Batch: A2 **Roll No.: 1911027**

Experiment No. : 07

Grade: AA / AB / BB / BC / CC / CD /DD

Signature of the Staff In-charge with date

Title: Fuzzification methods and operations on fuzzy set.

Objective: To implement fuzzification methods and perform operations on fuzzy sets.

Expected Outcome of Experiment:

CO4 : Apply basics of Fuzzy logic and neural networks.

Books/ Journals/ Websites referred:

- J.S.R.Jang, C.T.Sun and E.Mizutani, "Neuro-Fuzzy and Soft Computing", PHI, 2004, Pearson Education 2004.
- Davis E.Goldberg, "Genetic Algorithms: Search, Optimization and Machine Learning", Addison Wesley, N.Y., 1989.
- S. Rajasekaran and G.A.V.Pai, "Neural Networks, Fuzzy Logic and Genetic Algorithms", PHI, 2003.
- http://library.thinkquest.org/C007395/tqweb/history.html.

Pre Lab/ Prior Concepts:

A fuzzy set is a pair (U, m) where U is a set and $m: U \to [0, 1]$.

For each $x \in U$, the value $m(x)$ is called the **grade** of membership of x in (U, m). For a finite set $U = \{x_1, \ldots, x_n\}$, the fuzzy set (U, m) is often denoted by $\{m(x_1)/x_1, \ldots, m(x_n)/x_n\}$.

Let $x \in U$. Then x is called **not included** in the fuzzy set (U, m) if $m(x) = 0$, x is called **fully included** if $m(x) = 1$, and x is called a **fuzzy member** if $0 < m(x) < 1$. The set $\{x \in U \mid m(x) > 0\}$ is called the **support** of (U, m) and the set $\{x \in U \mid m(x) = 1\}$ is called its **kernel**. The function m is called the **membership function** of the fuzzy set (U, m).

Implementation Details:

Membership functions can either be chosen by the user arbitrarily, based on the user's experience (MF chosen by two users could be different depending upon their experiences, perspectives, etc.) or be designed using machine learning methods (e.g., artificial neural networks, genetic algorithms, etc). There are different shapes of membership functions; triangular, trapezoidal, piecewise-linear, Gaussian, bell-shaped, etc.

Triangular function:

$$\text{Triangle}(x; a,b,c) = \begin{cases} 0 & x <= a \\ \dfrac{x-a}{b-a} & a <= x <= b \\ \dfrac{c-x}{c-b} & b <= x <= c \\ 0 & c <= x \end{cases}$$

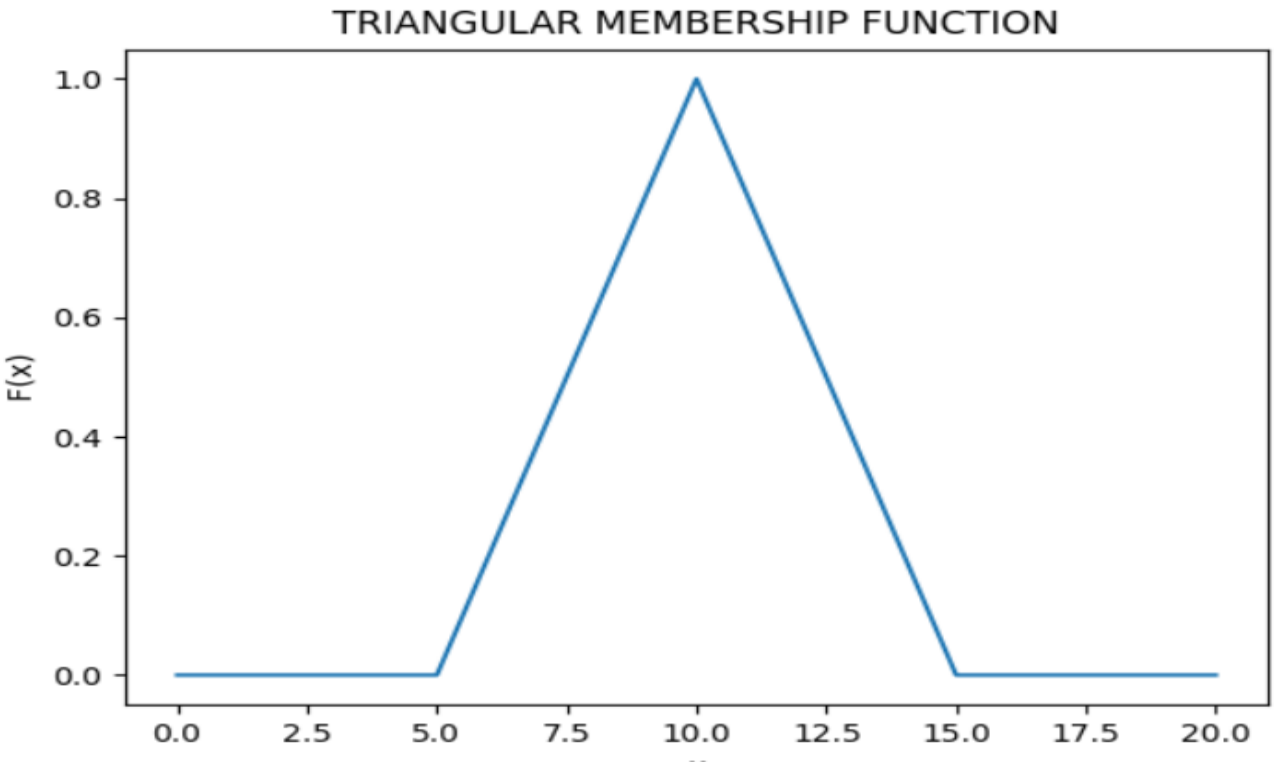
Implementation details:
Code:

```
import matplotlib.pyplot as plt
print("------------------------------------------------------------------------------------------------")
print("\t\t\t\tTRIANGULAR MEMBERSHIP FUNCTION")
print("------------------------------------------------------------------------------------------------")
temp=input("Enter value of a,b,c : ").split()
a,b,c=int(temp[0]),int(temp[1]),int(temp[2])
temp=input("Enter input value X : ").split()
print("------------------------------------------------------------------------------------------------")
x=[]
for i in temp:
    x.append(int(i))
print("INPUT (X) : ",end="")
for i in x:
    print(i," ",end="")
print("\n------------------------------------------------------------------------------------------------")
output=[]
```

```
for i in x:
    if(i<a):
        output.append(0)
    elif(i>=a and i<=b):
        output.append((i-a)/(b-a))
    elif(i>=b and i<=c):
        output.append((c-i)/(c-b))
    elif(i>c):
        output.append(0)
print("OUTPUT : ",end="")
for i in output:
    print(i," ",end="")
print("\n-------------------------------------------------------------------------------------
-------------------------------------------------")
f = output
plt.plot(x, f)
plt.xlabel(' x ')
plt.ylabel(' F(x) ')
plt.title('TRIANGULAR MEMBERSHIP FUNCTION')
plt.show()
```

Output:

```
-----------------------------------------------------------------------------------------------------------------------
                              TRIANGULAR MEMBERSHIP FUNCTION
-----------------------------------------------------------------------------------------------------------------------
Enter value of a,b,c : 5 10 15
Enter input value X : 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
-----------------------------------------------------------------------------------------------------------------------
INPUT (X) : 0  1  2  3  4  5  6  7  8  9  10  11  12  13  14  15  16  17  18  19  20
-----------------------------------------------------------------------------------------------------------------------
OUTPUT : 0  0  0  0  0  0.0  0.2  0.4  0.6  0.8  1.0  0.8  0.6  0.4  0.2  0.0  0  0  0  0  0
-----------------------------------------------------------------------------------------------------------------------
```



Ramp function:

Implementation details:

L-Ramp:

L- Ramp (x : a,b)

$$\begin{cases} 0 & x <= a \\ \dfrac{x-a}{b-a} & a <= x <= b \\ 1 & x >= b \end{cases}$$

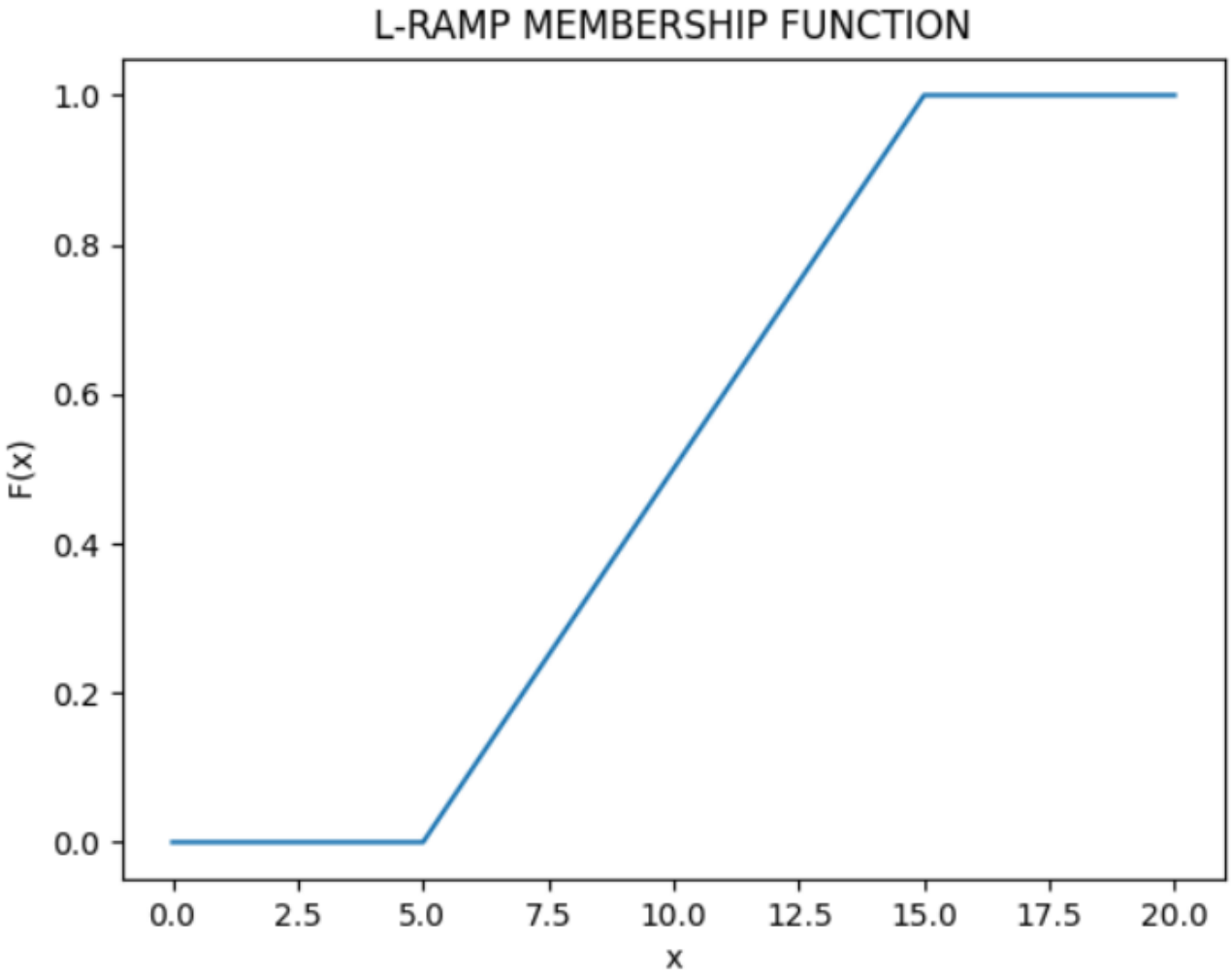
Code:

```
import matplotlib.pyplot as plt
print("------------------------------------------------------------------------------------
-----------------------------------------------")
print("\t\t\t\tL-RAMP MEMBERSHIP FUNCTION")
print("------------------------------------------------------------------------------------
-----------------------------------------------")
temp=input("Enter value of a,b : ").split()
a,b=int(temp[0]),int(temp[1])
temp=input("Enter input value X : ").split()
```

```
print("-------------------------------------------------------------------------------------
-------------------------------------------------")
x=[]
for i in temp:
    x.append(int(i))
print("INPUT (X) : ",end="")
for i in x:
    print(i," ",end="")
print("\n-------------------------------------------------------------------------------------
-------------------------------------------------")
output=[]
for i in x:
    if(i<a):
        output.append(0)
    elif(i>=a and i<=b):
        output.append((i-a)/(b-a))
    elif(i>b):
        output.append(1)
print("OUTPUT : ",end="")
for i in output:
    print(i," ",end="")
print("\n-------------------------------------------------------------------------------------
-------------------------------------------------")
f = output
plt.plot(x, f)
plt.xlabel(' x ')
plt.ylabel(' F(x) ')
plt.title('L-RAMP MEMBERSHIP FUNCTION')
plt.show()
```

Output:

```
-------------------------------------------------------------------------------------------------------------------
                              L-RAMP MEMBERSHIP FUNCTION
-------------------------------------------------------------------------------------------------------------------
Enter value of a,b : 5 15
Enter input value X : 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
-------------------------------------------------------------------------------------------------------------------
INPUT (X) : 0  1  2  3  4  5  6  7  8  9  10  11  12  13  14  15  16  17  18  19  20
-------------------------------------------------------------------------------------------------------------------
OUTPUT : 0  0  0  0  0  0.0  0.1  0.2  0.3  0.4  0.5  0.6  0.7  0.8  0.9  1.0  1  1  1  1  1
-------------------------------------------------------------------------------------------------------------------
```



R-Ramp:

L- Ramp (x : a,b)

$$\begin{cases} 0 & x<=a \\ \dfrac{b-x}{b-a} & a<=x<=b \\ 1 & x>= b \end{cases}$$

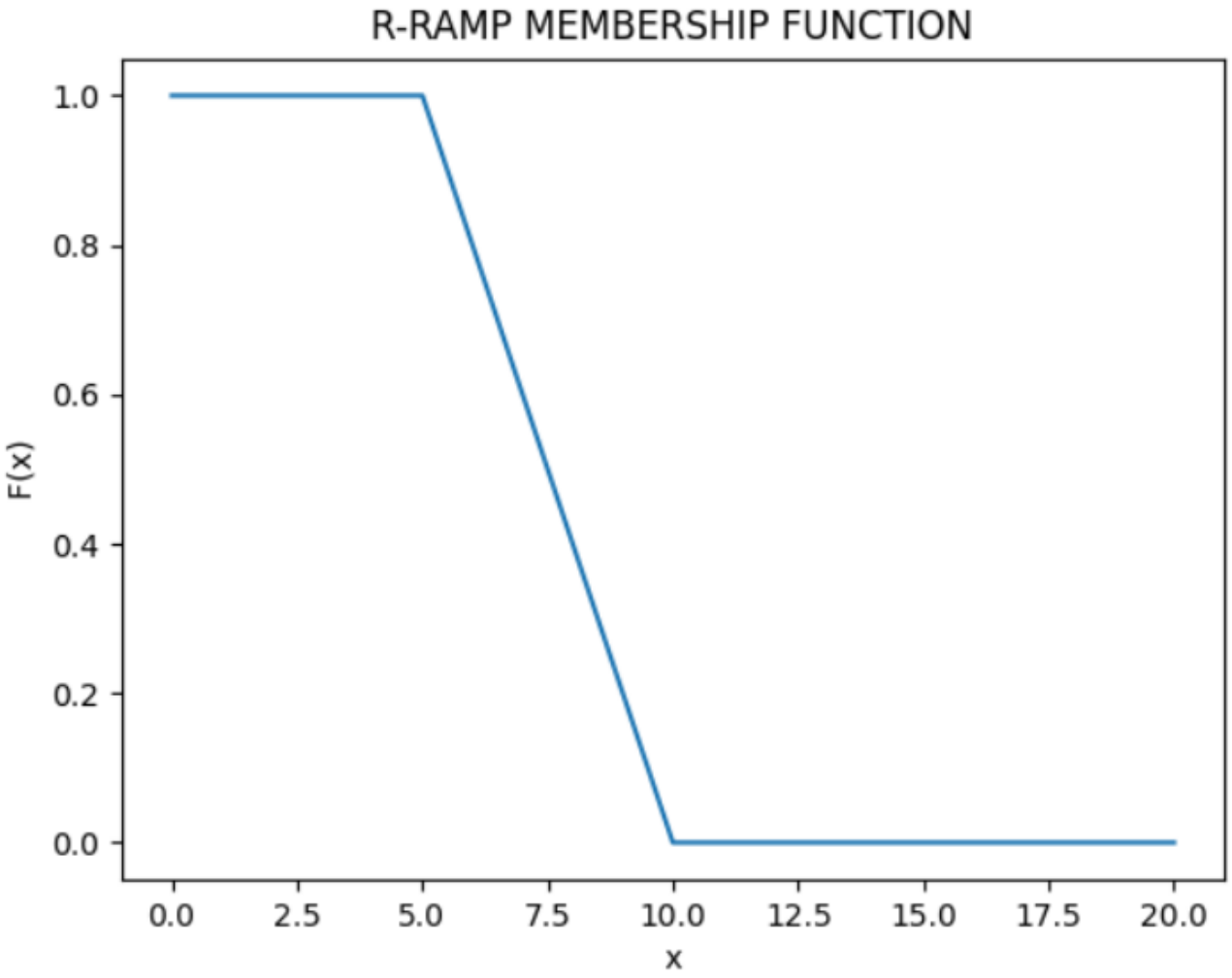
Code:

```
import matplotlib.pyplot as plt
print("-------------------------------------------------------------------------------------------------")
print("\t\t\t\tR-RAMP MEMBERSHIP FUNCTION")
```

```
print("-------------------------------------------------------------------------------------------------")
temp=input("Enter value of a,b : ").split()
a,b=int(temp[0]),int(temp[1])
temp=input("Enter input value X : ").split()
print("-------------------------------------------------------------------------------------------------")
x=[]
for i in temp:
    x.append(int(i))
print("INPUT (X) : ",end="")
for i in x:
    print(i," ",end="")
print("\n-------------------------------------------------------------------------------------------------")
output=[]
for i in x:
    if(i<a):
        output.append(1)
    elif(i>=a and i<=b):
        output.append((b-i)/(b-a))
    elif(i>b):
        output.append(0)
print("OUTPUT : ",end="")
for i in output:
    print(i," ",end="")
print("\n-------------------------------------------------------------------------------------------------")
f = output
plt.plot(x, f)
plt.xlabel(' x ')
plt.ylabel(' F(x) ')
plt.title('R-RAMP MEMBERSHIP FUNCTION')
plt.show()
```

Output:

```
-------------------------------------------------------------------------------------------------
                          R-RAMP MEMBERSHIP FUNCTION
-------------------------------------------------------------------------------------------------
Enter value of a,b : 5 10
Enter input value X : 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
-------------------------------------------------------------------------------------------------
INPUT (X) : 0  1  2  3  4  5  6  7  8  9  10  11  12  13  14  15  16  17  18  19  20
-------------------------------------------------------------------------------------------------
OUTPUT : 1  1  1  1  1  1.0  0.8  0.6  0.4  0.2  0.0  0  0  0  0  0  0  0  0  0  0
-------------------------------------------------------------------------------------------------
```



Trapezoidal function:

$$\text{Trapezoid}(x : a,b,c,d) = \begin{cases} 0 & x <= a \\ \dfrac{x-a}{b-a} & a <= x <= b \\ \dfrac{d-x}{d-c} & c <= x <= d \\ 0 & d <= x \end{cases}$$

Implementation details:

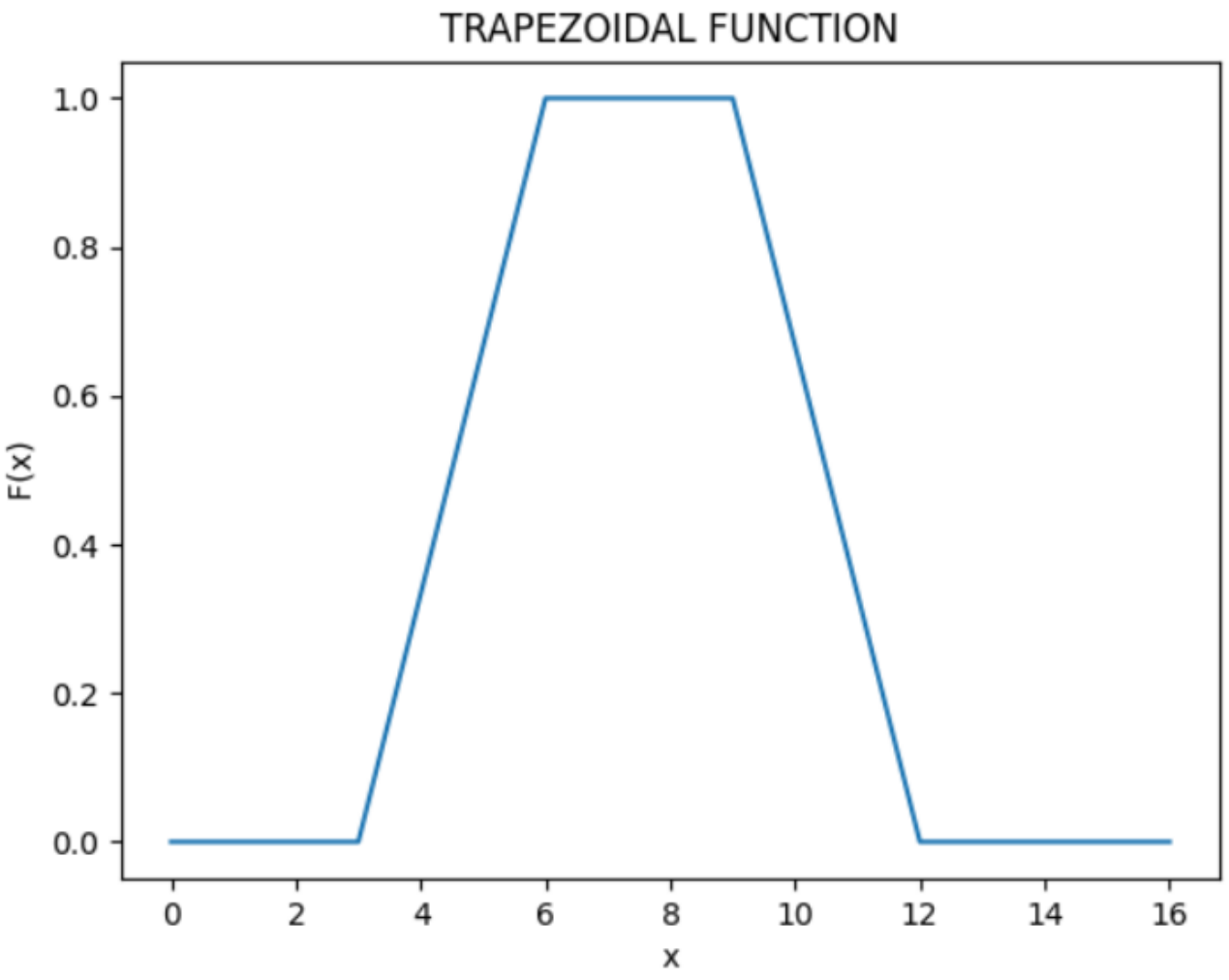
Code:

```
import matplotlib.pyplot as plt
print("-------------------------------------------------------------------------------------
-------------------------------------------------------")
print("\t\tTRAPEZOIDAL FUNCTION")
print("-------------------------------------------------------------------------------------
-------------------------------------------------------")
temp=input("Enter value of a,b,c,d : ").split()
a,b,c,d=int(temp[0]),int(temp[1]),int(temp[2]),int(temp[3])
temp=input("Enter input value X : ").split()
print("-------------------------------------------------------------------------------------
-------------------------------------------------------")
x=[]
for i in temp:
    x.append(int(i))
print("INPUT (X) : ",end="")
for i in x:
    print(i," ",end="")
print("\n-------------------------------------------------------------------------------------
-------------------------------------------------------")
output=[]
for i in x:
    if(i<a):
        output.append(0)
    elif(i>=a and i<=b):
        output.append((i-a)/(b-a))
    elif(i>=b and i<=c):
        output.append(1)
    elif(i>=c and i<=d):
        output.append((d-i)/(d-c))
    elif(i>d):
        output.append(0)
print("OUTPUT : ",end="")
for i in output:
    print(i," ",end="")
print("\n-------------------------------------------------------------------------------------
-------------------------------------------------------")
f = output
plt.plot(x, f)
plt.xlabel(' x ')
plt.ylabel(' F(x) ')
```

```
plt.title('TRAPEZOIDAL FUNCTION')
plt.show()
```

Output:

```
--------------------------------------------------------------------------------------------------------------------------------
                TRAPEZOIDAL FUNCTION
--------------------------------------------------------------------------------------------------------------------------------
Enter value of a,b,c,d : 3 6 9 12
Enter input value X : 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
--------------------------------------------------------------------------------------------------------------------------------
INPUT (X) : 0  1  2  3  4  5  6  7  8  9  10  11  12  13  14  15  16
--------------------------------------------------------------------------------------------------------------------------------
OUTPUT : 0  0  0  0.0  0.3333333333333333  0.6666666666666666  1.0  1  1  1  0.6666666666666666  0.3333333333333333  0.0  0  0  0  0
--------------------------------------------------------------------------------------------------------------------------------
```



There are two special cases of a trapezoidal function, which are called R-functions and L-functions:

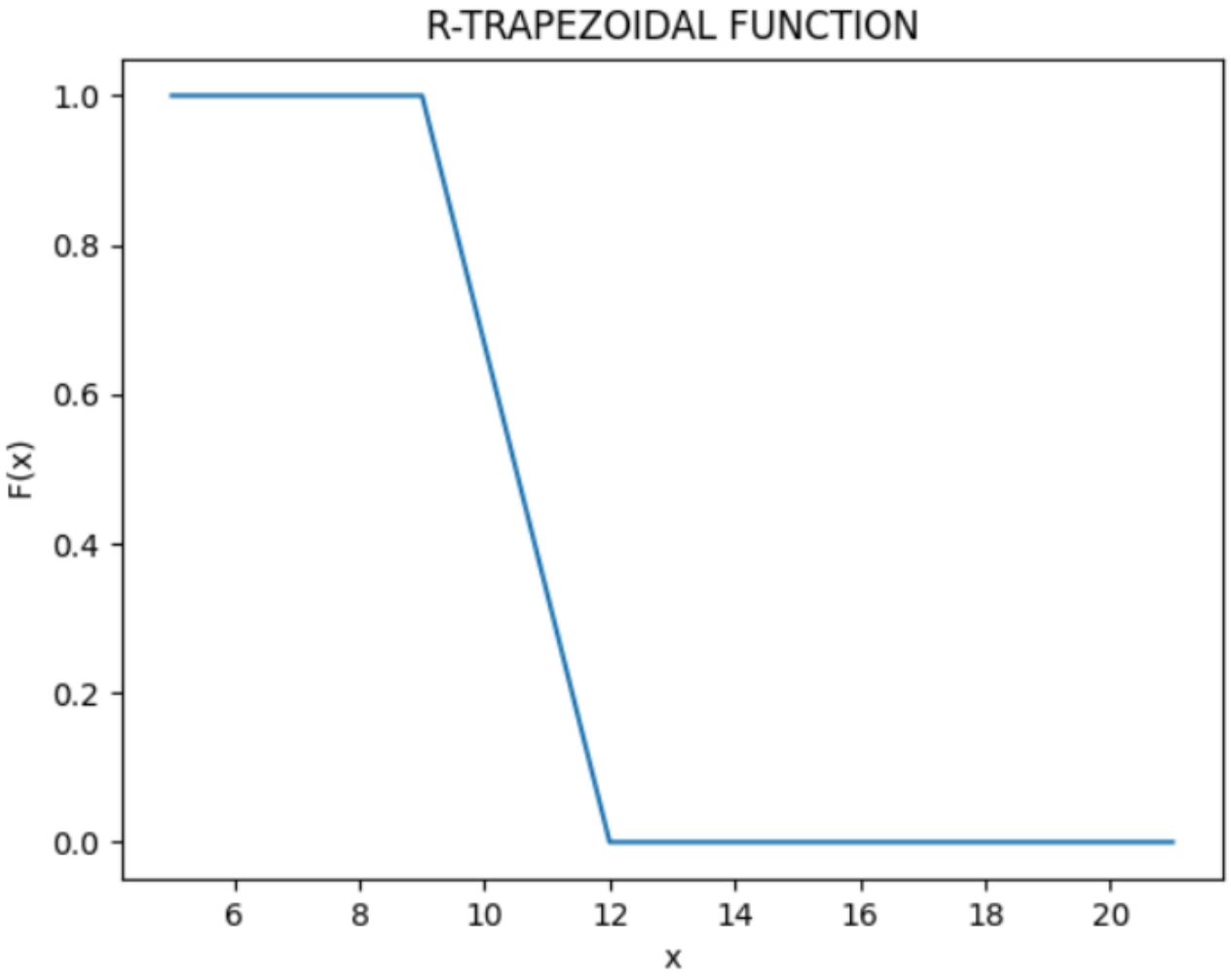
R-Function:
Implementation details:
Code:

```
import matplotlib.pyplot as plt
print("-------------------------------------------------------------------------------------
-----------------------------------------------")
print("\t\tR-TRAPEZOIDAL FUNCTION")
print("-------------------------------------------------------------------------------------
-----------------------------------------------")
temp=input("Enter value of c,d : ").split()
c,d=int(temp[0]),int(temp[1])
temp=input("Enter input value X : ").split()
print("-------------------------------------------------------------------------------------
-----------------------------------------------")
x=[]
for i in temp:
    x.append(int(i))
print("INPUT (X) : ",end="")
for i in x:
    print(i," ",end="")
print("\n-------------------------------------------------------------------------------------
-----------------------------------------------")
output=[]
for i in x:
    if(i>d):
        output.append(0)
    elif(i>=c and i<=d):
        output.append((d-i)/(d-c))
    elif(i<c):
        output.append(1)
print("OUTPUT : ",end="")
for i in output:
    print(i," ",end="")
print("\n-------------------------------------------------------------------------------------
-----------------------------------------------")
f = output
plt.plot(x, f)
plt.xlabel(' x ')
plt.ylabel(' F(x) ')
plt.title('R-TRAPEZOIDAL FUNCTION')
```

```
plt.show()
```

Output:

```
-------------------------------------------------------------------------------------------------------------------------------
                R-TRAPEZOIDAL FUNCTION
-------------------------------------------------------------------------------------------------------------------------------
Enter value of c,d : 9 12
Enter input value X : 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
-------------------------------------------------------------------------------------------------------------------------------
INPUT (X) : 5  6  7  8  9  10  11  12  13  14  15  16  17  18  19  20  21
-------------------------------------------------------------------------------------------------------------------------------
OUTPUT : 1  1  1  1  1.0  0.6666666666666666  0.3333333333333333  0.0  0  0  0  0  0  0  0  0  0
-------------------------------------------------------------------------------------------------------------------------------
```

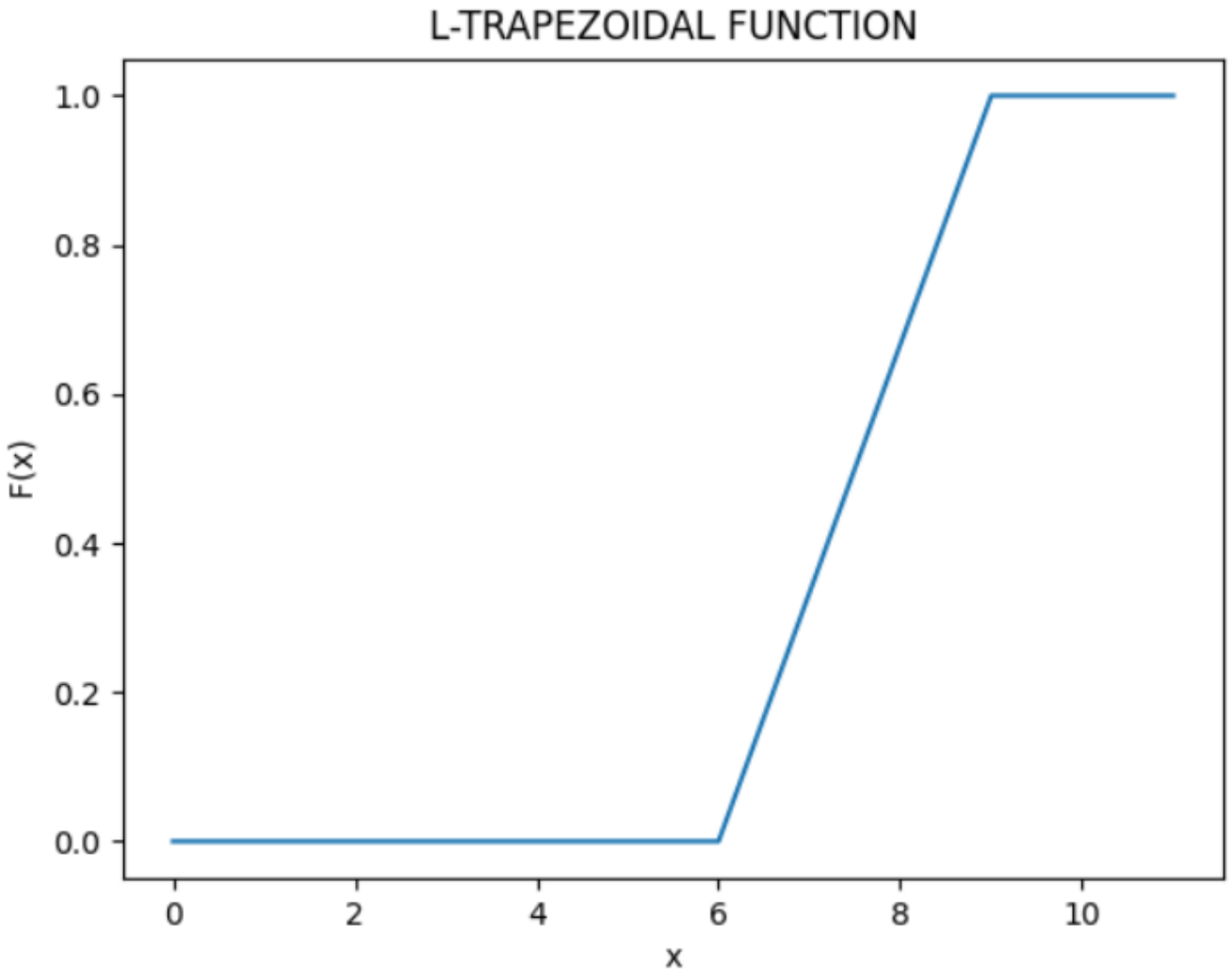


L-Function:
Implementation details:
Code:

```
import matplotlib.pyplot as plt
print("-------------------------------------------------------------------------------------------------")
print("\t\tL-TRAPEZOIDAL FUNCTION")
print("-------------------------------------------------------------------------------------------------")
temp=input("Enter value of a,b : ").split()
a,b=int(temp[0]),int(temp[1])
temp=input("Enter input value X : ").split()
print("-------------------------------------------------------------------------------------------------")
x=[]
for i in temp:
    x.append(int(i))
print("INPUT (X) : ",end="")
for i in x:
    print(i," ",end="")
print("\n-------------------------------------------------------------------------------------------------")
output=[]
for i in x:
    if(i<a):
        output.append(0)
    elif(i>=a and i<=b):
        output.append((i-a)/(b-a))
    elif(i>b):
        output.append(1)
print("OUTPUT : ",end="")
for i in output:
    print(i," ",end="")
print("\n-------------------------------------------------------------------------------------------------")
f = output
plt.plot(x, f)
plt.xlabel(' x ')
plt.ylabel(' F(x) ')
plt.title('L-TRAPEZOIDAL FUNCTION')
plt.show()
```

Output:

```
--------------------------------------------------------------------------------------------------------------------------------
                L-TRAPEZOIDAL FUNCTION
--------------------------------------------------------------------------------------------------------------------------------
Enter value of a,b : 6 9
Enter input value X : 0 1 2 3 4 5 6 7 8 9 10 11
--------------------------------------------------------------------------------------------------------------------------------
INPUT (X) : 0  1  2  3  4  5  6  7  8  9  10  11
--------------------------------------------------------------------------------------------------------------------------------
OUTPUT : 0  0  0  0  0  0  0.0  0.3333333333333333  0.6666666666666666  1.0  1  1
--------------------------------------------------------------------------------------------------------------------------------
```



Gaussian function:

Parameter { c, σ }

Gaussian (x ; c, σ) = $e^{-1/2((x-c)/\sigma)^2}$

Where, c is the center of MF, σ is the width of MF

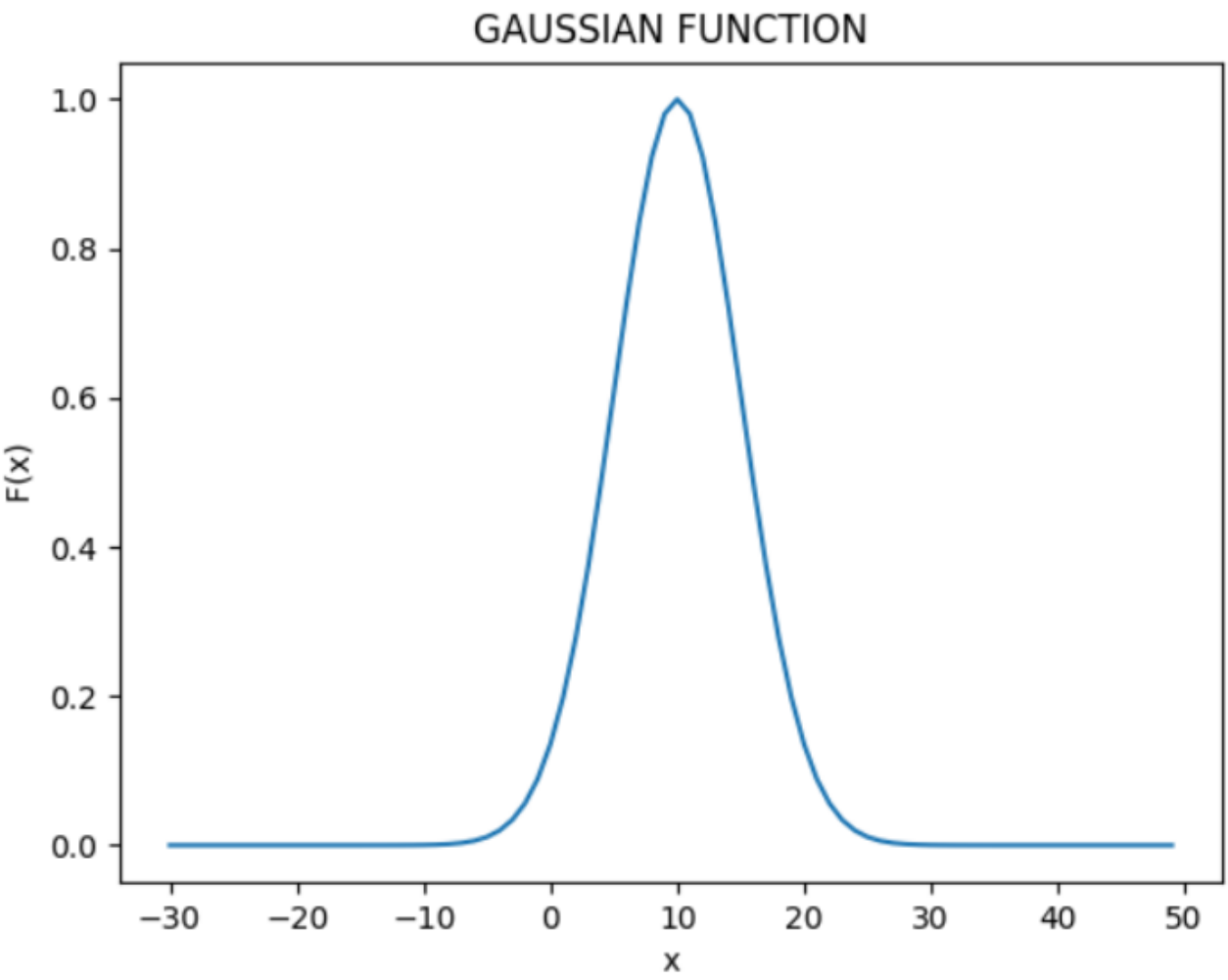
Implementation details:

Code:

```
import matplotlib.pyplot as plt
import math
print("-----------------------------------------------------------------")
print("\t\tGAUSSIAN FUNCTION")
print("-----------------------------------------------------------------")
temp=input("Enter value of c,sigma : ").split()
c,sigma=int(temp[0]),int(temp[1])
x=[]
for i in range(-30,50):
    x.append(int(i))
print("INPUT (X) : ",end="")
for i in x:
    print(i," ",end="")
print("\n-----------------------------------------------------------------")
output=[]
for i in x:
    out=math.exp(-1*(1/2)*((i-c)/sigma)**2)
    output.append(out)
print("OUTPUT : ",end="")
for i in output:
    print(i," ",end="")
print("\n-----------------------------------------------------------------")
f = output
plt.plot(x, f)
plt.xlabel(' x ')
plt.ylabel(' F(x) ')
plt.title('GAUSSIAN FUNCTION')
plt.show()
```

Output:

C=10 sigma=5


GAUSSIAN FUNCTION
F(x)
x
1.0
0.8
0.6
0.4
0.2
0.0
−30
−20
−10
0
10
20
30
40
50

Bell Shaped Function:

Parameter { a,b,c}

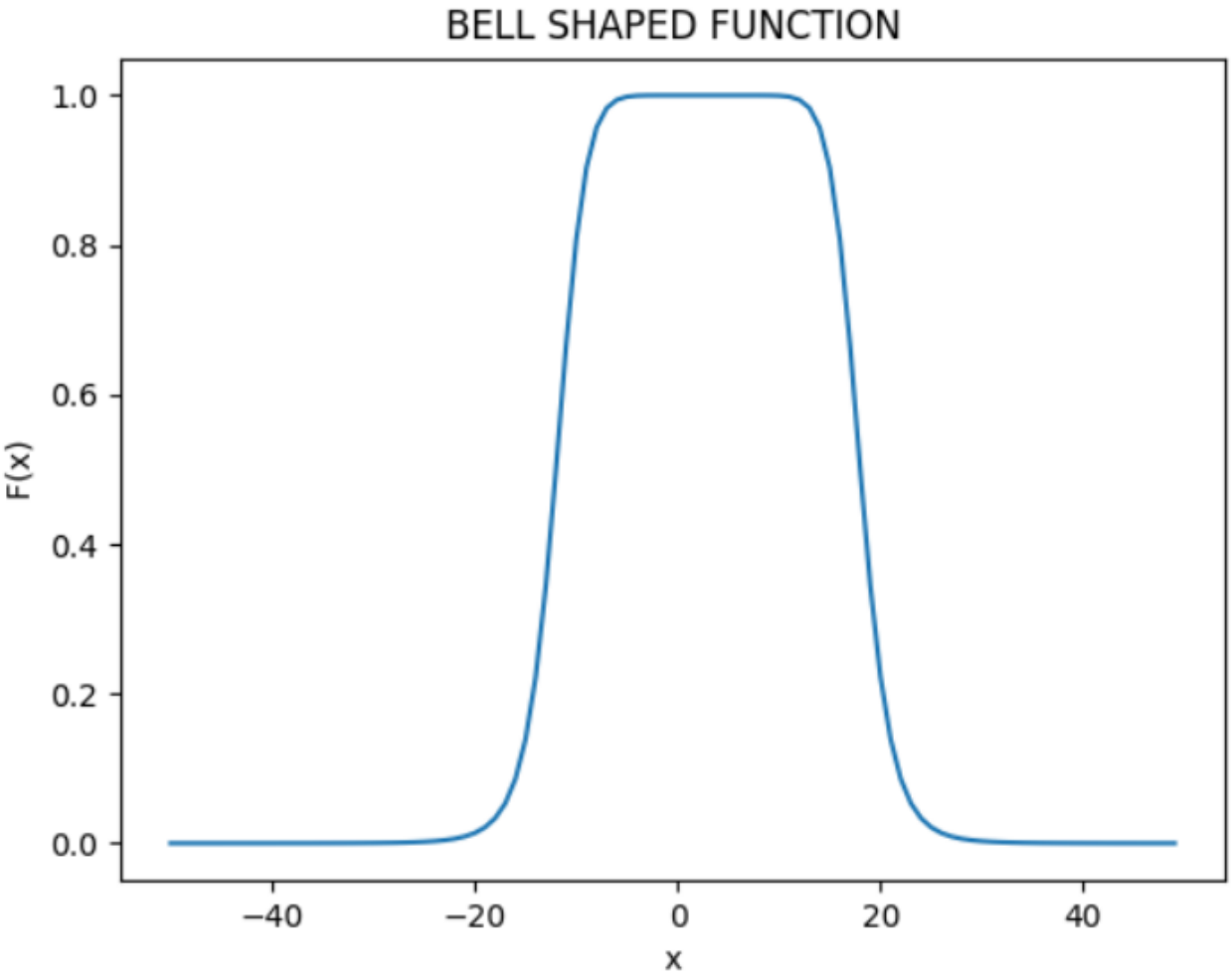
$$\text{Bell}(x: a,b,c) = \frac{1}{1 + \left| \frac{x - c}{a} \right|^{2b}}$$

Implementation details:

Code:

```
import matplotlib.pyplot as plt
import math
print("-----------------------------------------------------------------")
print("\t\tBELL SHAPED FUNCTION")
print("-----------------------------------------------------------------")
temp=input("Enter value of a,b,c : ").split()
a,b,c=int(temp[0]),int(temp[1]),int(temp[2])
x=[]
for i in range(-50,50):
    x.append(int(i))
print("INPUT (X) : ",end="")
for i in x:
    print(i," ",end="")
print("\n-----------------------------------------------------------------")
output=[]
for i in x:
    if(i>c):
        out=1/(1+((i-c)/a)**(2*b))
    else:
        out=1/(1+((c-i)/a)**(2*b))
    output.append(out)
print("OUTPUT : ",end="")
for i in output:
    print(i," ",end="")
print("\n-----------------------------------------------------------------")
f = output
plt.plot(x, f)
plt.xlabel(' x ')
plt.ylabel(' F(x) ')
plt.title('BELL SHAPED FUNCTION')
plt.show()
```

Output: a=15 b=3 c=5



FUZZY OPERATORS:

Union:

Consider 2 fuzzy sets denoted by F1 and F2. Consider Y be the union of F1 and F2, then for every member of F1 and F2

$$Y(x) = \max(F1(x), F2(x))$$

Union is denoted by : F1 U F2

For example:

Fuzzy set F1={‘a’: 0.3, ‘b’: 0.9, ‘c’: 1.1, ‘d’: 1.2, ‘e’: 0.4}

Fuzzy set F2={‘a’: 0.5, ‘b’: 0.2, ‘c’: 1.5, ‘d’: 0.2, ‘e’: 1.4}

F1 U F2={‘a’: 0.5, ‘b’: 0.9, ‘c’: 1.5, ‘d’: 1.2, ‘e’: 1.4}

Intersection:

Consider 2 fuzzy sets denoted by F1 and F2. Consider Y be the intersection of F1 and F2, then for every member of F1 and F2

$$Y(x) = \min(F1(x), F2(x))$$

Intersection is denoted by : F1 ∩ F2

For example:

Fuzzy set F1={‘a’: 0.3, ‘b’: 0.9, ‘c’: 1.1, ‘d’: 1.2, ‘e’: 0.4}

Fuzzy set F2={‘a’: 0.5, ‘b’: 0.2, ‘c’: 1.5, ‘d’: 0.2, ‘e’: 1.4}

F1 ∩ F2={‘a’: 0.3, ‘b’: 0.2, ‘c’: 1.1, ‘d’: 0.2, ‘e’: 0.4}

Complement:

Consider a fuzzy set denoted by F1. Consider Y be the complement of F1, then for every member of F1

$$Y(x) = 1 - F1(x)$$

For example:

Fuzzy set F1={‘a’: 0.3, ‘b’: 0.9, ‘c’: 0.1, ‘d’: 0.2, ‘e’: 0.4}

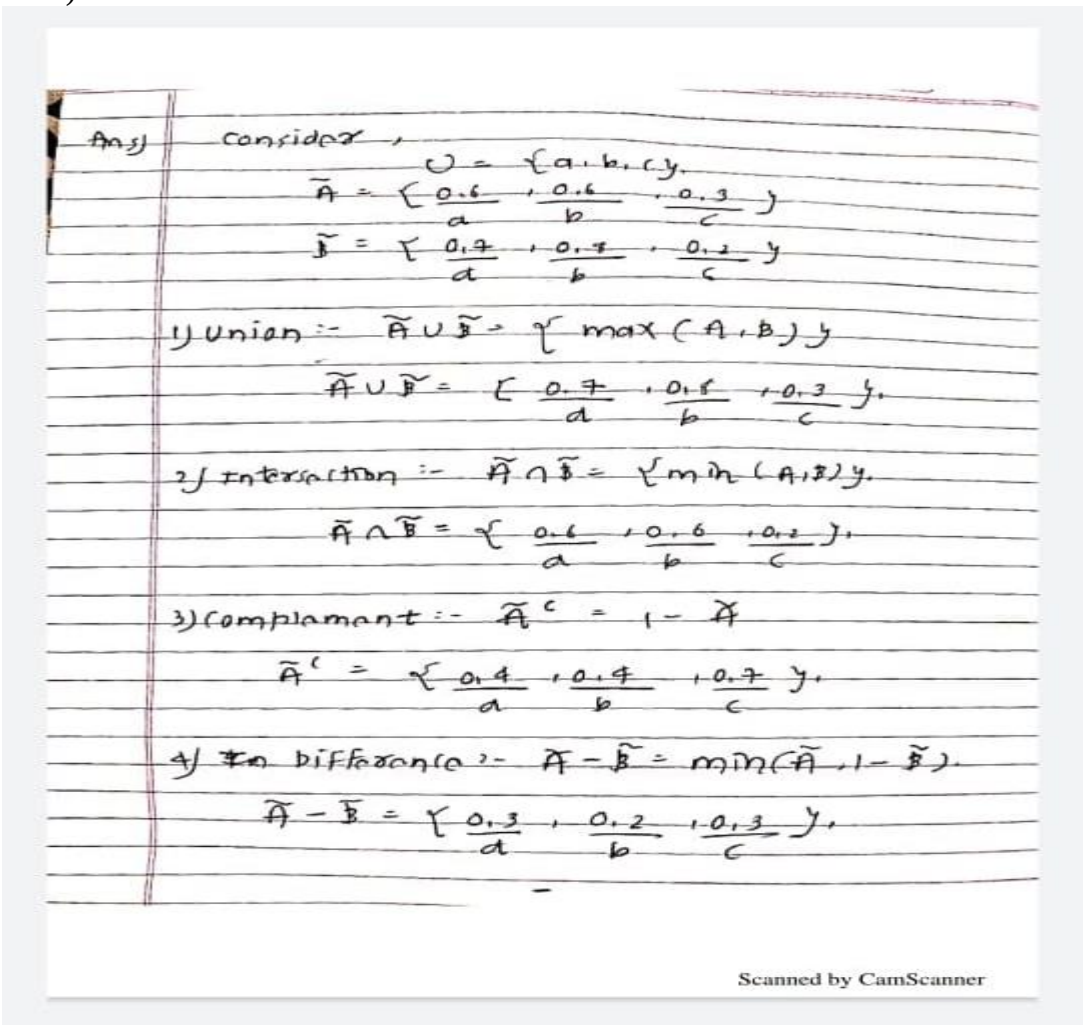
F1 Complement={‘a’: 0.7, ‘b’: 0.1, ‘c’: 0.9, ‘d’: 0.8, ‘e’: 0.6}

Conclusion: Thus, we have successfully implemented fuzzification and perform operations on fuzzy sets. Also implemented all the membership functions and plotted graph using python programming language.

Post Lab Descriptive Questions :

1. Take any example and explain all the operations on that.

ANS)



Date: 12 / 11 / 2021 **Signature of faculty in-charge**