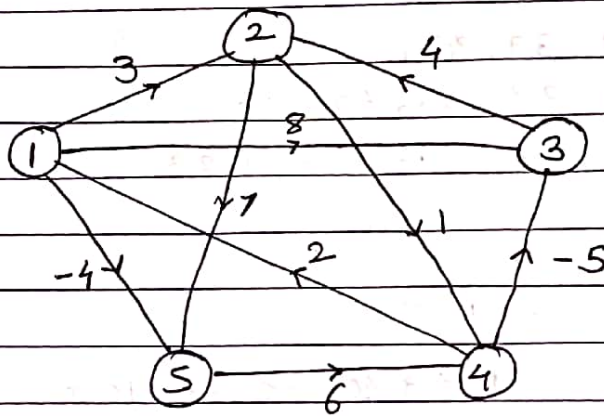


Floyd - Warshall - DP



$$A[i, j] = \min(A[i, j], A[i, k] + A[k, j])$$

A =

	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	-5	0	∞
5	∞	∞	∞	6	0

Path Matrix

P₀ :

-	1	1	-	1
-	-	-	2	2
-	3	-	-	-
4	-	4	-	-
-	-	-	5	-

Pass 1 : Vertex no. ① in between

A₁ :

0	3	8	∞	-4
∞	0	∞	1	7
∞	4	0	∞	∞
2	⑤	-5	0	②
∞	∞	∞	6	0

	1	2	3	4	5
$P_1 :$	1	-	1	1	-
	2	-	-	-	2
	3	-	3	-	-
	4	4	(1)	4	-
	5	-	-	-	5

Pass 2 : Vertex (2) in between

$A_2 :$	0	3	8	(4)	-4
	∞	0	∞	1	7
	∞	4	0	(5)	(11)
	2	5	-5	0	-2
	∞	∞	∞	6	0

$P_2 :$	-	1	1	(2)	1
	-	-	-	2	2
	-	3	-	(2)	(2)
	4	1	4	-	1
	-	-	-	5	-

Pass 3 : Vertex (3) in between

$A_3 :$	0	3	8	4	-4
	∞	0	∞	1	7
	∞	4	0	5	11
	2	(-1)	-5	0	-2
	∞	∞	∞	0	0

$P_3 :$	-	1	1	2	1
	-	-	-	2	2
	-	3	-	2	2
	4	(3)	4	-	1
	-	-	-	5	-

Pass 4: Vertex ④ in between

$$A_4 : \begin{bmatrix} 0 & 3 & -1 & 4 & -4 \\ \textcircled{3} & 0 & \textcircled{-4} & 1 & \textcircled{-1} \\ 7 & 4 & 0 & 5 & \textcircled{3} \\ 2 & -1 & -5 & 0 & -2 \\ \textcircled{8} & 5 & \textcircled{1} & 6 & 0 \end{bmatrix}$$

$$P_4 : \begin{bmatrix} - & 1 & 4 & 2 & 1 \\ 4 & - & 4 & 2 & 1 \\ 4 & 3 & - & - & 1 \\ 4 & 3 & 4 & - & 1 \\ 4 & 3 & 4 & 5 & - \end{bmatrix}$$

Pass 5: Vertex ⑤ in between

$$A_5 : \begin{bmatrix} 0 & 1 & -3 & -2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & -5 & 1 & 6 & 6 \end{bmatrix}$$

$$P_5 : \begin{bmatrix} - & 3 & 4 & 5 & 1 \\ 4 & - & 4 & 2 & 1 \\ 4 & 3 & - & 2 & 1 \\ 4 & 3 & 4 & \cancel{2} & 1 \\ 4 & 3 & 4 & 5 & - \end{bmatrix}$$

Path tracing : 1-2

For 1-2, cost is 3 according to original diag but acc to cost matrix it is 1

From path matrix $\rightarrow 1 \rightarrow 2$ is 3 $\Rightarrow 1-3-2$

From 1-3 \rightarrow value is 4 $\Rightarrow 1-4-3-2$

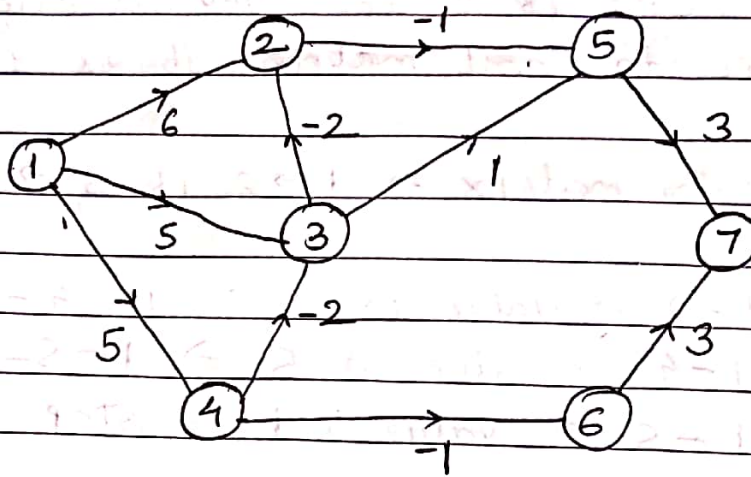
From 1-4 \rightarrow value is 5 $\Rightarrow 1-5-4-3-2$

For 1-5 \rightarrow value is 1 \Rightarrow STOP \uparrow

$$\text{dist}^2(2) = \min(\text{dist}^1(2), \min_i (\text{dist}^1[i] + \text{cost}[i, 2]))$$

Bellman Ford (Single Source, Dynamic)

-ve edges



No. of edges bet ⁿ	V →	1	2	3	4	5	6	7
1	1	0	6	5	5	∞	∞	∞
			1,2	1,3	1,4	-	-	-
2	2	0	3	3	5	5	4	∞
			1,3,2	1,4,3	1,4	1,2,5	1,4,6	-
3		0	1	3	5	2	4	7
			1,3,2	1,4,3,2	1,4,3			
4		0	1	3	5	0	4	3
5		0	1	3	5	0	4	3
6								

$$\text{dist}^2(2) = \min(6, 0+6, 6+0, 5-2, 5+\infty, \infty, \infty) \\ = 3$$

$$\text{dist}^2(3) = \min(5, 0+5, 6+\infty, 5+0, 5-2, \infty, \infty) \\ = 3$$

$$\text{dist}^2(4) = \min(5, 0+5, 6+\infty, \infty, 5, \infty, \infty) \\ = 5$$

Ex-

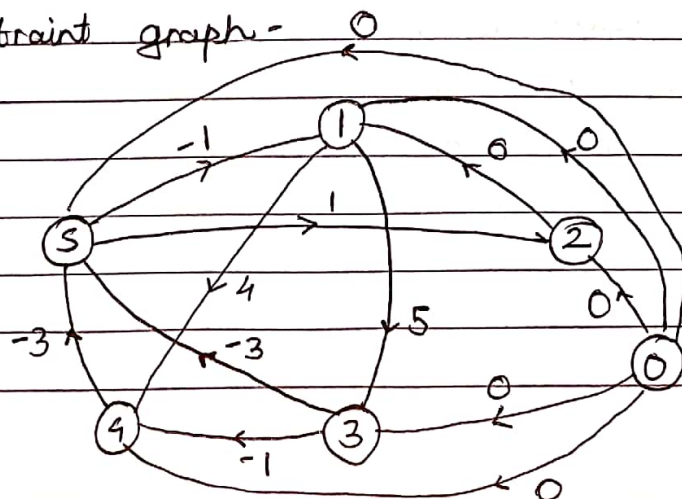
	A	x					
	1	-1	0	0	0	x_1	0
	1	0	0	0	-1	x_2	-1
	0	1	0	0	-1	x_3	1
	-1	0	1	0	0	x_4	5
	-1	0	0	1	0	x_5	4
	0	0	-1	1	0		-1
	0	0	-1	0	1		-3
	0	0	0	-1	1		-3

\leq

System of Difference constraints

$$\begin{aligned} x_1 - x_2 &\leq 0 & x_4 - x_1 &\leq 4 \\ x_1 - x_5 &\leq -1 & -x_3 + x_4 &\leq -1 \\ x_2 - x_5 &\leq 1 & -x_3 + x_5 &\leq -3 \\ -x_1 + x_3 &\leq 5 & -x_4 + x_5 &\leq -3 \end{aligned}$$

Constraint graph -



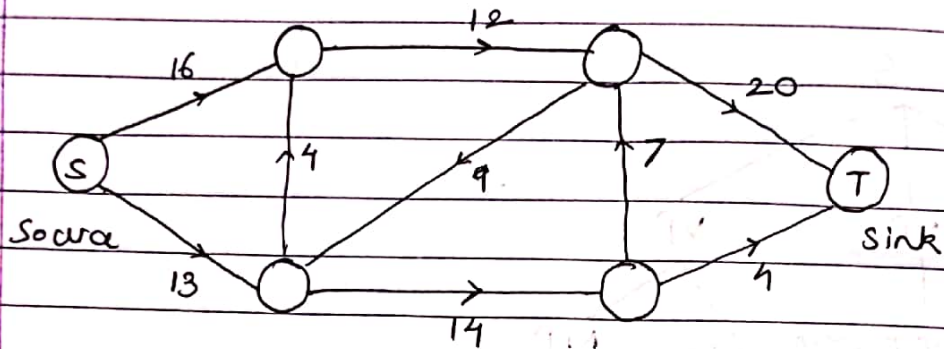
	0	1	2	3	4	5
1	0	0	0	0	0	0
2	0	-1	0	0	-1	-3
3	0	-4	-2	0	-1	-4
4	0	-5	-3	0	-1	-4

— x —

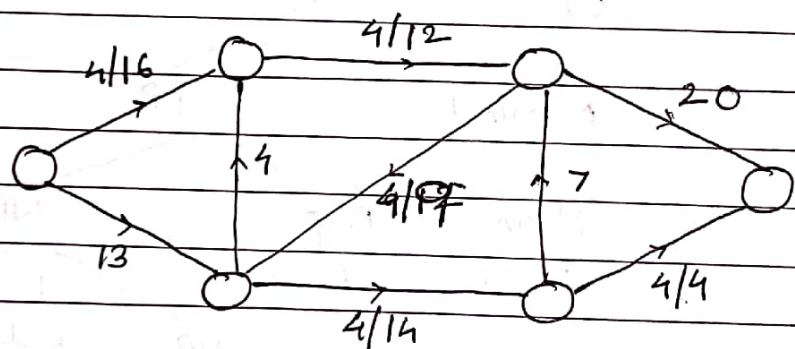
Syntax - Refers to the way words are arranged together, and the relationship between them.

Ex of Syntax tree-

Ford Fulkerson Algorithm for Max NW Flow (Network)

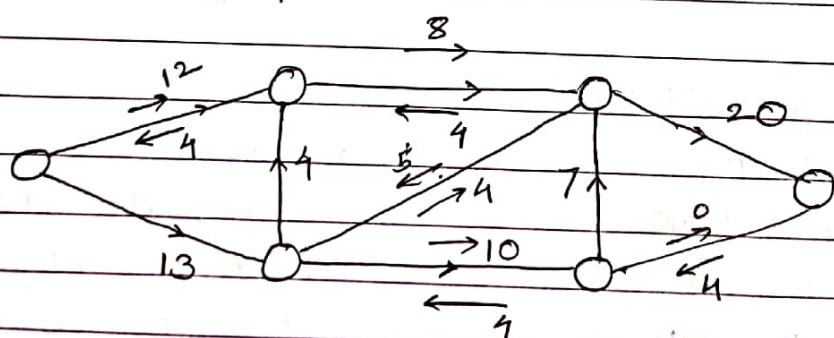


Going through the path $16 \rightarrow 12 \rightarrow 9 \rightarrow 14 \rightarrow 4$
 $\Rightarrow 4$ will be consumed

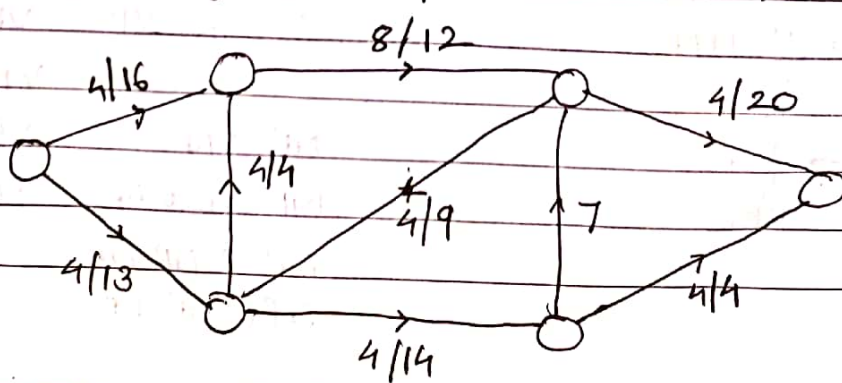


Flow = 4

Residual graph



Going through the path $13 \rightarrow 4 \rightarrow 12 \rightarrow 20$

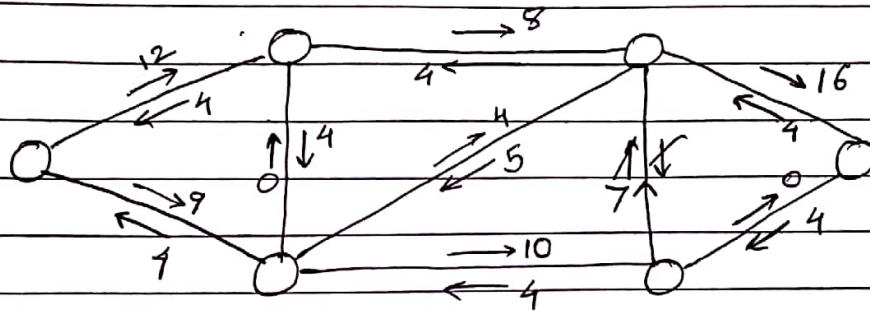


Flow = 8

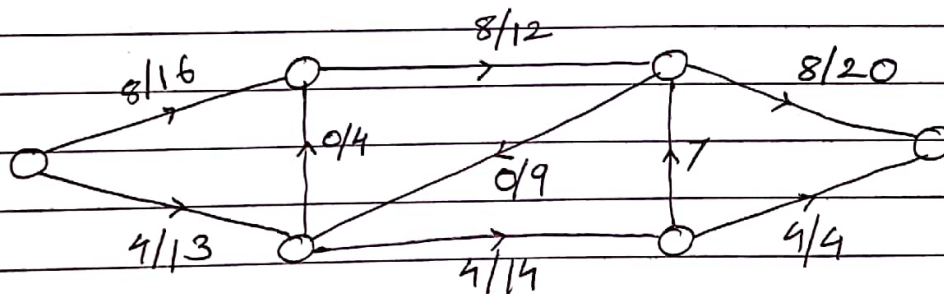
Concepts -

- Source / Sink
- Augmented Path - DFS
- Residual network
- Forward flow
- Backward flow

Residual graph



Going through $16 \rightarrow 4 \rightarrow 9 \rightarrow 20$



Flow = 12

Residual Gf

