CS4495/6495
Introduction to Computer Vision

2A-L2 *Filtering*

Matlab: Row, column vs. x, y

- Rows and Columns are not x,y

- When we say I(x,y), we usually mean the x and y that you learned in middle school. X goes to the right; Y goes up. In this case I(0,0) would be at the bottom left hand corner. Why we say “usually” I’ll talk about in a minute.

- When we say F(i,j) we usually mean ***row i and column j.*** Where rows go down and columns go over. So the origin would be at the top left of an image. So F(5,2) would be the pixel 5 over and down 2.

Matlab: Indexing from 1!

- Finally one more confusion to remind you of: in computer science our indexing typically starts with 0. This makes indexing much simpler. And since the target audience for this class are MSCS students I won't explain that.

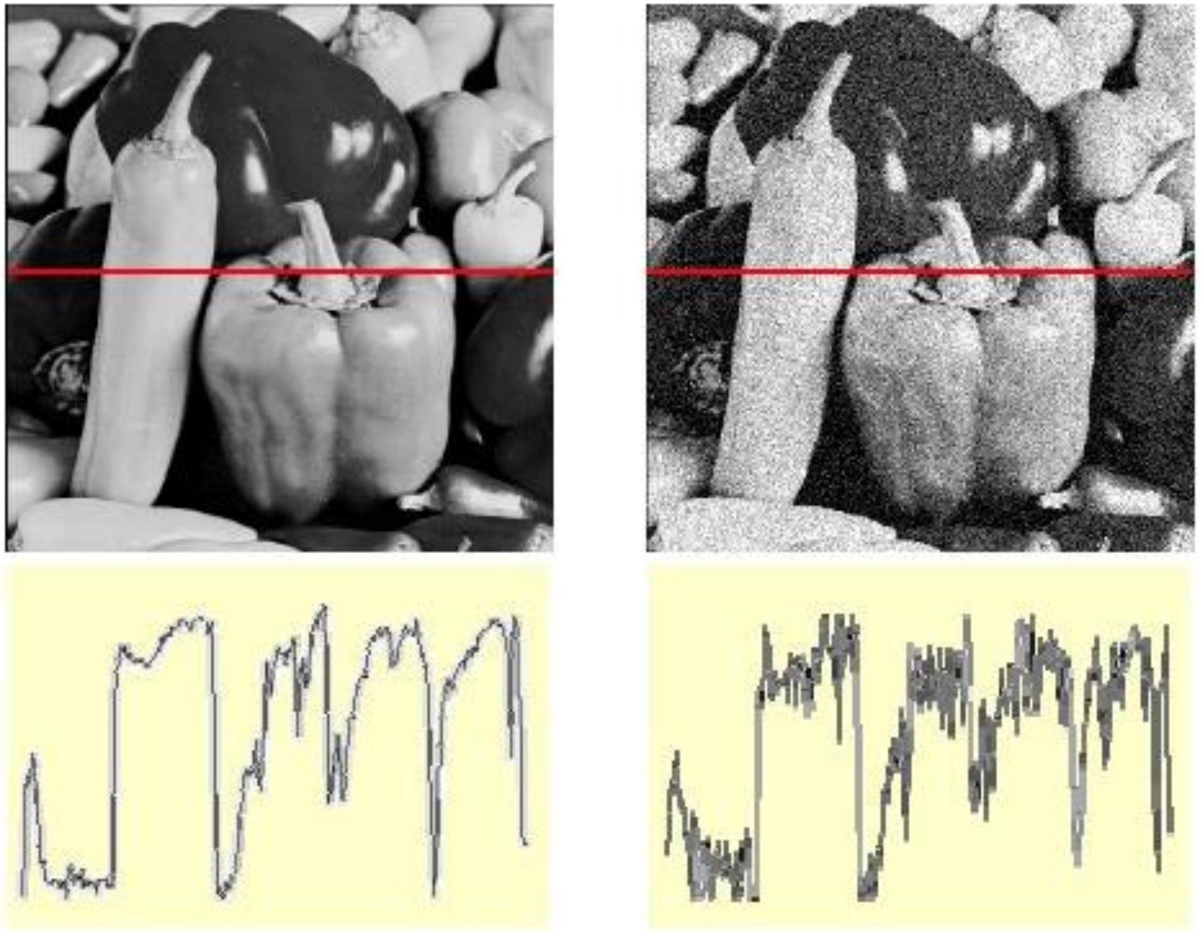
- But in Matlab the indexing starts at 1. So the origin of the image is F(1,1) – first row and first column. And when you display an array as an image it is at the top left of an array.

- This will no doubt inflict pain and suffering in a variety of your programs. Also, which way is "positive" y will also be confusing. In the I(x,y) notation positive Y usually means up, and in the F(i,j) – where i is the row value or the y value – positive Y goes down. Again, this will likely break your code somewhere during the course. So if your code is ding strange things relative to up and down, check your indexing.

Gaussian noise

```
noise =
    randn(size(im)).*sigma;
output = im + noise;
```

Fig: M. Hebert



$$f(x,y) = \overbrace{\tilde{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

Gaussian i.i.d. (“white”) noise:
$\eta(x,y) \sim \mathcal{N}(\mu,\sigma)$

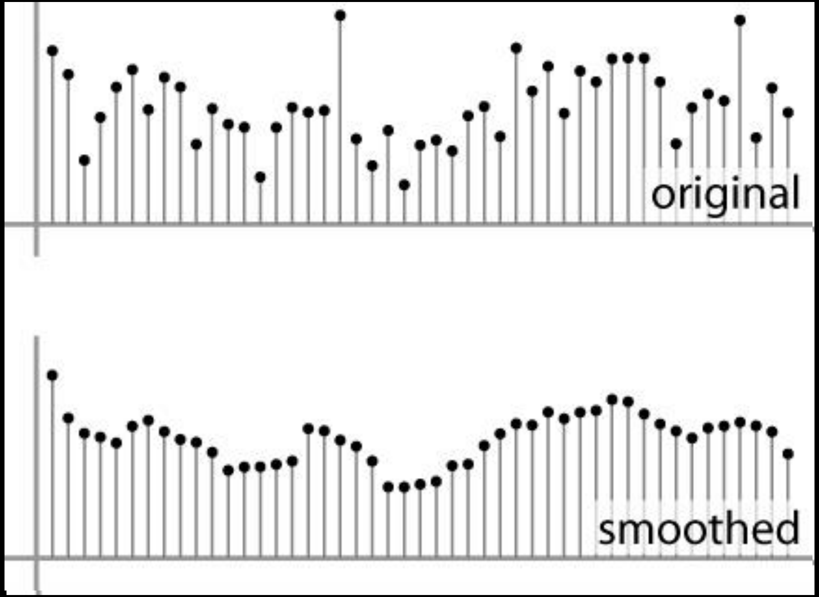
How do we remove noise?

Now I’m sure you all have a suggestion. The typical one is “Lets replace each pixel with the average of the values around it – ‘in the neighborhood’”.

Illustrating that in 1D it would look something like this:

First attempt at a solution – 1D

Replace each pixel with an average of all the values in its neighborhood – a *moving average:*



Source: S. Marschner

Thoughts

- OK, seems reasonable.   But the important question to ask is why was that an intuitive solution for many of you? What **assumptions** were you making about the image and the noise?

- A little thought and you'll realize that we have to make two key assumptions for the averaging method to make sense

Averaging assumptions

1. The “true” value of pixels are similar to the true value of pixels nearby.

2. The noise added to each pixel is done independently.

Quiz

If noise is just a function added to an image, we could remover the noise by subtracting the noise again – that is the operation is reversible:

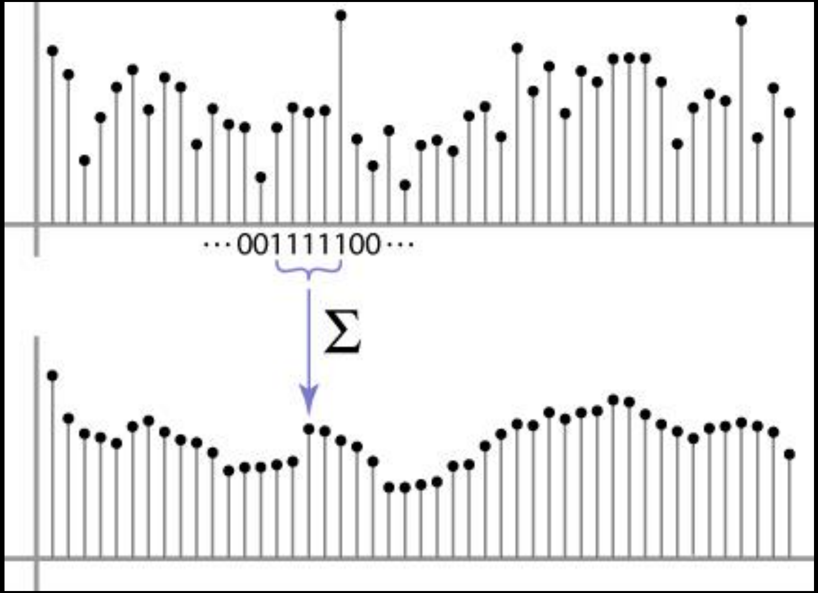
a) True

b) True but we don't know the noise function so we can't actually do the subtraction.

c) False. Additive noise destroys information in the image and there is no way to recover it.

Weighted Moving Average

- Can add weights to our moving average
- *Weights* [1, 1, 1, 1, 1] / 5



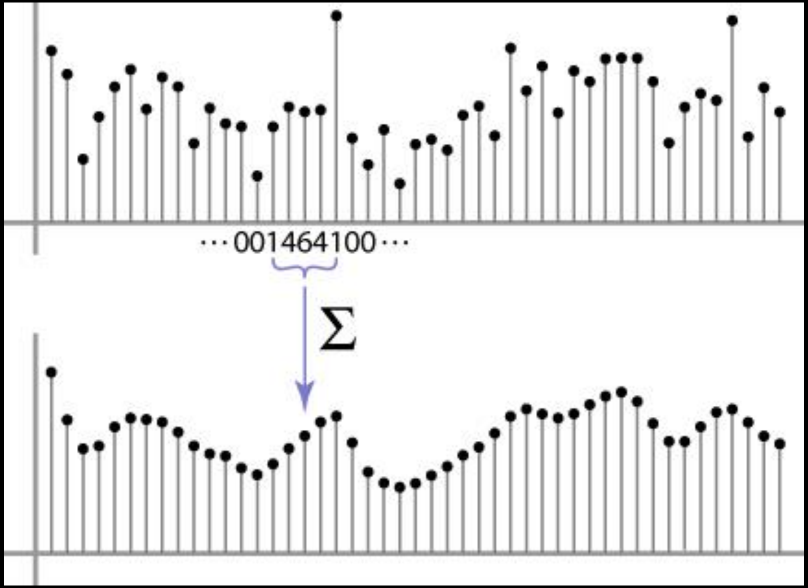
Source: S. Marschner

Weighted Moving Average

- Let’s examine those assumptions a bit more, especially the one about “smoothness”.
  - The basic idea is that nearby pixels have similar true underlying values.
  - Which, under pretty reasonable assumptions, implies that the closer a pixel is to some reference pixel, the more similar it would be.
  - So the more it should contribute to an average.

Weighted Moving Average

- Non-uniform weights [1, 4, 6, 4, 1] / 16



Source: S. Marschner

Quiz

To do the moving average computation the number of weights should be

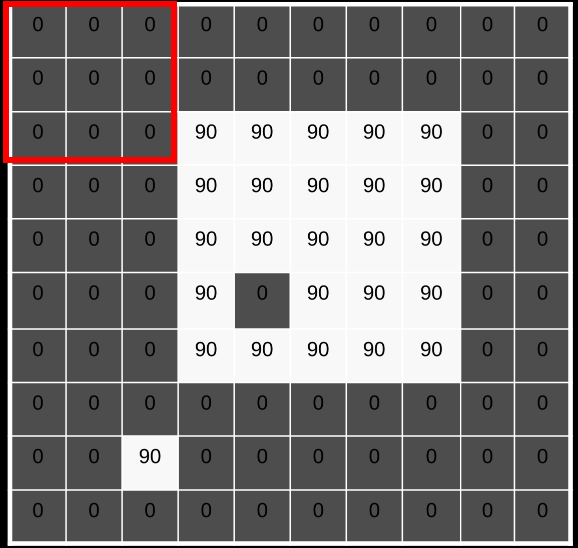
a) Odd – makes it easier to have a middle pixel

b) Even – that way the filter can be exactly symmetric around a pixel.

c) Either even or odd.

Moving Average In 2D

$F(x, y)$



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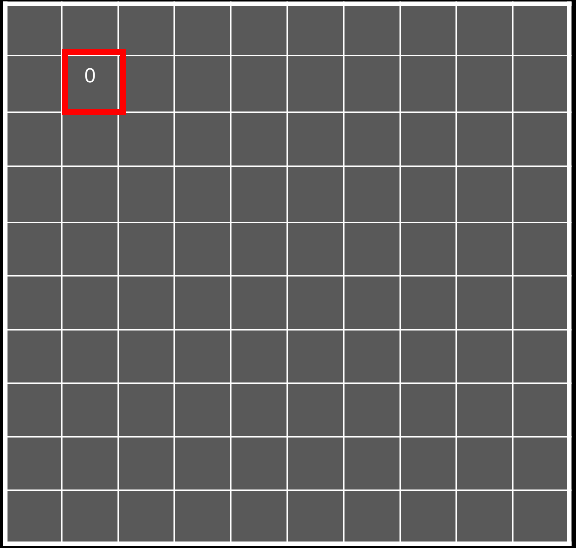
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

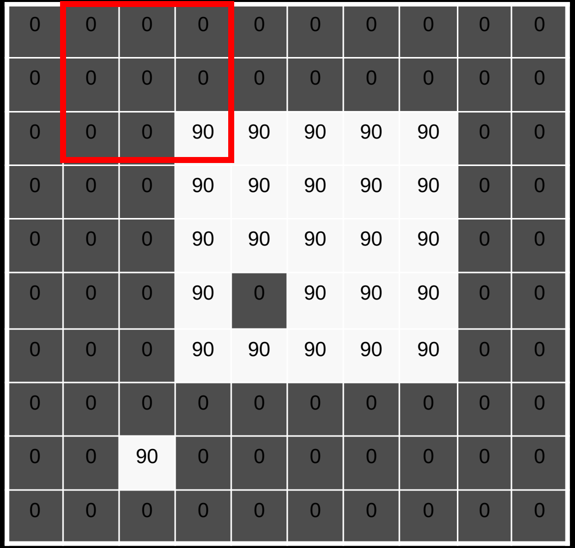


	0								

Source: S. Seitz

Moving Average In 2D

$F(x, y)$



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0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
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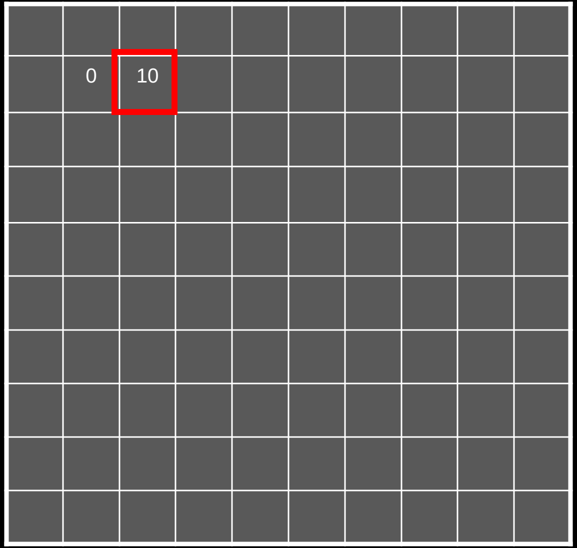
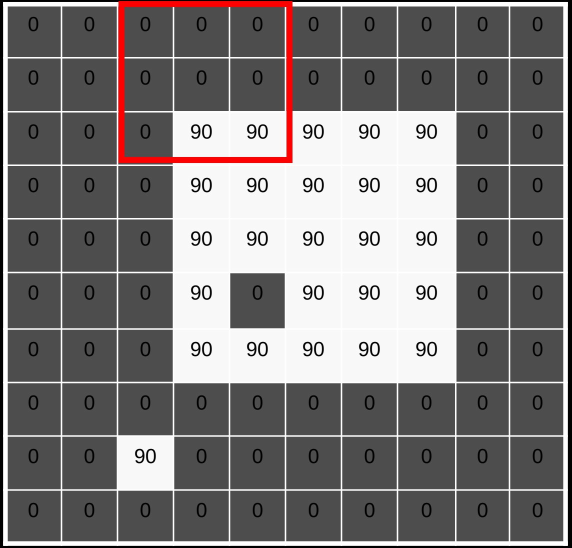
$G(x, y)$



	0	10							

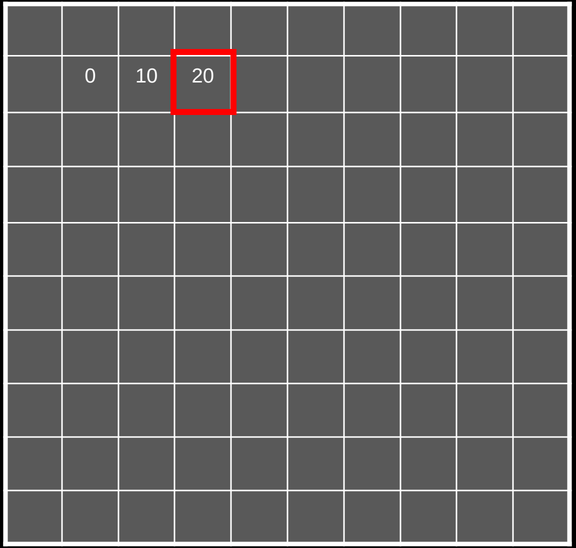
Moving Average In 2D

$F(x, y)$



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

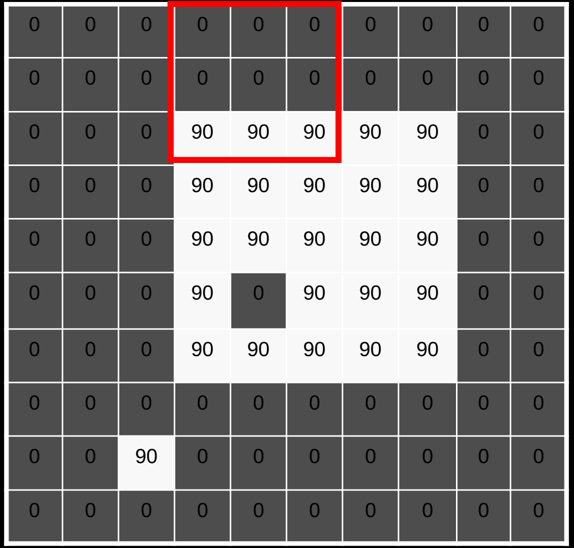
$G(x, y)$



	0	10	20						

Moving Average In 2D

$F(x, y)$



0	0	0	0	0	0	0	0	0	0
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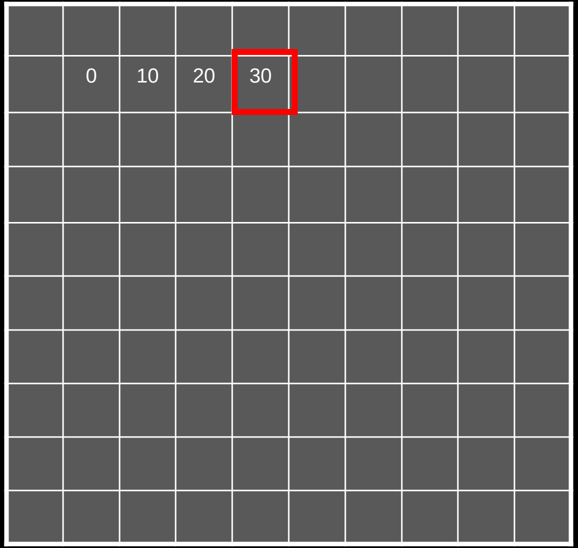
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

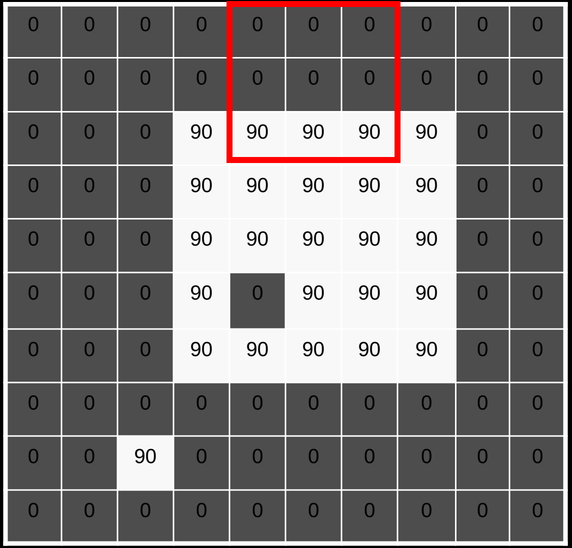
$G(x, y)$



	0	10	20	30					

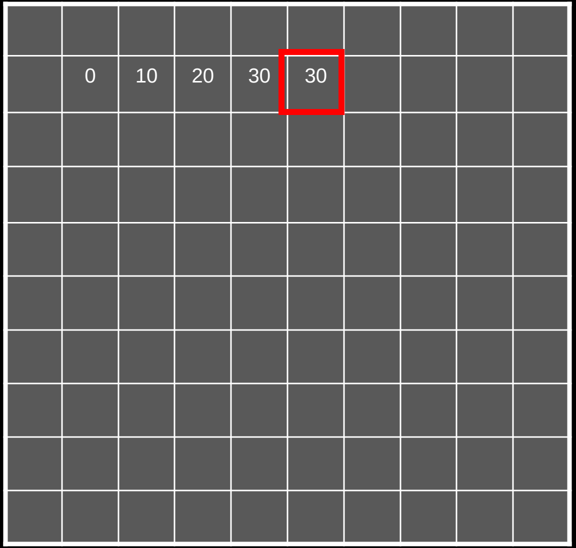
Moving Average In 2D

$F(x, y)$



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$



	0	10	20	30	30				

Moving Average In 2D

$F(x, y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(x, y)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Correlation filtering – uniform weights

Say the averaging window size is 2k+1 x 2k+1:

$$G[i,j] = \underbrace{\frac{1}{(2k+1)^2}}_{\textit{Uniform weight for each pixel}} \underbrace{\sum_{u=-k}^{k}\sum_{v=-k}^{k} F[i+u,j+v]}_{\textit{Loop over all pixels in neighborhood around image pixel F[i,j]}}$$

Correlation filtering – nonuniform weights

Now generalize to allow **different weights** depending on neighboring pixel's relative position:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} \underbrace{H[u,v]}_{\textit{Non-uniform weights}} F[i+u, j+v]$$

This is called ***cross-correlation***, denoted $G = H \otimes F$

Correlation filtering – nonuniform weights

Now generalize to allow **different weights** depending on neighboring pixel's relative position:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} \underbrace{H[u,v]}_{\textit{Non-uniform weights}} F[i+u, j+v]$$

The filter "***kernel***" or "**mask**" $H[u,v]$ is the matrix of weights in the linear combination.

Quiz

- The weights and shape of the filter are called the

a) Mask

b) Coefficients

c) Kernel

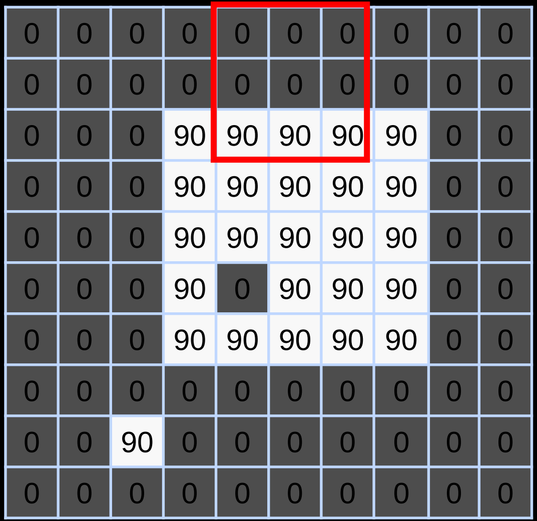
d) All of the above

What makes a good kernel or filter?

- Consider the example of a uniform (averaging) filter…

Averaging filter

$$F(x,y) \quad \otimes \quad H(u,v) = \quad G(x,y)$$



0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

"box filter"

?

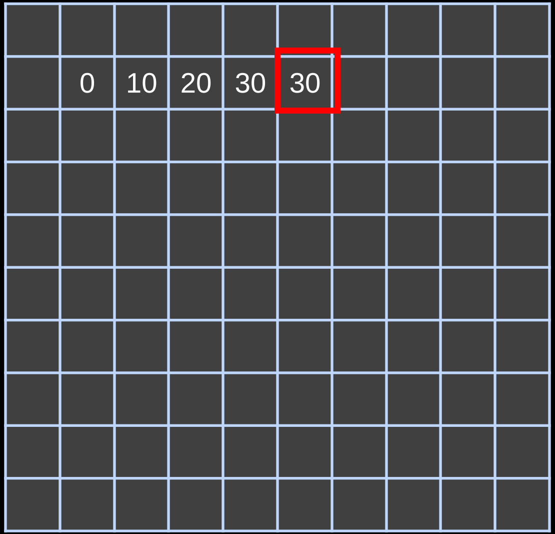
$$G = F \otimes H$$

Averaging filter

$$F(x,y) \quad \otimes \quad H(u,v) = \quad G(x,y)$$

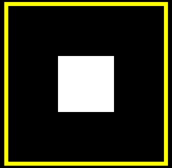
$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

"box filter"

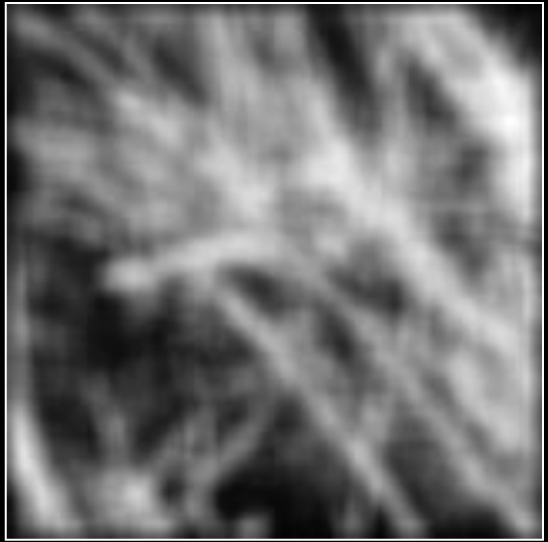


$$G = F \otimes H$$

Smoothing by box averaging



original

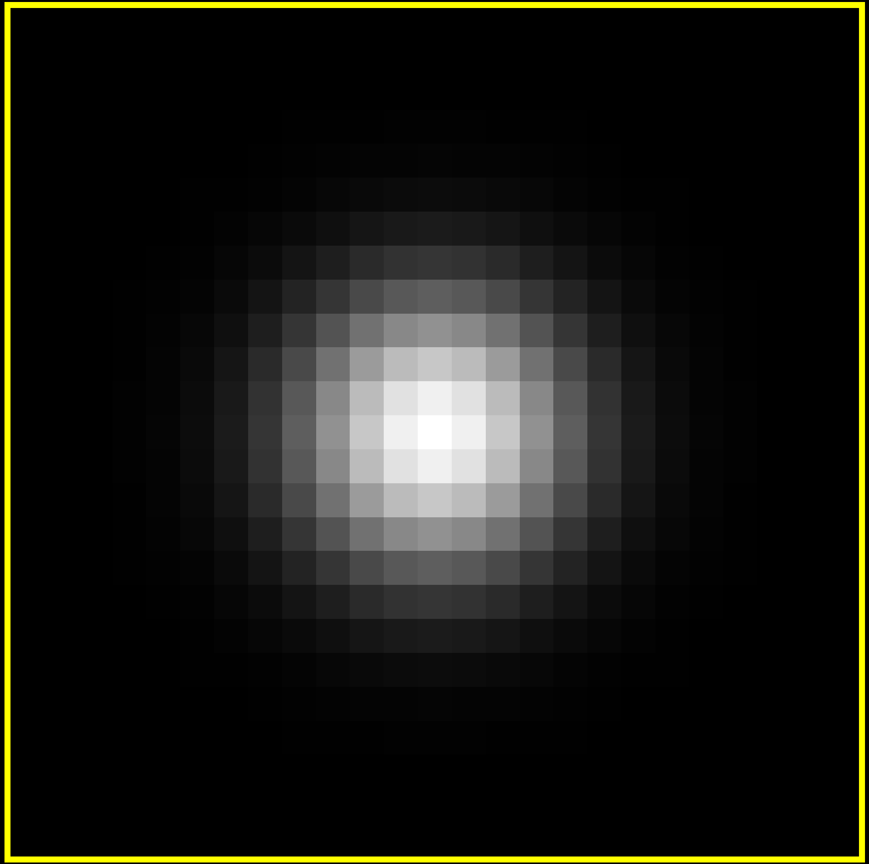


filtered

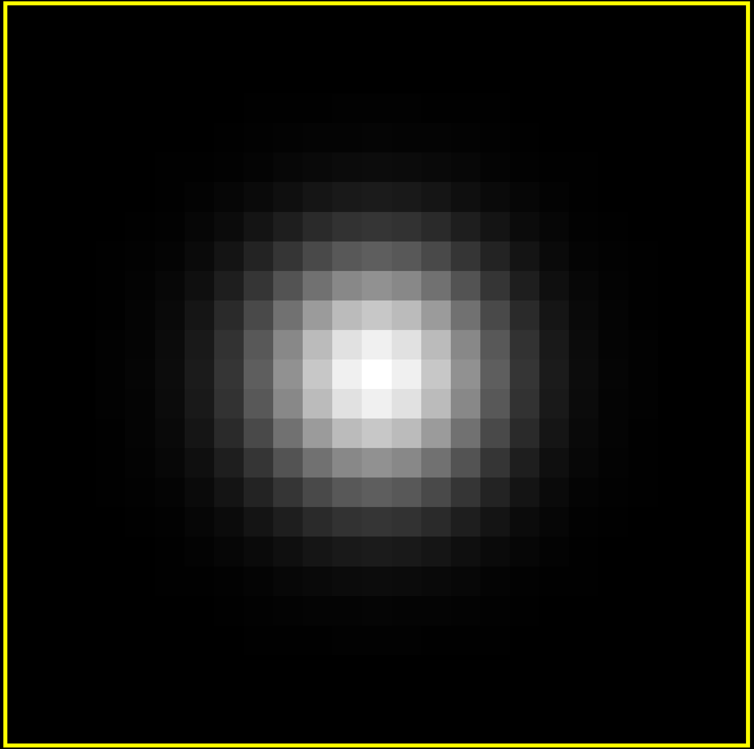
Averaging filter: Drawbacks

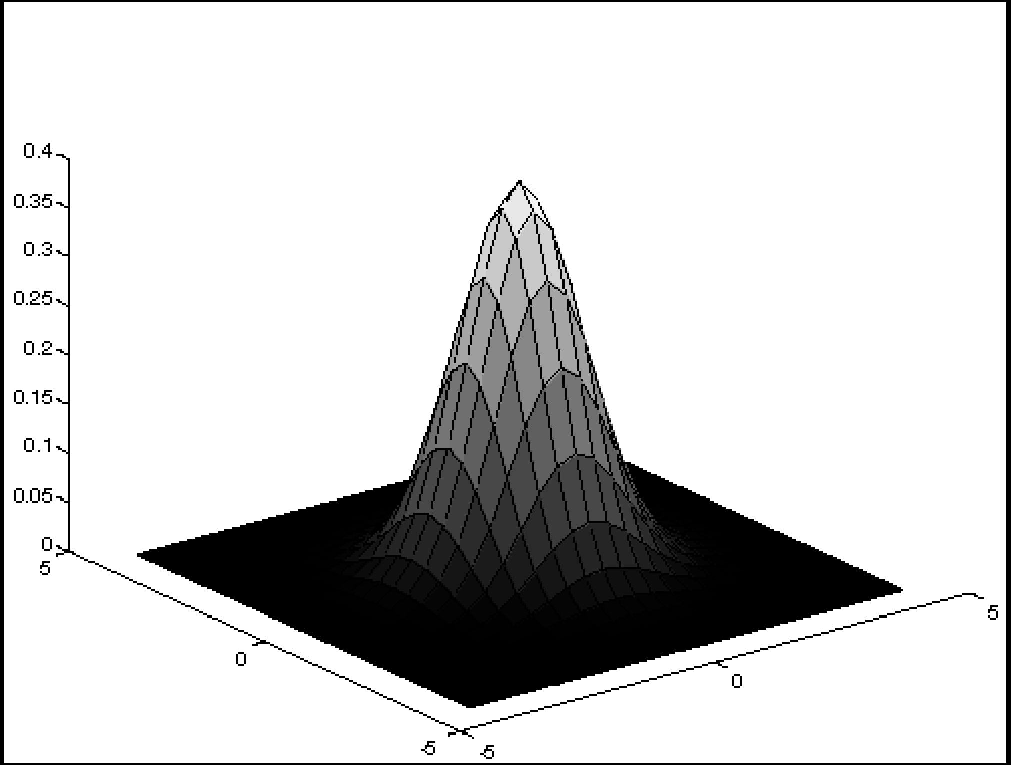
- The problem: squares aren’t smooth. And filtering an image with a filter that is not “smooth” seems wrong if we’re trying to “blur” the image. – (We’ll say more about what smooth really means in a few lectures.)

- So what’s the problem? To get a sense of what’s wrong, lets think about what a single spot of light viewed by an out of focus camera would look like.   The image would look something like this:



Blurry spot as a function





D. Forsyth

Quiz

To blur a single pixel into a “blurry” spot, we would need to need to filter the spot with a

a) 3x3 square of uniform weights
b) 11x11 square of uniform weights would be better since it’s bigger
c) Something that looks like a blurry spot – higher values in the middle, falling off to the edges.

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F(x, y)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F(x, y)$

$\frac{1}{16}$

1	2	1
2	4	2
1	2	1

$H(u, v)$

Gaussian filter

- Nearest neighboring pixels have the most influence

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

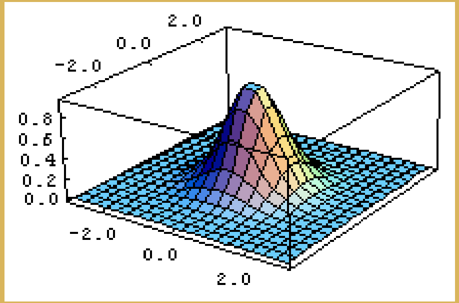
$F(x, y)$

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$H(u, v)$

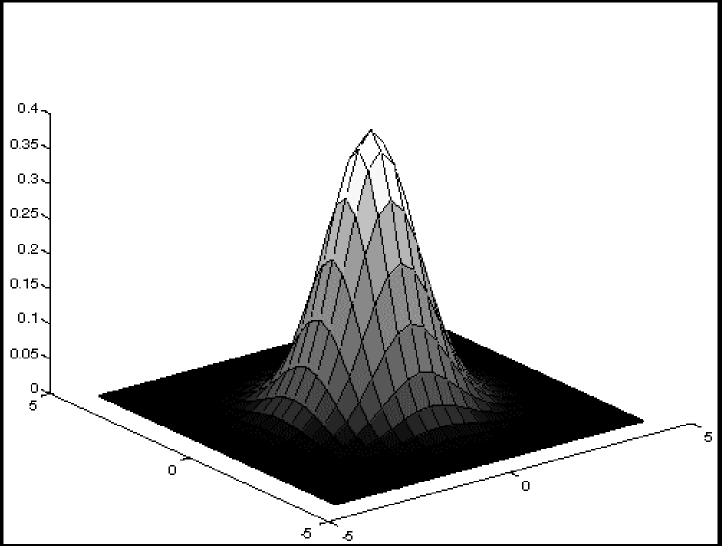
This kernel is an approximation of a Gaussian function:

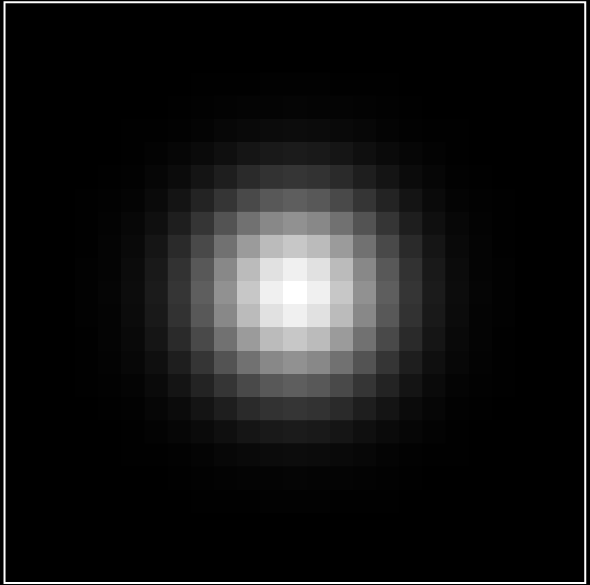
$$h(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



Source: S. Seitz

An Isotropic Gaussian





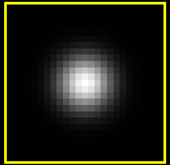
Smoothing kernel proportional to:

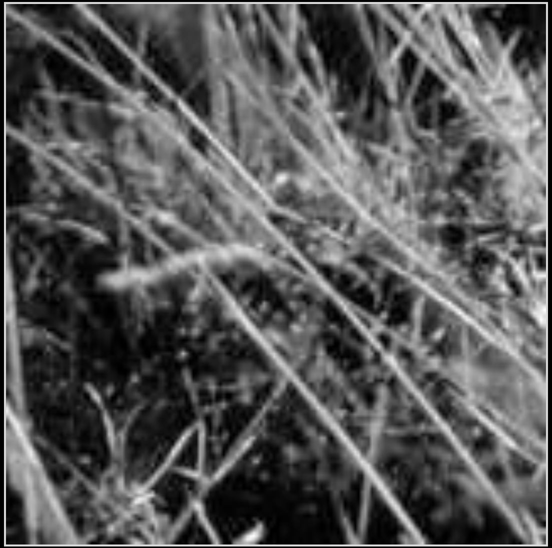
$$\exp\left(-\frac{(x^2 + y^2)}{2\sigma^2}\right)$$

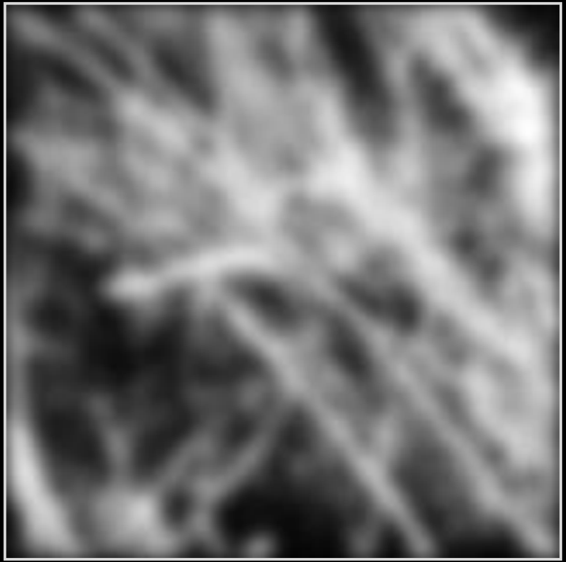
"circularly symmetric fuzzy blob"

D. Forsyth

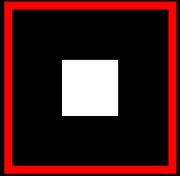
Smoothing with a Gaussian

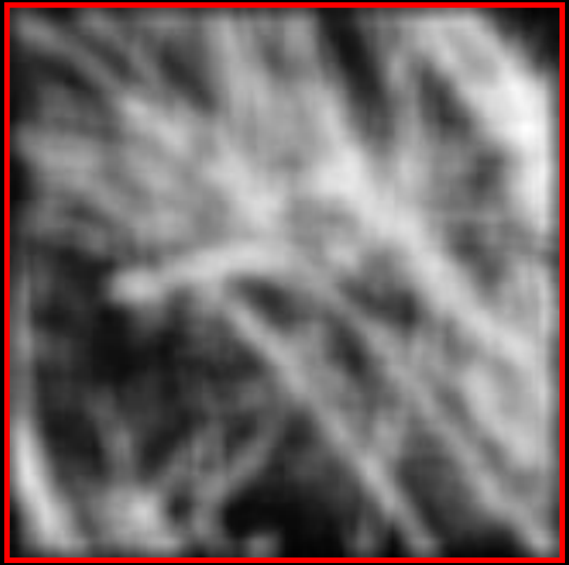




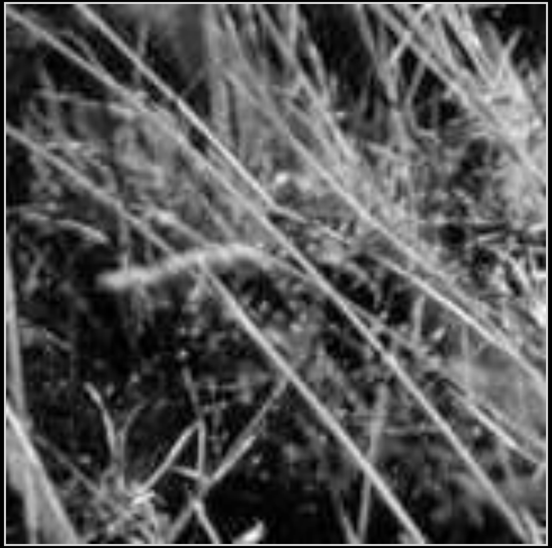
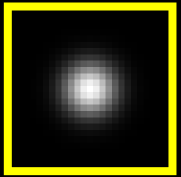
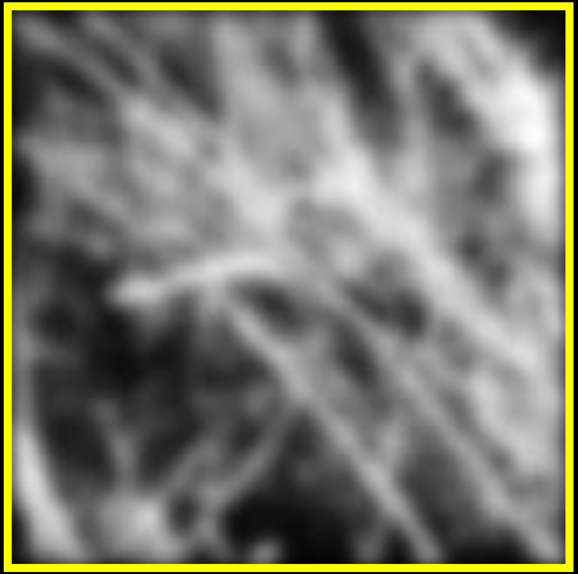


Smoothing with not a Gaussian





Smoothing with a Gaussian

Quiz

Gaussian filters are referred to as exponentials. The complete formula is:

a) is $$h(u,v) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

b) Or $$h(u,v) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

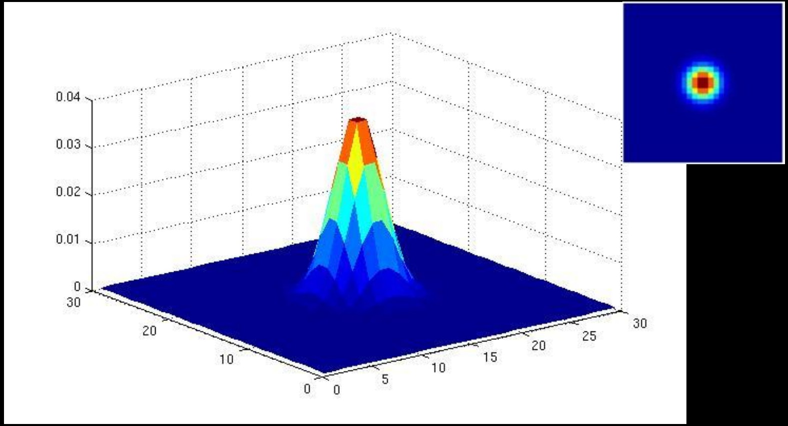
c) Or even: $$h(u,v) = \frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

Gaussian filter: Shape vs. size

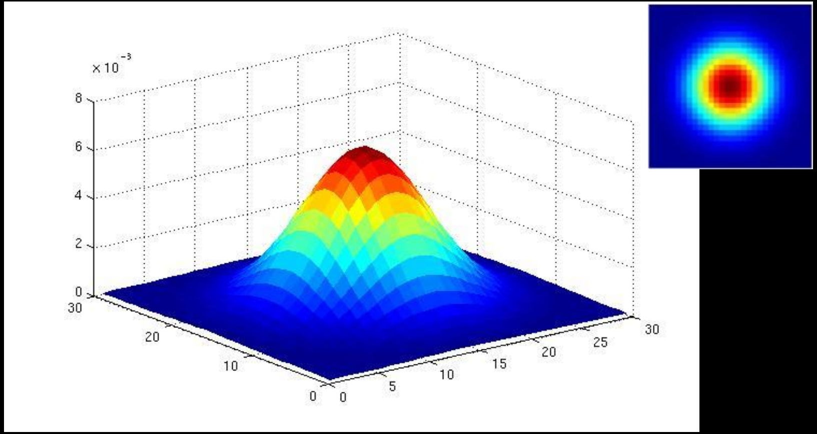
- The Gaussian we showed was isotropic – circular – so it had only one parameter (sigma).
- But that is much easier to say when we're doing continuous math than when working on a computer.
- On a computer, we need to talk about the *size* of the filter (3x3, 5x5, 11x11) and then the *shape* of the filter values (Gaussian distribution).

Gaussian filters

Variance (σ^2) or **standard deviation** (σ) – determines extent of smoothing



σ = 2 with 30 x 30 kernel

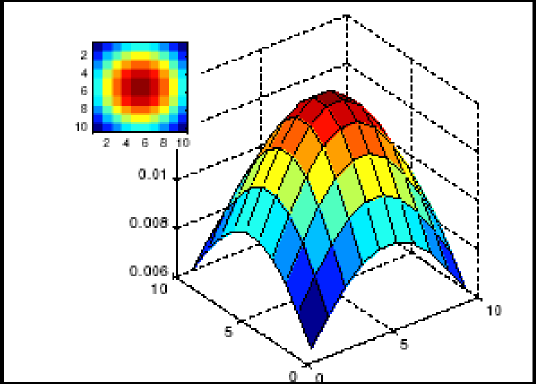


σ = 5 with 30 x 30 kernel

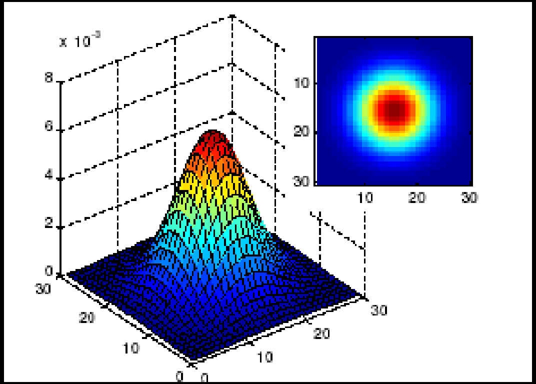
K. Grauman

Gaussian filters

Size of kernel or mask is *not* variance



σ = 5 with 10 x 10 kernel

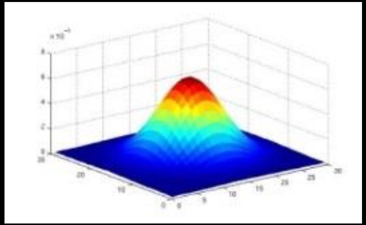
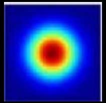
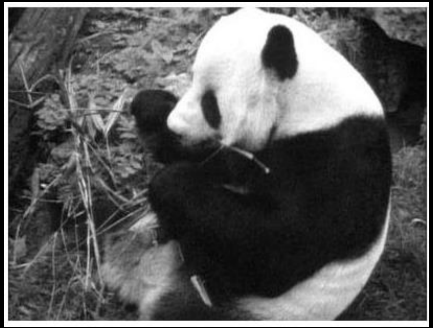


σ = 5 with 30 x 30 kernel

K. Grauman

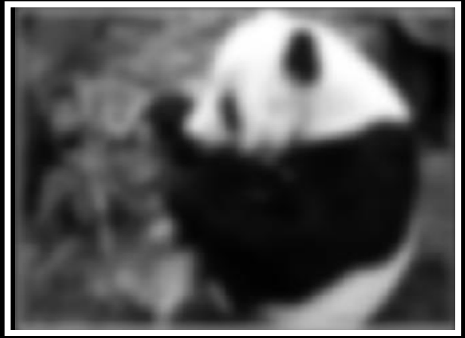
Matlab

```
>> hsize = 31;
>> sigma = 5;
>> h = fspecial('gaussian', hsize, sigma);

>> surf(h);

>> imagesc(h);

>> outim = imfilter(im, h);
>> imshow(outim);
```

im

$\longrightarrow$

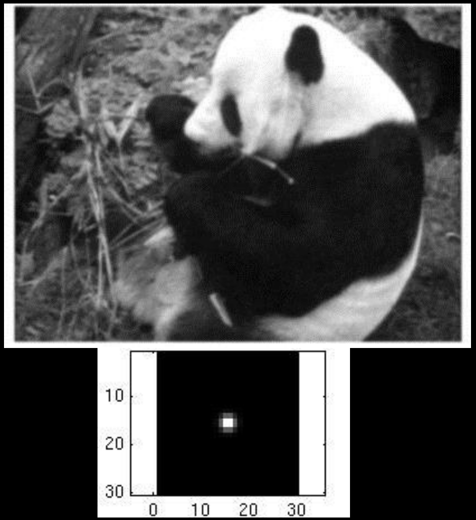


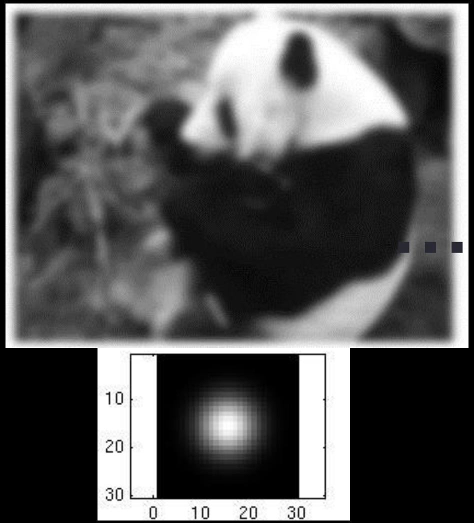
outim

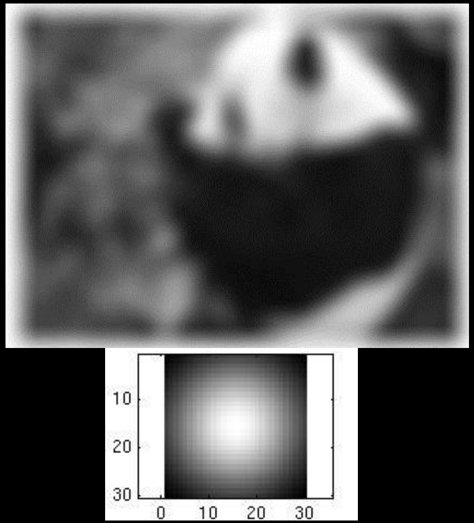
K. Grauman

Smoothing with a Gaussian

```
for sigma=1:3:10
 h = fspecial('gaussian‘,
 fsize, sigma);
 out = imfilter(im, h);
 imshow(out);
 pause;
end
```







Quiz

When filtering with a Gaussian, which is true:

a) The sigma is most important – it defines the blur kernel’s scale with respect to the image.

b) The kernel size is most important – because it defines the scale.

c) Altering the normalization coefficient does not effect the blur, only the brightness.

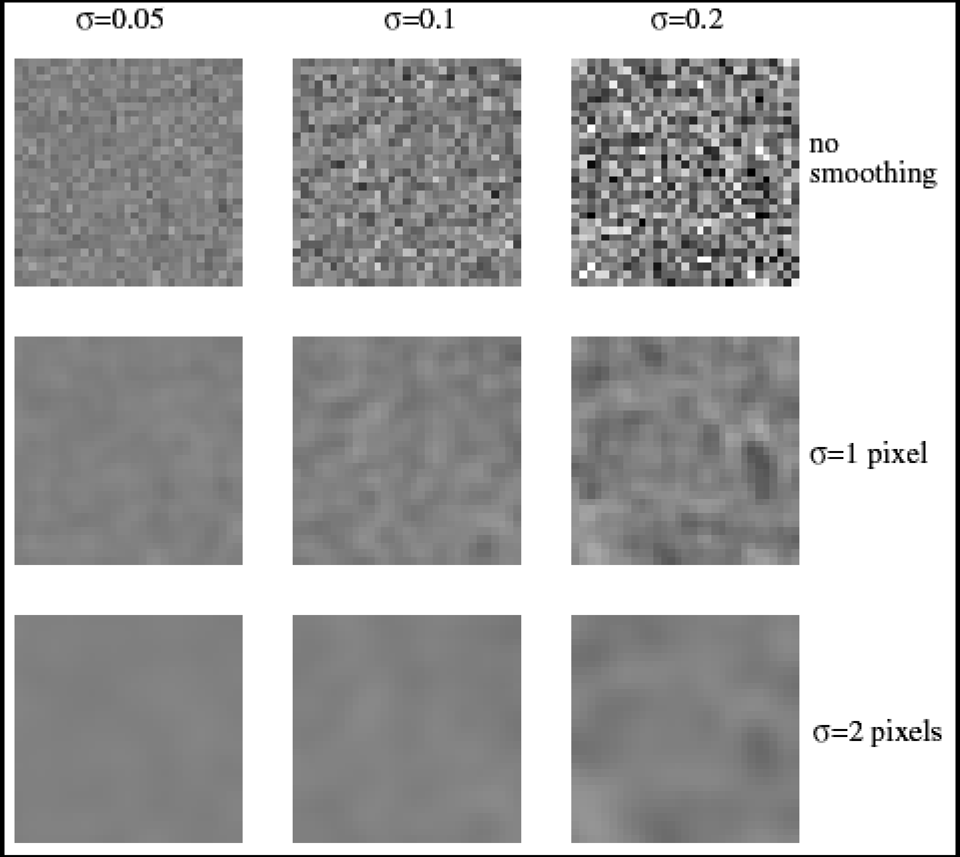
d) A and C.

A tale of two Gaussians

- Finally, a quick word of warning or clarification.
- We have just talked about sigma as a width of a the Gaussian filter. Last time we talked about ***sigma*** as the ***variance*** of a noise function.
- In one case – the filter – sigma is a width in space where as with noise it's a variance in ***value.*** The bigger the noise sigma was the more likely that large values of noise can be created.
- For noise it's usually sigma squared for variance; for filtering usually sigma for size.

Keeping the two Gaussians straight…

More Gaussian noise (like earlier) σ →



Wider Gaussian smoothing kernel σ →