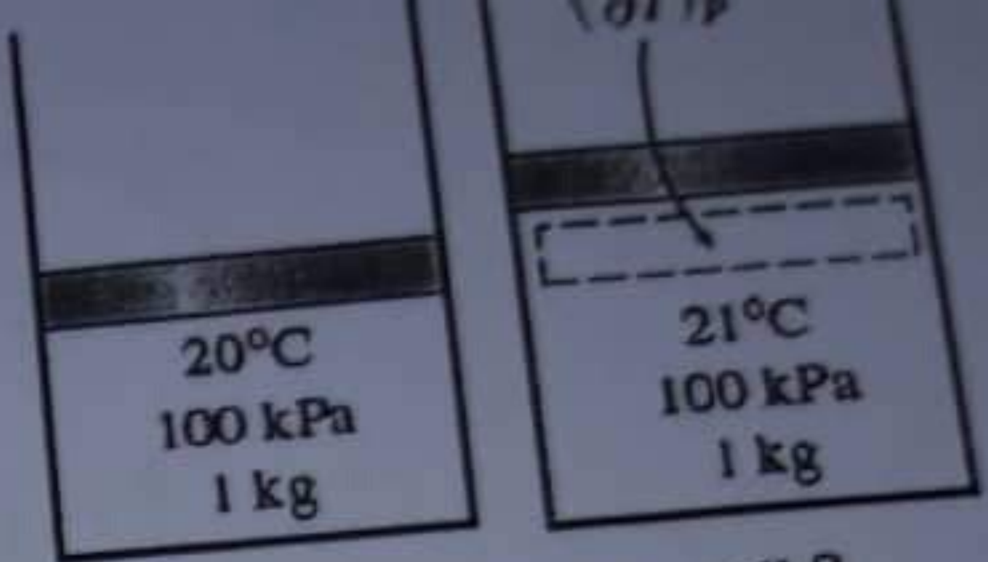
$$\beta \approx -\frac{1}{\rho}\frac{\Delta \rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \quad \text{At constant P}$$

$$\boxed{\rho_\infty - \rho = \rho\beta(T - T_\infty)} \text{ ---- (a)}$$



(b) A substance with a small β

اشتقاق

For an ideal gas $P = \rho RT$

$$\beta_{\text{ideal gas}} = \frac{1}{T}$$

(1) $PV = RT$

(2) $P\partial V = R\partial T$

(3) $\left(\frac{\partial U}{\partial T}\right)_P = \frac{R}{P}$

(4) $\beta = \frac{1}{V}\cdot\left(\frac{\partial V}{\partial T}\right)_P$

(5) $\beta = \frac{1}{V}\cdot\frac{R}{P}$

(6) $\beta_2 = \frac{R}{PV} \Rightarrow \frac{R}{RT}$

$$= \frac{1}{T}$$

Safawy

NATURAL/FREE CONVECTION

Free convection, also known as natural convection, is a phenomenon which occurs when a fluid or gas moves as a result of density changes which occur inside it, rather than in response to an external source of movement such as a fan or turbine. Natural convection can be seen, unsurprisingly, in nature, where it plays a role in a number of different natural processes, and it can also be seen in human-controlled settings, ranging from the kitchen to the chemistry lab. This differs from forced convection, in which movement is forced by the movement of a device such as a fan, as seen in a convection oven.

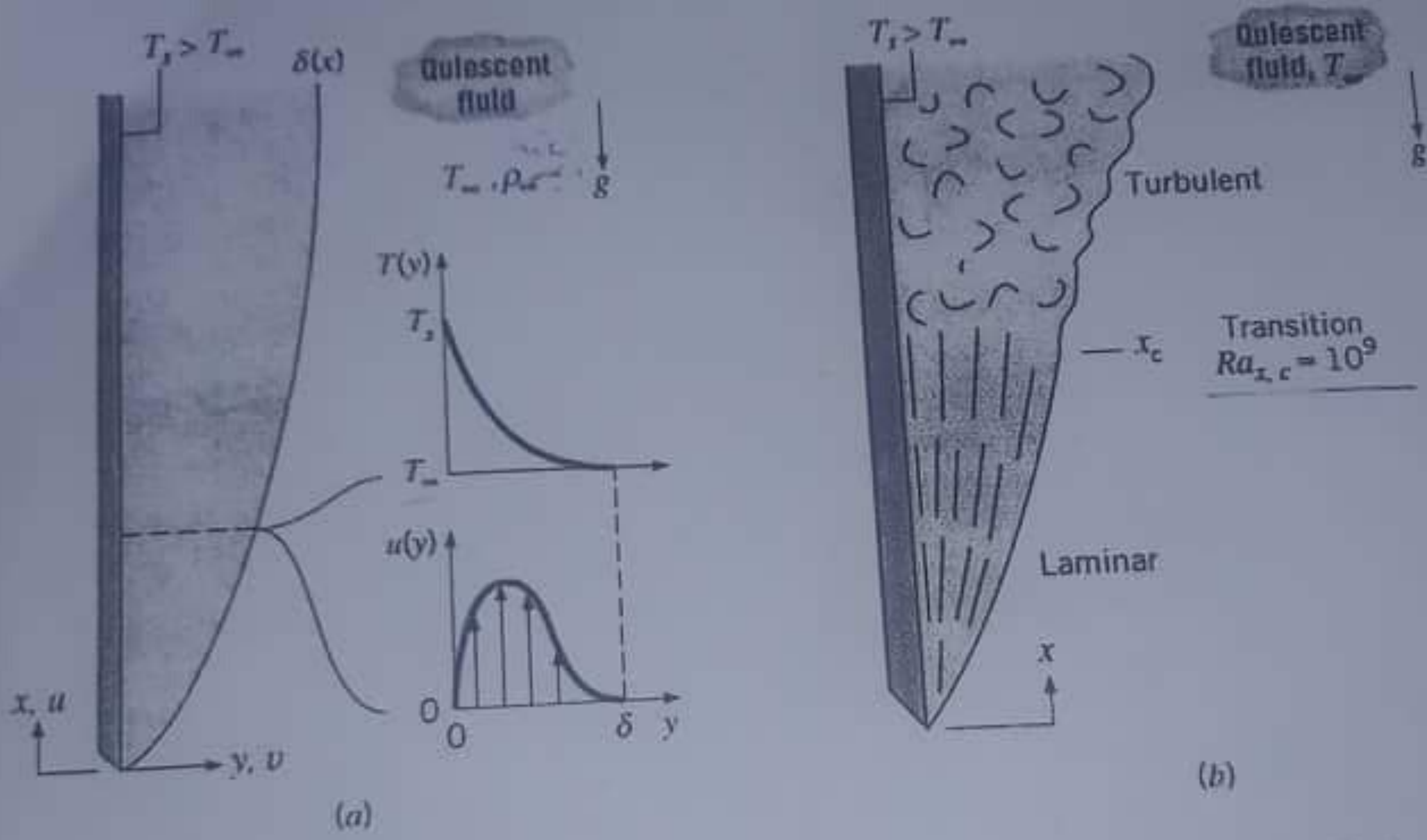
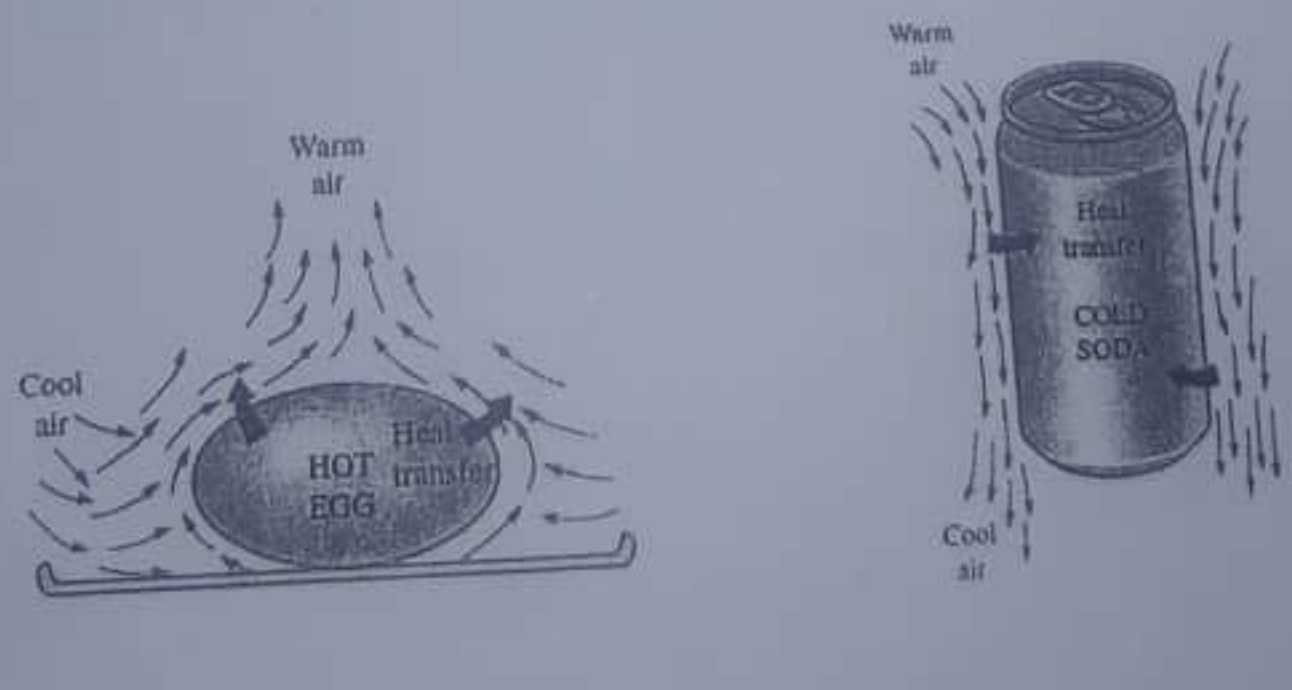


Figure 1 Boundary layer development on a heated vertical plate. (a) Velocity and temperature profiles in the boundary layer at the location x. (b) Boundary layer transitional flow conditions.



...ncy Force

...ward force exerted by a fluid on a body completely or partially immersed in it is called the ...ncy force. **The magnitude** of the buoyancy force is equal to the weight of the fluid displaced ...e body.

$F = \rho_{fluid} g V_{body}$

$$F_{net} = \boxed{W - F_{bouyancy}}$$
$$= \rho_{body} g V_{body} - \rho_{fluid} g V_{body}$$
$$= (\rho_{body} - \rho_{fluid}) g V_{body}$$

It is the buoyancy force that keeps the ships afloat in water ($W = F_{buoyancy}$) for floating objects

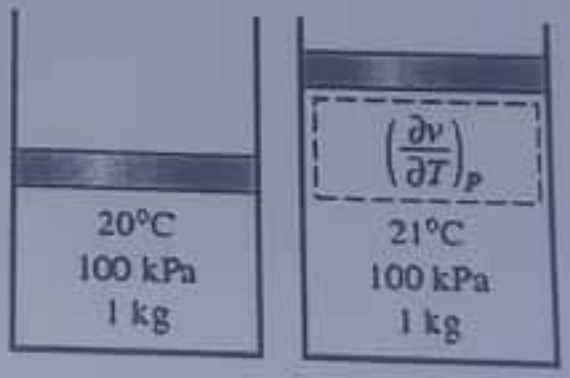
Volume Expansion Coefficient

The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure

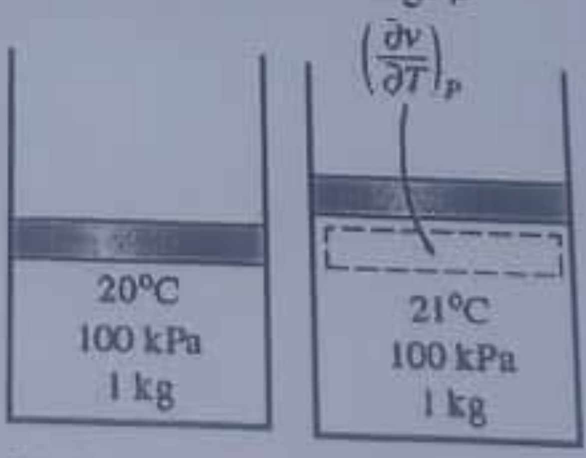
$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

$$\beta \approx -\frac{1}{\rho}\frac{\Delta \rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \quad \text{At constant P}$$

$$\boxed{\rho_\infty - \rho = \rho\beta(T - T_\infty)}$$ (a)



(a) A substance with a large β



(b) A substance with a small β

For an ideal gas $P = \rho RT$

$$\beta_{ideal\ gas} = \frac{1}{T}$$

$PV = RT$

$P\partial V = R\partial T$

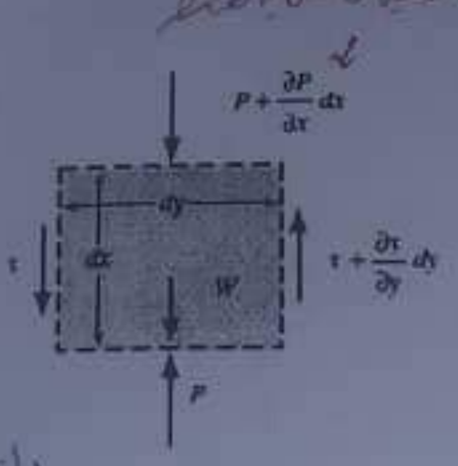
$\left(\frac{\partial U}{\partial T}\right)_P = \frac{R}{P}$

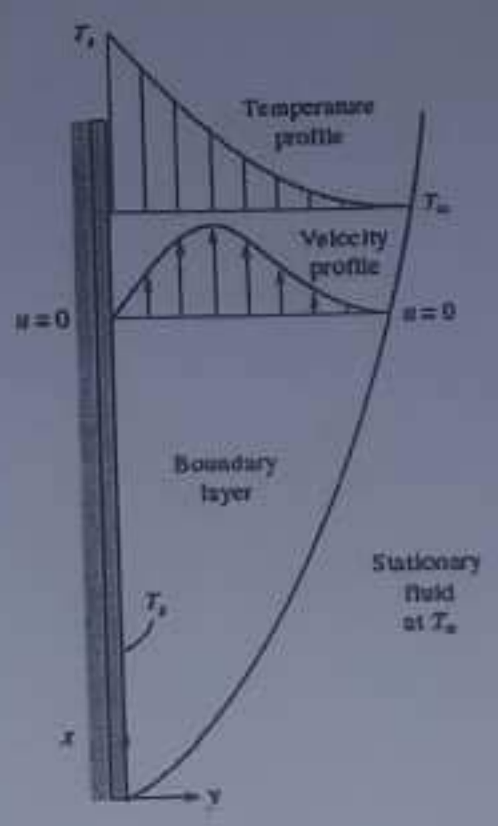
(4) $\beta = \frac{1}{V}\left(\frac{\partial U}{\partial T}\right)_P$

(5) $\beta = \frac{1}{V} \cdot \frac{R}{P}$

(6) $\beta = \frac{R}{PV} = \frac{R}{RT} = \frac{1}{T}$

Equation of Motion





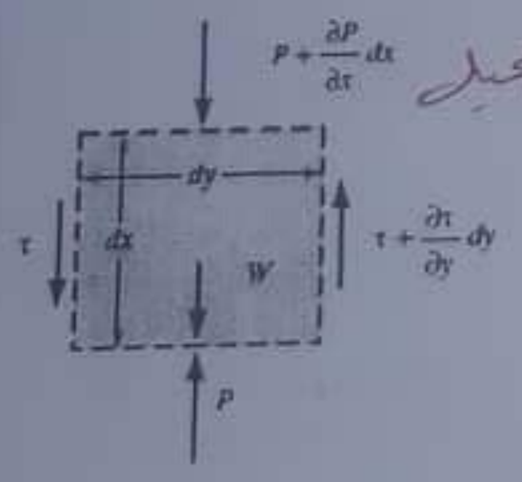
اشتقاق

Newton's 2nd law

Newton's 2nd law gives: $\delta m \cdot a_x = F_{surface,x} + F_{body,x}$

الكتلة = كثافة * الحجم

Mass $\delta m = \rho(dx \cdot dy \cdot 1)$



التعجيل

Acceleration

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

Forces

$$F_x = \left(\frac{\partial \tau}{\partial y}dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x}dx\right)(dy \cdot 1) - \rho g(dx.dy.1)$$

$$= \left(\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g\right)(dx \cdot dy \cdot 1)$$

$$\tau = \mu\left(\partial u / \partial y\right)$$

Momentum Equation

هذه المعادلة لا تحفظ

$$\boxed{\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g}$$

The x-momentum equation for the quiescent field outside the boundary layer can be found by applying the above equation as u = 0

$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g$$

The y-momentum equation results: $\frac{\partial P}{\partial y} = 0$ $\qquad P = P(x) = P_\infty(x)$

$$\frac{\partial P}{\partial x} = \frac{\partial P_\infty}{\partial x} = -\rho_\infty g$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g$$

$$\boxed{\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)} \qquad \boxed{\rho_\infty - \rho = \rho\beta(T - T_\infty)}$$

Grashof Number

$$x^* = \frac{x}{L_c} \quad y^* = \frac{y}{L_c} \quad u^* = \frac{u}{V} \quad v^* = \frac{v}{V} \quad \text{and} \quad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

(g) التعجيل الأرضي

$\frac{1}{T}$ (β)

(L) الطول

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right]\frac{T^*}{Re_L^2} + \frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$\boxed{Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}}$$

اللزوجة

$\nu = \frac{\mu}{\rho}$

The Grashof number Gr is a measure the relative of magnitudes of the buoyancy force and the opposing viscous force acting on the fluid.

Limits

For a vertical plate

* $Gr < 10^9$	Laminar
* $Gr > 10^9$	Turbulent

$Gr_L/Re_L^2 \ll 1$ Forced convection dominates

$Gr_L/Re_L^2 \gg 1$ Free convection dominates

$Re = \frac{\rho V t}{\eta}$

Nu for Free Convection

$Pr = \frac{\mu C_p}{k}$

$$\boxed{Nu = \frac{hL_c}{k}} = C(Gr_L\, Pr)^n = CRa_L^n$$

fluid

$$\boxed{Ra_L = Gr_L\, Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr$$

0.7 fluid Pr
0.6 Pr

Pr

Values of n and C depend on geometry of the surface and flow regime

The value of n is usually ¼ for laminar flow and 1/3 for turbulent flow. The value of the constant C is normally less than 1.

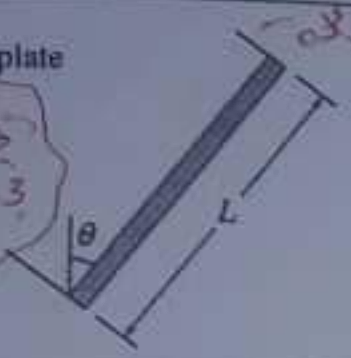
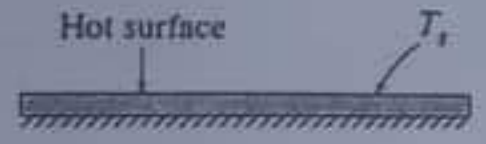
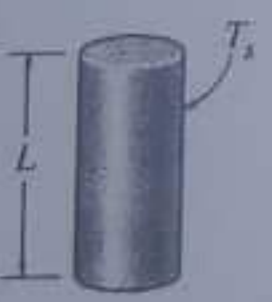
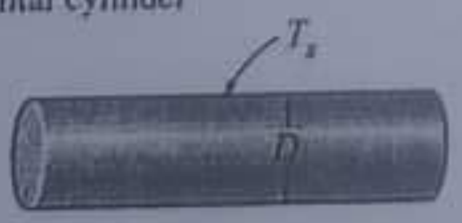
Boundary Layer Thickness:
For laminar flow of gases (Pr =0.7), the boundary layer thickness ($\delta_v \approx \delta_T$) can be estimated using the expression:

$$\frac{\delta}{x} = 6(Gr/4)^{-1/4}$$

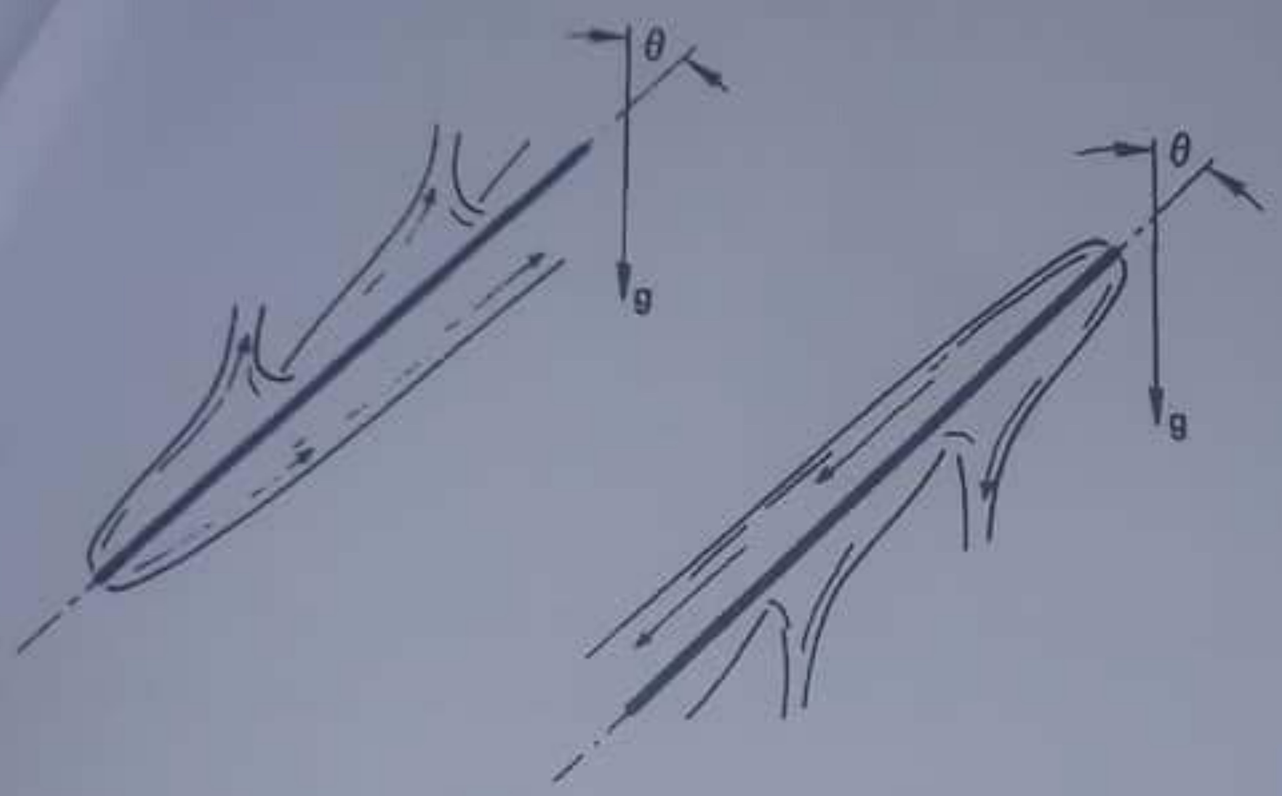
$$Pr = 0.7, Ra_L \le 10^9$$

h ثابتة في الـ free
متغيرة في الـ force

...1 Empirical correlations for the average Nusselt number for natural convection over surfaces.

Geometry	Characteristic length L_c	Range of Ra	Nu $= \frac{hL_c}{k}$
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ $Nu = 0.1Ra_L^{1/3}$ *Churchill-Chu correlation* $Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for $Ra < 10^9$
Horizontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate) Hot surface  (*b*) Lower surface of a hot plate (or upper surface of a cold plate)  Hot surface	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$Nu = 0.54Ra_L^{1/4}$ $Nu = 0.15Ra_L^{1/3}$ $Nu = 0.27Ra_L^{1/4}$
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	*Churchill-Chu correlation* $Nu = \left\{0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$
Sphere 	D	$Ra_D \leq 10^{11}$ $(Pr \geq 0.7)$	*Churchill correlation* $Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$

clined Plates:



(a) A hot inclined plate (b) A cold inclined plate

Figure Natural convection flows on the upper and lower surfaces of inclined plates.

2. Horizontal Plates:

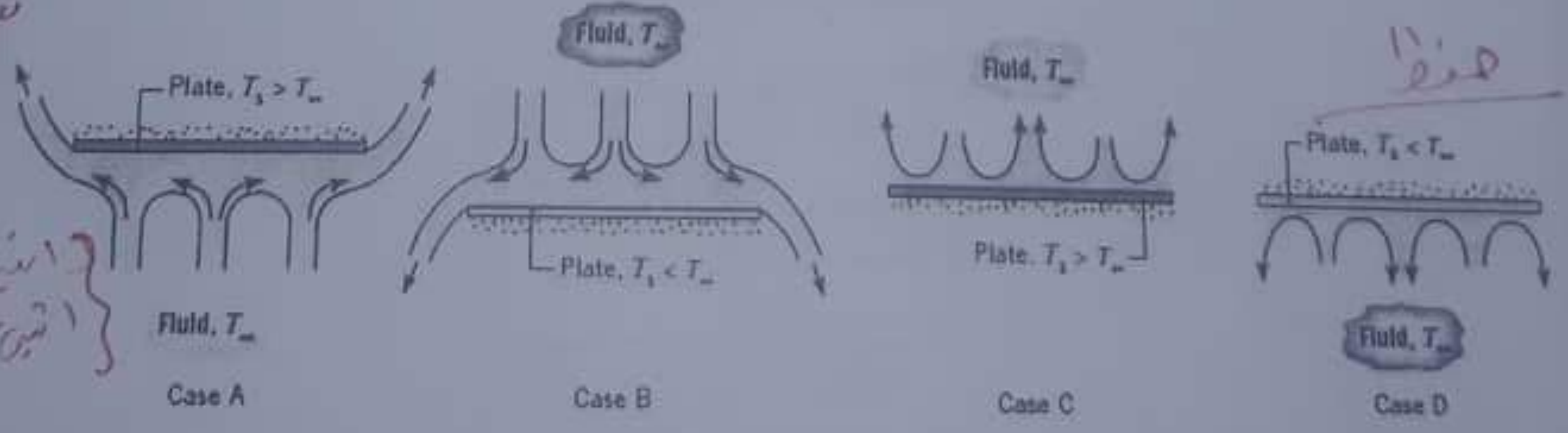


Figure Free convection buoyancy-driven flows for hot ($T_s > T_\infty$) and cold ($T_s < T_\infty$) horizontal plates:
Case A — hot surface facing downwards, Case B — cold surface facing upwards, Case C — hot surface facing upwards, and Case D — cold surface facing downwards.

3. Vertical Cylinders:

The relations for vertical plates can also be used for vertical cylinders if the boundary layer thickness is much less than the cylinder diameter.

4. Horizontal Cylinder:

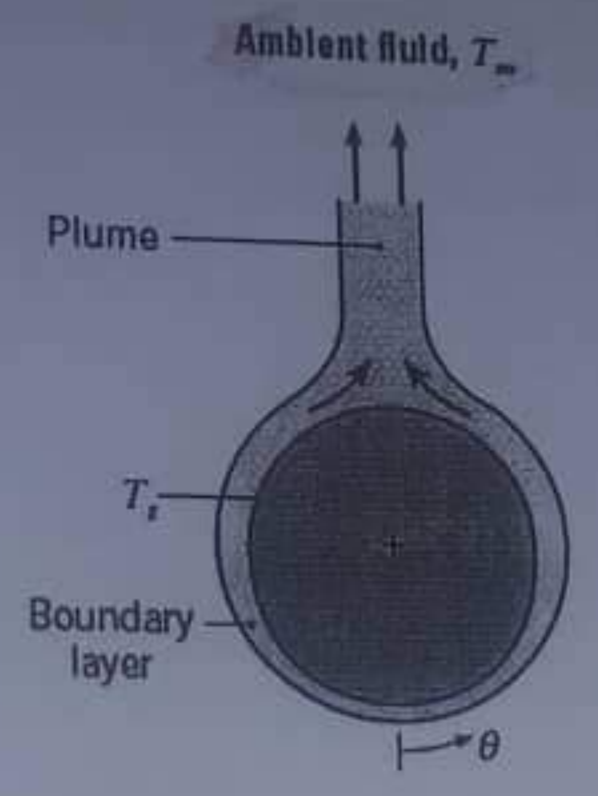


Figure Natural convection flow over a horizontal hot cylinder.

Combined Free and Forced Convection:

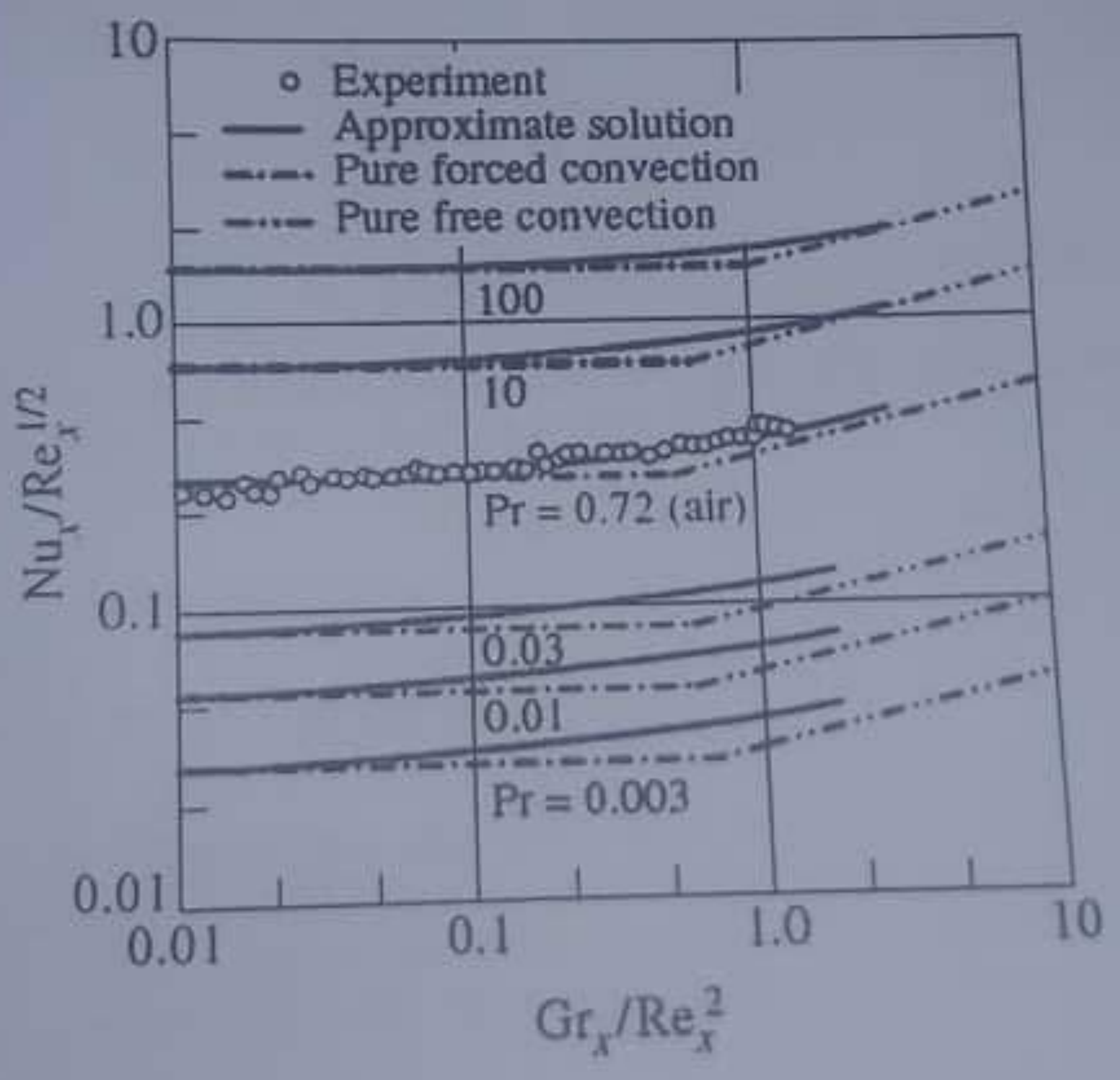


Figure Variation of the local Nusselt number Nu_x for combined natural and forced convection from a hot isothermal vertical plate.

Note:

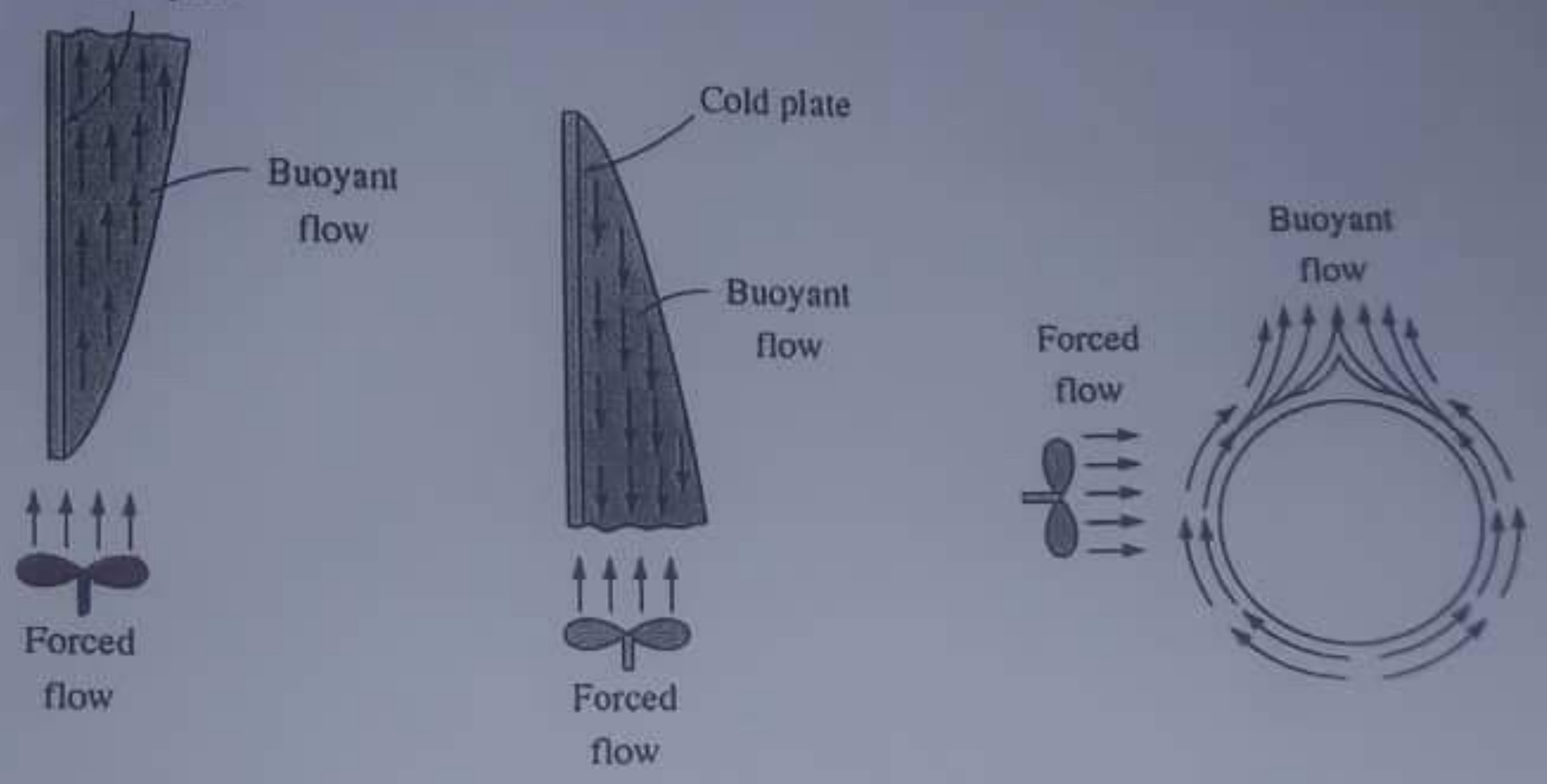
For a given fluid, the parameter **Gr/Re^2** represents the importance of natural convection relative to forced convection:

free

$Gr/Re^2 < 0.1$ natural convection is negligible. force

$0.1 < Gr/Re^2 < 10$ neither is negligible. (force+free)

$Gr/Re^2 > 10$ forced convection is negligible. free



(a) Assisting flow (b) Opposing flow (c) Transverse flow

Figure Natural convection can enhance or inhibit heat transfer, depending on the relative directions of buoyancy-induced motion and the forced convection motion.

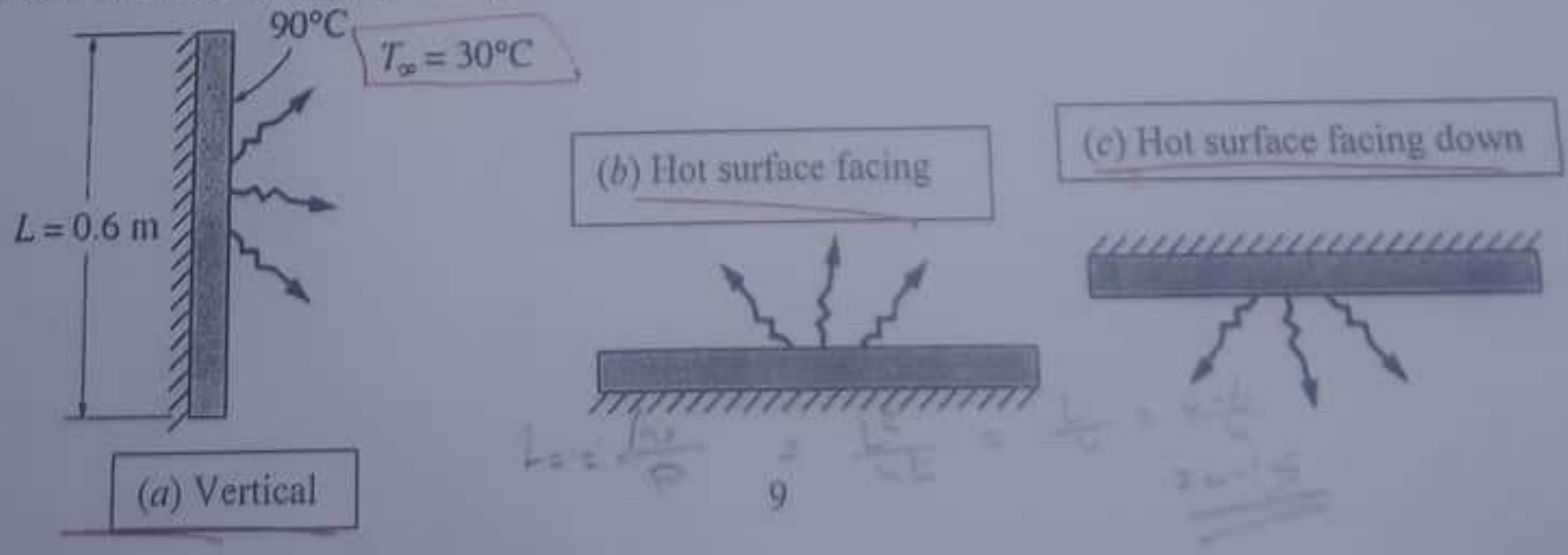
$$Nu_{combined} = (Nu^n_{forced} \pm Nu^n_{natural})^{1/n}$$

+: for assisting and transverse flows

-: for opposing flow

$3 < n < 4$ and $n = 3$ for vertical sufaces.

Example 1: Consider a 0.6-m X 0.6-m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 90°C, while the other side is insulated, as shown in Figure. Determine the rate of heat transfer from the plate by natural convection if the plate is (a) vertical, (b) horizontal with hot surface facing up, and (c) horizontal with hot surface facing down.



$L_c = \frac{A_s}{P}$

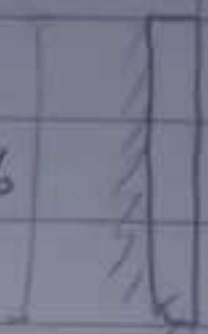
$A = L^2$

$P = 4L$

Ex/1

(a)

$$T_f = \frac{T_s + T_\infty}{2} = \frac{90+30}{2} = 60\,C^\circ$$



نحولها لل (k)

$$B = \frac{1}{T_{f\,(k)}}$$

From table

$Pr = 0.722$

$k = 0.028\ W/m\cdot C^\circ$

$\nu = 1.896 \times 10^{-5}\ m^2/s$

$$B = \frac{1}{333\,(k)}$$

$$Gr = \frac{gB(T_s - T_\infty)L^3}{\nu^2}$$

$$Ra_L = Gr \times Pr$$

$$Ra_L = \frac{(9.81)(\frac{1}{333})(90-30)(0.6)^3}{1.896 \times 10^{-5}}(0.722)$$

$$= 7.656 \times 10^8$$

$$Nu = \left\{0.825 + \frac{0.387\,Ra_L^{1/6}}{\left(1 + (0.492/Pr)^{9/16}\right)^{8/27}}\right\}^2$$

$Nu = 113.4$

$Nu = \frac{hL}{k}$

$h = \frac{k}{L} Nu = \frac{0.02808}{0.6} (113.4)$

$= 5.306 \ W/m^2 \, C°$

$Q = h A_s (T_s - T_\infty)$ $\quad A_s = L^2$

$Q = 5.306 (0.36)(90 - 30)$ $\quad = 0.6 \times 0.6$

$= 115 \ W$

(13)

$L_c = \frac{A_s}{P}$

$= \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6}{4} = 0.15$

$Ra_L = \frac{g \beta (T_s - T_\infty) L_c^3}{\nu^2} Pr$

$= \frac{9.8 \times \frac{1}{333} (90 - 30) \, 0.15^3}{1.896 \times 10^{-5}} = 0.72$

$Ra_L = 1.196 \times 10^7$

$Nu = 0.15 \, Ra_L^{1/3} = 31.76$

$Nu = \frac{hL}{k} \rightarrow h = \frac{k}{L} Nu$

$= 5.946$

التاريخ / /

موضوع:

$A_s = L^2 = 0.6^2$

$Q = h A_s (T - T_\infty)$

$= 5.946 \times (0.6)^2 \times (90 - 30)$

$= 128$

(C)

$Nu = 0.27 \, Ra_L^{1/4}$

$= 0.27 (1.196 \times 10^7)^{1/4}$

$= 15.86$

$h = \frac{k}{L_c} Nu$

$= \frac{0.028}{0.15} (15.86)$

$= 2.973 \quad W/m^2 C$

$Q = h A_s (T_s - T_\infty)$

$= 64.2 \quad W$

RAMY

$q = h A_s (T_s - T_\infty)$

$T_f = \frac{T_s - T_\infty}{2}$

التاريخ / /

الموضوع:

$T_f \Rightarrow k$

$\beta_c \Rightarrow \nu$

EX/9

$D \qquad T_\infty$

$\epsilon = 0.1$

$q_{total} = q_{rad} + q_{conv}$

$T = \frac{165 + 23}{2} = \frac{188}{2} = 94$

$T_f = 367$

$\beta = \frac{1}{T_f} = \frac{1}{367} = 2.725 \times 10^{-3}$

$k = 0.0313$

$\nu = 22.8 \times 10^{-6}$

$Pr = 0.697$

$q = h \underbrace{A_s}_{\pi D} (T_s - T_\infty) + \epsilon \pi D \sigma (T_s^4 - T_\infty^4)$

L هو يساوي (1)

هنا نشتغل حسب ال D القطر

$Ra_D = Gr \cdot Pr$

$= \frac{g \beta (T_s - T_\infty) D^3}{\nu^2} Pr$

$= 5.073 \times 10^6$

$Nu = \left[0.6 + \frac{0.387 Ra^{1/6}}{\left(1 + (0.559/Pr)^{9/16}\right)^{8/27}} \right]^2$

RAMY

with a
W/m · °C,
walls that face the
the average temperatu
window is measured to be 5°C
outdoors is -5°C, determine
(a) the convection heat transfer coeff
rface of the window,
rate of total heat transfer thr
mbined natural conve
ient on the outer
lect the the

التاريخ / /

$Nu = 23.3$

$h = \frac{k}{D} Nu$

$= 7.29 \ W/m^2 K$

التاريخ / /

الموضوع: المنحة الاولى

سؤال الاول

Q/1

1) $T_f = \frac{T_s + T_\infty}{2} = \frac{20+70}{2} = 45 C^\circ$

$T_k = 45 + 273$

$= 318$

$k = 0.02699$ $\quad$ $Pr = 0.7241$

$\nu = 1.749 \times 10^{-5}$ $\quad$ $\beta = \frac{1}{T_f} = \frac{1}{318}$

$Ra_D = \frac{g\beta (T_s - T_\infty) D^3}{\nu^2} Pr$

$= 1.869 \times 10^6$

$Nu = \left[0.6 + \frac{0.387 \, Ra_D^{1/6}}{(1 + (0.559/Pr)^{9/16})^{8/27}} \right]^2$

$= 17.4$

$h = \frac{k}{D} Nu = 5.869 \ W/m^2 . C^\circ$

$A_s = \pi D L = 1.508 \ m^2$

$q = h A_s (T - T_\infty)$

$= (5.869)(1.508)(70 - 20)$

$= 443 \ W$

RAMY