Homework 1: Regular Languages: Automata and Expressions

CS 212 Nature of Computation
Habib University

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General instructions

- For drawing finite automata, see this TikZ guide or the JFLAP tool. Hand drawn diagrams will not be accepted.
- Please ensure that your solutions are neatly formatted and organized, and use clear and concise language.
- Please consult Canvas for a rubric containing the breakdown of points for each problem.
- For all the problems below, $\Sigma = \{a, b\}$.
- Some of the problems below make use of the following count function.

$$n_a(w) = \text{the number of occurences of } a \text{ in } w, \text{ where } a \in \Sigma, w \in \Sigma^*.$$

Problems

1. $\boxed{\text{15 points}}$ List 2 members and 2 non-members of the language, $(a \cup ba \cup bb)\Sigma^*$.

Solution: Members:

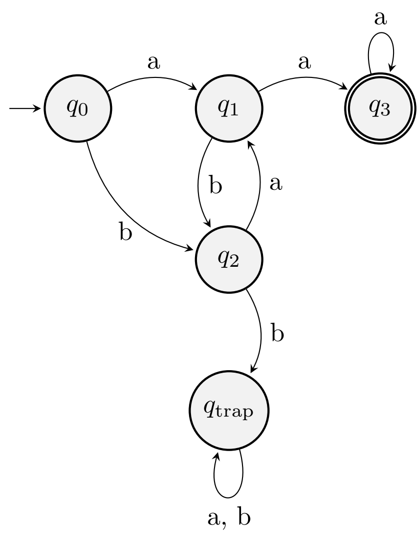
- `aab`: This string starts with a and is followed by any string from Σ^*.
- `bbaa`: This string starts with bb and is followed by any string from Σ^*.

Non-members:

- ε (empty string): The empty string is not included in $(a \cup ba \cup bb)$.
- `b`: This string is not included in $(a \cup ba \cup bb)$.

2. $\boxed{\text{20 points}}$ Provide the state diagram of a simplified DFA that recognizes the language,

$$A = \{w \in \Sigma^* \mid n_a(w) \geq 2, n_b(w) \leq 1\}.$$



Solution:

3. $\boxed{\text{30 points}}$ Given the languages, A and B, we derive the language, $C = \{w \in A \mid w \in B\}$. Prove or disprove the following claim.

Claim 1. *If A and B are regular languages, then so is C.*

Solution: We have to prove a language C has members : w is member of A given that it is member of B, where A and B are some regular languages then so is C. To accomplish this we can construct a DFA that accepts language C to complete the proof.

let $C' = (Q, \sigma, \delta, q_0, F)$ be a DFA that accepts language C. Language A is accepted by DFA A' and language B is accepted by DFA B' Let $A' = (Q_A, \sigma, \delta_A, q_{0A}, F_A)$ and $B' = (Q_B, \sigma, \delta_B, q_{0B}, F_B)$

We can construct C as follows:

- $Q = (q_A, q_B) \mid q_A \in Q_A, q_B \in Q_B$ so the new state set Q is the cartesian product of the state sets of A and B.
- $\sigma = \sigma_A = \sigma_B$ (since w $\in \Sigma*$ and $\Sigma = \{a, b\}$ which is same for A and B)
- $\delta((q_A, q_B), a) = (\delta_A(q_A, a), \delta_B(q_B, a))$

- $q_0 = (q_{0A}, q_{0B})$
- $F = F_A \times F_B$ the start states and end states of C' will be pair of start states and end states of A' and B' respectively.

Now we have to prove that C' is a DFA that accepts language C.

- C is a DFA because it has a finite set of states, a finite set of input symbols, a transition function, a start state, and a set of end states.
- C accepts language C because it accepts all strings w that are accepted by A and B.

Therefore, C is a DFA that accepts language C, so the claim is true.

4. $\boxed{\text{35 points}}$ Given the languages, A and B, we define the following operation.

$$A \smile_a B = \{u \in A \mid \exists v \in B \ni n_a(u) = n_a(v)\}$$

Prove or disprove the following claim.

Claim 2. *The class of regular languages is closed under $\smile_a$.*

Solution: To prove the claim, we need to show that the class of regular languages is closed under the $\smile_a$ operation. To do this, we need to show that for any two regular languages A and B, the language $A \smile_a B$ is also regular. For this proof, if we can construct a DFA that accepts the language $A \smile_a B$. Let $A' = (Q_A, \Sigma, \delta_A, q_{0A}, F_A)$ and $B' = (Q_B, \Sigma, \delta_B, q_{0B}, F_B)$ be the DFAs that accept languages A and B, respectively. However since the resulting language should should have equal number of a's from A and B in both strings so DFA cannot cater to dynamic nature and hence an NFA is required. Moreover, we have seen that an NFA can be converted to a DFA using subset construction. Let C' be an NFA that accepts C where C = $A \smile_a B$

- $Q = \{(q1, q2) | q1 \in Q_A, q2 \in Q_B\ \}$ or we can say that the new state set Q is the cartesian product of the state sets of A and B i.e. $Q = Q_A \times Q_B$
- $\Sigma_\epsilon = \Sigma_A \cup \epsilon = \Sigma_B \cup \epsilon$
  For q_a and $q_b \in$ Q and t $\in \Sigma_\epsilon$

$$\delta((q_A, q_B), t) = \begin{cases} \{(\delta_A(q_a, t), \delta_B(q_B, t))\} when \text{ t = a} \\ \{(\delta_A(q_a, b), q_b)\} when \text{ t = b} \\ \{(q_a, \delta_b(q_b, a),)\} when \text{ t} = \epsilon \end{cases}$$

- $q_0 = (q_{0_A}, q_{0_B})$

- $F = F_A \times F_B$

  To enunciate the start state of C' is pair of start state of A' and B',
  End state F is also cartesian of end state of A' and B'
  $\delta(Q, t)$ is union of all possible states where on a given $\Sigma \in \Sigma_\epsilon$ transition function of A' and B' respectively can transition to.

  $L(C)$ is regular because it is accepted by an NFA and as per theorem 1.39 of our text book every NFA has a corresponding DFA, thus language C is regular. Therefore, the claim is true. Hence, the class of regular languages is closed under the $\smile_a$ operation.