SDM

ASSIGNMENT-5

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RNo.: 17CS60R83

Sol^n 1:- Given

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 \\ 4 & 4 & 4 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 2 & 2 \end{bmatrix}$$

⇒ To find CUR decomposition

- If column 0 & 1 & rows 1 & 2 are selected.

⇒ Frobenius norm of matrix A

$$\Rightarrow \|A\|_f^2 = 243$$

⇒ Probability (0^{th} column) $= \sqrt{r * P_{0c}}$

$$= \sqrt{2 * 0.210} = 0.648$$

where $r \Rightarrow$ no of col. selected from A

$P_{0c} \Rightarrow$ Probality of 0^{th} column

$$\Rightarrow \frac{\sum_i a_{i0}^2}{\|A\|_f^2}$$

⇒ $P(1^{st}$ column$) = \sqrt{r P_{1c}} = \sqrt{2 * 0.210}$

$$= 0.648$$

$\Rightarrow P(1^{st}\text{ raw}) = \sqrt{rP_{1r}} = \sqrt{2 * 0.111}$
$= 0.471$

$\Rightarrow P(2^{nd}\text{ raw}) = \sqrt{rP_{2r}} = \sqrt{2 * 0.197}$
$= 0.628$

Therefore

$$C = \begin{bmatrix} 1/0.648 & 1/0.648 \\ 3/0.648 & 3/0.648 \\ 4/0.648 & 4/0.648 \\ 5/0.648 & 5/0.648 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1.54 & 1.54 \\ 4.63 & 4.63 \\ 6.17 & 6.17 \\ 7.72 & 7.72 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Similarly $R = \begin{bmatrix} 6.36 & 6.36 & 6.36 & 0 & 0 \\ 6.36 & 6.36 & 6.36 & 0 & 0 \end{bmatrix}$

First we will compute U

As $W \Rightarrow$ Interaction matrix of selected coloumn & rows.

So $W = \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix}$

Applying SVD to W :-

$$W = X \Sigma Y^T$$

where Y is eigen vector of $W^T W$.

- Now $W^T W \Rightarrow \begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 25 & 25 \\ 25 & 25 \end{bmatrix}$$

- Finding eigen values of $W^T W$.

ie $\Rightarrow |W^T W - \lambda I| = 0$

$$\Rightarrow \begin{bmatrix} 25-\lambda & 25 \\ 25 & 25-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (25-\lambda)^2 - 625 = 0$$

$$\Rightarrow \lambda^2 + 625 - 50\lambda - 625 = 0$$

$$\Rightarrow \lambda(\lambda - 50) = 0$$

So Eigen Values $\Rightarrow$ 50, 0.

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Now finding eigen vector :-

Case 1 :- when $\lambda = 0$

$$\begin{bmatrix} 25 & 25 \\ 25 & 25 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$25x + 25y = 0$$

Therefore unit eigen vector $\Rightarrow \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

Case 2 :- When $\lambda = 50$

$$\begin{bmatrix} 25 & 25 \\ 25 & 25 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 50 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$25x + 25y = 50x$$
$$25x + 25y = 50y$$

Therefore unit eigen vector $\Rightarrow \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

So the Y matrix will be.

$$Y = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

Similarly X matrix will be eigen vector of WW^T

$$WW^T = \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 18 & 24 \\ 24 & 32 \end{bmatrix}$$

- Finding eigen values of WW^T.

ie $\begin{bmatrix} 18-\lambda & 24 \\ 24 & 32-\lambda \end{bmatrix} = 0$

ie $(32-\lambda)(18-\lambda) - 576 = 0$

$576 - 18\lambda - 32\lambda + \lambda^2 - 576 = 0$

$\lambda^2 - 50\lambda = 0$

$\lambda(\lambda - 50) = 0$

Therefore eigen values of $WW^T \Rightarrow 50, 0$.

Now finding eigen vector:-

Case 1 :- When $\lambda = 50$, eigen vector $\rightarrow$

$$\begin{bmatrix} 18 & 24 \\ 24 & 32 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 50 \begin{bmatrix} x \\ y \end{bmatrix}$$

$18x + 24y = 50x$

$\Rightarrow$ $24x + 32y = 50y$

$\Rightarrow y = \frac{4}{3}x$.

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So unit vector corresponding $\lambda = 50$.

$$\Rightarrow \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$$

Case 2 :- For $\lambda = 0$.

$$\begin{bmatrix} 18 & 24 \\ 24 & 32 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$18x + 24y = 0$$

ie $y = -\frac{3}{4}x$

Unit vector corresponding to $\lambda = 0$

$$\Rightarrow \begin{bmatrix} 0.8 \\ -0.6 \end{bmatrix}$$

Therefore

$$X = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$$

Also Σ is square root of eigen value of $W^T W$ or $W W^T$

therefore,

$$\Sigma = \begin{bmatrix} 7.071 & 0 \\ 0 & 0 \end{bmatrix}$$

→ Therefore $W = X \Sigma Y^T$

$$W = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix} \begin{bmatrix} 7.071 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.7071 & 0.707 \\ 0.707 & -0.707 \end{bmatrix}$$

- Computing Σ^+

$$\Sigma^+ = \begin{bmatrix} \frac{1}{7.071} & 0 \\ 0 & 0 \end{bmatrix}$$

- Computing U.

$$U = Y (\Sigma^+)^2 X^T$$

$$U = \begin{bmatrix} 0.7071 & 0.7071 \\ 0.7071 & -0.7071 \end{bmatrix} \begin{bmatrix} 0.02 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.00848 & 0.01131 \\ 0.00848 & 0.01131 \end{bmatrix}$$

So Final CUR decomposition ⇒

$$C \Rightarrow \begin{bmatrix} 1.54 & 1.54 \\ 4.63 & 4.63 \\ 6.17 & 6.17 \\ 7.72 & 7.72 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad U \Rightarrow \begin{bmatrix} 0.00848 & 0.01131 \\ 0.00848 & 0.01131 \end{bmatrix}$$

$$R \Rightarrow \begin{bmatrix} 6.36 & 6.36 & 6.36 & 0 & 0 \\ 6.36 & 6.36 & 6.36 & 0 & 0 \end{bmatrix}$$