Assignment – 1 (Linear Algebra)

Q.1. Prove or disprove:

(a). The set of all vectors (v_1, v_2, v_3) in $\Re^3$ such that $v_1 + v_2 = 0$ is a subspace of $\Re^3$.

(b). The set of all vectors (v_1, v_2, v_3, v_4) in $\Re^4$ such that $v_1 - v_2 = 0$, $v_3 = 5v_1$, $v_4 = 0$ is a subspace of $\Re^4$.

Q.2. Are the following set of vectors linearly independent or dependent?

(a). $(3,-2,0,4)$, $(5,0,0,1)$, $(-6,1,0,1)$, $(2,0,0,3)$

(b). $(3,4,7)$, $(2,0,3)$, $(8,2,3)$, $(5,5,6)$

(c). $(1,1,0)$, $(1,0,0)$, $(1,1,1)$

Q.3. Find the rank and nullity of the following matrices

(a). $A = \begin{bmatrix} 8 & 2 & 5 \\ 16 & 6 & 29 \\ 4 & 0 & -7 \end{bmatrix}$ (b). $A = \begin{bmatrix} 8 & 0 & 4 \\ 0 & 2 & 0 \\ 4 & 0 & 2 \\ 0 & 4 & 0 \end{bmatrix}$ (c). $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$

Q.4. Solve the following system of equations by using Gauss Elimination Method :

(a) $3.0x + 6.2y = 0.2$

$2.1x + 8.5y = 4.3$

(b) $3x + 2y + z = 3$

$2x + y + z = 0$

$6x + 2y + 4z = 6$

(c) $3x + 2y + 2z - 5w = 8$

$0.6x + 1.5y + 1.5z - 5.4w = 2.7$

$1.2x - 0.3y - 0.3z + 2.4w = 2.1$

Q.5. Determine the inverse of the following matrices by Gauss-Jordan Method :

(a). $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & -1 & 2 \\ 2 & 4 & 11 \end{bmatrix}$ (b). $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix}$ (c). $A = \begin{bmatrix} 1 & 2 & -9 \\ -2 & -4 & 19 \\ 0 & -1 & 2 \end{bmatrix}$

Q.6. Apply the Gauss-Jacobi and Gauss-Seidel iteration (3 steps) to the following system, starting from $(1,1,1)$

(a). $10\,x + y + z = 6$ (b). $8x + 2y - z = 185.8$

$x + 10\,y + z = 6$ $x + 9\,y - 2\,z = 49.1$

$x + y + 10\,z = 6$ $x + y + 6\,z = -\,61.3$

Q.7. Find the eigenvalues and eigenvectors of the following matrices:

(a). $A = \begin{bmatrix} 5 & -2 \\ 9 & -6 \end{bmatrix}$ (b). $A = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix}$ (c). $A = \begin{bmatrix} -3 & 0 & 4 & 2 \\ 0 & 1 & -2 & 4 \\ 2 & 4 & -1 & -2 \\ 0 & 2 & -2 & 3 \end{bmatrix}$

Q.8. Verify Cayley-Hamilton theorem for the following matrices:

(a). $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, (b). $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

Q.9. Find the inverse of matrix A using Cayley-Hamilton's theorem:

(a). $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$, (b). $A = \begin{bmatrix} 3 & 2 & 4 \\ 4 & 3 & 2 \\ 2 & 4 & 3 \end{bmatrix}$, (c). $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$.

Q.10. Let $\hat{A} = P^{-1}AP$. Prove that A and $\hat{A}$ have the same eigenvalues. Also, prove that if x is an eigenvector of A, then $y = P^{-1}x$ is an eigenvector of $\hat{A}$. Here, A and P are the following matrices.

(a). $A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$, $P = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ (b). $A = \begin{bmatrix} 4 & 0 & 0 \\ 12 & -2 & 0 \\ 21 & -6 & 1 \end{bmatrix}$, $P = \begin{bmatrix} 4 & 0 & 6 \\ 0 & 2 & 0 \\ 6 & 0 & 10 \end{bmatrix}$.

Q.11. Diagonalize following matrices by using Similar Transformations:

(a). $A = \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$, (b). $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$, (c). $A = \begin{bmatrix} 3 & 10 & -15 \\ -18 & 39 & 9 \\ -24 & 40 & -15 \end{bmatrix}$.

Q.12 Which of the following are Linear Transformations:

(a) $T:\Re^2 \to \Re^2$ defined by $T(x,y)=(x+y,x)$

(b) $T:\Re^2 \to \Re^2$ defined by $T(x,y)=(xy,x)$

(c) $T:\Re^2 \to \Re^2$ defined by $T(x,y)=(x+1,y+2)$

(d) $T:\Re^3 \to \Re^2$ defined by $T(x,y,z)=(x+y+z,2x+3y+4z)$

(e) $T:\Re^3 \to \Re^2$ defined by $T(x,y,z)=(|x|,y+z)$

(f) $T:\Re^2 \to \Re^3$ defined by $T(x,y)=(x+3,2y,x+y)$

(g) $T:\Re^3 \to \Re^3$ defined by $T(x,y,z)=(x+y,x-z,2x-y+z)$