

# Curtin University – Department of Computing

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# ASSIGNMENT 2 REPORT

*DATE: 30/07/2022*

*COURSE TITLE:  
Fundamental Concepts of  
Cryptography (ISEC2000)*

*STUDENT'S NAME: Hussain Mehdi*

*STUDENT ID: 20337270*

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## Introduction

This report is for Assignment 2 and contains answers to the questions asked along with brief on running the RSA code.

## Question 1:

The Euclidean algorithm is based on the following assertion. Given two integers  $a, b$ , ( $a > b$ ),

$$\gcd(a, b) = \gcd(b, a \bmod b).$$

Prove the assertion (1) mathematically. (Note that proof by example is NOT appropriate here)

To mathematically prove  $\gcd(a, b) = \gcd(b, a \bmod b)$ :

$a \bmod b$  is the remainder when  $a$  is divided by  $b$ .  $\therefore$  For some integer  $c$ , the statement is  $a - bc$  and it is between 0 and  $b - 1$ .

Lets let  $f = \gcd(a, b)$

$f$  divides both  $a$  and  $b$  therefore, it divides both  $a - bc$  and  $b$  which in turn divides  $\gcd(b, a \bmod b)$ .

Let  $g = \gcd(b, a \bmod b)$ , where  $g$  divides both  $b$  and  $a - bc$ . Which means that it divides both  $a$  and  $b$  and so it divides  $\gcd(a, b)$  respectively.

[\(math.stackexchange.com, 2012\)](https://math.stackexchange.com/)

Alternative Proof:

As per the division algorithm, there exists some  $x$  and  $r < b$  such that  $a = bx + r$  :

$$r = a \bmod b \text{ and } r = a - bx$$

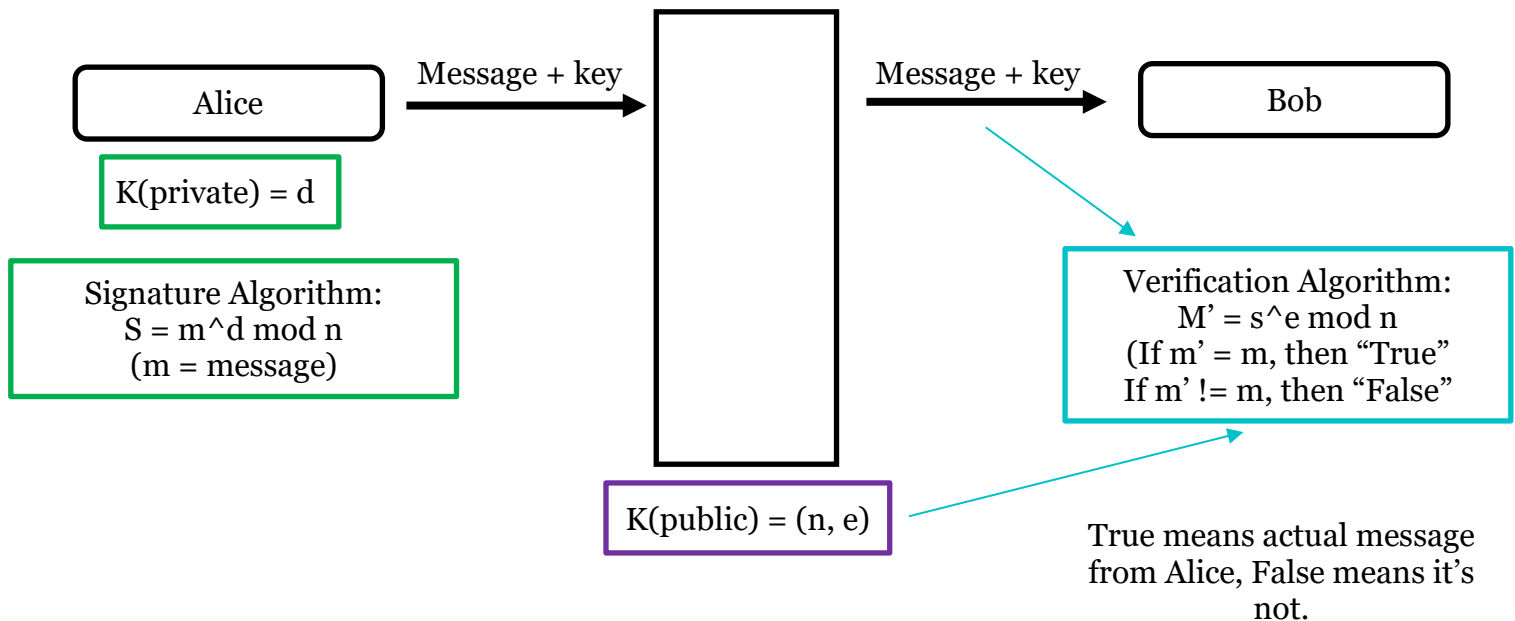
Any divisor (common) of  $a$  and  $b$  will also be a common divisor of  $a \bmod b$ . This is particularly the case with  $\gcd(a, b)$  is a divisor of  $r$  and of  $b$ , which implies that  $\gcd(a, b) \mid \gcd(b, r)$ .

Therefore,  $\gcd(a, b) = \gcd(b, r)$  which in-turn means that  $\gcd(a, b) = \gcd(b, a \bmod b)$

[\(quora.com, 2016\)](https://quora.com/)

## Question 2:

Assuming that Alice signed a document  $m$  using the RSA signature scheme. (You should describe the RSA signature structure first with a diagram and explain the authentication principle). The signature is sent to Bob. Accidentally Bob found one message  $m_0$  ( $m \neq m'$ ) such that  $H(m) = H(m')$ , where  $H()$  is the hash function used in the signature scheme. Describe clearly how Bob can forge a signature of Alice with such  $m'$ .



Sender (Alice) has a private key which is denoted by 'd' and a public key which has  $(n, e)$ . Private key is only known by Alice, but public key is known by everyone. Alice takes the message ( $m$ ) and puts it in the signature algorithm which takes input of the message and private key ( $d$ ). The signature does modular exponentiation by taking message to the power of  $d$  multiplied by mod  $n$ . Alice is the only one to theoretically produce her message algorithm as she is the only one who knows  $m$  and  $d$ . The result of signature algorithm is the signature. The Message and the signature are sent to Bob through a channel (ideally with encryption).

Once Bob receives the message, he checks it using the verification algorithm which takes in the message, signature to the power of  $e$  and mod  $n$  (known by the public). If  $m'$  is equal to the message, then True is returned. If not, False is returned and this means that the sender is not authentic, and the message might have been changed.

Hash collisions although not very common can and do occur. A hash for one message can result in the same hash for a totally different message. In this case, Bob was able to find a message that was signed by Alice that had the same hash as another message. The hash function applied is no secret. Bob can attempt an Existential forgery using a chosen message attack. Bob sends  $m$  to Alice and obtains  $\text{SigK}(H(m))$ . After which,  $(m, \text{SigK}(h(m)))$  is a valid signed message.

Reference Material: [\(UoA, 2020\)](#)

### Question 3:

Below are the RSA Code Implementation Screenshots along with description.

Files:

- RSA.py
- primeGen.py

Running Code:

- python3 RSA.py

RSA.py File code snippets:

```

1  #####
2  # Name: Hussain Mehdi                                     #
3  # Student ID: 20337270                                    #
4  # Purpose: Implementation of RSA Algorithm                #
5  #####
6  import random
7  import json
8  import primeGen
9
10
11 # METHOD: Euclid's algorithm function for GCD
12 def gcd(a, b):
13     while b != 0:
14         a, b = b, a % b
15     return a
16
17
18 # METHOD: Euclid's extended algorithm function for multiplicative inverse
19 def multi_inverse(e, phi):
20     d = 0
21     x1 = 0
22     x2 = 1
23     y1 = 1
24     temp_phi = phi
25
26     while e > 0:
27         temp1 = temp_phi // e
28         temp2 = temp_phi - temp1 * e
29         temp_phi = e
30         e = temp2
31
32         x = x2 - temp1 * x1
33         y = d - temp1 * y1
34
35         x2 = x1
36         x1 = x
37         d = y1
38         y1 = y
39
40     if temp_phi == 1:
41         return d + phi
42

```

## primeGen.py File code snippet:

```
1 #####
2 # Name: Hussain Mehdi #
3 # Student ID: 20337270 #
4 # Purpose: Prime Number Generation for RSA #
5 # Source: GeeksForGeeks (https://www.geeksforgeeks.org/how-to-generate-large-prime-numbers-for-rsa-algorithm/) #
6 #####
7
8 import random
9
10 # Pre-generated primes
11 prePrimeList = [2, 3, 5, 7, 11, 13, 17, 19, 23, 29,
12                 31, 37, 41, 43, 47, 53, 59, 61, 67,
13                 71, 73, 79, 83, 89, 97, 101, 103,
14                 107, 109, 113, 127, 131, 137, 139,
15                 149, 151, 157, 163, 167, 173, 179,
16                 181, 191, 193, 197, 199, 211, 223,
17                 227, 229, 233, 239, 241, 251, 257,
18                 263, 269, 271, 277, 281, 283, 293,
19                 307, 311, 313, 317, 331, 337, 347, 349]
20
21 # METHOD: Random n Bit Generator
22 def nRandomBit(n):
23     return random.randrange(2**(n-1)+1, 2**n - 1)
24
25 # METHOD: Getting lower prime and testing
26 def getLowPrime(n):
27     while True:
28         # Get random number
29         pc = nRandomBit(n)
30         # Test divisibility by prePrimeList
31         for divisor in prePrimeList:
32             if pc % divisor == 0 and divisor**2 <= pc:
33                 break
34         else: return pc
35
36 # METHOD: Miller Rabin Primality Test for prime
37 def millerPass(mrc):
38     maxDivisionsByTwo = 0
39     ec = mrc-1
40     while ec % 2 == 0:
41         ec >>= 1
```

## RSA-test.txt Test file snippet:

```
1 ##### Beginning of the questions #####
2 \uplevel{\Large \textbf{Question answering}}
3 \begin{questions}
4 \question[10] The Euclidean algorithm is based on the following assertion. Given two integers  $a$ ,  $b$ , ( $a > b$ ),
5
6 \begin{equation}\label{gcd}
7 \text{gcd}(a, b) = \text{gcd}(b, a \bmod b).
8 \end{equation}
9 Prove the assertion (\ref{gcd}) \textbf{mathematically}. (Note that proof by example is NOT appropriate here)
10
11 \question[20] Assuming that Alice signed a document  $m$  using the RSA signature scheme. (You should describe the RSA signature structure first with a diagram and explain the authentication process)
12
13 \vspace{0.5in}
14 \fullwidth{\Large \textbf{Programming}}
15 \question[50] Implement the RSA algorithm (C/C++, Java, Python). The requirements are as follows:
16 \begin{itemize}
17 \item Implement each component as a separate function, such as key schedule, prime test, the extended Euclidean algorithm, binary modular exponentiation, and so on.
18 \item Implement both encryption and decryption of RSA. Encryption takes a txt file as input and output another txt file containing ciphertext (use hexadecimal for easy readability). Decryption takes a ciphertext file as input and output a txt file containing the original message.
19 \item Your code should encrypt and decrypt standard keyboard characters, including letters, numbers, and symbols.
20 \item The prime numbers  $p$  and  $q$  should be larger than  $2^{64}$ . (you are allowed to use libraries to handle large numbers, such as BigInteger in Java)
21 \item The strategy of source coding (converting characters to integers in RSA) is up to you. You can encrypt one or more characters at a time, but make sure the constraint  $m < n$  is satisfied.
22 \item Use the provided file \text{RSA-test.txt} to test your code.
23 \end{itemize}
24 After implementing your code, please \textbf{answer the following questions} in your report:
25 \begin{parts}
26 \part[10] What are the lessons you learned, and difficulties you met, in the process of implementing RSA?
27 \part[10] Describe what you have done for source coding and decoding.
28 \end{parts}
29 \end{questions}
```







## public\_keys.txt (Generated File):

```

public_keys.txt
1 2534548305018681858572592885252351494956235118802801809981350335972752042145798920246797933270468964831377113613078625994056290414853129450279840869213
2 1420738595075918324915755871855769720182473178901974654075980321269506146369446640661359823337150549302369776850633948755948323853526541731029419366799

```

## Demo Decryption:

```
PS C:\Users\Hussain\Desktop\Crypto-Assignment-2> python3 .\RSA.py
```

```
----- RSA Algorithm -----
```

```
Choose one of the following:
```

```
Enter (1) for Encrypt or (2) for Decrypt: 2
```

```
Enter filename to decrypt data from: encrypted.txt
```

```
Opening encrypted file for decryption: encrypted.txt
```

```
Loading Keys from file...
```

```
File has been decrypted!
```

```
===== END =====
```

```
PS C:\Users\Hussain\Desktop\Crypto-Assignment-2> 
```

## Decrypted File (decrypted.txt)

```

decrypted.txt
1 %%%----- Beginning of the questions -----
2 \uplevel{\Large \textbf{Question answering}}
3 \begin{questions}
4 \question[10] The Euclidean algorithm is based on the following assertion. Given two integers  $a$ ,  $b$ , ( $a > b$ ),
5
6 \begin{equation}\label{gcd}
7 \quad \text{gcd}(a, b) = \text{gcd}(b, a \% b, \text{mod } \%, b).
8 \end{equation}
9 Prove the assertion (\ref{gcd}) \textbf{mathematically}. (Note that proof by example is NOT appropriate here)
10
11 \question[20] Assuming that Alice signed a document  $m$  using the RSA signature scheme. (You should describe the RSA signature structure first with a diagram and explain the authentication process)
12
13 \vspace{0.5in}
14 \fullwidth{\Large \textbf{Programming}}
15 \question[50] Implement the RSA algorithm (C/C++, Java, Python). The requirements are as follows:
16 \begin{itemize}
17 \item Implement each component as a separate function, such as key schedule, prime test, the extended Euclidean algorithm, binary modular exponentiation, and so on.
18 \item Implement both encryption and decryption of RSA. Encryption takes a txt file as input and output another txt file containing ciphertext (use hexadecimal for easy readability). Decryption takes a ciphertext file as input and output a plaintext file.
19 \item Your code should encrypt and decrypt standard keyboard characters, including letters, numbers, and symbols.
20 \item The prime numbers  $p$  and  $q$  should be larger than  $2^{64}$ . (You are allowed to use libraries to handle large numbers, such as BigInteger in Java)
21 \item The strategy of source coding (converting characters to integers in RSA) is up to you. You can encrypt one or more characters at a time, but make sure the constraint  $m < n$  is satisfied
22 \item Use the provided file \text{RSA-test.txt} to test your code.
23 \end{itemize}
24 After implementing your code, please \textbf{answer the following questions} in your report:
25 \begin{parts}
26 \part[10] What are the lessons you learned, and difficulties you met, in the process of implementing RSA?
27 \part[10] Describe what you have done for source coding and decoding.
28 \end{parts}
29 \end{questions}

```

### **3a) What are the lessons you learned, and difficulties you met, in the process of implementing RSA?**

I learned about Prime number generation and the complexity behind accurately generating large prime numbers. Additionally, I learned about Public and Private Key pairs and techniques of generating them. Lastly, I learned about the security and complexity associated with RSA and importance of large key sizes.

The difficulty I faced when implementing RSA was the generation of prime numbers for Key Pair generation. I tried multiple techniques however, efficiently generating prime number greater than  $2^{64}$  was difficult. I was able to figure out an effective method as explained on GeeksForGeeks website for efficient and quick generation of RSA Prime Numbers.

Lastly, I faced a little issue with time management due to multiple submission within the same week.

### **3b) Describe what you have done for source coding and decoding.**

I start off by generating prime numbers which are required for the generation of key pairs. The primes are sent to the key generation function (`gen_key_pair()`) where  $\phi$ ,  $e$ ,  $\gcd$  and multiplicative inverse are all calculated and Public Key ( $e, n$ ) and Private Key ( $d, n$ ) are generated.

#### Encryption Function (Source Coding):

The encryption function takes in plain text and checks for public key file.  $N$  and  $e$  are read from the file and put into the function. We use `ord()` to convert each character to its ASCII value and `pow()` function to do the calculation of  $a^b \bmod n$ . This is repeated for each digit in the plain text and an array of bytes is returned.

#### Decryption Function (Decoding):

The decryption function takes in cipher text and searches for the private keys file.  $D$  and  $n$  are read from the file and put into the function. The `pow()` function is used and  $a^b \bmod n$  is done, the result is casted to an `str()` and stored in a list. The list is then parsed and converted to `int`, the result of which is converted to a character using `chr()` function. The array of bytes is then returned as a string and written to a file.



## References

<https://www.geeksforgeeks.org/how-to-generate-large-prime-numbers-for-rsa-algorithm/>

<https://math.stackexchange.com/questions/59147/why-gcda-b-gcdb-a-bmod-b-understanding-euclidean-algorithm>

<https://www.quora.com/What-are-the-proofs-for-gcd-a-b-gcd-b-a-mod-b>