CS4495/6495
Introduction to Computer Vision

3C-L1 *Extrinsic camera calibration*

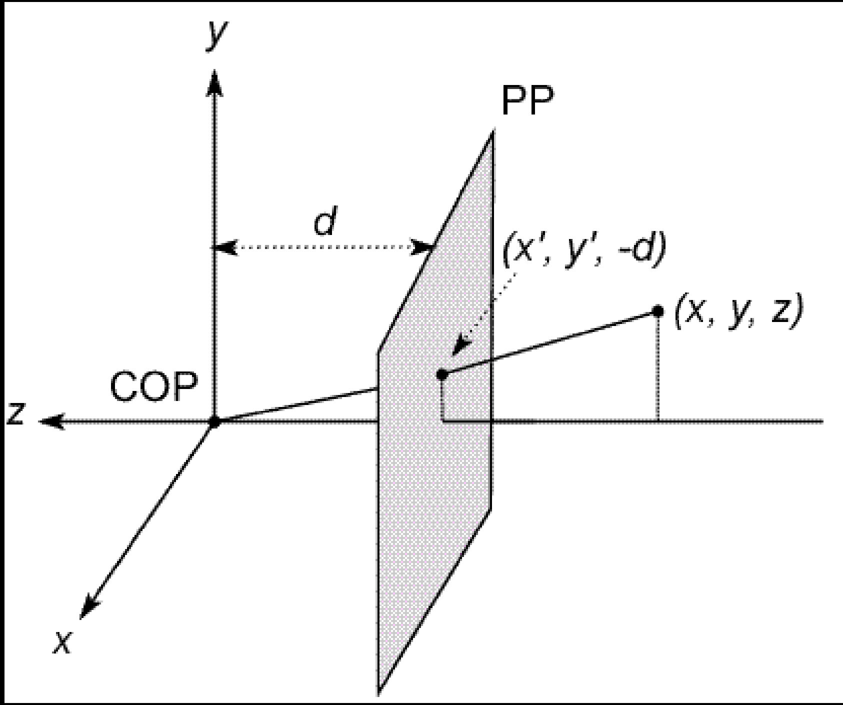
Recall: Modeling projection

Projection equations

- Compute intersection with Perspective Projection of ray from (x,y,z) to COP
- Derived using similar triangles

$$(X, Y, Z) \rightarrow \left(-d\frac{X}{Z}, -d\frac{Y}{Z}, -d\right)$$

(assumes normal Z negative – we'll change later)



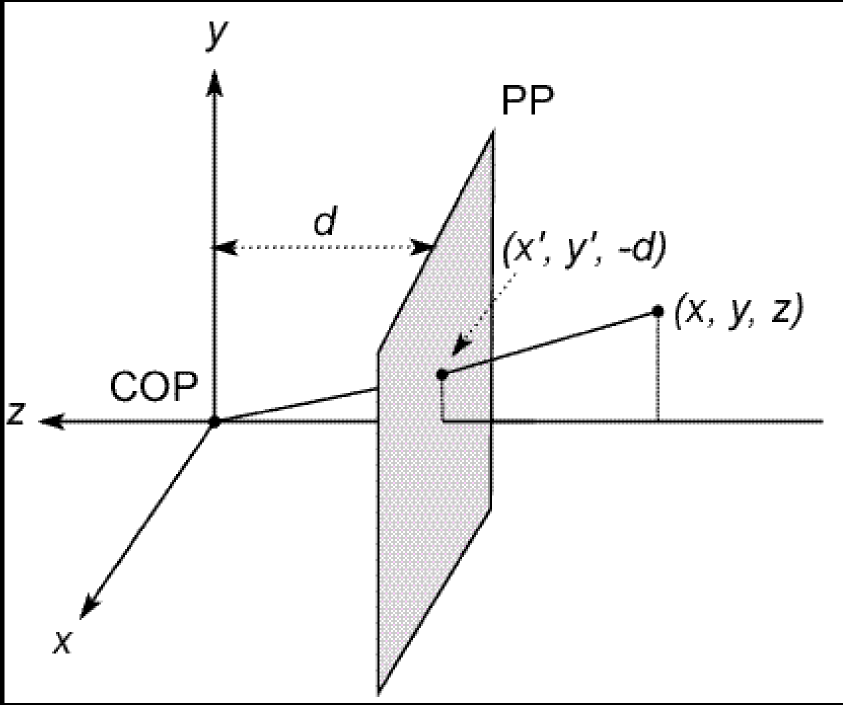
Recall: Modeling projection

Projection equations

$$(X, Y, Z) \rightarrow (-d\frac{X}{Z}, -d\frac{Y}{Z}, -d)$$

We get the projection by throwing out the last coordinate:

$$(x', y') = (-d\frac{X}{Z}, -d\frac{Y}{Z})$$



Recall: Homogeneous coordinates

Is this a linear transformation?

- No – division by the (not constant) Z is non-linear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image (2D) coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene (3D) coordinates

Recall: Homogeneous coordinates

Converting *from* homogeneous coordinates:

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

(this makes homogenous coordinates invariant under scale)

Recall: Perspective Projection

- Projection is a matrix multiply using homogeneous coordinates (and |z|):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ |z| \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ |z/f| \end{bmatrix} \Rightarrow \left(f\frac{x}{|z|}, f\frac{y}{|z|} \right)$$

$$\Rightarrow (u, v)$$

S. Seitz

But…

- In all this discussion we have the notion of a camera’s coordinate system – an origin and an orientation.
- We put the center of projection at this origin and the optic axis down the z axis.
- So to do geometric reasoning about the world we need to relate the coordinate system of the world to that of the camera and the image.
- Today we’ll do from the world to the camera, and next time from the camera to the image.

Geometric Camera calibration

- We want to use the camera to tell us things about the world.
    - So we need the relationship between coordinates in the world and coordinates in the image: ***geometric camera calibration***
- For reference see Forsyth and Ponce, sections 1.2 and 1.3.

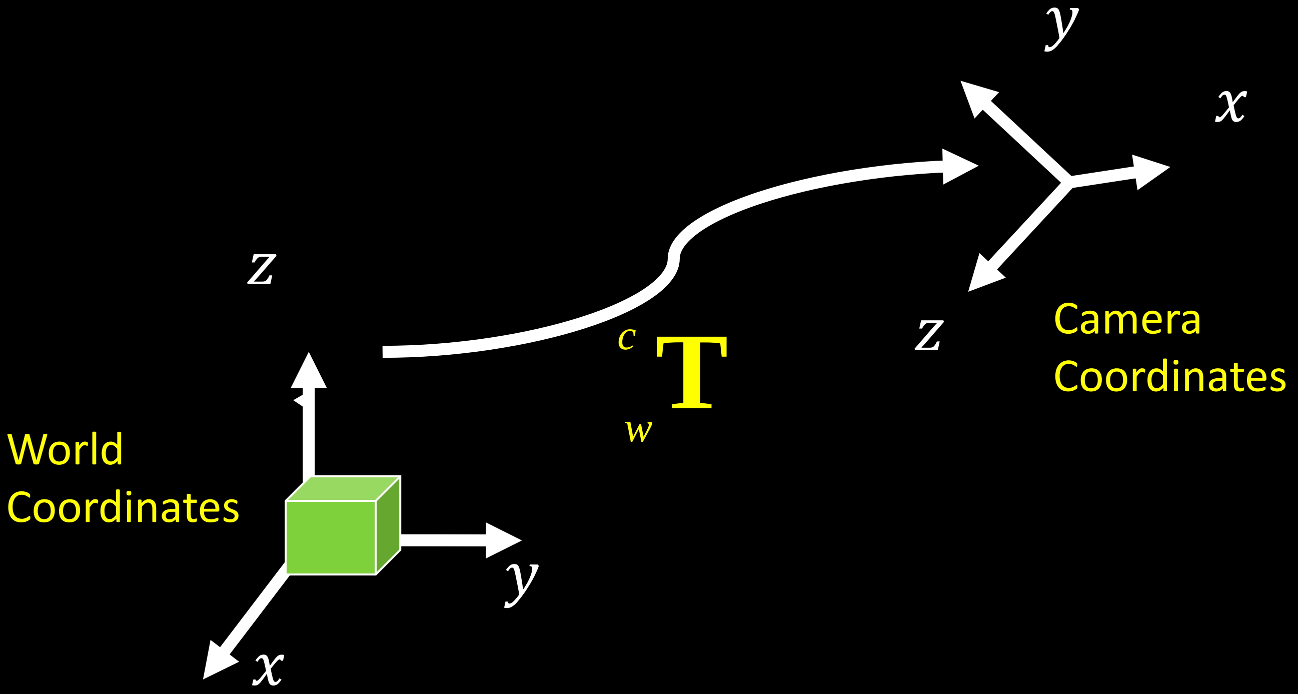
Geometric Camera calibration

Composed of 2 transformations:

- From some (arbitrary) world coordinate system to the camera's 3D coordinate system. *Extrinisic parameters (or camera pose)*

- From the 3D coordinates in the camera frame to the 2D image plane via projection.
  Intrinisic parameters

Camera Pose



Quiz

How many degrees of freedom are there in specifying the extrinsic parameters?

a) 5

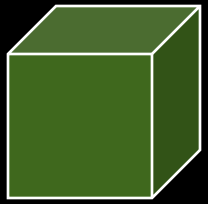
b) 6

c) 3

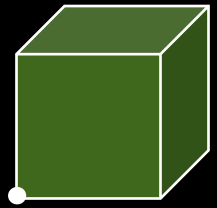
d) 9

Rigid Body Transformations

Need a way to specify the six degrees-of-freedom of a rigid body. Why 6?

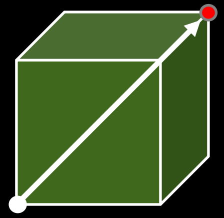


A rigid body is a collection of points whose positions relative to each other can't change



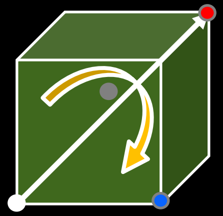
Fix one point, 3 DOF

$\boxed{3}$



Fix second point, 2 more DOF (must maintain distance constraint)

$\boxed{+2}$

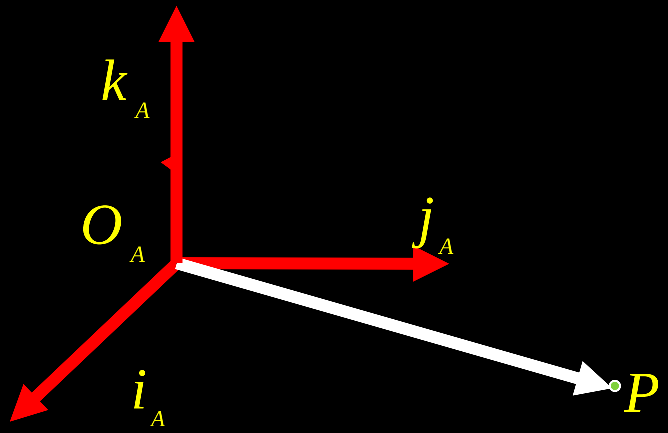


Third point adds 1 more DOF, for rotation around line

$\boxed{+1}$

Notation (from F&P)

- Superscript references coordinate frame
- ^{A}P is coordinates of P in frame A
- ^{B}P is coordinates of P in frame B



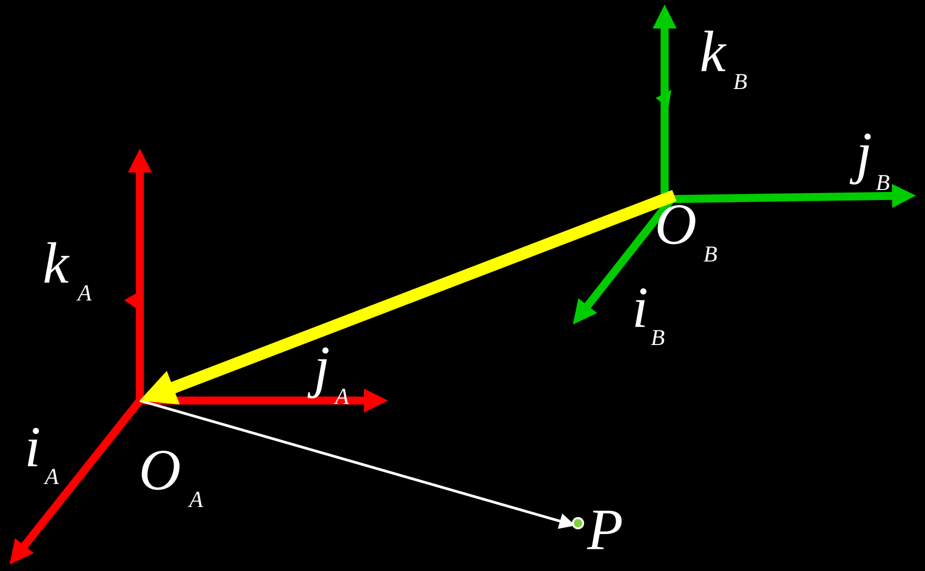
$$^{A}P = \begin{pmatrix} ^{A}x \\ ^{A}y \\ ^{A}z \end{pmatrix} \Leftrightarrow \overline{OP} = \left(^{A}x \cdot \overline{i_A} \right) + \left(^{A}y \cdot \overline{j_A} \right) + \left(^{A}z \cdot \overline{k_A} \right)$$

Translation Only

$$^{B}P = {}^{A}P + {}^{B}(O_A)$$

or

$$^{B}P = {}^{B}(O_A) + {}^{A}P$$



Translation

- Using homogeneous coordinates, translation can be expressed as a matrix multiplication.

$$ {}^{B}P = {}^{A}P + {}^{B}O_{A} $$

3x3 identity

$$ \boxed{\begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} = \begin{bmatrix} I & {}^{B}O_{A} \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix}} $$

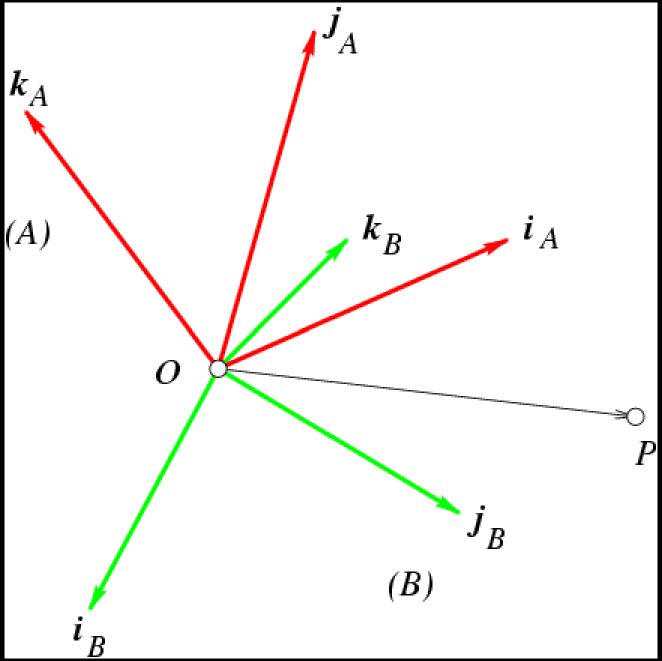
(Translation is commutative)

Rotation

$$\overrightarrow{OP} = \begin{pmatrix} i_A & j_A & k_A \end{pmatrix} \begin{pmatrix} {}^{A}x \\ {}^{A}y \\ {}^{A}z \end{pmatrix} = \begin{pmatrix} i_B & j_B & k_B \end{pmatrix} \begin{pmatrix} {}^{B}x \\ {}^{B}y \\ {}^{B}z \end{pmatrix}$$

$$ {}^{B}P = {}^{B}_{A}R \; {}^{A}P $$

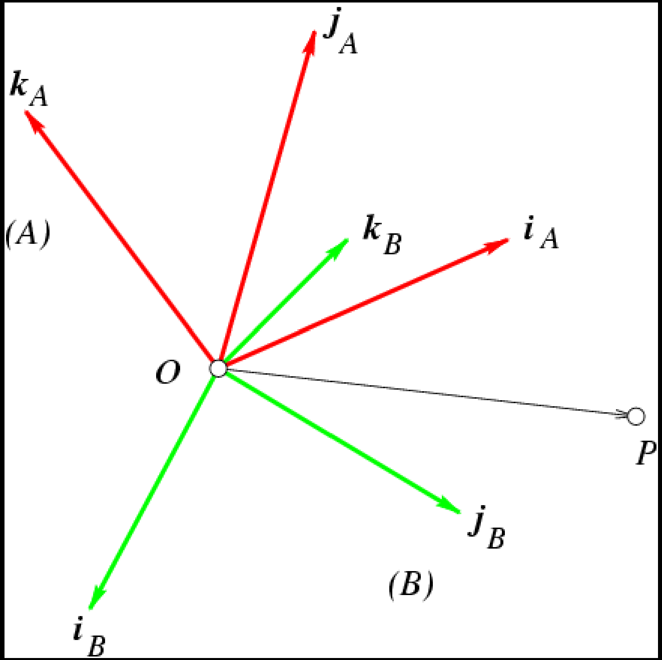
${}^{B}_{A}R$ *means describing frame A in the coordinate system of frame B*



Rotation

$$
{}^{B}_{A}R = \begin{bmatrix} \mathbf{i}_A.\mathbf{i}_B & \mathbf{j}_A.\mathbf{i}_B & \mathbf{k}_A.\mathbf{i}_B \\ \mathbf{i}_A.\mathbf{j}_B & \mathbf{j}_A.\mathbf{j}_B & \mathbf{k}_A.\mathbf{j}_B \\ \mathbf{i}_A.\mathbf{k}_B & \mathbf{j}_A.\mathbf{k}_B & \mathbf{k}_A.\mathbf{k}_B \end{bmatrix}
$$

$$
= \begin{bmatrix} {}^{\mathbf{B}}\mathbf{i}_{\mathbf{A}} & {}^{\mathbf{B}}\mathbf{j}_{\mathbf{A}} & {}^{\mathbf{B}}\mathbf{k}_{\mathbf{A}} \end{bmatrix}
$$



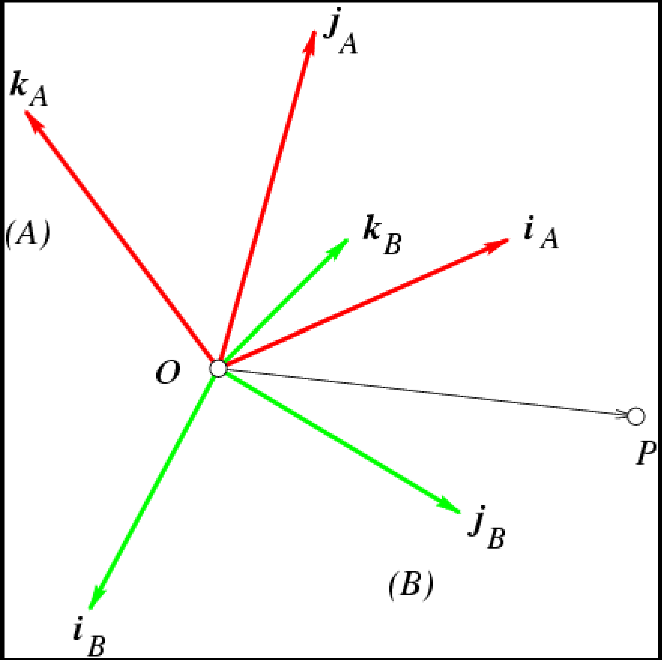
The columns of the rotation matrix are the axes of frame A expressed in frame B. Why?

Rotation

$$
{}^{B}_{A}R = \begin{bmatrix} \mathbf{i}_A.\mathbf{i}_B & \mathbf{j}_A.\mathbf{i}_B & \mathbf{k}_A.\mathbf{i}_B \\ \mathbf{i}_A.\mathbf{j}_B & \mathbf{j}_A.\mathbf{j}_B & \mathbf{k}_A.\mathbf{j}_B \\ \mathbf{i}_A.\mathbf{k}_B & \mathbf{j}_A.\mathbf{k}_B & \mathbf{k}_A.\mathbf{k}_B \end{bmatrix}
$$

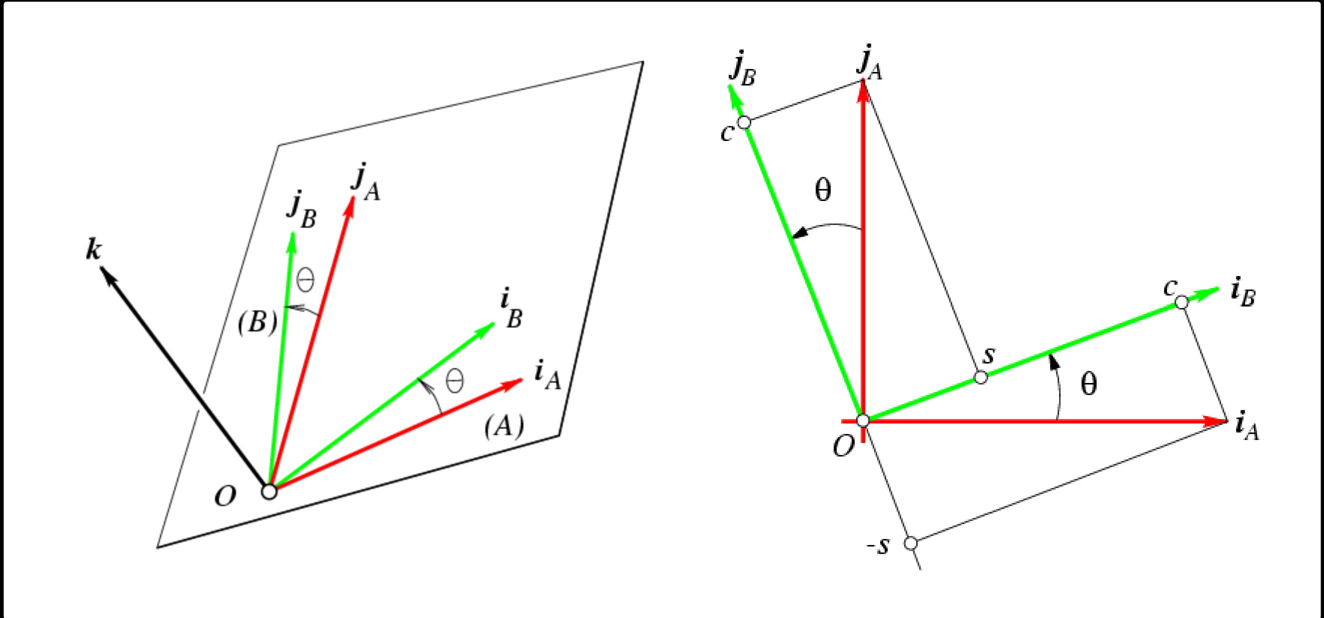
$$
= \begin{bmatrix} {}^{\mathbf{B}}\mathbf{i}_{\mathbf{A}} & {}^{\mathbf{B}}\mathbf{j}_{\mathbf{A}} & {}^{\mathbf{B}}\mathbf{k}_{\mathbf{A}} \end{bmatrix}
$$

$$
= \begin{bmatrix} {}^{\mathbf{A}}\mathbf{i}_{\mathbf{B}}{}^{T} \\ {}^{\mathbf{A}}\mathbf{j}_{\mathbf{B}}{}^{T} \\ {}^{\mathbf{A}}\mathbf{k}_{\mathbf{B}}{}^{T} \end{bmatrix}
$$



Orthogonal matrix!
Why?

Example: Rotation about z axis



What is the rotation matrix?

$$R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Combine 3 to get arbitrary rotation

- Euler angles: Z, X’, Z’’
- Or heading, pitch roll: world Z, new X, new Y …
- Or roll, pitch and yaw …
- Or Azimuth, elevation, roll…
- Three basic matrices: order matters, but we’ll not focus on that

Combine 3 to get arbitrary rotation

$$R_X(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{bmatrix}$$

$$R_Z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_Y(\kappa) = \begin{bmatrix} \cos(\kappa) & 0 & -\sin(\kappa) \\ 0 & 1 & 0 \\ \sin(\kappa) & 0 & \cos(\kappa) \end{bmatrix}$$

Rotation in homogeneous coordinates

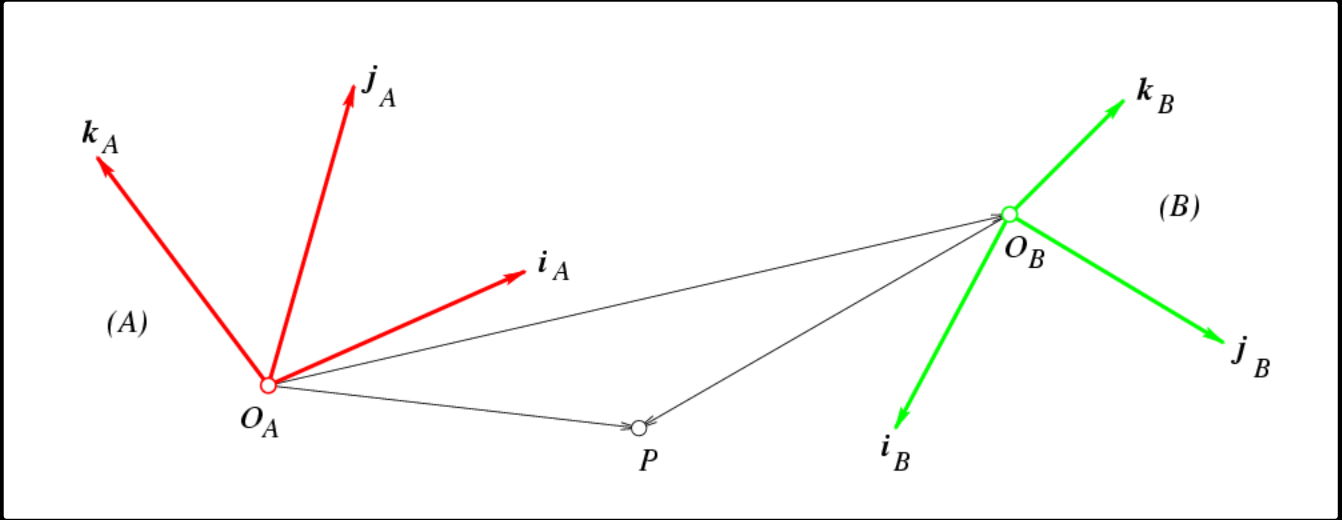
- Using homogeneous coordinates, rotation can be expressed as a matrix multiplication.

$$ {}^{B}P = {}^{B}_{A}R \, {}^{A}P $$

$$ \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{B}_{A}R & 0 \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} $$

- Rotation is ***not*** commutative

Rigid transformations



$$^{B}P = {}^{B}_{A}\boldsymbol{R}\,{}^{A}P + {}^{B}O_{A}$$

Rigid transformations (con't)

Unified treatment using homogeneous coordinates:

$$\begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & {}^{B}O_{A} \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} {}^{B}_{A}R & 0 \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} {}^{B}_{A}R & {}^{B}O_{A} \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix}$$

Rigid transformations (con't)

And even better:

$$\begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{B}_{A}R & {}^{B}O_{A} \\ \mathbf{0}^{T} & 1 \end{bmatrix} \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = {}^{B}_{A}T \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix}$$

Invertible!

so

$$\begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = {}^{A}_{B}T \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} = \left({}^{B}_{A}T \right)^{-1} \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix}$$

Translation and rotation

From frame A to B:

Non-homogeneous (“regular) coordinates

$$ {}^{B}\vec{p} = {}^{B}_{A}R \; {}^{A}\vec{p} + {}^{B}_{A}\vec{t} $$

3x1 translation vector

3x3 rotation matrix

Translation and rotation

Homogeneous coordinates:

$$ {}^{B}\vec{p} = {}^{B}_{A}T \; {}^{A}\vec{p} $$

$$ {}^{B}\vec{p} = \begin{pmatrix} \begin{pmatrix} & & \\ & {}^{B}_{A}R & \\ & & \end{pmatrix} & \begin{matrix} | \\ {}^{B}_{A}\vec{t} \\ | \end{matrix} \\ 0 \quad 0 \quad 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} $$

Homogenous coordinates allows us to write coordinate transforms as a single matrix!

From World to Camera

Rotation from world to camera frame

Translation from world to camera frame

$$ {}^{C}\vec{p} = {}^{C}_{W}R \; {}^{W}\vec{p} + {}^{C}_{W}\vec{t} $$

Point now in camera frame

Point in world frame

Non-homogeneous coordinates

From World to Camera

$$\begin{pmatrix} \\ {}^{C}\vec{p} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} - & - & - \\ - & {}^{C}_{W}R & - \\ - & - & - \end{matrix}} & \boxed{\begin{matrix} | \\ {}^{C}_{W}\vec{t} \\ | \end{matrix}} \\ 0 \quad 0 \quad 0 & 1 \end{pmatrix} \begin{pmatrix} \\ {}^{W}\vec{p} \\ \\ \\ \end{pmatrix}$$

Homogeneous coordinates

From world to camera is the ***extrinsic*** *parameter matrix (4x4)*

(sometimes 3x4 if using for next step in projection – not worrying about inversion)

Quiz

How many degrees of freedom are there in the 3x4 extrinsic parameter matrix?

a) 12
b) 6
c) 9
d) 3

From World to Camera

$$\begin{pmatrix} \\ {}^{C}\vec{p} \\ \\ \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} - & - & - \\ - & {}^{C}_{W}R & - \\ - & - & - \end{matrix}} & \boxed{\begin{matrix} | \\ {}^{C}_{W}\vec{t} \\ | \end{matrix}} \\ 0 \quad 0 \quad 0 & 1 \end{pmatrix} \begin{pmatrix} \\ {}^{W}\vec{p} \\ \\ \end{pmatrix}$$

Homogeneous coordinates

*From world to camera is the **extrinsic** parameter matrix (4x4)*

(sometimes 3x4 if using for next step in projection – not worrying about inversion)