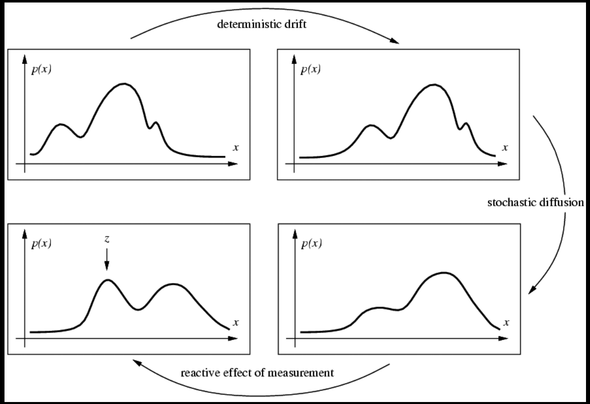
CS4495/6495
Introduction to Computer Vision

7C-L1 *Bayes filters*



Recall: Tracking with dynamics

Key idea: Given a model of expected motion, predict where objects will occur in next frame, even before seeing the image

Goals:

- Do less work looking for the object, restrict the search.
- Get improved estimates since measurement noise is tempered by smoothness, dynamics priors.

Recall: Tracking with dynamics

Key idea: Given a model of expected motion, predict where objects will occur in next frame, even before seeing the image

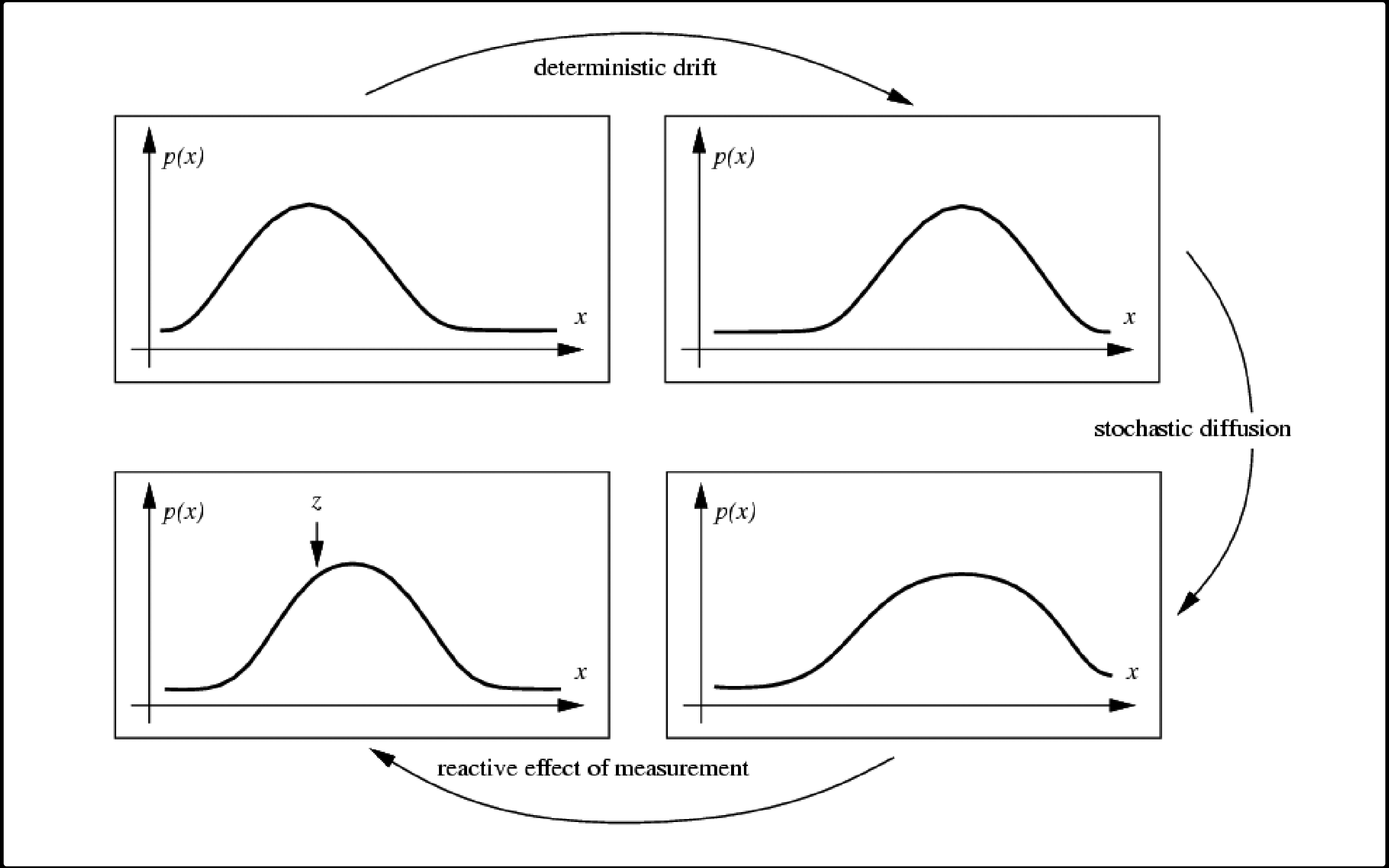
Assumption – continuous (modeled) motion patterns:

- Objects do not disappear and reappear in different places in the scene
- Camera is not moving instantly to new viewpoint
- Gradual change in pose between camera and scene

The Kalman filter

- A method for tracking *linear dynamical models* in *Gaussian noise* contexts (dynamics and measurements).
- Predicted/corrected *state densities* are *Gaussian*
  - You only need to maintain the mean and covariance
  - The calculations are easy (all the integrals can be done in closed form)

Propagation of Gaussian densities



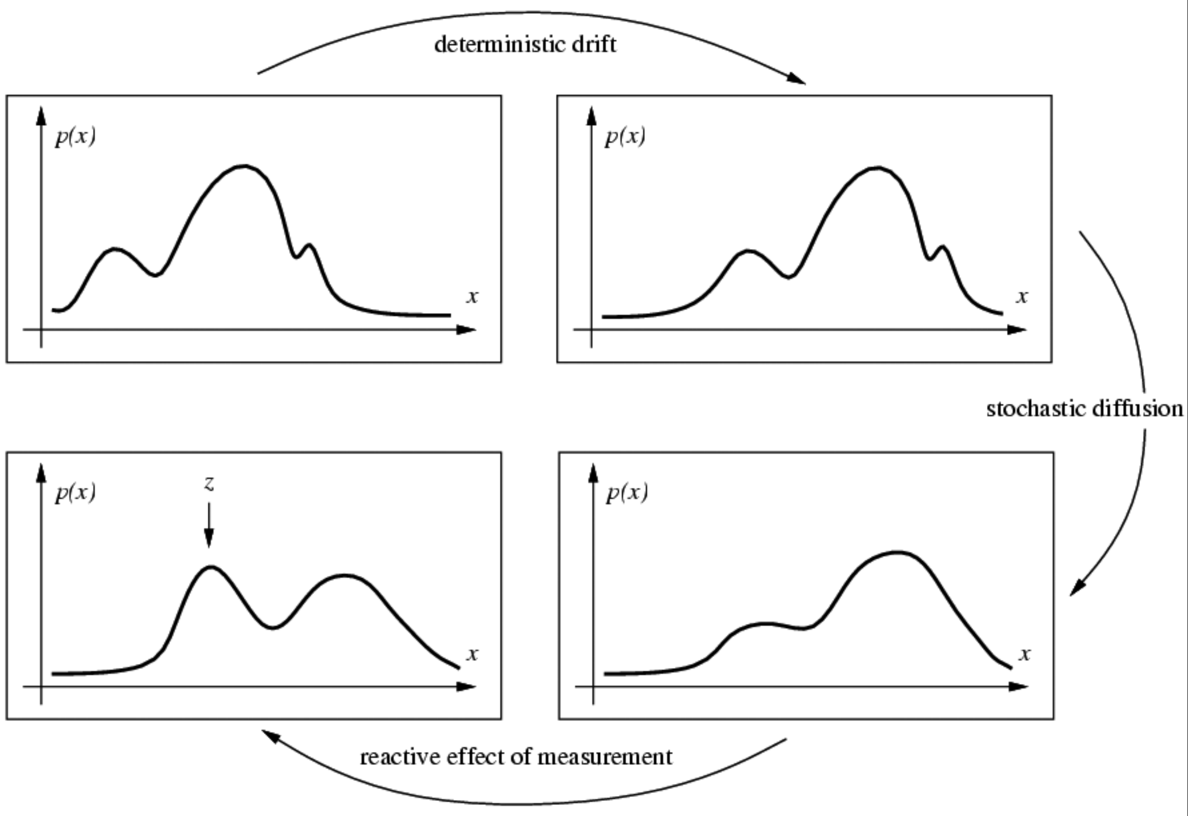
Kalman filter pros and cons

- Pros
  - Simple updates, compact and efficient
- Cons
  - Unimodal distribution, only single hypothesis
  - Restricted class of motions defined by linear model
    - Extensions called “Extended Kalman Filtering”
- *So what might we do if not Gaussian?*

Some old(er) examples of tracking

Isard and Blake CONDENSATION tracking

Propagation of general densities

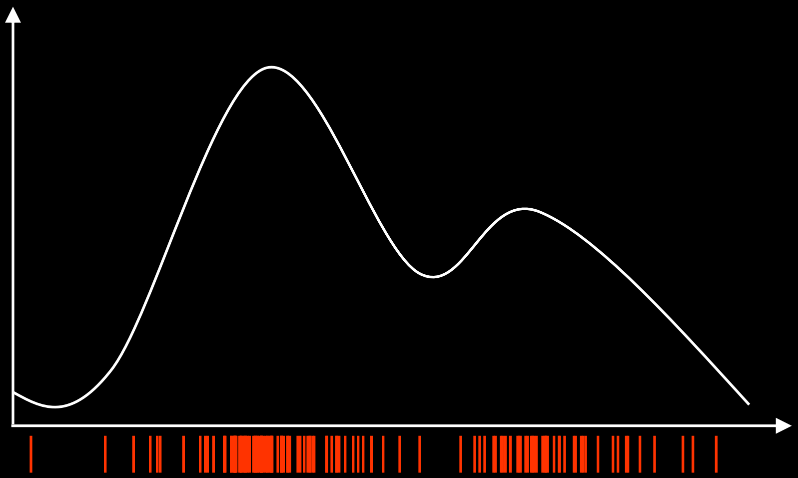


Before we go any further…

In particle filtering, the measurements are written as z_t and not as y_t

- So we'll start seeing z's

Particle Filters: Basic Idea



Density is represented by both **where** the particles are and their **weight**.

$p(x = x_0)$is now probability of drawing an *x* with value (really close to) x_0.

set of *n* (weighted) particles X_t

Goal: $p(x_t \in X_t) \approx p\left(x_t \middle| z_{\{1 \ldots t\}}\right)$ with equality when $n \to \infty$

Bayes Filters: Framework

Given

1. Prior probability of the system state $p(x)$
2. Action (dynamical system) model:

$$p(x_t|u_{t-1}, x_{t-1})$$

Bayes Filters: Framework

3. Sensor model (likelihood) $p(z|x)$
4. Stream of observations z and action data u:

$$data_t = \{u_1, z_2 \ldots, u_{t-1}, z_t\}$$

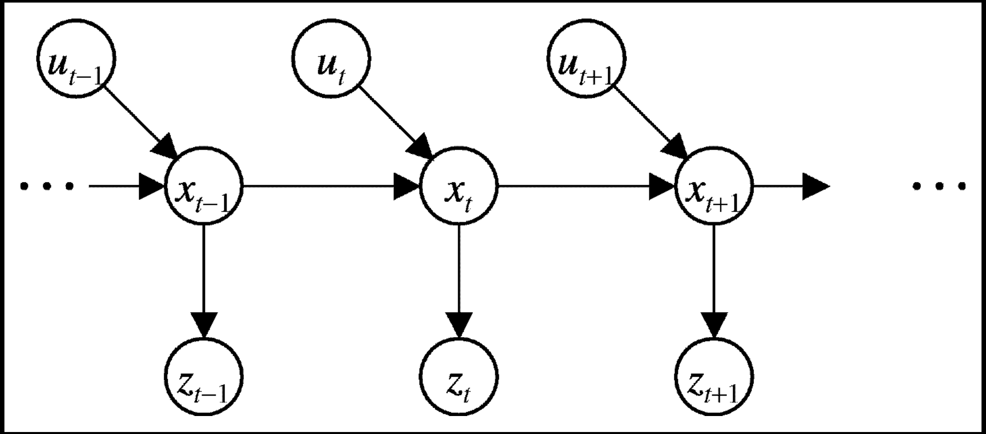
Bayes Filters: Framework

Wanted

- Estimate of the state *X* at time *t*
- The posterior of the state is also called ***belief***:

$$Bel(x_t) = P(x_t \mid u_1, z_2 \ldots, u_{t-1}, z_t)$$

Graphical Model Representation



Underlying Assumptions

- Static world
- Independent noise

$$p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t \mid x_t)$$

$$p(x_t \mid x_{1:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$

Bayes Rule reminder

prior before measurement

$$p(x \mid z) = \frac{p(z \mid x)\boxed{p(x)}}{p(z)}$$

$$= \eta\, p(z \mid x)\, p(x)$$

$$\propto p(z \mid x)\, p(x)$$

Bayes Filters

z = observation
u = action
x = state

$$Bel(x_t) = P(x_t \mid u_1, z_2 \ldots, u_{t-1}, z_t)$$

Bayes $$= \eta \; P(z_t \mid x_t, u_1, z_2, \ldots, u_{t-1}) \; P(x_t \mid u_1, z_2, \ldots, u_{t-1})$$

Likelihood **Prior**

Bayes Filters

z = observation
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x = state

$$Bel(x_t) = P(x_t \mid u_1, z_2 \ldots, u_{t-1}, z_t)$$

Bayes $$= \eta \; P(z_t \mid x_t, u_1, z_2, \ldots, u_{t-1}) \; P(x_t \mid u_1, z_2, \ldots, u_{t-1})$$

Sensor Ind $$= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_2, \ldots, u_{t-1})$$

Bayes Filters

z = observation
u = action
x = state

Sensor Ind $= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_2, \ldots, u_{t-1})$

Bayes Filters

z = observation
u = action
x = state

Sensor Ind $= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_2, \ldots, u_{t-1})$

Total Probability of "Prior" $= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_1, z_2, \ldots, u_{t-1}, x_{t-1}) \cdot$

$$P(x_{t-1} \mid u_1, z_2, \ldots, u_{t-1}) \, dx_{t-1}$$

Markov $= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_{t-1}, x_{t-1}) \; P(x_{t-1} \mid u_1, z_2, \ldots, u_{t-1}) \, dx_{t-1}$

Bayes Filters

z = observation
u = action
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Markov $= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_{t-1}, x_{t-1}) \; P(x_{t-1} \mid u_1, z_2, \ldots, u_{t-1}) \; dx_{t-1}$

Bayes Filters

z = observation
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$$\text{Markov} = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_{t-1}, x_{t-1}) \; P(x_{t-1} \mid u_1, z_2, \ldots, u_{t-1}) \; dx_{t-1}$$

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int \underbrace{P(x_t \mid u_{t-1}, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}}_{\text{prediction } \textit{before} \text{ taking measurement}}$$