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Speech & Language Processing
↳ Most of the content from NLP

Keywords -
Word Embedding, Regular expressions, NLP, Tokenization

—x—

O - notation
(Upper Bound) $0 \le f(n) \le cg(n)$

Ω - notation
(Lower Bound) $0 \le cg(n) \le f(n)$

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* Better to think an algo in a recursive manner
→ To get an eqn and solve to find time complexity.

- Simple observation
- Recursive eqn (by substitution)
- Master's theorem
- ~~Co~~ Solving by Recursion tree

Q. $t_n = 2t_{n-1} + n$ —— ①
Replace n by $n+1$:
$t_{n+1} = 2t_n + n + 1$
$\Rightarrow$ $2t_{n+1} = 4t_n + 2n + 2$ —— ②

$t_n = 2t_{n-1} + n$ —— ①

$t_{n-1} = 2t_{n-2} + n-1$

$t_{n-2} = 2t_{n-3} + n-2$

⋮

$t_{n-k} = 2t_{n-k-1} + n-k$

$t_n + t_{n-1} + t_{n-2} + \cdots t_{n-k}$
$= 2(t_{n-1} + t_{n-2} + \cdots t_{n-k-1}) +$
$n + n-1 + n-2 + \cdots n-k$

$\Rightarrow t_n + (t_{n-1} + t_{n-2} + \cdots t_{n-k})$
$= 2(t_{n-1} + t_{n-2} + \cdots t_{n-k}) + 2t_{n-k-1}$
$+ \; nk - \frac{k(k+1)}{2}$

$\Rightarrow$

$t_{n-1} + t_{n-2} + \cdots t_{n-k} + 2t_{n-k-1}$

$= \frac{k(k+1)}{2} - nk$

$= \frac{k^2 + k - 2nk}{2}$

$n = k+1$

$\Rightarrow t_k + t_{k-1} + t_{k-2} + \cdots t_1 + 2t_0$

$= -\frac{k(k+1)}{2}$

$t_n = 2t_{n-1} + n$ — ①

$2t_{n+1} = 4t_n + 2n + 2$ — ②

Replacing n by $n+2$ in ①

$t_{n+2} = 2t_{n+1} + n + 2$ — ③

Add ① & ③

$t_n + t_{n+2} = 2t_{n-1} + 2t_{n+1} + n + n + 2$

$\Rightarrow$ $\cancel{2}\, t_{n+2} - \cancel{2}\, t_{n+1} + t_n - 2t_{n-1} = 2n + 2$ — ④

Subtract ② from ④

$t_{n+2} - 2t_{n+1} + t_n - 2t_{n-1} - 2t_{n+1} + 4t_n = 2n + 2 - 2n - 2$

$\Rightarrow$ $t_{n+2} - 4t_{n+1} + 5t_n - 2t_{n-1} = 0$

$x^3 - 4x^2 + 5x - 2 = 0$
Roots $= 1, 1, 2$

$t_n = C_1 2^n + C_2 1^n + C_3 n 1^n$

$t_n = O(2^n)$

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Q. $T(n) = 4T(n/2) + n$ —— ①

$T\left(\frac{n}{2}\right) = 4T\left(\frac{n}{4}\right) + \frac{n}{2}$ —— ②

$T(n) = 4\left[4T\left(\frac{n}{4}\right) + \frac{n}{2}\right] + n$

$= 16T\left(\frac{n}{4}\right) + 2n + n = 16T\left(\frac{n}{4}\right) + 3n$

$= 16\left[4T\left(\frac{n}{8}\right) + \frac{n}{4}\right] + 3n$

$= 64T\left(\frac{n}{8}\right) + 7n$

$64\left[4T\left(\frac{n}{16}\right) + \frac{n}{8}\right] + 7n$

$256T\left(\frac{n}{16}\right) + 15n$

$T(n) = 4^K T\left(\frac{n}{2^K}\right) + (2^K - 1)n$

$T(n) = 4^K T\left(\frac{n}{2^K}\right) + (2^K - 1)n$

$n = 2^K$

$T(n) = (2^2)^K T(1) + (n-1)n$

$= n^2 + n^2 - n = 2n^2 - n$

$T(n) = O(n^2)$

Ma'am's Method :

Replacing n by 2^K - $T(2^K) = 4T(2^{K-1}) + 2^K$

Putting, $t_K = T(2^K) = 4t_{K-1} + 2^K$ —— ①

Putting $K = K-1$, $t_{K-1} = 4t_{K-2} + 2^{K-1}$ —— ②

Multiplying ② by 2

$2t_{K-1} - 8t_{K-2} = 2 \cdot 2^{K-1} = 2^K$ —— ③

Subtract ③ from ①

$t_K - 6t_{K-1} + 8t_{K-2} = 0 \Rightarrow x^2 - 6x + 8 = 0$

Roots $(2, 4)$ $T(2^K) = C_1 2^K + C_2 4^K$

$= C_1 n + C_2 n^2$

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Q. $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$T\left(\frac{n}{2}\right) = 4T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2$

$T(n) = 4\left[4T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2\right] + n^2$

$= 16T\left(\frac{n}{4}\right) + 2n^2$

$T(n) = 16\left[4T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2\right] + 2n^2$

$= 64T\left(\frac{n}{8}\right) + n^2 + 2n^2 = 64T\left(\frac{n}{8}\right) + 3n^2$

$T(n) = 64\left[4T\left(\frac{n}{16}\right) + \left(\frac{n}{8}\right)^2\right] + 3n^2$

$= 256T\left(\frac{n}{16}\right) + 4n^2$

$T(n) = 256\left[4T\left(\frac{n}{32}\right) + \left(\frac{n}{16}\right)^2\right] + 3n^2$

$= 1024T\left(\frac{n}{32}\right) + 4n^2$

$T(n) = 4^K T\left(\frac{n}{2^K}\right) + Kn^2$

$2^K = n \Rightarrow K = \log n$

$T(n) = n^2 T(1) + n^2 \log n$

$T(n) = O(n^2 \log n)$

Q. $T(n) = 7T\left(\frac{n}{2}\right) + 3n^2$

$T(n) = 7\left[7T\left(\frac{n}{4}\right) + 3\left(\frac{n}{2}\right)^2\right] + 3n^2$

(Will get difficult by Old School Method)

Putting $n = 2^k$, $T(2^k) = t_k$

$t_k = 7t_{k-1} + 3 \cdot 2^{2k}$ —— ①

Putting $k = k-1$

$t_{k-1} = 7t_{k-2} + 3 \cdot 2^{2(k-1)}$ —— ②

Multiplying ② by 4

$4t_{k-1} = \cancel{+4} \; 28t_{k-2} + 3 \cdot 2^{2k}$ —— ③

Subtract ③ from ③

$4t_{k-1} - 28t_{k-2} - 3 \cdot 2^{2k} - t_k + 7t_{k-1} + 3 \cdot 2^{2k} = 0$

$-t_k + 11t_{k-1} - 28t_{k-2} = 0$

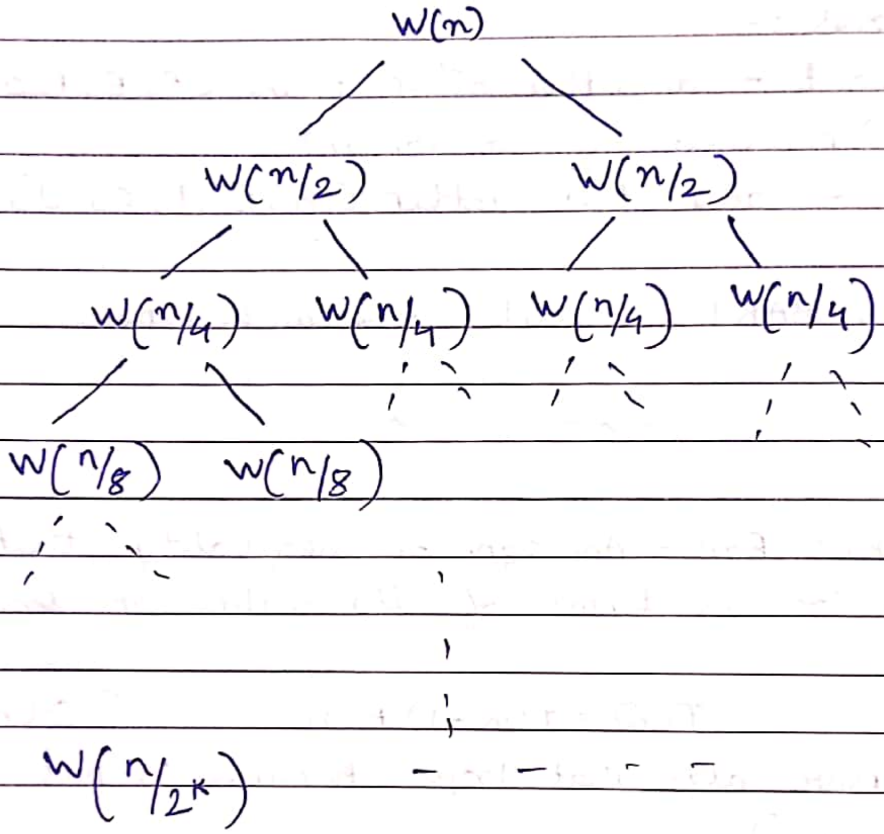
$\Rightarrow t_k - 11t_{k-1} + 28t_{k-2} = 0$

$x^2 - 11x + 28 = 0$ Roots $= 4, 7$

$t_k = C_1 \cdot 4^k + C_2 \cdot 7^k = C_1 n^2 + C_2 7^{\log n}$

$T(n) = O(7^{\log n})$

Q. $W(n) = 2W(n/2) + n^2$



n^2

$\frac{n^2}{2}$

$4\left(\frac{n}{4}\right)^2 = \frac{n^2}{4}$

$8\left(\frac{n}{8}\right)^2 = \frac{n^2}{8}$

$\frac{n^2}{2^K}$

$$T(n) = n^2 + \frac{n^2}{2} + \frac{n^2}{4} + \frac{n^2}{8} + \cdots + \frac{n^2}{2^K}$$

$$= n^2\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^K}\right]$$

$$= \cancel{n^2\left[\frac{1(1-1/2)}{(1-1/2)^n}\right]}$$

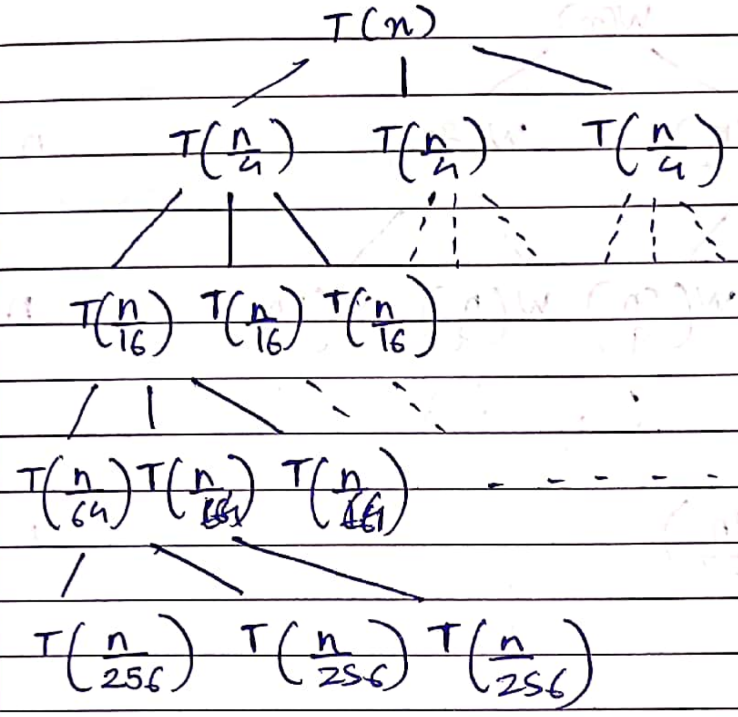
$$= n^2\left[\frac{(1-1/2)}{1-\frac{1}{2^n}}\right] = n^2 \cdot \frac{1/2}{2^n - 1} \cdot 2^n$$

$$= \cancel{\theta(n^2 2}$$

$$\theta(2^n)$$

Q. $T(n) = 3T(n/4) + cn^2$

$\cancel{(4)^2}$
$(4^2)^2 = 16^2$
$(4^2)^3 = 64^2$



cn^2

$3 \times c\left(\frac{n}{4}\right)^2 = \frac{3cn^2}{4^2}$

$9 \times c\left(\frac{n}{16}\right)^2 = \frac{9cn^2}{16^2}$

$27 \times c\left(\frac{n}{64}\right)^2$

$= \frac{27cn^2}{64^2}$

- - - - - - - - - - - $3^K c\left(\frac{n^2}{16^K}\right)^K$

$= cn^2\left(\frac{3}{16}\right)^K$

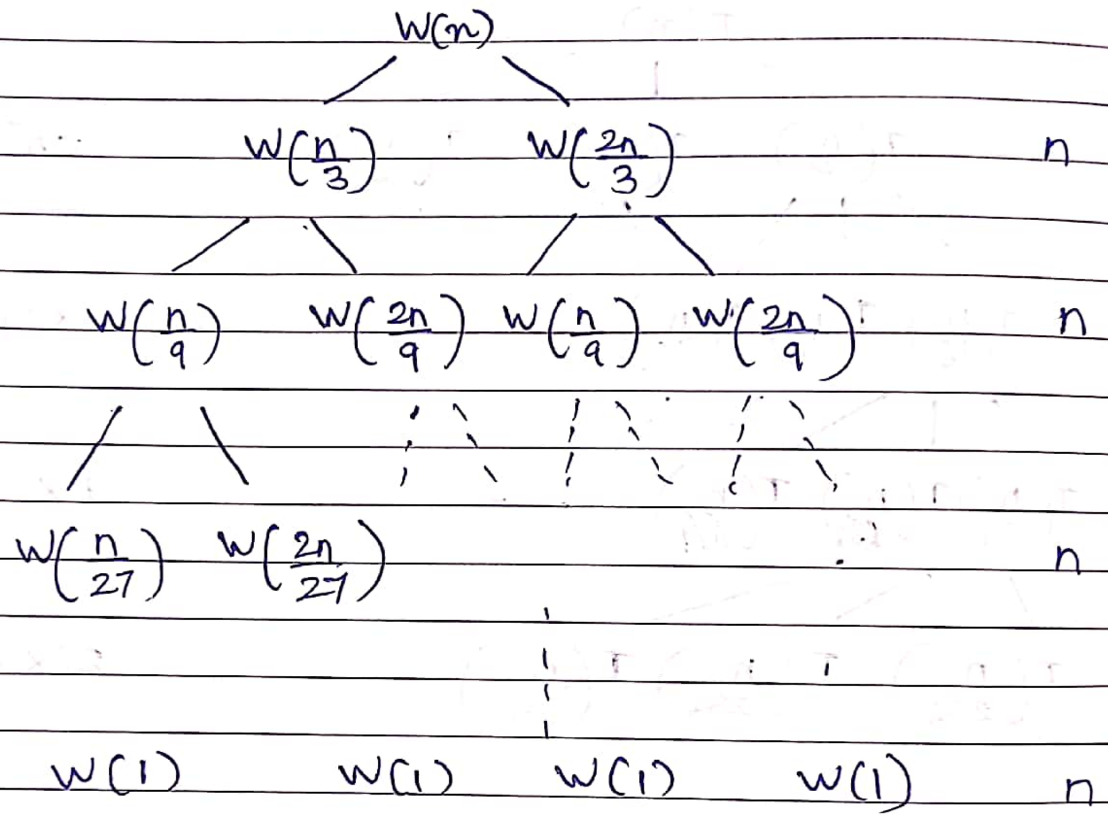
$$T(n) = cn^2 + cn^2\left(\frac{3}{16}\right) + cn^2\left(\frac{3}{16}\right)^2 + cn^2\left(\frac{3}{16}\right)^3 + \cdots + cn^2\left(\frac{3}{16}\right)^K$$

$$= cn^2\left[1 + \frac{3}{16} + \left(\frac{3}{16}\right)^2 + \left(\frac{3}{16}\right)^3 + \cdots \left(\frac{3}{16}\right)^K\right]$$

$$= cn^2\left[\frac{1}{1 - 3/16}\right] = \frac{16}{13}cn^2$$

$$= \Theta(n^2)$$

Q. $W(n) = W(n/3) + W(2n/3) + n$



$T(n) = n + n + n + \cdots + n$ (K times)

$= nK = n\log n$

$= \theta(n\log n)$

MASTER'S METHOD

$T(n) = aT\left(\frac{n}{b}\right) + f(n)$, $a \geq 1$, $b > 1$ & $f(n) > 0$

Idea: Compare $f(n)$ with $n^{\log_b a}$

NP - Non Deterministic Polynomial class of problems.

P : Class P problem that can be solved in polynomial time.

* Verification is easier than finding a solution (NP)

NP : A decision prob where instances.

NP - complete : An NP problem X for which it is possible to reduce any other NP problem Y to X in polynomial time.

- It means that we can solve Y quickly if we know how to solve X quickly.

Precisely, Y is reducible to X if there is a polynomial time algo 'f' to transform instances x of X to instances y = f(x).

* It can be shown that every NP problem can be reduced to 3-SAT - Karp's Theorem

Discussed in Last lecture :-
Spell correction & Min Edit distance

DP algo -

Initialization
$D(i,0) = i$
$D(0,j) = j$
Recurrence relation :
For each $i = 1 \dots M$
For each $j = 1 \dots N$

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 & \text{(delete)} \\ D(i,j-1) + 1 & \text{(insertion)} \\ D(i-1,j-1) + \begin{cases} 2 ; & \text{if } X(i) \neq Y(j) \\ 0 ; & \text{if } X(i) = Y(j) \end{cases} & \text{(subs)} \end{cases}$$

Termination :
$D(N,M)$ is distance

Min edit with Backtrace -

n	9	8	9	10	11	12	11	10	9	
o	8	7	8	9	10	11	10	9	8	
i	7	6	7	8	9	10	9	8	9	
t	6	5	6	7	8	9	8	9	10	
n	5	4	5	6	7	8	9	10	11	
e	4	3	4	5	6	7	8	9	10	
t	3	4	5	6	7	8	7	8	9	
n	2	3	4	5	6	7	8	7	8	
i	1	2	3	4	5	6	7	6	7	
#	0	1	2	3	4	5	6	7	8	9
	#	e	x	e	c	u	t	i	o	n

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
```

TC : $O(nm)$, SC : $O(nm)$

Backtrace : $O(n+m)$

Weighted Minimum Edit Distance - when there are ~~pos~~ some errors have more possibility then others.

NOISY CHANNEL

(Example for NB)

5 features : x_1, x_2, x_3, x_4, x_5

3 classes : Y_1, Y_2, Y_3

$$\cancel{P(A)}\ P(E_1|A) = \frac{P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + \cdots}$$

An observation x of the misspelled word

Find the correct word w,

$$\hat{w} = \underset{w \in V}{\operatorname{argmax}}\ P(w|x)$$

$$= \underset{w \in V}{\operatorname{argmax}}\ \frac{P(x|w)\,P(w)}{P(x)}$$

$$= \underset{w \in V}{\operatorname{argmax}}\ P(x|w)\,P(w)$$

Q.1 $T(n) = 9T(n/3) + n$

$a = 9,\ b = 3,\ f(n) = n$

$\Rightarrow n^K \log^P n = n \Rightarrow K = 1$

$a > b^K \quad \{9 > 3^1\}$

$\Theta(n^{\log_3 9}) = \Theta(n^2)$

Q.2 $T(n) = T(2n/3) + 1$

$a = 1,\ b = 3/2,\ f(n) = 1$

$n^K \log^P n = 1 \Rightarrow K = 0$

$a = b^K \quad p = 0$

$T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

$= \Theta(n^{\log_{3/2} 1} \log n) = \Theta(\log n)$

Q.3 $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$n^K \log^P n = n\log n$

$\Rightarrow K = 1,\ p = 1$

$a < b^K,\ p \geq 0$

$T(n) = \Theta(n^K \log^P n) = \Theta(n\log n)$

Q.4 ~~$T(n/2$~~ $T(n) = 8T(n/2) + \Theta(n^2)$

$a = 8,\ b = 2,\ f(n) = \Theta(n^2)$

$n^K \log^P n = n^2 \Rightarrow K = 2,\ p = 0$

$a < b^K$

$T(n) = \Theta(n^{\log_b a}) = \Theta(n^{\log_2 8}) = \Theta(n^3)$

Q.5 $T(n) = 7T(n/2) + \Theta(n^2)$

$a = 7,\ b = 2,\ n^K \log^P n = n^2$

$K = 2,\ p = 0$

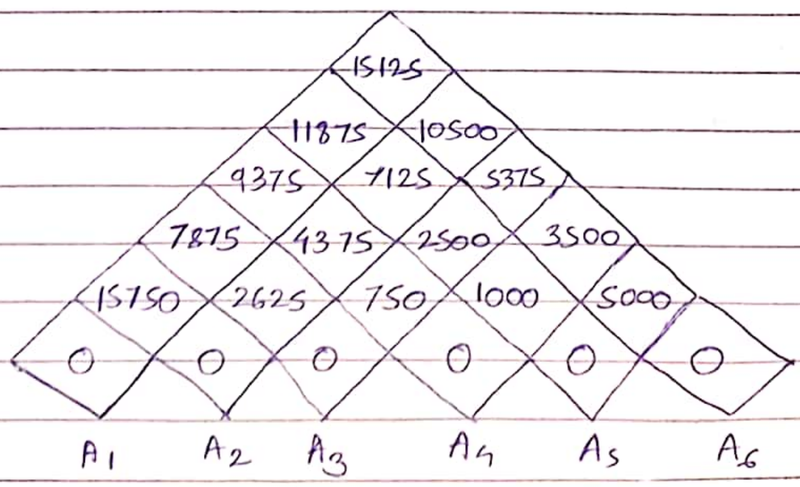
$a < b^K,\ T(n) = \Theta(n^K \log^P n)$

$= \Theta(n^2)$

Criteria	DP	D&C	Greedy
Divide into subparts	Y	Y	N
Independent Subproblems	N	Y	N
Guaranteed optimal sol^n	Y	Y	N
Choose best at current step	N	Y	Y
Suitable for NP, NP-H class probs	Y	N	Y
Bottom up/ Top down	Both	Top-down	-

Problem :

A_1	A_2	A_3	A_4	A_5	A_6
30×35	35×15	15×5	5×10	10×20	20×25



$A \times B = C$ No. of calculations $= pqr$

$p\times q \quad q\times r \quad p\times r$

$$\min \begin{cases} (A_1A_2)A_3 = 15750 + 2250 = 18000 \\ A_1(A_2A_3) = 5250 + 2625 = 7875 \end{cases}$$

$$\min \begin{cases} (A_2A_3)A_4 = 2625 + 1750 = 4375 \\ (A_2)(A_3A_4) = 5250 + 750 = 6000 \end{cases}$$

$$\min \begin{cases} (A_3A_4)A_5 = 750 + 3000 = 3750 \\ A_3(A_4A_5) = 1500 + 1000 = 2500 \end{cases}$$

$$\min \begin{cases} (A_4A_5)A_6 = 1000 + 2500 = 3500 \\ A_4(A_5A_6) = 1250 + 5000 = 6250 \end{cases}$$

$$\min \begin{cases} (A_1A_2A_3)A_4 = 7875 + 1500 = 9375 \\ (A_1A_2)(A_3A_4) = 15750 + 750 + 4500 = 21000 \\ A_1(A_2A_3A_4) = 10500 + 4375 = 14875 \end{cases}$$

$$\min \begin{cases} (A_2A_3A_4)A_5 = 4375 + 7000 = 11375 \\ (A_2A_3)(A_4A_5) = 2625 + 1000 + 3500 = 7125 \\ A_2(A_3A_4A_5) = 9000 + 2500 = 11500 \end{cases}$$

$$\min \begin{cases} (A_3A_4A_5)A_6 = 2500 + 7500 = 10000 \\ (A_3A_4)(A_5A_6) = 750 + 5000 + 3750 = 9500 \\ A_3(A_4A_5A_6) = 1875 + 3500 = 5375 \end{cases}$$

$$\min \begin{cases} (A_1A_2A_3A_4)A_5 = 9375 + 6000 = 15375 \\ (A_1A_2A_3)(A_4A_5) = 7875 + 1000 + 3000 = 11875 \\ (A_1A_2)(A_3A_4A_5) = 15750 + 2500 + 9000 = 27250 \\ A_1(A_2A_3A_4A_5) = 21000 + 7125 = 28125 \end{cases}$$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & 20 & 30 & 10 & 11 \\ 2 & 15 & \infty & 16 & 4 & 2 \\ 3 & 3 & 5 & \infty & 2 & 4 \\ 4 & 19 & 6 & 18 & \infty & 3 \\ 5 & 16 & 4 & 7 & 16 & \infty \end{array}$$

TSP Branch & Bound

reduced row
reduced column
reduced matrix

Brute-force complexity - $n!$

reduced row (from every row find min val & subtract it from all elements)

$$\begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix}$$

$L = 10 + 2 + 2 + 3 + 4 + 1 + 3$
$= 25$

reduced column

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

path is from 1 to 2
(i) make all row 1 as ∞
(ii) all incoming to 2 as ∞
(iii) make 2 to 1 as ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

cost of node 2 = cost of node 1 + cost of edge (1,2) + reduction cost
$= 25 + 10 + 0 = 35$

path from 1 to 3

i) make all row 1 as ∞

ii) all incoming to 3 as ∞

iii) make 3 to 1 as ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12-11 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 13-11 & 3 & \infty & \infty & 0 \\ 11-11 & 0 & \infty & 12 & \infty \end{bmatrix}$$

cost of node 3 = cost of node 1 + cost of edge (1,3) + reduction cost

= 25 + 17 + 11 = 53

path from 1 to 4

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

cost of node 4

= 25 + 0 + 0

= 25

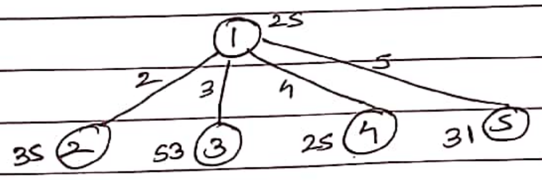
path from 1 to 5

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12^{-2} & \infty & 11^{-2} & 2^{-2} & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15^{-3} & 3^{-3} & 12^{-3} & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

cost of node 5

= 25 + 1 + 5

= 31



Path from 4 to 2

(i) make all row 4 as ∞

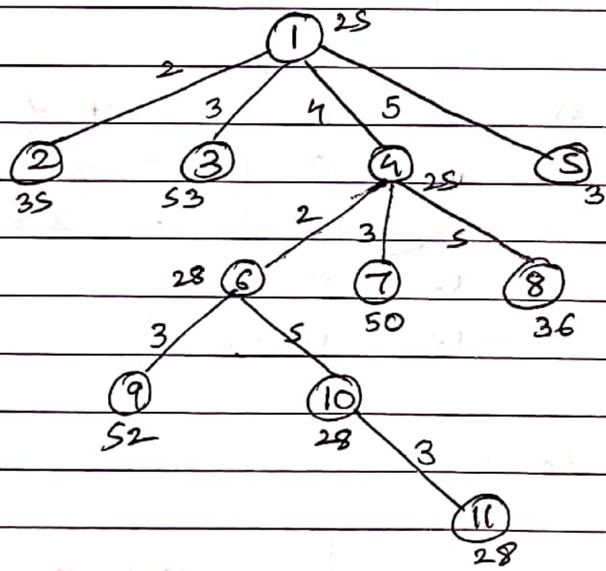
(ii) all incoming to 2 as ∞

(iii) make 4 to 2 as ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

cost of node 4 = cost of node 2 + cost of edge (4, 2) + reduction cost

$= 25 + 3 + 0 = 28$



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TSP by Dynamic programming

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 2 & 9 & 10 \\ 2 & 1 & 0 & 6 & 4 \\ 3 & 15 & 7 & 0 & 8 \\ 4 & 6 & 3 & 12 & 0 \end{array}$$

consider source 1.

Coming back to city 1

$g(2,\phi) = 1$, $g(3,\phi) = 15$, $g(4,\phi) = 6$

$K = 1$ (with one vertex in betn)

Set $\{2\}$ = $g(3,\{2\})$, $g(4,\{2\})$
$C_{32} + g(2,\phi)$ $\quad C_{42} + g(2,\phi)$

Set $\{3\}$ = $g(2,\{3\})$, $g(4,\{3\})$
$C_{23} + g(3,\phi)$ $\quad C_{43} + g(3,\phi)$

Set $\{4\}$ = $g(2,\{4\})$, $g(3,\{4\})$
$C_{24} + g(4,\phi)$ $\quad C_{34} + g(4,\phi)$

More clear written Below

Set $\{2\}$ = $g(3,\{2\}) = C_{32} + g(2,\phi) = 7+1 = 8$
$g(4,\{2\}) = C_{42} + g(2,\phi) = 3+1 = 4$

Set $\{3\}$ = $g(2,\{3\}) = C_{23} + g(3,\phi) = 6+15 = 21$
$g(4,\{3\}) = C_{43} + g(3,\phi) = 12+15 = 27$

Set $\{4\}$ = $g(2,\{4\}) = C_{24} + g(4,\phi) = 4+6 = 10$
$g(3,\{4\}) = C_{34} + g(4,\phi) = 8+6 = 14$

$P(3,\{2\}) = 2$, $P(4,\{2\}) = 2$
$P(2,\{3\}) = 3$, $P(4,\{3\}) = 3$
$P(2,\{4\}) = 4$, $P(3,\{4\}) = 4$

$K = 2$ (2 vertex in betⁿ)

Set $\{2,3\} = g(4, \{2,3\})$

$= \min \{ C_{42} + g(2, \{3\}), C_{43} + g(3, \{2\}) \}$

$= \min(3 + 21, 12 + 8) = 20$, $P(4, \{2,3\}) = 2$

Set $\{2,4\} = g(3, \{2,4\})$

$= \min \{ C_{32} + g(2, \{4\}), C_{34} + g(4, \{2\}) \}$

$= \min(7 + 10, 8 + 4) = 12$, $P(3, \{2,4\}) = 4$

Set $\{3,4\} = g(2, \{3,4\})$

$= \min \{ C_{23} + g(3, \{4\}), C_{24} + g(4, \{3\}) \}$

$= \min(6 + 14, 4 + 27) = 20$, $P(2, \{3,4\})$ =

$K = 3$

Set $\{2,3,4\} = g(1, \{2,3,4\})$

$= \min \{ C_{12} + g(2, \{3,4\}) +$
$C_{13} + g(3, \{2,4\}) +$
$C_{14} + g(4, \{2,3\}) \}$

$= \min(2 + 20, 9 + 12, 10 + 20)$

$= 21$, $P(1, \{2,3,4\}) = 3$

Successor of node 1 : $P(1, \{2, 3, 4\}) = 3$

— " — node 3 : $P(3, \{2, 4\}) = 4$

— " — node 4 : $P(4, \{2\}) = 2$

$P(2, \phi) = 1$

[[1 — 2 — 4 — 3 — 1]]

0/1 Knapsack by Branch & Bound

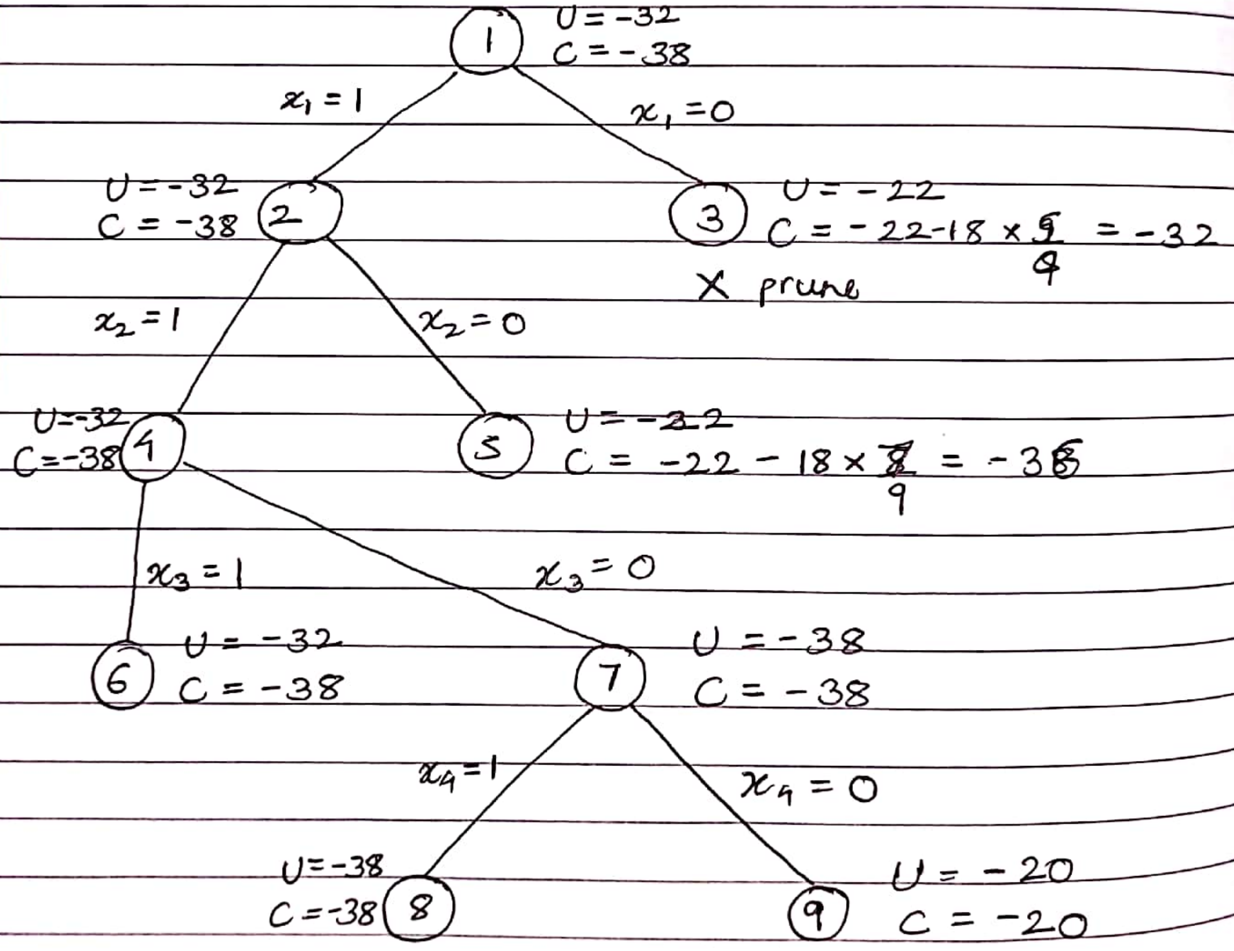
$n = 4$, $m = 15$

Profit = $\{10, 10, 12, 18\}$

Weight = $\{2, 4, 6, 9\}$

Initial upper bound
$U = -32$

Cost function
$C = -32 - 18 \times \frac{3}{9} = -38$



Sol^n - $\{1, 1, 0, 1\}$, Max profit = 38

$n = 4$, $m = 10$ $\qquad U =$

$P = \{40, 42, 25, 12\}$ $\qquad C =$

$\omega = \{4, 7, 5, 3\}$

Table Continued from (Dt. 01/02/23)

Criteria	Backtracking	Branch & Bound
Divide into Sub	N	N
Independent Sub	N	N
Guaranteed optimal	Y	Y
Choose best at Current step	N	Y
Suitable for NP, NP-H class problems	Y	Y
Bottom-up / Top-down	Top-down	Top-down

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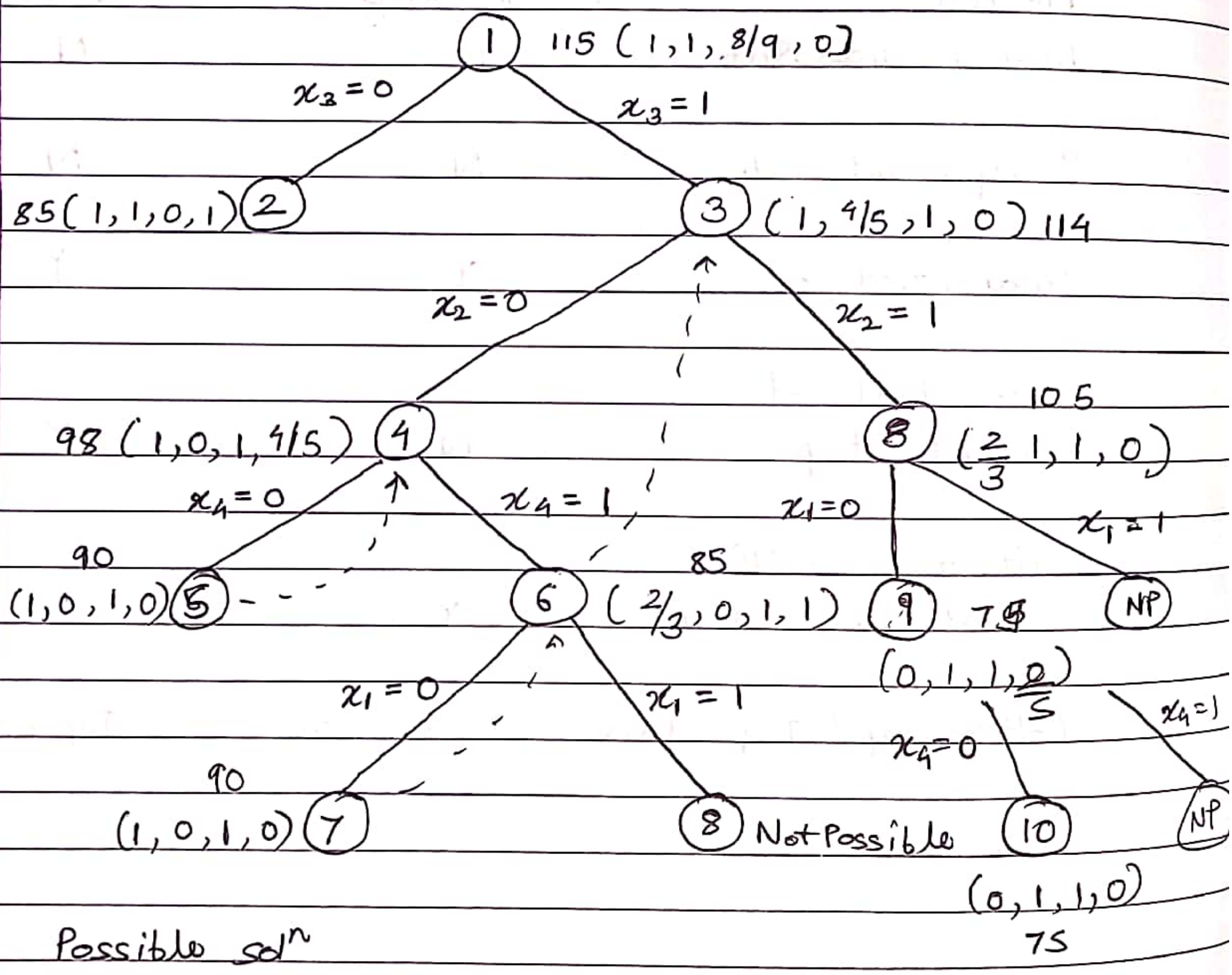
0/1 Knapsack - Backtracking (DFS)

$m = 16$, $n = 4$

$P = \{45, 30, 45, 10\}$

$W = \{3, 5, 9, 5\}$

[$P/W = [15, 6, 5, 2]$]



Possible sol^n

85 - {1, 1, 0, 1}

55 - {0, 0, 1, 1}

90 - {1, 0, 1, 0}

75 - {0, 1, 1, 0}

0/1 Knapsack - BT H/W

$m = 110$, $n = 8$

$P = \{11, 21, 31, 33, 43, 53, 55, 65\}$

$W = \{1, 11, 21, 23, 33, 43, 45, 55\}$

$P/W = [11, 1.91, 1.47, 1.43, 1.3, 1.23, 1.22, 1.18]$

~~1~~

$n = 4$, $m = 16$

$P = \{40, 30, 50, 10\}$

$W = \{2, 5, 10, 5\}$

$P/W = [20, 6, 5, 2]$