

mae_598_hw1.py

```

1  from scipy.optimize import minimize
2
3  def objective(x):
4      x1=x[0]
5      x2=x[1]
6      x3=x[2]
7      x4=x[3]
8      x5=x[4]
9      obj_1= (x1-x2)**2+(x2+x3-2)**2+(x4-1)**2+(x5-1)**2
10     return obj_1
11
12 def constraint1(x):
13     return x[0]+(3*x[1])
14
15 def constraint2(x):
16     return x[2]+x[3]-(2*x[4])
17
18 def constraint3(x):
19     return x[1]-x[4]
20
21 x0=[1,-5,-3,2,9]
22
23 b=(-10,10)
24 bnds=(b,b,b,b,b)
25 con1={'type':'eq','fun':constraint1}
26 con2={'type':'eq','fun':constraint2}
27 con3={'type':'eq','fun':constraint3}
28 cons=[con1,con2,con3]
29
30 sol = minimize(objective,x0,method='SLSQP',bounds=bnds,constraints=cons)
31 print(sol)

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njev: 7
(my_packages) PS C:\Users\hbhavnav\Documents\Hussain\ASU\collision detection> python -u "c:\Users\hbhavnav\Documents\Hussain\ASU\collision detection\mae_598_hw1
.py"
message: Optimization terminated successfully
success: True
status: 0
  fun: 4.093023273402283
   x: [-7.675e-01  2.558e-01  6.279e-01 -1.162e-01  2.558e-01]
  nit: 7
   jac: [-2.047e+00 -1.860e-01 -2.233e+00 -2.232e+00 -1.488e+00]
 nfev: 43
 njev: 7

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In the above program we have found the minimum of the objective function to be 4.09302327.

The initial guess is shown in the vector $x_0=[1,-5,-3,2,9]$

The minimum value does not change even if we change the initial guesses. I tried many different initial values within the given range and always converged to the same answer

Q2) Let x and $b \in \mathbb{R}^n$ be vectors and $A \in \mathbb{R}^{n \times n}$ be a square matrix. Define $f: \mathbb{R}^n \rightarrow \mathbb{R}$ as $f(x) = b^T x + x^T A x$

a) what are the gradient and Hessian of $f(x)$ with respect to x ?

Ans) $\nabla f(x) = \frac{\partial f(x)}{\partial x}$

we know the derivative of $b^T x = b$

+ the derivative of $x^T A x = A x + A^T x$

$\therefore \boxed{\nabla f(x) = b + A x + A^T x} \rightarrow$ This is the gradient.

$H(x) = \frac{\partial^2 f(x)}{\partial x^2} \rightarrow$ This is the second derivative

$\boxed{H(x) = A + A^T} \rightarrow$ This is the Hessian.

Derive the first & second order Taylor's series approx of $f(x)$ at $x=0$. Are these approximations exact.

$$f(x) \approx f(x_0) + \nabla f(x_0)^T (x - x_0) + \frac{1}{2} (x - x_0)^T H(x_0) (x - x_0)$$

for $f(x) = b^T x + x^T A x$

$f(x_0) = 0.$

$$\nabla f(x_0) = b + Ax_0 + A^T x_0 \\ = b$$

$$Ax_0 = 0$$

$$A^T x_0 = 0.$$

$$H(x_0) = A + A^T$$

$$f(x) \approx 0 + b^T x + \frac{1}{2} x^T [A + A^T] x \rightarrow \text{Taylor's Approximation}$$

The approximation will be exact as.

every other term after the second order term will be 0.

3] Let $A \in \mathbb{R}^{n \times n}$ be a square matrix.

a) what are the necessary and sufficient conditions for A to be positive definite?

Ans: 1) All the eigen values of A must be positive
 $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n > 0$

2) Determinants at every level of matrix should be positive
 $|1 \times 1|, |2 \times 2|, \dots, |n \times n| > 0$

3) All pivots must be positive

4) $x^T A x$ must be positive

b) What are the necessary and sufficient conditions for A to have full rank?

Ans The matrix is full rank if all columns are linearly independent.

This also means that the null space only has the $\{0\}$ vector.

c) If there exists $y \in \mathbb{R}^n$ & $y \neq 0$ such that $A^T y = 0$, then what are the conditions for $Ax = b$ to have a solution for x ?

Ans If $A^T y = 0$ then we can say that matrix A is not full rank & a null space exists for a solution x to exist, b must be in the column space of the linearly independent columns of A . $[Ax = b \text{ can have a solution } x. \text{ where } x \neq 0]$

Q4) Types of food $\rightarrow N_1, N_2, N_3, \dots, N_n$
Types of nutrition $\rightarrow M_1, M_2, M_3, \dots, M_n$

$a_{ij} \rightarrow$ quantity of nutrition j in food i

$c_i \rightarrow$ cost of food i (unit price)

$b_j \rightarrow$ Minimum nutrition type j for a month.

The function that we need to minimize is of the cost variable as we want to satisfy the nutrition requirements for the lowest cost.

minimize for function $f = x_1 c_{N_1} + x_2 c_{N_2} + \dots + x_n c_{N_n}$

x

$x_1, x_2, x_3, \dots, x_n \rightarrow$ This is the quantity of each food type that we require.

Constraints: $x_1 a_{N_1 M_1} + x_2 a_{N_2 M_1} + \dots + x_n a_{N_n M_1} \geq b_{M_1}$

This is the constraint only for nutrition type M_1

For each nutrition type we will have the same equation but M_1 will be replaced by M_2, M_3, \dots, M_n .

Additionally the quantity of food $x_1, x_2, \dots, x_n \geq 0$
need to be non-negative.