mae_598_hw1.py

```python
from scipy.optimize import minimize

def objective(x):
    x1=x[0]
    x2=x[1]
    x3=x[2]
    x4=x[3]
    x5=x[4]
    obj_1= (x1-x2)**2+(x2+x3-2)**2+(x4-1)**2+(x5-1)**2
    return obj_1

def constraint1(x):
    return x[0]+(3*x[1])

def constraint2(x):
    return x[2]+x[3]-(2*x[4])

def constraint3(x):
    return x[1]-x[4]

x0=[1,-5,-3,2,9]

b=(-10,10)
bnds=(b,b,b,b,b)
con1={'type':'eq','fun':constraint1}
con2={'type':'eq','fun':constraint2}
con3={'type':'eq','fun':constraint3}
cons=[con1,con2,con3]

sol = minimize(objective,x0,method='SLSQP',bounds=bnds,constraints=cons)
print(sol)
```

```
(my_packages) PS C:\Users\hbhavnag\Documents\Hussain\ASU\collision detection> python -u "c:\Users\hbhavnag\Documents\Hussain\ASU\collision detection\mae_598_hw1.py"
 message: Optimization terminated successfully
 success: True
  status: 0
     fun: 4.093023273402283
       x: [-7.675e-01  2.558e-01  6.279e-01 -1.162e-01  2.558e-01]
     nit: 7
     jac: [-2.047e+00 -1.860e-01 -2.233e+00 -2.232e+00 -1.488e+00]
    nfev: 43
    njev: 7
```

In the above program we have found the minimum of the objective function to be 4.09302327.

The initial guess is shown in the vector x0=[1,-5,-3,2,9]

The minimum value does not change even if we change the initial guesses. I tried many different initial values within the given range and always converged to the same answer

Hw-1 MAE - 598 - Design Optimization

Q2) Let x and $b \in R^n$ be vectors and $A \in R^{n \times n}$ be a square matrix. Define $f: R^n \to R$ as $f(x) = b^T x + x^T A x$

a) what are the gradient and Hessian of $f(x)$ with respect to x?

Ans) $\nabla f(x) = \dfrac{\partial f(x)}{\partial x}$

we know the derivative of $b^T x = b$

& the derivative of $x^T A x = Ax + A^T x$

$\therefore \boxed{\nabla f(x) = b + Ax + A^T x} \rightarrow$ This is the gradient.

$H(x) = \dfrac{\partial^2 f(x)}{\partial x^2} \rightarrow$ This is the second derivative

$\boxed{H(x) = A + A^T} \rightarrow$ This is the Hessian.

Derive the first & second order Taylor's Series. approx of $f(x)$ at $x=0$. Are these approximations exact.

$f(x) \approx f(x_0) + \nabla f(x_0)^T (x - x_0) + \frac{1}{2}(x-x_0)^T H(x_0)(x - x_0)$

for $f(x) = b^T x + x^T A x$

$f(x_0) = 0.$

$$\nabla f(x_0) = b + Ax_0 + A^T x_0 \qquad Ax_0 = 0$$

$$= b \qquad A^T x_0 = 0.$$

$$H(x_0) = A + A^T$$

$$\boxed{f(x) \approx 0 + b^T x} \rightarrow \text{first order.}$$

$$\boxed{f(x) \approx 0 + b^T x + \frac{1}{2} x^T [A + A^T] x} \rightarrow \text{Taylors Second order Approximation}$$

The approximation will be exact as.
every other term. after the second order term will be 0

3] Let $A \in R^{n \times n}$ be a square matrix.

a) what are the necessary and sufficient conditions for A to be positive definite?

Ans: 1) All the eigen values of A must be positive

$\lambda_1, \lambda_2, \lambda_3 \ldots\ldots \lambda_n > 0$

2) Determinents at every level of matrix should be positive

$|1 \times 1|, |2 \times 2|, \ldots\ldots |n \times n| > 0$

3) All pivots must be positive

4) $x^T A x$ must be positive

b) What are the necessary and sufficient conditions for A to have full rank?

Ans The matrix is full rank if all columns are linearly Independant.

This also means that the null space ~~&~~ only has the {0} vector.

Also the eigen values will be non zero.

c) If there exists $y \in R^n$ & $y \neq 0$ such that $A^T y = 0$, then what are the conditions for $Ax = b$ to have a solution for x?

Ans If $A^T y = 0$ then we can say that matrix A is not full rank & a null space exists for a solution x to exist, b must be in the column space of the linearly Independant columns. of A. [$Ax = b$ can have a solution x. ~~when~~ where $x \neq 0$]

Q4) Types of food $\rightarrow N_1, N_2, N_3 \ldots\ldots N_n$

Types of nutrition $\rightarrow M_1, M_2, M_3 \ldots\ldots M_n$

$a_{ij} \rightarrow$ quantity of nutrition j in food i

$c_i \rightarrow$ ~~$\$$~~ cost of food i (unit price)

$b_j \rightarrow$ Minimum nutrition of type j for a month.

The function that we need to minimize is of the cost variable as we want to satisfy the nutrition requirements for the lowest cost.

minimize c for function $\boxed{f = x_1 c_{N_1} + x_2 c_{N_2} + \ldots x_n c_{N_n}}$

$x_1, x_2, x_3 \ldots x_n \rightarrow$ This is the quantity of each food type that we require.

Constrains : $\boxed{x_1 a_{N_1 M_1} + x_2 a_{N_2 M_1} \ldots x_n a_{N_n M_1} \geq b_{M_1}}$

This is the constraint only for nutrition type M_1

For each nutrition type we will have the same equation but M_1 will be replaced by $M_2, M_3 \ldots M_n$.

Additionally the quantity of food $\boxed{x_1, x_2 \ldots x_n \geq 0}$ need to be non-negative.