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Estimating counterfactual Covid-19 trajectories in Sweden under a hypothetical lockdown with Neural CDEs

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Abstract

Counterfactual estimation via synthetic controls is a recent development in causal inference. Despite its popularity, this method only allows for regular time series aligned in time and for synthetic controls expressed as linear combinations of observed control units. We propose to use a continuous-time alternative that models the counterfactual trajectory explicitly using the formalism of *neural controlled differential equations* (CDEs), that recently emerged as a leading machine learning technology to learn with irregularly sampled, misaligned, multivariate time series. In particular, we use neural CDEs to quantify the extent to which a hypothetical lockdown would have limited the spread of COVID-19 infections. We focus on the example of Sweden - a European country that did not implement a lockdown during the first wave of COVID-19 - and construct counterfactual lockdown trajectories for both the *reproduction rate* and *daily deaths*. Finally, we compare the estimated counterfactual daily-deaths-trajectory generated by the neural CDE with the trajectory produced by the state-of-the-art epidemiology software `epidemia` using the counterfactual reproduction-rate-trajectory produced by a second neural CDE. Our counterfactual daily-deaths-trajectory resembles very closely the one produced by `epidemia`, demonstrating the effectiveness of neural CDEs to create counterfactual scenarios that recover biologically inspired modelling.

1 Introduction

Throughout the COVID-19 pandemic, policy makers have used various tools to limit the spread of the virus. The main measure adopted by many governments across the globe has been the imposition of a lockdown. A lockdown is certainly a useful measure when it comes to limit the spread of the virus, but it entails very high economic costs. Therefore, it is essential to quantify both the costs and the benefits of lockdowns. In this study we focus on the latter and quantify the extent to which a lockdown limits the spread of COVID-19 infections and deaths. Here we specifically focus on the country of Sweden as it stands out from its European peers in that its government opted against a lockdown even though the exposure to COVID-19 in Sweden was not systematically different from the rest of Europe. Still, the Swedish authorities merely advised—rather than ordered—citizens to adjust their behavior in the face of the pandemic. In order to quantify the lockdown effect, we benchmark the actual developments in Sweden against a counterfactual lockdown scenario.

1.1 Non-technical summary

We ask the following question:

Problem statement *What would be the counterfactual trajectory of the reproduction rate and daily deaths in Sweden had the lockdown been applied during the same period it was applied in other similar EU countries?*

In *counterfactual causality* the goal is to answer the question of *what would have happened to the outcome of interest if a different intervention had been applied*. One way to answer this question is to define a control group of comparable units such as countries, regions, individuals, patients etc. in order to approximate the (counterfactual) trajectory of the target unit outcome had a different intervention been applied. We are interested in the case where a target unit (in this case a country) undergoes an intervention of interest at a particular point in time T, and then remains exposed to this intervention at all times afterwards $t > T$. Both pre-treatment $(t < T)$ and post-treatment $(t > T)$ outcomes are assumed to be available. We ask whether we can *infer the counterfactual trajectory had the unit not been exposed to the intervention* using a population of control units never exposed to the intervention.

In section 2 we provide a summary of the synthetic control methods, explain its limitations and outline our neural CDE model and associated learning algorithm that is able to generate counterfactual trajectories in continuous-time. In section 3 we run two experiments: based on a donor pool of similar European countries, we construct two control units, one the for the reproduction-rate and another for daily deaths; these control units behave similarly to Sweden before it is put under lockdown for several weeks; then we fit two neural CDE models to learn the control dynamics on the pre-lockdown period between the control units and Sweden; the two outcomes (see Figure 2) in the post-lockdown phase approximate the counterfactual lockdown scenarios for Sweden. Finally, we benchmark our model by feeding the estimated counterfactual trajectory for the reproduction rate into a state-of-the-art epidemiology software `epidemia` [1], developed by researchers at Imperial College London and detailed in the Appendix, to produce a biologically inspired trajectory for daily deaths. We find that the trajectory produced by `epidemia` closely matches the daily-deaths-trajectory produced by our second neural CDE model, showing how the neural CDE is able to generate realistic counterfactual scenarios that recover biologically inspired modelling.

2 Generating counterfactual trajectories with neural CDEs

2.1 Synthetic controls

The *synthetic control method* [2, 3] is a recent innovation in causal inference to solve this counterfactual problem. It uses a weighted combination of control units to inform the behaviour before the intervention takes place, and then uses the learnt relationship between the control units and the unit of interest to drive the behaviour of the latter forward in time after the intervention. Synthetic controls have become popular in the fields of policy analysis due to their simplicity and transparency. They have been used in many real-world applications such as quantifying the effect of taxes on the consumption of cigarettes [4] or in biomedical sciences to estimate the effect of public health interventions [5, 6]. In this project a unit will be a country.

Consider n units $Y_1, ..., Y_n$ where each unit Y_i is represented by a *multivariate time series*

$$Y_i = ((t_1, Y_{i,t_1}), (t_2, Y_{i,t_2}), ..., (t_m, Y_{i,t_n})) \qquad (1)$$

that is a collection of points $Y_{i,t_j} \in \mathbb{R}^d$ with corresponding time-stamps $t_i \in \mathbb{R}$ such that $t_0 < ... < t_n$. Suppose one unit $(i = 1)$ does NOT undergo an *intervention* (in our case, Sweden does not enter a lockdown) at time $T \in (t_0, t_m)$, while all other units $Y_2, ..., Y_n$ do undergo the intervention and act as control group.

Let $Y^1_{i,t}$ be the *potential outcome* for unit i at time t in the scenario where the intervention did occur, and analogously let $Y^0_{i,t}$ be the corresponding potential outcome assuming the intervention did not occur. The observed outcome for unit i at time t is denoted by $Y_{i,t}$ and satisfies the following relation

$$Y_{1,t} = \begin{cases} Y^0_{1,t} & \text{if } t < T \\ (Y^1_{1,t} - Y^0_{1,t}) & \text{if } t \geq T \end{cases} \tag{2}$$

$$Y_{i,t} = \begin{cases} Y^0_{i,t} & \text{if } t < T \\ Y^1_{i,t} & \text{if } t \geq T \end{cases}, \quad i = 2, ..., n \tag{3}$$

The quantity $c_1(t) := Y^1_1(t) - Y^0_1(t)$ is called the *causal effect of the intervention* on unit 1 at time t. Synthetic control methods make the simple assumption that there exist weights $w_2, ..., w_n$ such that for any $t \in [t_0, T]$ before intervention the trajectory $Y_{1,t}$ can be written as a weighted average of observed control outcomes

$$Y_{1,t} = \sum_{i=2}^{n} w_i Y_{i,t} \tag{4}$$

and then, for every $t > T$ after the intervention, uses this approximation to estimate the fictitious trajectory. The time series defined by $\sum_{i=2}^{n} w_i Y_{i,t}$ is called *synthetic control*.

The classical synthetic control models assumes the time series $Y_1, ..., Y_n$ are aligned and regularly sampled from an underlying continuous process. In this way, synthetic controls can be interpreted as a discrete approximation of the latent counterfactual path of the treated country. However, this setup breaks down if the control time series are misaligned or irregularly sampled, which is often the case in real-world applications. Furthermore, for complex dynamics, the linear model of equation (4) is too restrictive so not plausible.

2.2 Neural CDEs for counterfactual trajectories

To remedy these two issues, each control unit is interpolated (using cubic splines) into a *continuous path* $Y_i : [t_0, t_m] \to \mathbb{R}^d$. We are interested in estimating the counterfactual path (or trajectory) of the first unit $Y^1_{1,t}$ after intervention at time $t > T$. Again, $Y^1_{1,t}$ represents the fictitious scenario that Sweden enters a lockdown at the same time T as all other countries $Y_2, ..., Y_n$ used as control units. Let $\boldsymbol{Y} = (Y_2, ..., Y_n) : [t_0, t_n] \to \mathbb{R}^{d \times (n-1)}$ be the $(n-1) \times d$ dimensional path (i.e. a trajectory with $(n-1) \times d$ channels) that includes all $n-1$ control paths. We model the dynamics of the synthetic controls using the mathematical formalism of a *controlled differential equation* (CDE) [7]

$$dY_{1,t} = f(Y_{1,t})d(Y_{2,t}, ..., Y_{n,t}), \quad Y_{1,t_0} = y_{1,t_0} \tag{5}$$

where y_{1,t_0} is an initial value of the trajectory of unit 1, and f is a *latent vector field* that is learnt from data before the intervention. Equation (5) relates an "infinitesimal displacement" in Y_1 to an infinitesimal displacement in all other control units $Y_2, ..., Y_n$ (for more mathematical details see for example [7]). Of one integrates both sides of equation (5), one arrives to the following more explicit expression

$$Y_{1,t} = y_{1,t_0} + \int_{t_0}^{t} f(Y_{1,t}) d\boldsymbol{Y}_t \tag{6}$$

where $\boldsymbol{Y}_t = (Y_{2,t}, ..., Y_{n,t})$ is the *augmented trajectory* obtained by stacking all channels of the trajectories Y_i and representing the global temporal behaviour of all control units $Y_2, ..., Y_n$. The integral in equation (6) is a classical Riemann integral (learnt in high school) and "$f(Y_{1,s})d\boldsymbol{Y}_s$" is understood as matrix-vector multiplication. Following [8, 9], we will parametrize the vector field f by a neural network (as we shall explain in the next section). This model is capable of processing irregularly aligned data and may be evaluated at any point in time. A representation of our model is depicted in Figure 1. Furthermore, this model may be trained efficiently with existing *adjoint backpropagation* algorithms from [8] and is easy to implement. We will explicitly outline the training procedure in the sequel. We note that equation (6) may be interpreted as a non-linear continuous-time extension to the linear discrete-time synthetic $\sum_{i=2}^{n} w_i Y_{i,t}$ of equation (4).

Model *Neural CDEs* are the continuous-time analogous of recurrent neural networks (RNNs) and are defined in [8, 9] so that the latent vector field f in equation (6) is parametrized by a neural network f_θ. Neural CDEs generalize the popular *Neural ODEs* formulation of [10], allowing for time-varying initial conditions [8, 9].

Let $\xi_\theta : \mathbb{R}^d \to \mathbb{R}^\ell$ be a neural network that embeds the observations into a ℓ-dimensional latent state, i.e. $z_t := \xi_\theta(Y_{1,t})$. Let $f_\theta : \mathbb{R}^\ell \to \mathbb{R}^{(n-1)\times\ell}$ be a neural network parametrizing the latent vector field f of equation

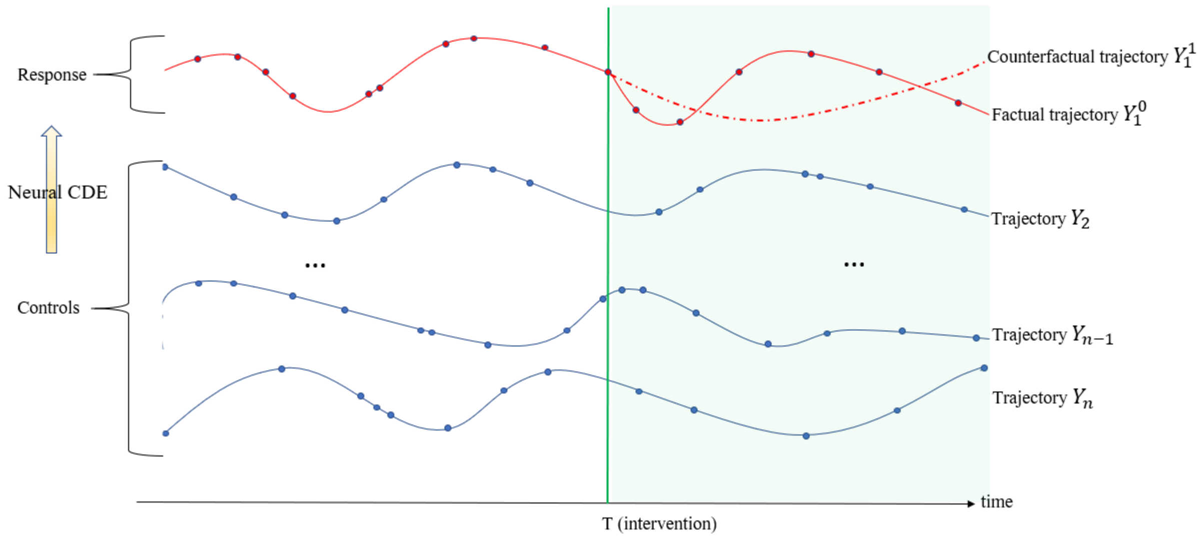

Fig. 1. Continuous-time synthetic control $y_{1,t_0} + \int_{t_0}^{t} f(Y_{1,t})d\boldsymbol{Y}_t$ depends continuously on control paths over time and naturally accommodates for misaligned observations and complex dynamics. The latent vector field f is learned before the intervention T and used after intervention to construct the counterfactual trajectory. The dots represent observations available in the data.

(6), and let $\phi_\theta : \mathbb{R}^\ell \to \mathbb{R}^d$ be a neural network that defines the observation mechanism projecting the latent space into the observation space to recover an estimate of the counterfactual path $\hat{y}_{1,t} := \phi_\theta(z_t)$. In summary

$$z_{t_0} = \xi_\theta(y_{1,t_0}), \quad z_t = z_{t_0} + \int_{t_0}^{t} f_\theta(z_s)d\boldsymbol{Y}_s, \quad Y_{1,t} = \phi_\theta(z_t), \quad \text{for } t \in (t_0, t_m] \tag{7}$$

That is – just like an RNN – we have an evolving hidden state z, which is fed into ϕ_θ a to produce an output Y_1. This formulation is a universal approximator [8, Appendix B].

Learning algorithm For each estimate of the neural networks $\xi_\theta, f_\theta, \phi_\theta$, the latent trajectory is obtained by evaluating the model of equation (7) forward in time via a call to any numerical ODE solver

$$\hat{z}_{t_1}, ..., \hat{z}_{t_k} = \text{ODESolve}(f_\theta, \hat{z}_{t_0}, (t_1, \boldsymbol{Y}_{t_1}), ..., (t_k, \boldsymbol{Y}_{t_k})) \tag{8}$$

The goodness of the nets $\xi_\theta, f_\theta, \phi_\theta$ is then quantified via a loss function $\mathcal{L} : \mathbb{R}^{d\times k} \times \mathbb{R}^{d\times k} \to \mathbb{R}$ that compares the reconstructed trajectory $\hat{y}_{1,t} = \phi_\theta(z_t)$ with the observed trajectory $Y_{1,t}$ for $t < T$ before the intervention. An efficient gradient descent algorithm is then used to update all parameters as in [8–10]. We note that the backpropagation is carried out efficiently without the need to backpropagate through the ODE solver (via the *adjoint method* presented in [10]). This method is straightforward to implement using pre-existing tools, as demonstrated in the code implementation attached to this manuscript. We made use of the python library `torchdiffeq` [10].

3 Experiments

Rationale for Control Countries Our first step is to choose a group of suitable control countries from which we can infer Sweden's hypothetical trajectory. Since we will be modelling death data, it is important that we choose countries with similar life expectancy and GDP per capita, which are important metrics in evaluating the social and economic infrastructure within a country. Furthermore, we need to pick countries that imposed a lockdown during the same time period. Luckily for us, in March 2020 many European countries imposed a lockdown within the same two-week period. Therefore, we are able to create a list of 11 countries which we will use as our control group: [Austria, Belgium, Denmark, France, Germany, Greece, Italy, Holland, Norway, Portugal, Spain]. Within our control group, the lengths of lockdowns differed, so we take an average lockdown length of 8 weeks.

Data Gathering We will primarily use data from the owid dataset; from which we take reproduction rate and death data. We normalise death data using the population data to achieve deaths per capita. Considerations were made into the use of testing data. However, since we are only looking at death data, there is no need to consider testing as it is safe to assume that all who died from COVID were known to have had it. Furthermore, discussions were carried out into whether we would want to look at much more specific measures, such as school closures or closures in the hospitality sector. However, most of these measures were usually put into place at the same time, in some form of large-scale intervention (such as a lockdown). Therefore, we decided that the most conclusive intervention we could model was a lockdown.

Data Pre-processing We use the neural CDE model (7) to estimate the counterfactual trajectory for 1) reproduction rate and 2) death rate in Sweden using respectively the same variables for the other control countries. For the reproduction rate we took the raw time series whilst for the death rate we used the raw number of deaths divided by the population of the respective country and smoothed the resulting series using an exponential moving average*. We followed [8] in approximating the underlying paths using cubic spline interpolations with knots at the observation times.

Architecture The integrated f_θ in the neural CDE (7) was taken to be a feed-forward neural network with a two hidden layers of size 10 and *elu* activation functions after each layer except from the output layer. The dimensionality of the hidden state ℓ governing the hidden dynamics z was taken to be 5 for both experiments. The activation function was chosen to be the *elu* function, although the *relu* performed similarly but was less stable in optimization with a larger variance across different runs of the algorithm. We did not explicitly tune hyperparameters (hidden layers, activation function, etc.) for performance. Further tuning could be done by cross-validation on the observed pre-lockdown trajectories if enough observations were given.

Optimization In both experiments we used the *Adam* optimizer as implemented in PyTorch. The ODE solver used to extrapolate the hidden state was taken to be the *fourth-order Runge-Kutta* with $3/8$ rule solver, as implemented by passing `method=rk4` to the `odeint adjoint` function of the `torchdiffeq` [8, 10] used also for adjoint back-propagation training.

Findings Our counterfactual neural CDE model suggests that a lockdown would have made the decrease in reproduction rate from linear to slightly sub-linear, as it can be observed in Figure 2, mimicking closely the behaviour of other countries. We find that a lockdown would have reduced the number of COVID-19 deaths in Sweden by approximately one fourth. However, we emphasise the fact that this is a qualitative statement. A more careful calibration and extra time is needed in order to get a more accurate quantitative estimate of the counterfactual trajectory. As it can be observed, our neural CDE model converges to a very good solution approximating almost exactly the pre-lockdown trajectory of Sweden using data from other countries.

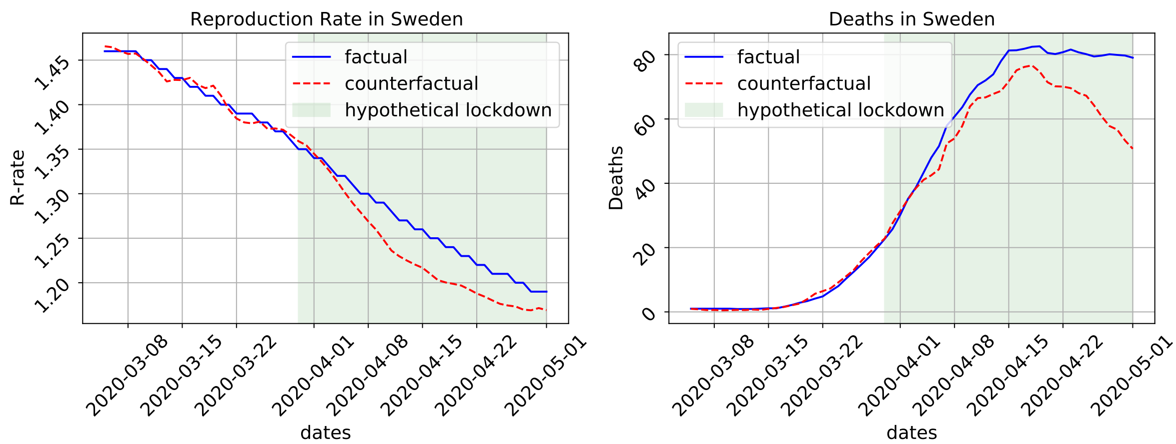

Fig. 2. Estimation of the counterfactual trajectories for reproduction rate (left) and deaths (right) in Sweden after an hypothetical lockdown.

A demo of the procedure can be found and is reproducible at https://colab.research.google.com/drive/1cnpNMLJgM8sAv2OuW24C8uP60U2TMkMN?usp=sharing. We also provide our full implementation in a zip file and attach an html version of the notebook to this manuscript.

Comparison with epidemia We finally benchmark the neural CDE model by feeding the estimated counterfactual reproduction-rate-trajectory into `epidemia` (see Appendix for technical details). The software `epidemia` produces a biologically inspired trajectory for daily deaths from the reproduction rate. As it can be seen from Figure 3, the trajectory produced by `epidemia` closely matches the daily-deaths-trajectory produced

*https://pandas.pydata.org/pandas-docs/stable/reference/api/pandas.DataFrame.ewm.html

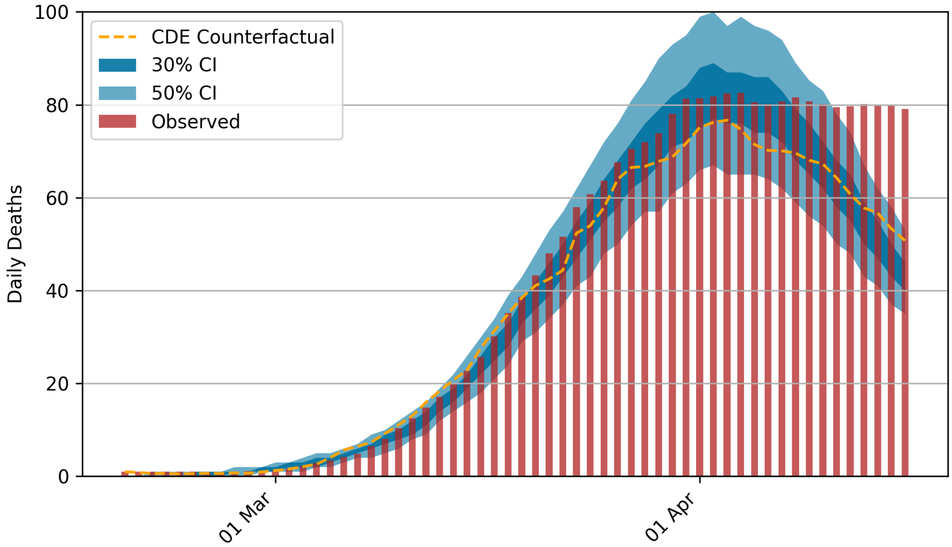

Fig. 3. Comparison of estimated counterfactual trajectories for deaths in Sweden after a hypothetical lockdown. The CDE trajectory falls within 50% confidence interval of the output trajectory from *epidemia*

by our second neural CDE model, showing how the neural CDE is able to generate realistic counterfactual scenarios that recover the output modelling from `epidemia` inspired from biological facts.

4 Conclusion

In this project we proposed to model counterfactual trajectories in continuous-time via a neural CDE model. We used neural CDEs to quantify the extent to which a hypothetical lockdown would have limited the spread of COVID-19 infections. We focus on the example of Sweden - a European country that did not implement a lockdown during the first wave of COVID-19 - and, based on a donor pool of similar European countries, we constructed counterfactual lockdown trajectories for both the reproduction-rate and daily deaths curves (see Figure 2). Finally, to benchmark our model we fed the estimated counterfactual trajectory for the reproduction rate into a state-of-the-art epidemiology software `epidemia` [1], to produce a biologically inspired trajectory for daily deaths. We find that the trajectory produced by `epidemia` closely matches the daily-deaths-trajectory produced by our second neural CDE model, showing how the neural CDE is able to generate realistic counterfactual scenarios that recover biologically inspired modelling.

Limitations and future work The CDE model only produces one single counterfactual trajectory and does not produce confidence intervals on the predictions. A more useful model would be able to sample from the underlying counterfactual distribution and produce accurate confidence intervals. As mentioned above, the results are purely qualitative. For more precise quantitative statements deeper calibration and analysis of the model is required. Furthermore, the CDE model does not use any additional features (mobility data, biological and environmental factors) in order to estimate the counterfactual trajectories. However, this could potentially be easily incorporated in the proposed formalism by augmenting the control $\boldsymbol{Y}$ in equation (7) by these additional time-dependent features. The CDE model also has advantages over mechanistic models such as `epidemia` in that it doesn't require assumptions about epidemiological parameters. This means we can more easily model other trajectories such as hospitalisation cases without prior estimation of distribution of times from infection-to-observation.

5 References

1. Bhatt, J. A. S., Gandy, A., Mishra, S., Unwin, J., Flaxman, S. & Samir. *epidemia: Modeling of Epidemics using Hierarchical Bayesian Models* 2020.
2. Abadie, A., Diamond, A. & Hainmueller, J. Synthetic control methods for comparative case studies: Estimating the effect of California's tobacco control program. *Journal of the American statistical Association* **105,** 493–505 (2010).

3. Abadie, A. & Gardeazabal, J. The economic costs of conflict: A case study of the Basque Country. *American economic review* **93,** 113–132 (2003).
4. Abadie, A. Using synthetic controls: Feasibility, data requirements, and methodological aspects. *Journal of Economic Literature* (2019).
5. Bouttell, J., Craig, P., Lewsey, J., Robinson, M. & Popham, F. Synthetic control methodology as a tool for evaluating population-level health interventions. *J Epidemiol Community Health* **72,** 673–678 (2018).
6. Pieters, H., Curzi, D., Olper, A. & Swinnen, J. Effect of democratic reforms on child mortality: a synthetic control analysis. *The Lancet Global Health* **4,** e627–e632 (2016).
7. Lyons, T. J., Caruana, M. & Lévy, T. *Differential equations driven by rough paths* (Springer, 2007).
8. Kidger, P., Morrill, J., Foster, J. & Lyons, T. Neural controlled differential equations for irregular time series. *arXiv preprint arXiv:2005.08926* (2020).
9. Morrill, J., Salvi, C., Kidger, P., Foster, J. & Lyons, T. Neural RDEs for long time series via the log-ode method. *arXiv preprint arXiv:2009.08295* (2020).
10. Chen, R. T., Rubanova, Y., Bettencourt, J. & Duvenaud, D. Neural ordinary differential equations. *arXiv preprint arXiv:1806.07366* (2018).

A Epidemia

The *epidemia* package [1] provides a mechanistic model to estimate the number of infections over time within a population. It estimates the reproduction-rate by linking observations such as deaths backwards to the number of infections weeks prior, using an appropriate distribution of times from infection to death, along with a previously estimated infection-fatality-ratio. The details of the infection process can be found on the *epidemia* website.

We model the expected values y_t of the daily deaths, accounting for the uncertainty associated with data. The expected number of daily deaths are modelled as $y_t = \mathbb{E}[Y_t]$. The actual number of weekly deaths Y_t are assumed to follow a negative binomial distribution with mean y_t and variance $y_t + \frac{y_t^2}{\phi}$, where ϕ follows a positive half normal distribution, i.e.

$$
\begin{aligned}
&Y_t \sim \text{Negative Binomial}\left(y_t,\ d_{t,m} + \frac{y_t^2}{\phi}\right),\\
&\phi \sim \mathcal{N}(10,\, 2).
\end{aligned}
$$

We use the negative binomial distribution to allow for observed over-dispersion of counts around the mean value. In the case of the negative binomial distributions, the variance of $Y_{t,m}^{(1)}$ around the expected number of deaths $y_t^{(1)}$ scales linearly with ${y_{t,m}^{(1)}}^2$. These expected daily deaths are linked mechanistically to the reproduction-rate, as detailed on the *epidemia* website.

We first calibrate the model by estimating the reproduction-rate as a weekly random walk. This adjusts the epidemiological parameters associated with Sweden needed for the infection process, such as the infection-fatality-rate. We use our estimates for the reproduction-rate from the CDE model to simulate counterfactual scenarios for the infection process, which is directly linked to the trajectory for the death rate. Comparison of this counterfactual death rate from *epidemia* with the CDE counterfactual death rate demonstrate that the CDE counterfactual death rate falls within the 50% confidence interval given by *epidemia*.