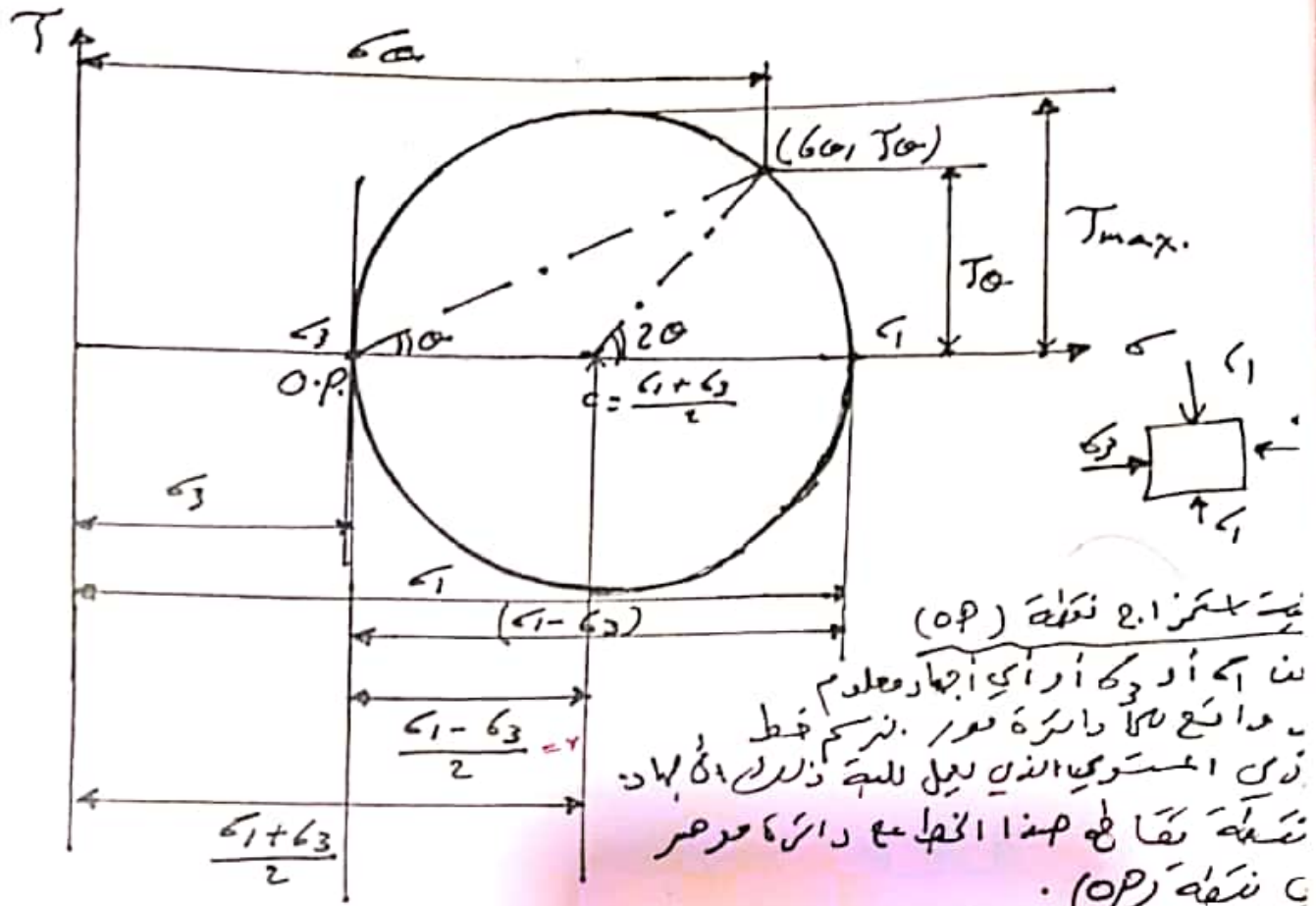


Mohr Circle:-

LEC 14
7/3/2016



نقطة (op) نقطة مركز الدائرة (O.P.)
نقطة (σ, τ) أي أن أي اتجاه معلوم
نقطة (σ, τ) أي أن أي اتجاه معلوم
نقطة (σ, τ) أي أن أي اتجاه معلوم
نقطة (σ, τ) أي أن أي اتجاه معلوم

From the Graphical of Mohr circle:..

σ_1 = The major principle stresses at $\tau = 0$

σ_3 = minor principle stresses at $\tau = 0$

$(\frac{\sigma_1 - \sigma_3}{2})$ = radius of Mohr circle = τ_{max} .

$(\frac{\sigma_1 + \sigma_3}{2})$ = center of Mohr circle.

$\sigma_1 - \sigma_3$ = deviator stress = σ_d .

Origin of plane "op"

Is a point on Mohr circle with the following property. A line through "op" and any point (like A) of the Mohr circle will be parallel to the plane on which the stresses given by (A) act.

Ex 1:- For the element shown, find the stresses on the θ -A by using Mohr circle.

Sol:- since $\tau = 0$, the stresses given is principle stress

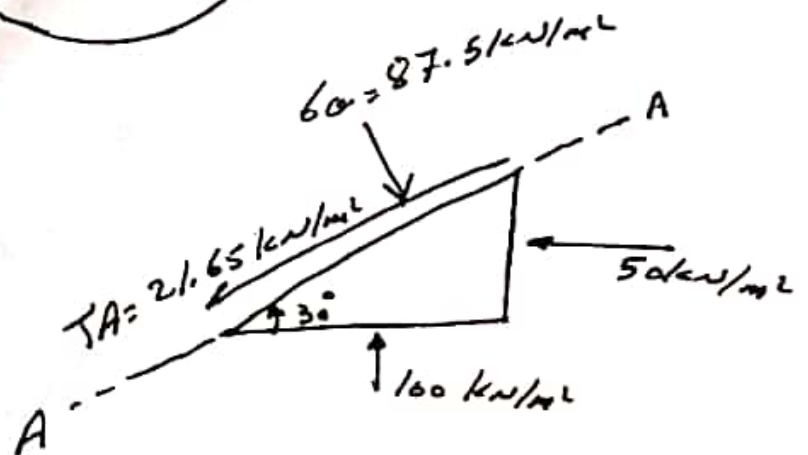
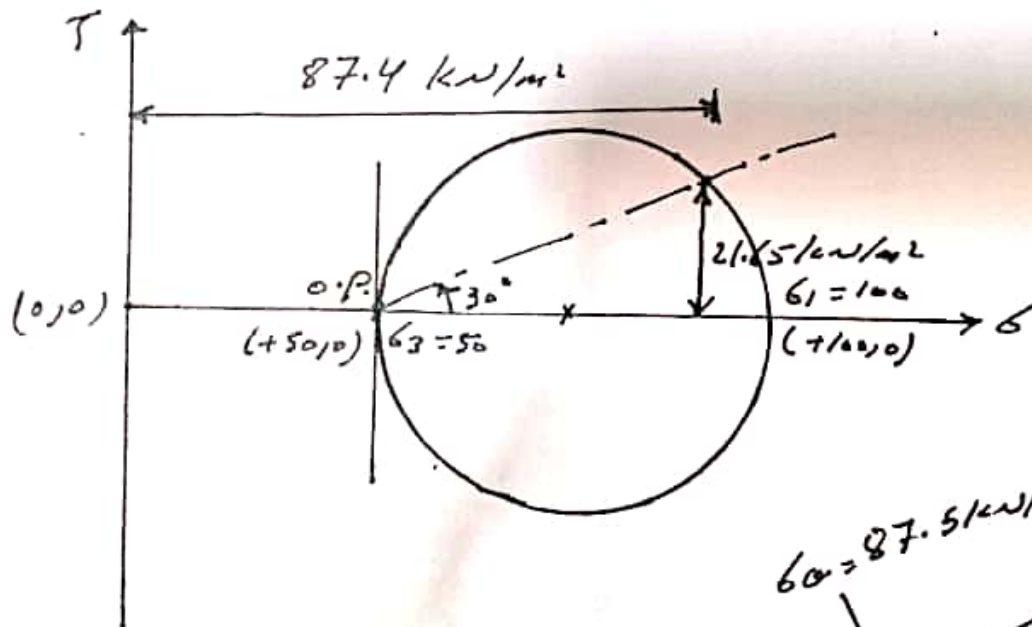
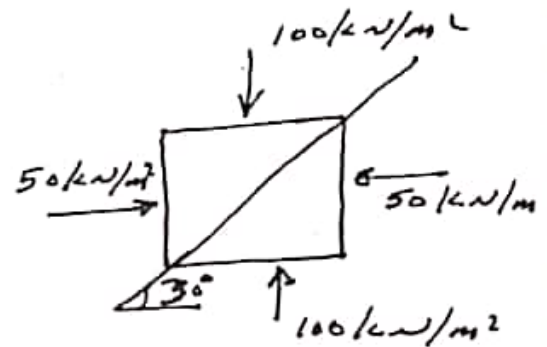
$$\sigma_1 = 100 \text{ kN/m}^2$$

$$\sigma_3 = 50 \text{ kN/m}^2 \text{ and } \theta = 30^\circ$$

From the drawing of Mohr circle.

$$\sigma_\theta = 87.4 \text{ kN/m}^2$$

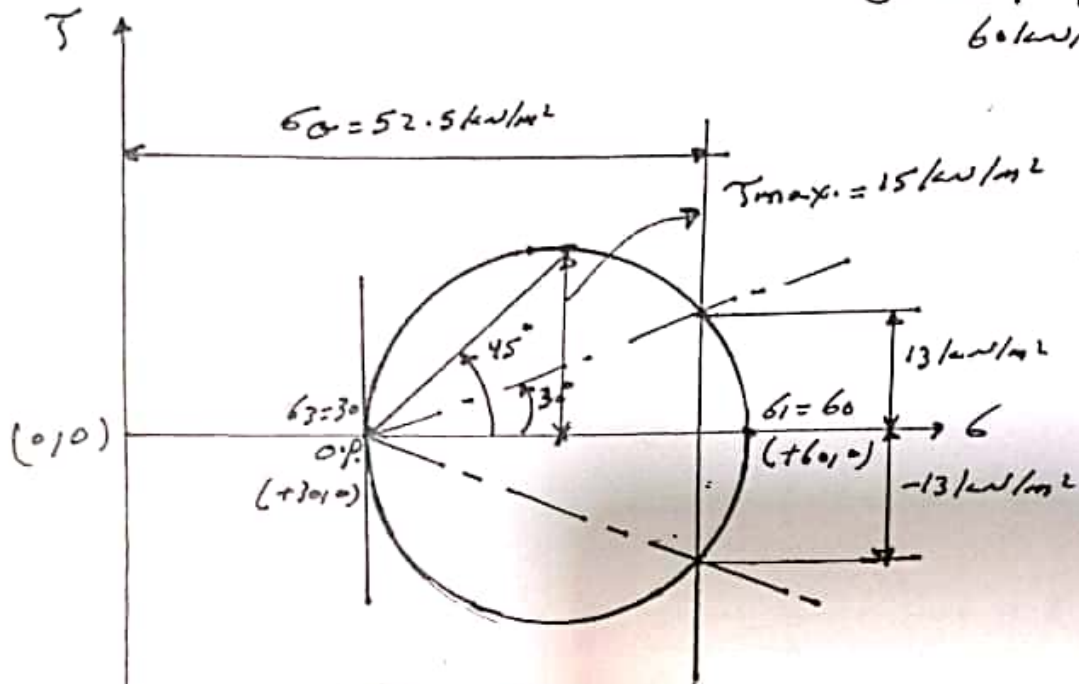
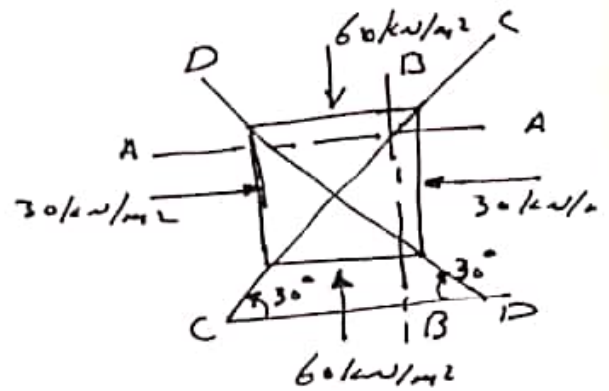
$$\tau_\theta = 21.65 \text{ kN/m}^2$$



Ex 2:- For the element shown, find the stresses on the planes A-A, B-B, C-C, D-D

Sol:-

since $\tau = 0$ so the stresses given is principle stresses at all planes



at Plane A-A:-

$$\sigma_1 = 60 \text{ kN/m}^2, \sigma_3 = 30 \text{ kN/m}^2$$

From the drawing of Mohr circle:-

$$\sigma_\sigma = 60 \text{ kN/m}^2, \tau = 0 \text{ kN/m}^2$$

at plane B-B:- $\sigma_1 = 60 \text{ kN/m}^2, \sigma_3 = 30 \text{ kN/m}^2$

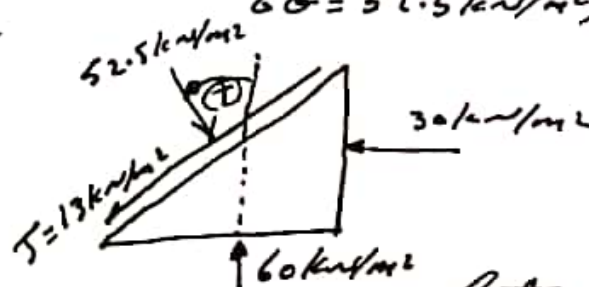
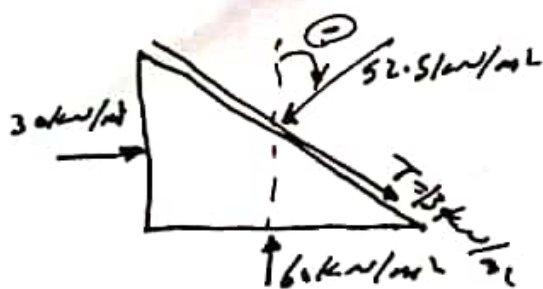
$$\sigma_\sigma = 30 \text{ kN/m}^2, \tau = 0 \text{ kN/m}^2$$

at plane C-C:- $\sigma_1 = 60 \text{ kN/m}^2, \sigma_3 = 30 \text{ kN/m}^2$

$$\sigma_\sigma = 52.5 \text{ kN/m}^2, \tau = 13 \text{ kN/m}^2$$

at plane D-D:- $\sigma_1 = 60 \text{ kN/m}^2, \sigma_3 = 30 \text{ kN/m}^2$

$$\sigma_\sigma = 52.5 \text{ kN/m}^2, \tau = -13 \text{ kN/m}^2$$



(22)

Dr. Hussein T. Mahabrah

$$u = u_s + u_e \quad (u_e = \Delta b) / \Delta b = \Delta \bar{b} + \Delta u$$

In this stage: $\Delta u = u_e = \Delta b$ — $\Delta \bar{b} = \text{Zero}$.

And (it's very short stage).

Stage (2):-

Drainage stage:- as water flows out of soil forced the hydraulic gradient caused by increase in P.W.P. The excess in (P.W.P.) (u_e) will gradually decreased. The effective stress will increased simultaneously and the soil volume will decreased due to particles movement close together.

Here, at any time this stage $u = u_s + u_e$, ($0 < u_e < u_{e0}$), and $\Delta b > \Delta \bar{b} > 0$, $\Delta b = \Delta \bar{b} + \Delta u$ ($\Delta u = u_e$).

this stage is very long stage.

the process of gradual reduction of excess pore water pressure with time is called (dissipation of excess P.

Stage (3)

Drained condition:- the of soil when the excess pore water pressure has been dissipated completely ($u_e = 0$).

in this case $\Delta b = \Delta \bar{b}$

this condition lasts forever provided that no change in total stress or location of ground water table take place. this process is called (consolidation in soil) when soil particles movement occurs in one direction only it's called (one dimensioned consolidation)..

$\Delta \sigma$	Δu	$\Delta \bar{\sigma}$	Time
10	10	0	$t = 0$
10	8	2	$t_1 > 0$
10	6	4	$t_2 > t_1$
After many times
10	0	10	$t = \infty$

$$\sigma = \bar{\sigma} + u \longrightarrow \Delta \sigma = \Delta \bar{\sigma} + \Delta u$$

when the total stress (σ) on saturated soil is changed (increased) by a certain value $\Delta \sigma$. The soil will pass through three distinguished stages.

① before (σ) is changed, the pore water pressure at point is equal to the static pressure (u_s) which is given by the location of groundwater table ($u = u_s$).

② after loading (application of $\Delta \sigma$), the p.w.p. will no longer be equal to the static value. There ($u = u_s + u_e$).

u_e = excess pore water pressure is the increase in p.w. above the static value.

Stage (1):-

At the instant of total stress increase (at time = $t = 0$) all change in total stress will be carried by water as an increase in pressure above the static value. This means that at this time ($u_e = \Delta \sigma = u_{e0}$ = initial excess pore water pressure).

In this stage soil is said to be in the undrained condition.
undrained condition:- the condition of saturated soil when a stress increment ($\Delta \sigma$) is carried completely by water as an increase in pressure called initial excess pore water pressure (u_{e0}) (no dissipation of water pressure has taken place).

Signature

Response of Effective stress to change in Total stress:

suppose a surface surcharge ($\Delta \sigma$) is placed on the surface of laterally confined soil. ($\Delta \sigma$) is infinity extent surface load applied instantaneously (the time duration for load increase = Zero).

⊗ Before loading (initially $u = u_s$).

⊗ immediately after loading at time = t_0 soil particles try to move closer together. But the water filling the voids resist this movement as result, the pressure water will increase above the static pressure (u_s) the increase in p.w.p. above the static value called excess pore water pressure (u_e).

At time = t_0 the excess in pore water pressure is called (initial excess) (p.w.p. (u_e)). at this time

($u_{e0} = \Delta \sigma = q$) hence $\Delta \bar{\sigma} = 0$.


As time proceeds — water escapes out of soil pores — pressure in water decrease — soil particles now can move close together — inter particles forces increase — effective stresses increase.

⊗ Before loading ($u = u_s$)

at time = t_0 $u = u_s + u_{e0} = u_s + \Delta \sigma$ so, ($u_{e0} = \Delta \sigma$)

$$u = u_s + u_e \quad (0 \leq u_e \leq u_{e0}).$$

⊗ after long time the excess p.w.p will become zero and so the p.w.p will be equal to the static pressure again $u = u_s$ (after very long time). At this time water movement stops and particles movement stop too and the increase in total stress ($\Delta \sigma$) will be carried by the mineral skeleton entirely, as an increase in effective stress ($\Delta \bar{\sigma}$).


T. Nahabh

Ex 3:- For the element shown, find the normal and shear stresses on the plane 1-1?

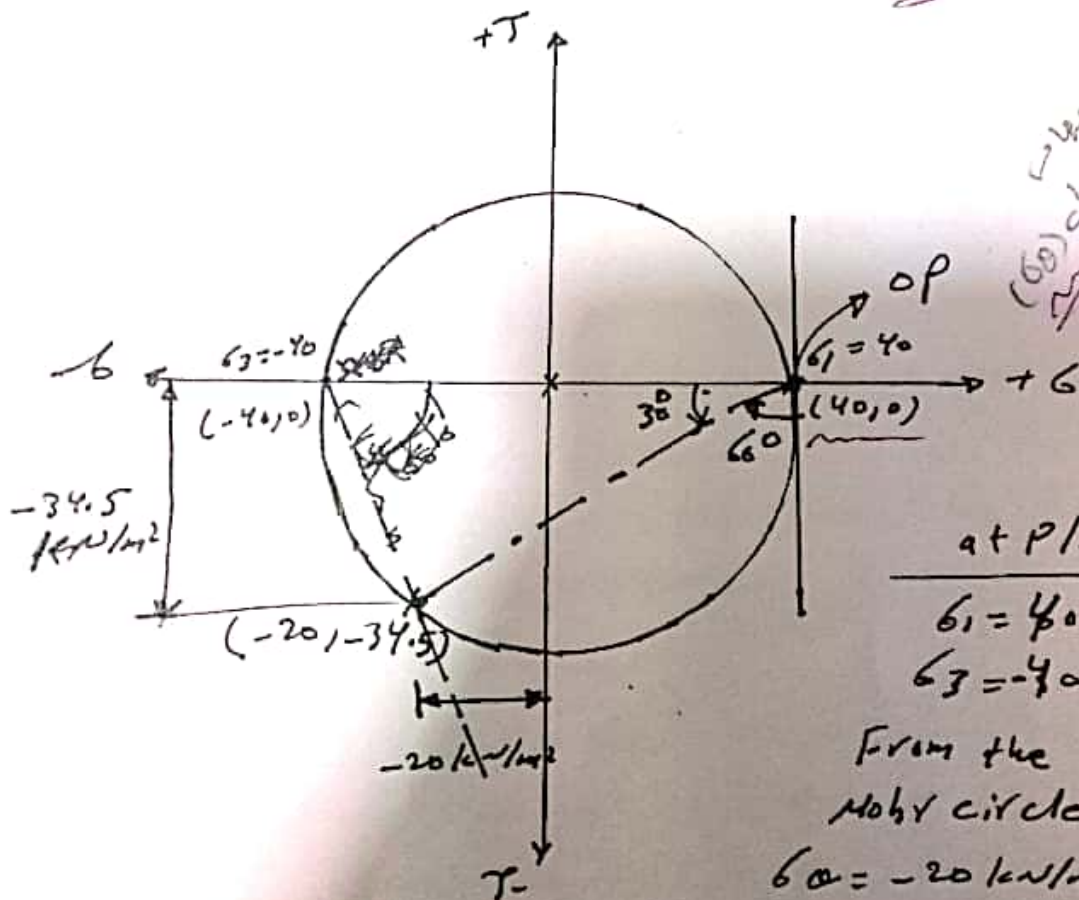
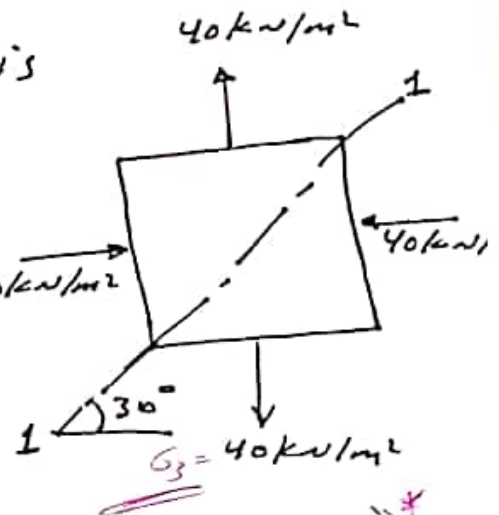
Sol:- Since $T=0$ so, the stresses given is principle stresses

$$\sigma_1 = 40 \text{ kN/m}^2$$

$$\sigma_3 = -40 \text{ kN/m}^2$$

$$\theta = -60^\circ$$

$\sigma_1 = 40 \text{ kN/m}^2$
 $\sigma_3 = -40 \text{ kN/m}^2$



at Plane 1-1

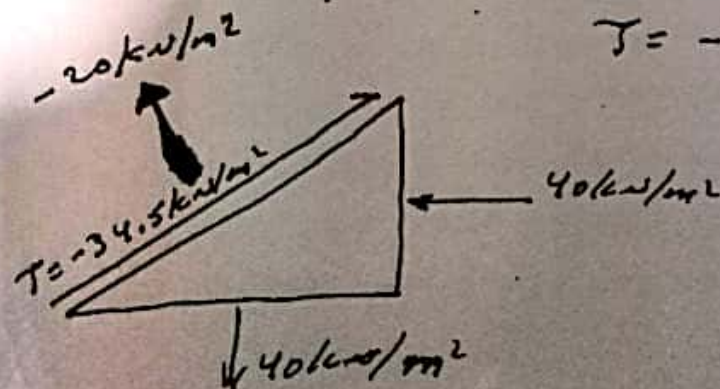
$$\sigma_1 = 40 \text{ kN/m}^2$$

$$\sigma_3 = -40 \text{ kN/m}^2$$

From the drawing of Mohr's circle:-

$$\sigma_c = -20 \text{ kN/m}^2$$

$$\tau = -34.5 \text{ kN/m}^2$$

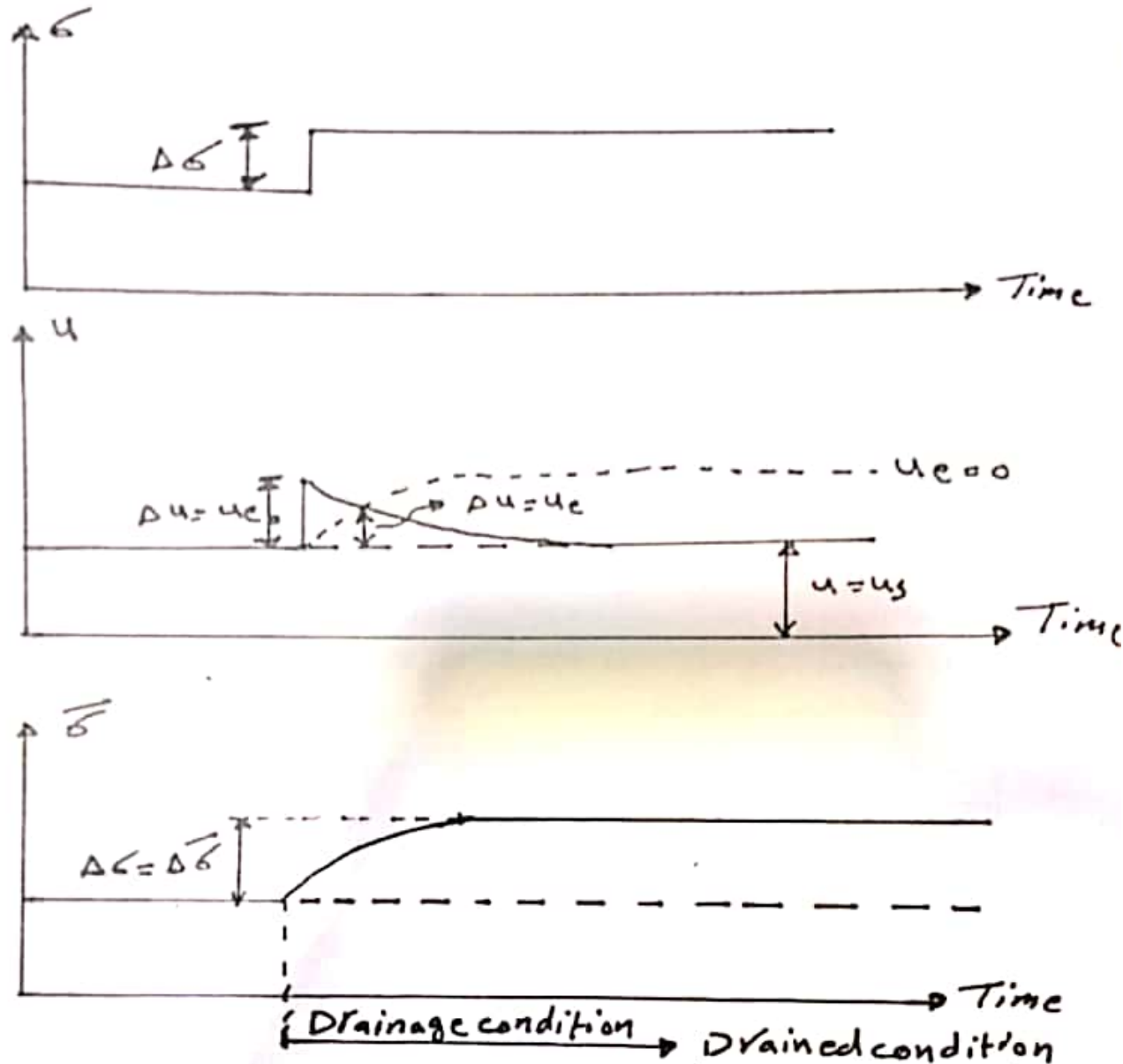


(74)

Dr. Hussein T. Mahabeh

Consolidation :-

The process of gradual squeezing of water out of soil accompanied by reduction in soil volume and increase of effective stress under an increase in total stress.



Time required to accomplish any percent of consolidation process depends on the following factors:-

- ① Time increases with increasing the volume of water which must be expelled out of soil during consolidation.

$t \propto \text{volume of water}$

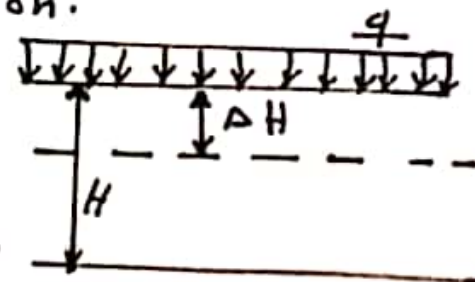
$$\text{volume of water} = m \times \Delta\sigma \times H$$

where: m = soil compressibility

$\Delta\sigma$ = change (increase in total stress).

H = thickness of consolidated layer.

$$t \propto m \times \Delta\sigma \times H.$$



Dr. Hussain T. Nahab

Example: For soil profile shown in figure, Calculate the final consolidation in clay layer.

Sol:-

$$① \quad s_{ef} = \frac{\Delta e}{1+e_0} \times H = \frac{e_0 - e_f}{1+e_0} \times H$$

$$s_{ef} = \frac{1.83 - 1.4}{1 + 1.4} \times (11.6 - 7.3) = 0.653 \text{ m}$$

$$s_{ef} = m_v \times \Delta \bar{\sigma} \times H$$

$$\Delta \bar{\sigma} = \Delta \sigma = q = \gamma_{fill} \times H_{fill}$$

$$= 22 \times 4.5 = 99 \text{ kN/m}^2$$

$$s_{ef} = 1.5348 \times 10^{-3} \times 99 \times (11.6 - 7.3) = 0.653 \text{ m}$$

$$s_{ef} = \frac{C_c}{1+e_0} \times H \times \log \frac{\bar{\sigma} + \bar{\sigma}_v}{\bar{\sigma}_v}$$

$\bar{\sigma}_v = \sigma_v - u$ at the middle of the clay layer before fill application

$$\bar{\sigma}_v = (7.3 - 2) \times 18.22 + \left(\frac{11.6 - 7.3}{2} \right) \times 16.34 = 131.7 \text{ kN/m}^2$$

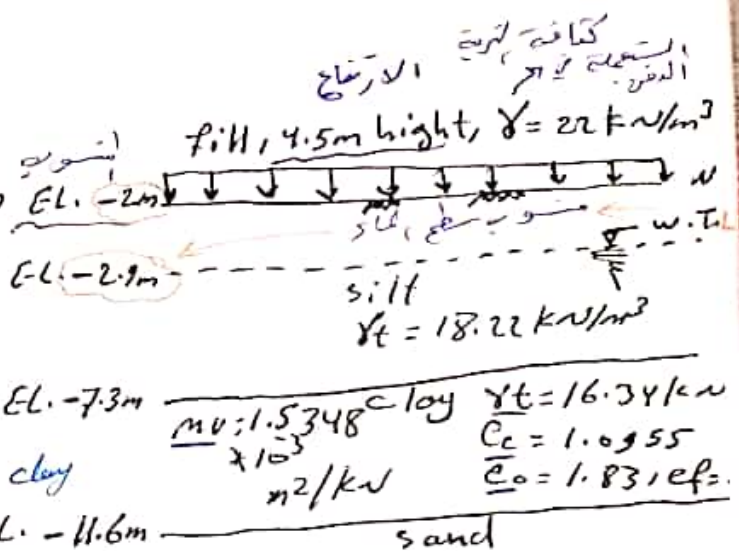
$$u = \left[(7.3 - 2.9) + \left(\frac{11.6 - 7.3}{2} \right) \right] \times 10 = 65.54 \text{ kN/m}^2$$

$$\bar{\sigma}_v = 131.7 - 65.54 = 66.14 \text{ kN/m}^2$$

$$s_{ef} = \frac{1.0955}{1 + 1.83} \times (11.6 - 7.3) \times \log \frac{66.14 + 99}{66.14} = 0.653 \text{ m}$$

② Degree of consolidation ($u\%$):-

It is the ratio of the amount of the dissipated excess pore water pressure (any pressure in water above the static pore pressure), to initial excess pore water pressure, for an element of soil located at depth (z) below the top surf of the clay layer at a specific time.



where:

s_{ef} = final consolidation settlement. الهبوط النهائي

e_0 = initial void ratio

Δe = change in void ratio $= e_0 - e_f$

e_f = final void ratio.

H = thickness of compressed clayey soil.

m_v = coefficient of volume compressibility for the stress range $\Delta \bar{\sigma}$ change in effective stress between initial and final conditions.

$$\Delta \bar{\sigma} = \bar{\sigma}_{vp} - \bar{\sigma}_{vo}$$

C_c = Compression index. مؤشر الانضغاطية

C_r = reloading index. مؤشر إزالة الضغط

$\bar{\sigma}_{vo}$ = initial effective stress.

$\bar{\sigma}_{vm}$ = maximum pre-consolidation stress.

* The degree of consolidation can be represented by the following terms.

1- in terms of void ratio: $u_z\% = \frac{e_0 - e_t}{e_0 - e_f} \times 100\%$

where: e_0, e_f = initial and final void ratio respectively.

e_t = void ratio at the time in question. نسبة الفراغ بالوقت المحدد

2- in term of effective stress: $u_z\% = \frac{\bar{\sigma}_{vt} - \bar{\sigma}_{vo}}{\bar{\sigma}_{vp} - \bar{\sigma}_{vo}} \times 100\%$

where: $\bar{\sigma}_{vp}, \bar{\sigma}_{vo}$ = final and initial effective stress respectively.

$\bar{\sigma}_{vt}$ = effective stress at the time in question.

3- in term of excess pore water pressure: $u_z\% = \frac{u_i - u_e}{u_i} \times 100\%$

OR $u_z\% = 1 - \frac{u_e}{u_i}$, u_i = initial excess p.w.p. $= \Delta \bar{\sigma}$
 u_e = excess p.w.p. at time in question.

(*) Settlement After a specific Time (s_{et}): الهبوط بعد وقت محدد

$$s_{et} = U_{ave} \times s_{ef}$$

U_{ave} = (average degree of consolidation) = the degree of consolidation for layer (average) irrespective of the depth (z).

$T_v = \frac{\pi}{4} \times U_{ave}^2$, T_v = time factor $\left\{ \frac{C_v \times t}{d^2} \right\}$, C_v = coefficient of consolidation. t = time in question.

d = whole thickness of clay layer.

1. Normally and over consolidated soils:-

soil types according to stress history:-

A - Normally Consolidated soil (N.C.C.):-

soil is designated by N.C.C. represent the soil at which the existing effective vertical stress is the largest stress experienced by the soil at the present time and in the past, thus:-

For N.C.C. $\bar{\sigma}_{vm} = \bar{\sigma}_{v0}$

initial effective stress ($\bar{\sigma}_{v0}$) = existing or present time vertical effective stress.

maximum pre-consolidation stress ($\bar{\sigma}_{vm}$) = the maximum effective stress which has ever been subjected to the soil specimen and completely consolidated under its effect.

B - over consolidated soil (O.C.C.):-

soil is designated by (O.C.C.) refer to the soil experienced a stress in past larger than the existing vertical stress acting at the present time.

For (O.C.C.) $\bar{\sigma}_{vm} > \bar{\sigma}_{v0}$

over consolidation ratio (O.C.R.):-

It is defined as, $O.C.R. = \frac{\bar{\sigma}_{vm}}{\bar{\sigma}_{v0}}$

if (O.C.R.) = 1 $\bar{\sigma}_{vm} = \bar{\sigma}_{v0}$ --- then the soil is N.C.C.

if (O.C.R.) > 1 $\bar{\sigma}_{vm} > \bar{\sigma}_{v0}$ --- then the soil is O.C.C.

② Consolidation Settlement:-

To calculate the final consolidation settlement (i.e. at $t = \infty$), we can use one of the following correlation:-

1- $S_{ef} = \frac{\Delta e}{1+e_0} \times H$

2- $S_{ef} = m_v \times \Delta \bar{\sigma} \times H$

3- For N.C.C., $S_{ef} = \frac{C_c}{1+e_0} \times H \times \log \frac{\Delta \bar{\sigma} + \bar{\sigma}_{v0}}{\bar{\sigma}_{v0}}$

4- For O.C.C., check:

a- if $\Delta \bar{\sigma} + \bar{\sigma}_{v0} \leq \bar{\sigma}_{vm}$ then:

$S_{ef} = \frac{C_r}{1+e_0} \times H \times \log \frac{\Delta \bar{\sigma} + \bar{\sigma}_{v0}}{\bar{\sigma}_{v0}}$

b- if $\Delta \bar{\sigma} + \bar{\sigma}_{v0} > \bar{\sigma}_{vm}$, then:

$S_{ef} = \frac{C_c}{1+e_0} \times H \times \log \frac{\bar{\sigma}_{vm}}{\bar{\sigma}_{v0}} + \frac{C_r}{1+e_0} \times H \times \log \frac{\Delta \bar{\sigma} + \bar{\sigma}_{v0}}{\bar{\sigma}_{vm}}$

2- Time decrease with increasing the rate by which water is expelled

$$t \propto \frac{1}{\text{velocity of water}}$$

$$\text{velocity} = ki = k \times \frac{\Delta u}{H}$$

where: $\Delta u = u_0$ = initial excess in pore water pres.

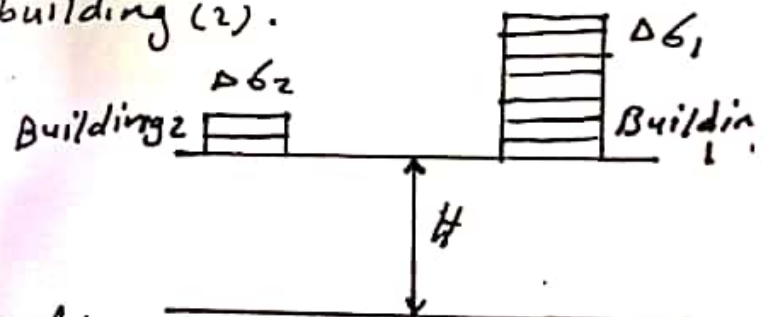
$$t \propto \frac{m \times \Delta u \times H}{k \times \frac{\Delta u}{H}}$$

$$t \propto \frac{m \times \Delta u \times H}{k \times \frac{\Delta u}{H}} \quad \therefore t \propto \frac{m H^2}{k}$$

The time required to accomplish any percent of consolidation process is called (hydro dynamic time lag). This time is:

- 1- increase with increasing soil compressibility.
- 2- increase very rapidly with the increasing the thickness of layer (H).
- 3- decreases with increasing soil permeability.
- 4- independent of magnitude of the change in total stress ($\Delta \sigma$)

Time required to complete consolidation (to complete settlement) is the same for both buildings. 1- ten story 2- two story. But the volume change of soil (settlement) is much higher for building (1) than for building (2).



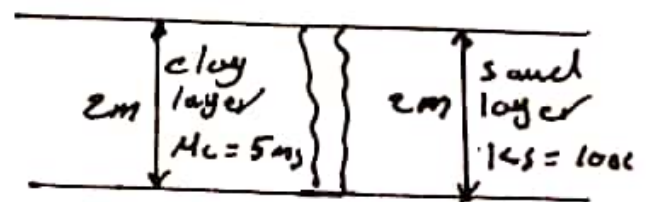
Ex: if three months are required to achieve same consolidation sand layer. what is time required to get the same consolidation percent in the clay layer?

$$\text{Sol: } \frac{t_c}{t_s} = \frac{\left(\frac{m H_c^2}{k}\right)_c}{\left(\frac{m H_s^2}{k}\right)_s}$$

$$\frac{t_c}{t_s} = \left(\frac{m}{k}\right)_c \times \left(\frac{H_c}{H_s}\right)^2$$

$$\frac{t_c}{t_s} = \frac{5 m_s}{k_c} \times \frac{1000 k_c}{m_s} = 5000$$

$$t_c = 5000 \times t_s = \frac{5000 \times 3}{12} = 1250 \text{ years.}$$



[Signature]