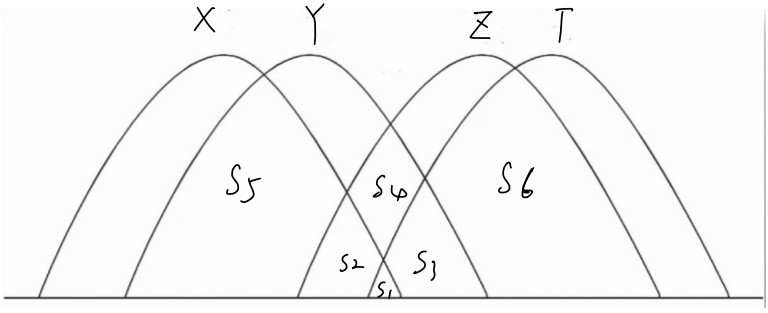
$I(X;Z) + I(Y;T)$

$= S_2 + S_\varphi + S_3 + S_\varphi$

$\because S_1 \geq 0$

$\therefore$ 不等式成立

②

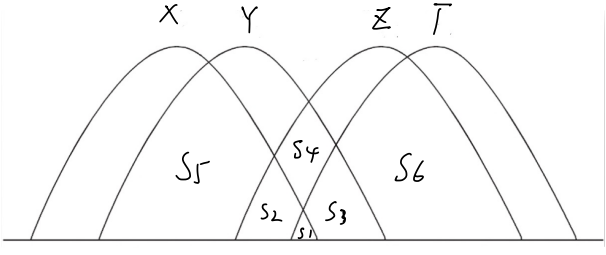


$I(X;T) + I(Y;Z) = 2S_1 + S_2 + S_3 + S_4$

$I(X;Y) + I(Z;T) = S_5 + S_2 + S_1 + S_6 + S_3 + S_1$

$S_4 \geq S_5 + S_6$ 无法比较大小，故不等式不恒成立

③



$I(X;Y) + I(Z;T) = S_5 + S_2 + S_1 + S_6 + S_3 + S_1$

$I(X;Z) + I(Y;T) = S_2 + S_1 + S_1 + S_3$

∴ 不等式成立.