CS4495/6495
Introduction to Computer Vision

3D-L3 *Fundamental matrix*

Weak calibration

Main idea:

- Estimate epipolar geometry from a (redundant) set of point correspondences between two uncalibrated cameras

From before: Projection matrix

$$\begin{bmatrix} w x_{im} \\ w y_{im} \\ w \end{bmatrix} = \mathbf{K}_{\text{int}} \boldsymbol{\Phi}_{ext} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

From before: Projection matrix

$$\begin{bmatrix} w\,x_{im} \\ w\,y_{im} \\ w \end{bmatrix} = \mathbf{K}_{\text{int}} \boldsymbol{\Phi}_{ext} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\boldsymbol{\Phi}_{ext} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & -\mathbf{R}_1^{\mathrm{T}}\mathbf{T} \\ r_{21} & r_{22} & r_{23} & -\mathbf{R}_2^{\mathrm{T}}\mathbf{T} \\ r_{31} & r_{32} & r_{33} & -\mathbf{R}_3^{\mathrm{T}}\mathbf{T} \end{bmatrix}$$

From before: Projection matrix

$$\begin{bmatrix} w\,x_{im} \\ w\,y_{im} \\ w \end{bmatrix} = \mathbf{K}_{\text{int}}\,\mathbf{\Phi}_{ext} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\mathbf{K}_{\text{int}} = \begin{bmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

Note: Invertible, scale x and y, assumes no skew

From before: Projection matrix

$$\begin{bmatrix} w\,x_{im} \\ w\,y_{im} \\ w \end{bmatrix} = \mathbf{K}_{int}\,\mathbf{\Phi}_{ext} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\mathbf{p}_{im} = \mathbf{K}_{int}\underbrace{\mathbf{\Phi}_{ext}\,\mathbf{P}_w}_{\mathbf{p}_c}$$

$$\boxed{\mathbf{p}_{im} = \mathbf{K}_{int}\,\mathbf{p}_c}$$

Uncalibrated case

For a given camera: $\mathbf{p}_{im} = \mathbf{K}_{int}\mathbf{p}_c$

And since invertible: $\mathbf{p}_c = \mathbf{K}_{int}^{-1}\mathbf{p}_{im}$

Uncalibrated case

So, for **two** cameras (left and right):

$$\mathbf{p}_{c,left} = \mathbf{K}_{int,left}^{-1}\,\mathbf{p}_{im,left}$$

$$\mathbf{p}_{c,right} = \underbrace{\mathbf{K}_{int,right}^{-1}}_{\text{Internal calibration matrices, one per camera}}\,\mathbf{p}_{im,right}$$

Uncalibrated case

$$\mathbf{p}_{c,right} = \mathbf{K}^{-1}_{int,right}\,\mathbf{p}_{im,right} \qquad \mathbf{p}_{c,left} = \mathbf{K}^{-1}_{int,left}\,\mathbf{p}_{im,left}$$

From before, the ***essential*** *matrix* ***E.***

$$\boxed{\mathbf{p}_{c,right}^{\mathrm{T}}\,\mathbf{E}\,\mathbf{p}_{c,left} = 0}$$

$$\left(\mathbf{K}^{-1}_{int,right}\mathbf{p}_{im,right}\right)^{\mathrm{T}}\mathbf{E}\left(\mathbf{K}^{-1}_{int,left}\mathbf{p}_{im,left}\right) = 0$$

Uncalibrated case

$$\left(\mathbf{K}_{int,right}^{-1}\mathbf{p}_{im,right}\right)^{\mathrm{T}}\mathbf{E}\left(\mathbf{K}_{int,left}^{-1}\mathbf{p}_{im,left}\right)=0$$

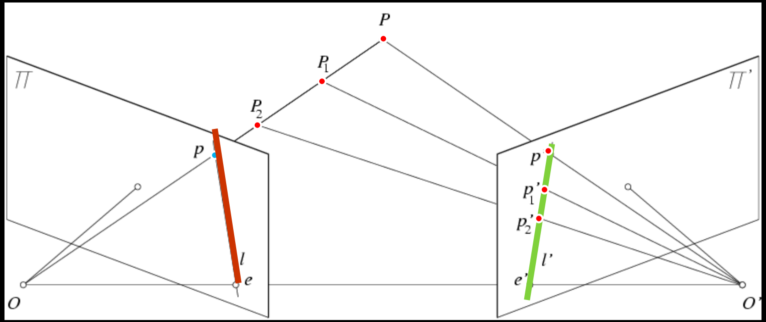
$$\mathbf{p}_{im,right}^{\mathrm{T}}\underbrace{\left(\left(\mathbf{K}_{int,right}^{-1}\right)^{T}\mathbf{E}\,\mathbf{K}_{int,left}^{-1}\right)}\mathbf{p}_{im,left}=0$$

"Fundamental matrix": $\mathbf{F}$

$$\boxed{\mathbf{p}_{im,right}^{\mathrm{T}}\mathbf{F}\,\mathbf{p}_{im,left}=0} \quad or \quad \mathbf{p}^{T}\mathbf{F}\,\mathbf{p}'=0$$

Properties of the Fundamental Matrix

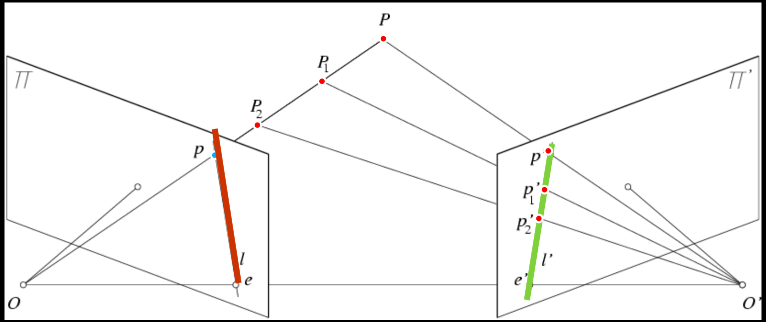
$$\mathbf{p}^T \mathbf{F} \mathbf{p}' = 0$$



$\mathbf{l} = \mathbf{F}\mathbf{p}'$ is the epipolar *line* in the *p* image associated with p’

Properties of the Fundamental Matrix

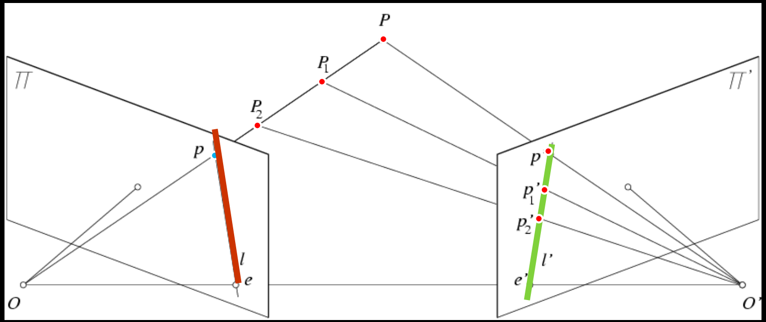
$$\mathbf{p}^T \mathbf{F} \mathbf{p}' = 0$$



$\mathbf{l}' = \mathbf{F}^T \mathbf{p}$ is the epipolar line in the prime image associated with p

Properties of the Fundamental Matrix

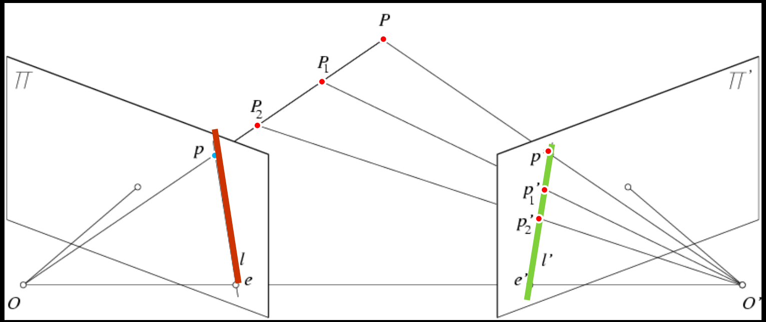
$$\mathbf{p}^T \mathbf{F} \mathbf{p}' = 0$$



Epipoles found by $Fp' = 0$ and $F^T p = 0$

Properties of the Fundamental Matrix

$$\mathbf{p}^T \mathbf{F} \mathbf{p}' = 0$$



F is singular (mapping from homogeneoues 2-D point to 1-D family so rank 2 – more later)

Fundamental matrix

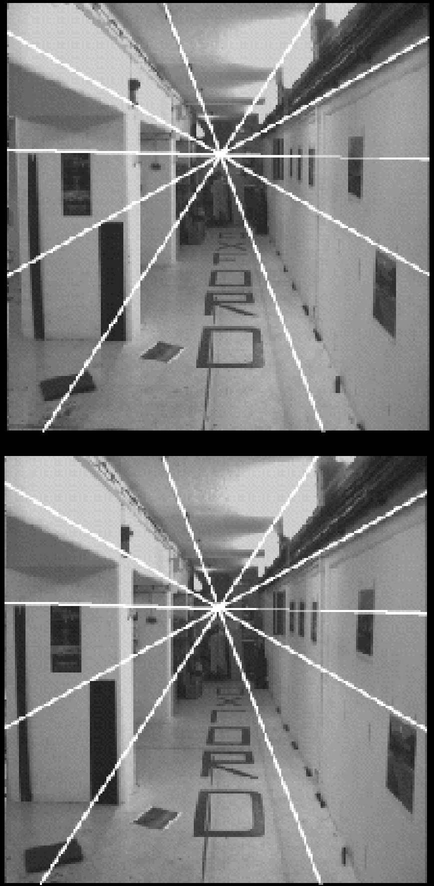
- Relates pixel coordinates in the two views
- More general form than essential matrix:

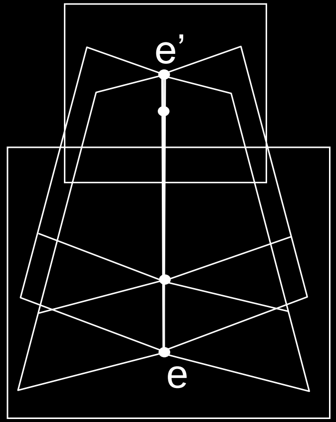
  We remove the need to know intrinsic parameters

Fundamental matrix

- If we estimate fundamental matrix from correspondences in pixel coordinates, can reconstruct epipolar geometry without intrinsic or extrinsic parameters.

Different Example: Forward motion





courtesy of Andrew Zisserman

Computing F from correspondences

$$\mathbf{p}_{im,right}^{T}\,\mathbf{F}\,\mathbf{p}_{im,left} = 0$$

Each point correspondence generates ***one*** constraint on F

$$\begin{bmatrix} u' & v' & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0$$

Computing F from correspondences

$$\begin{bmatrix} u' & v' & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0$$

Multiply out:

$$\begin{bmatrix} u'_1 u_1 & u'_1 v_1 & u'_1 & v'_1 u_1 & v'_1 v_1 & v'_1 & u_1 & v_1 & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = \mathbf{0}$$

Computing F from correspondences

Collect N of these:

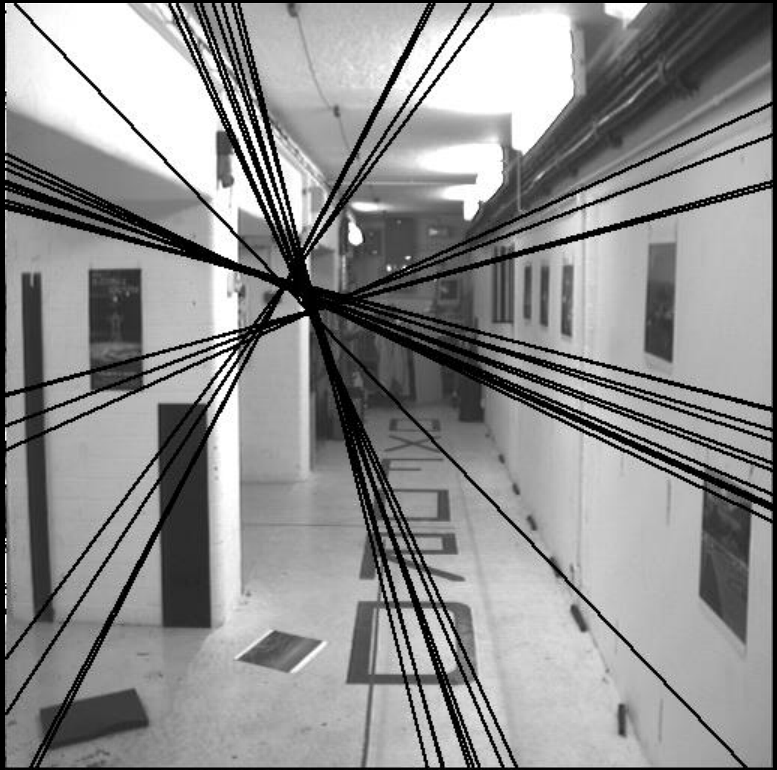
$$\begin{bmatrix} u'_1 u_1 & u'_1 v_1 & u'_1 & v'_1 u_1 & v'_1 v_1 & v'_1 & u_1 & v_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u'_n u_n & u'_n v_n & u'_1 & v'_n u_n & v'_n v_n & v'_n & u_n & v_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = \mathbf{0}$$

And solve for ***f*** the elements of F**….**

The (in)famous “eight-point algorithm”

$$
\begin{bmatrix}
250906.36 & 183269.57 & 921.81 & 200931.10 & 146766.13 & 738.21 & 272.19 & 198.81 & 1.00 \\
2692.28 & 131633.03 & 176.27 & 6196.73 & 302975.59 & 405.71 & 15.27 & 746.79 & 1.00 \\
416374.23 & 871684.30 & 935.47 & 408110.89 & 854384.92 & 916.90 & 445.10 & 931.81 & 1.00 \\
191183.60 & 171759.40 & 410.27 & 416435.62 & 374125.90 & 893.65 & 465.99 & 418.65 & 1.00 \\
48988.86 & 30401.76 & 57.89 & 298604.57 & 185309.58 & 352.87 & 846.22 & 525.15 & 1.00 \\
164786.04 & 546559.67 & 813.17 & 1998.37 & 6628.15 & 9.86 & 202.65 & 672.14 & 1.00 \\
116407.01 & 2727.75 & 138.89 & 169941.27 & 3982.21 & 202.77 & 838.12 & 19.64 & 1.00 \\
135384.58 & 75411.13 & 198.72 & 411350.03 & 229127.78 & 603.79 & 681.28 & 379.48 & 1.00
\end{bmatrix}
\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0
$$

Just solving for F…

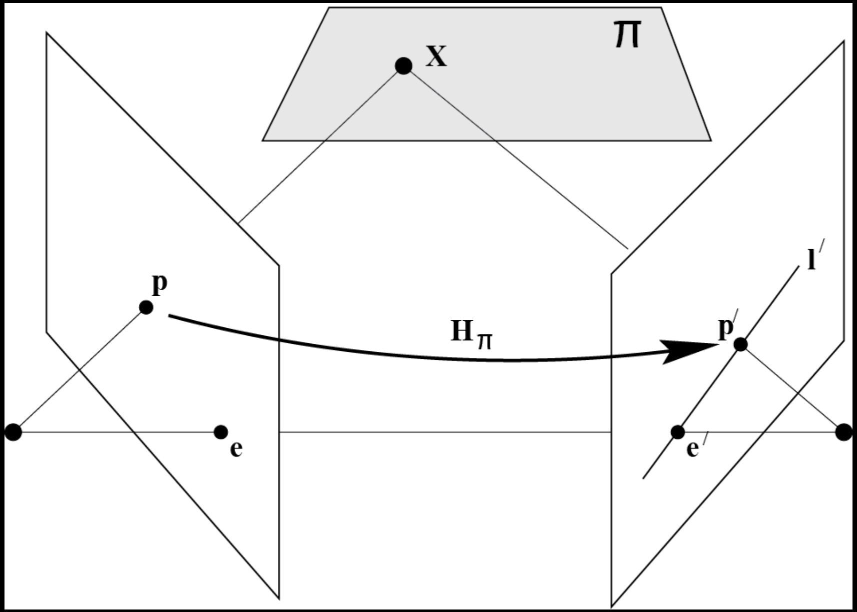


Rank of F

- Assume we know the homography H_π that maps from Left to Right (Full 3x3)

$$\mathbf{p}' = \mathbf{H}_\pi \mathbf{p}$$

- Let line $\mathbf{l}'$ be the epiloar line corresponding to $\mathbf{p}$ – goes through epipole $\mathbf{e}'$



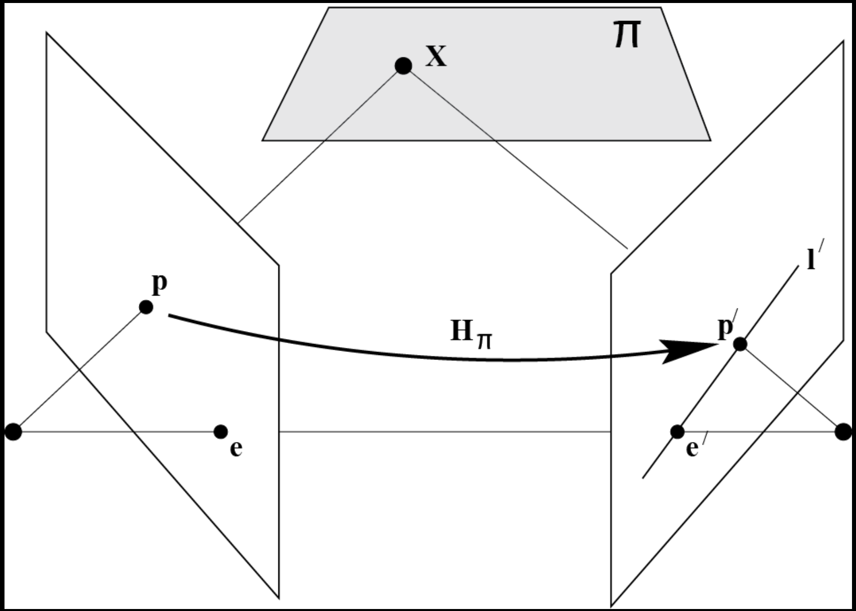
Rank of F

- Let line **l'** be the epiloar line corresponding to **p** – goes through epipole **e'**

$$\mathbf{l}' = \mathbf{e}' \times \mathbf{p}'$$

$$= \mathbf{e}' \times \mathbf{H}_{\pi}\mathbf{p}$$

$$= \boxed{[\mathbf{e}']_{\times}\mathbf{H}_{\pi}}\,\mathbf{p}$$



But **l'** is the epipolar line for p: $\mathbf{l}' = \boxed{\mathbf{F}}\,\mathbf{p}$

Rank of F is rank of $[e']_x = 2$

Fix the linear solution

- Use SVD or other method to do linear computation for F
- Decompose F using SVD (not the same SVD):

$$\mathbf{F} = U D V^T$$

Fix the linear solution

- Use SVD or other method to do linear computation for F
- Decompose F using SVD (not the same SVD):

$$\mathbf{F} = U D V^{T}$$

- Set the last singular value to zero:

$$D = \begin{bmatrix} r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & t \end{bmatrix} \Rightarrow \hat{D} = \begin{bmatrix} r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

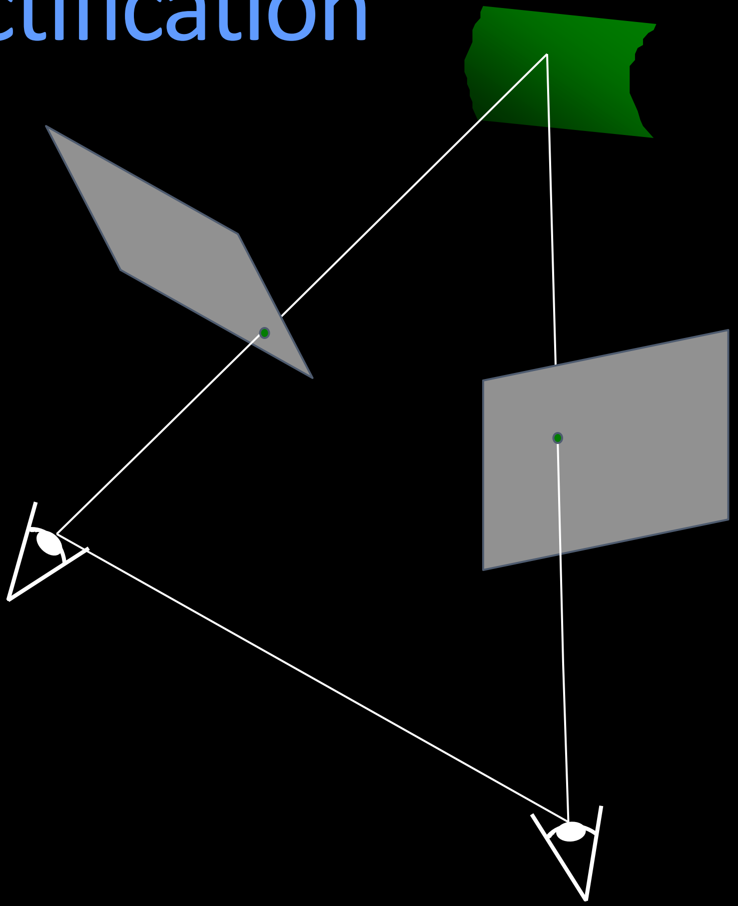
Fix the linear solution

- Estimate new F from the new $\hat{D}$

$$\hat{\mathbf{F}} = U \hat{D} V^T$$

That’s better…

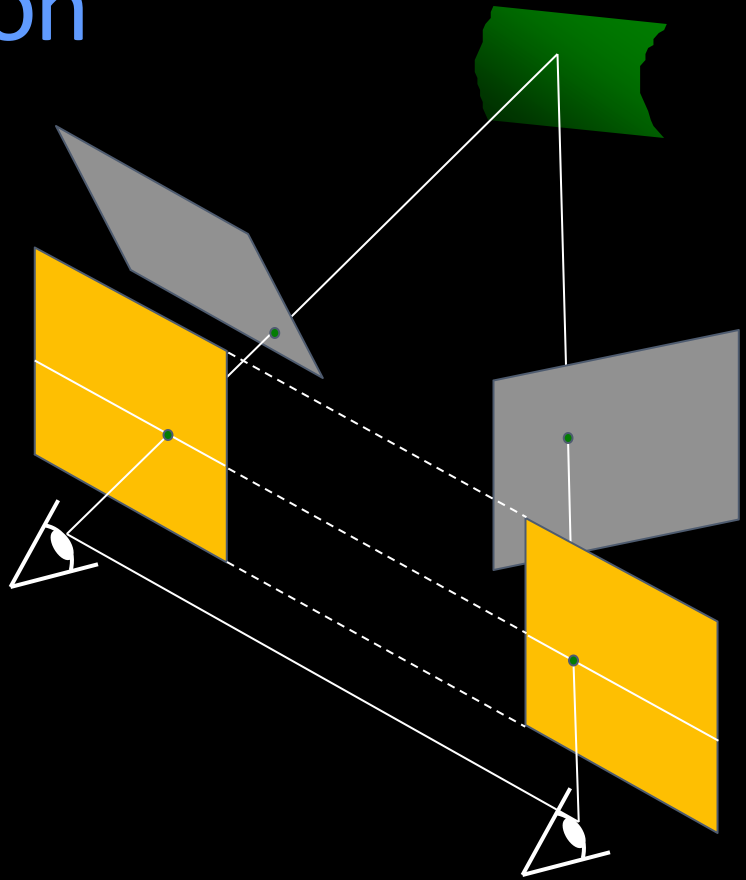
Stereo image rectification

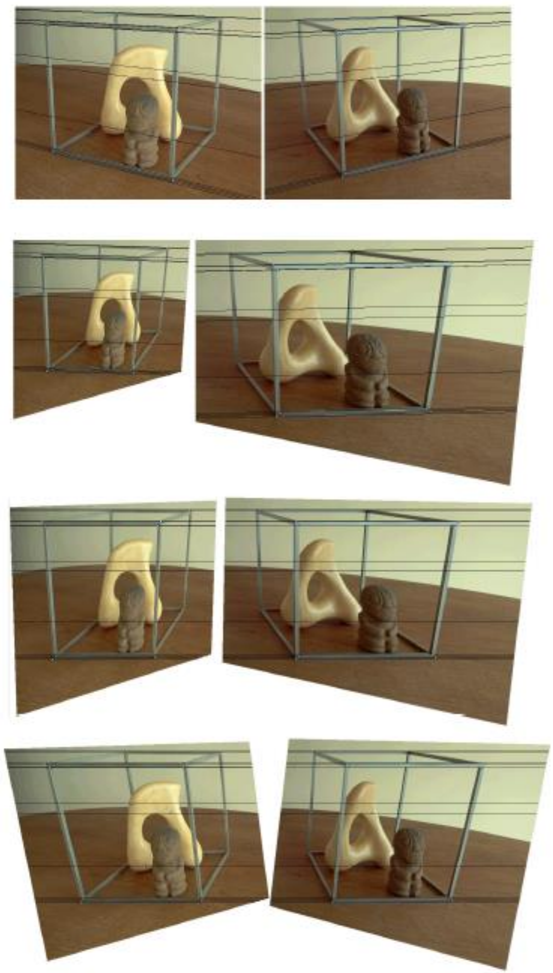


Stereo image rectification

- Reproject image planes onto a common plane parallel to the line between optical centers –each a homography
- Pixel motion is horizontal after this transformation

➢ C. Loop and Z. Zhang. [Computing Rectifying Homographies for Stereo Vision](). IEEE Conf. Computer Vision and Pattern Recognition, 1999.





(a) Original image pair overlayed with several epipolar lines.

(b) Image pair transformed by the specialized projective mapping H_p and H'_p. Note that the epipolar lines are now parallel to each other in each image.

(c) Image pair transformed by the similarity H_r and H'_r. Note that the image pair is now rectified (the epipolar lines are horizontally aligned).

(d) Final image rectification after shearing transform H_s and H'_s. Note that the image pair remains rectified, but the horizontal distortion is reduced.

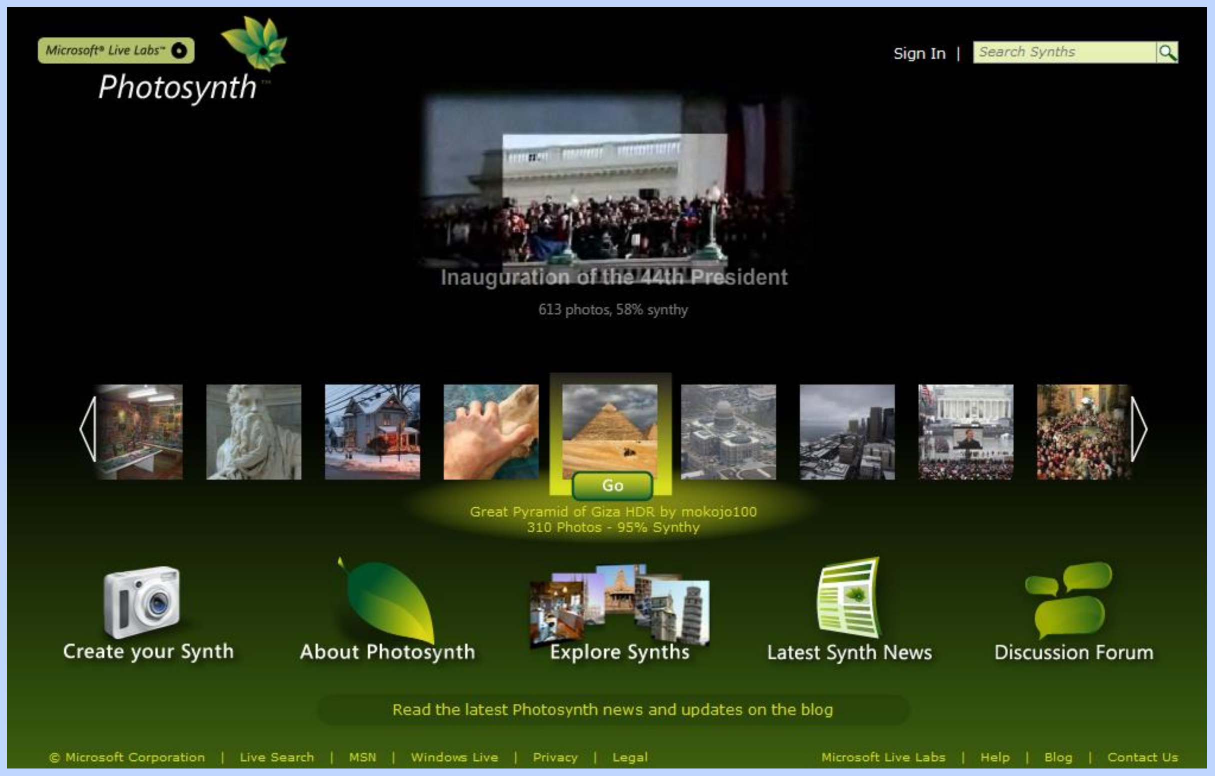
C. Loop and Z. Zhang, Computing Rectifying Homographies for Stereo Vision, IEEE Conf. Computer Vision and Pattern Recognition, 1999.

Photo synth

Noah Snavely, Steven M. Seitz, Richard Szeliski, "Photo tourism: Exploring photo collections in 3D," SIGGRAPH 2006

http://photosynth.net/

Photosynth.net



Based on Photo Tourism

by Noah Snavely, Steve Seitz, and Rick Szeliski

3D from multiple images

Building Rome in a Day: Agarwal et al. 2009

Summary

- For 2-views, there is a geometric relationship that define the relations between rays in one view to rays in the other – epipolar geometry.
- These relationships can be captured algebraicly as well:
  - Calibrated – Essential matrix
  - Uncalibrated – Fundamental matrix.
- This relation can be estimated from point correspondences.