Phys 512 Problem Set 1

Due on github Friday Sep 18 at 4 PM. You may discuss problems, but everyone must write their own code.

Problem 1: We saw in class how Taylor series/roundoff errors fight against each other when deciding how big a step size to use when calculating numerical derivatives. If we allow ourselves to evaluate our function f at four points ($x \pm \delta$ and $x \pm 2\delta$),
a) what should our estimate of the first derivative at x be? Rather than doing a complicated fit, I suggest thinking about how to combine the derivative from $x \pm \delta$ with the derivative from $x \pm 2\delta$ to cancel the next term in the Taylor series.
b) Now that you have your operator for the derivative, what should δ be in terms of the machine precision and various properties of the function? Show for $f(x) = \exp(x)$ and $f(x) = \exp(0.01x)$ that your estimate of the optimal δ is at least roughly correct.

Problem 2: Lakeshore 670 diodes (successors to the venerable Lakeshore 470) are temperature-sensitive diodes used for a range of cryogenic temperature measurements. They are fed with a constant 10 μA current, and the voltage is read out. Lakeshore provides a chart that converts voltage to temperature, available at https://www.lakeshore.com/products/categories/specification/temperature-products/cryogenic-temperature-sensors/dt-670-silicon-diodes, or you can look at the text file I've helpfully copied and pasted (lakeshore.txt). Write a routine that will take an arbitrary voltage and interpolate to return a temperature. You should also make some sort of quantitative (but possibly rough) estimate of the error in your interpolation as well (this is a common situation where you have been presented with data and have to figure out *some* idea of how to get error estimates).

Problem 3: Take $\cos(x)$ between $-\pi/2$ and $\pi/2$. Compare the accuracy of polynomial, cubic spline, and rational function interpolation given some modest number of points, but for fairness each method should use the same points. Now try using a Lorentzian $1/(1+x^2)$ between -1 and 1.

What should the error be for the Lorentzian from the rational function fit? Does what you got agree with your expectations when the order is higher (say n=4, m=5)? What happens if you switch from np.linalg.inv to np.linalg.pinv (which tries to deal with singular matrices)? Can you understand what has happened by looking at p and q?¡

Problem 4: One can work out the electric field from an infinitessimally thin spherical shell of charge with radius R by working out the field from a ring along its central axis, and integrating those rings to form a spherical shell. Use both your integrator and scipy.integrate.quad to plot the electric field from the shell as a function of distance from the center of the sphere. Make sure the range of your plot covers regions with $z < R$ and $z > R$. Make sure one of your

z values is R. Is there a singularity in the integral? Does quad care? Does your integrator? Note - if you get stuck setting up the problem, you may be able to find solutions to Griffiths problem 2.7, which sets up the integral.