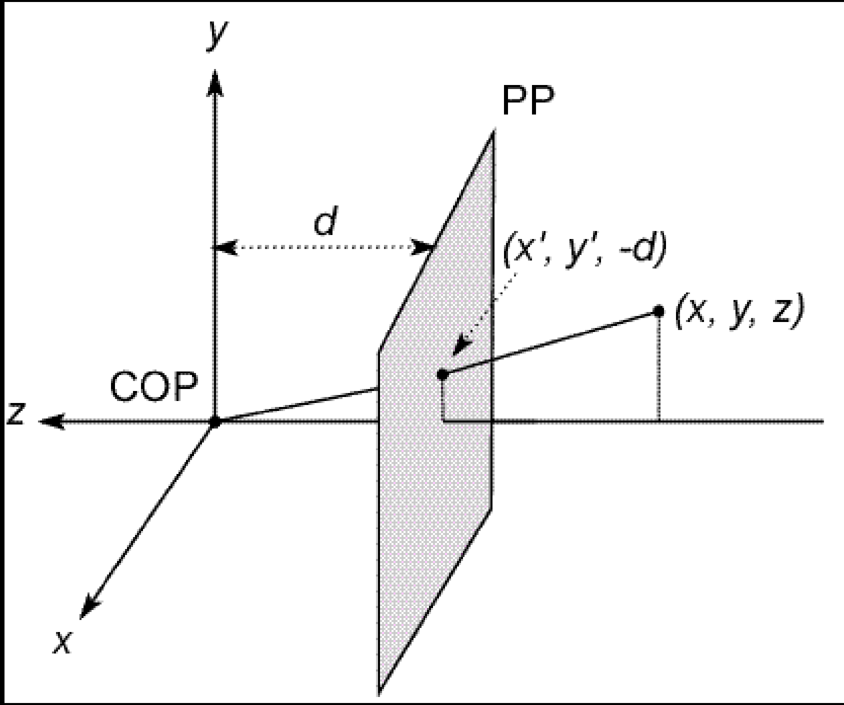
CS4495/6495
Introduction to Computer Vision

3A-L2 *Perspective imaging*

Modeling projection – coordinate system

- We will use the pin-hole model as an approximation
- Put the optical center (Center Of Projection) at the origin
- STANDARD (x,y) COORDINATE SYSTEM
- Put the image plane (Projection Plane) in front of the COP (why?)
- The camera looks down the ***negative*** z axis



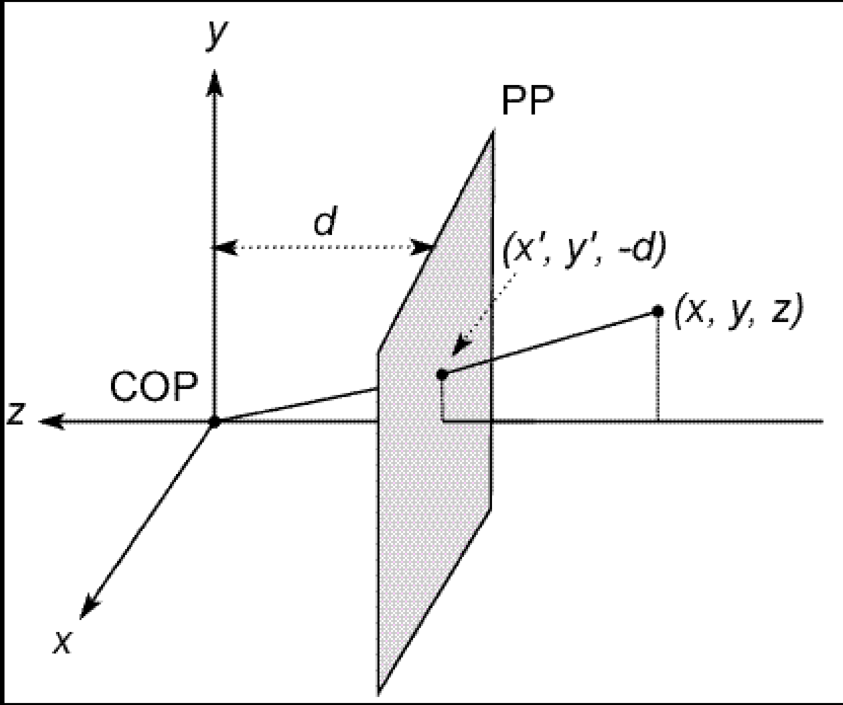
Modeling projection

Projection equations

- Compute intersection with Perspective Projection of ray from (x,y,z) to COP
- Derived using similar triangles

$$(X, Y, Z) \rightarrow \left(-d\frac{X}{Z}, -d\frac{Y}{Z}, -d\right)$$

(assumes normal Z negative – we'll change later)



Modeling projection

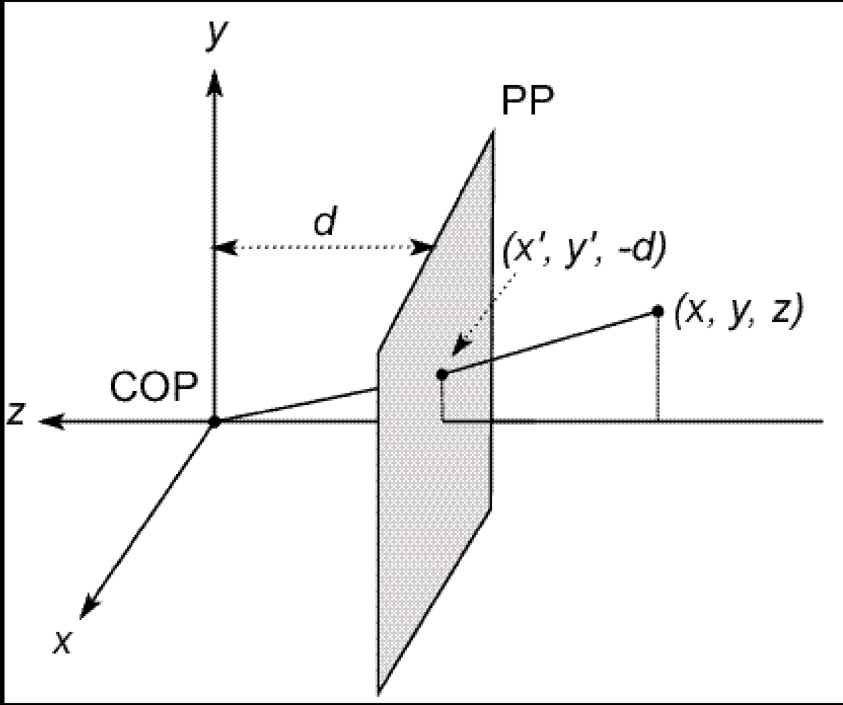
Projection equations

$$(X, Y, Z) \rightarrow (-d\frac{X}{Z}, -d\frac{Y}{Z}, -d)$$

We get the projection by throwing out the last coordinate:

$$(x', y') = (-d\frac{X}{Z}, -d\frac{Y}{Z})$$

Distant objects are smaller



Quiz

- When objects are very far away, the real X and real X can be huge. If I move the camera (the origin) those numbers hardly change. This explains:

a) Why the moon follows you.
b) Why the North Star is always North.
c) Why you can tell time from the Sun regardless of where you are?
d) All of the above.

Homogeneous coordinates

Is this a linear transformation?

- No – division by the (not constant) Z is non-linear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image (2D) coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene (3D) coordinates

Homogeneous coordinates

Converting *from* homogeneous coordinates:

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

(this makes homogenous coordinates invariant under scale)

Perspective Projection

- Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z} \right)$$

$$\Rightarrow (u, v)$$

S. Seitz

Perspective Projection

- How does scaling the projection matrix change the transformation?

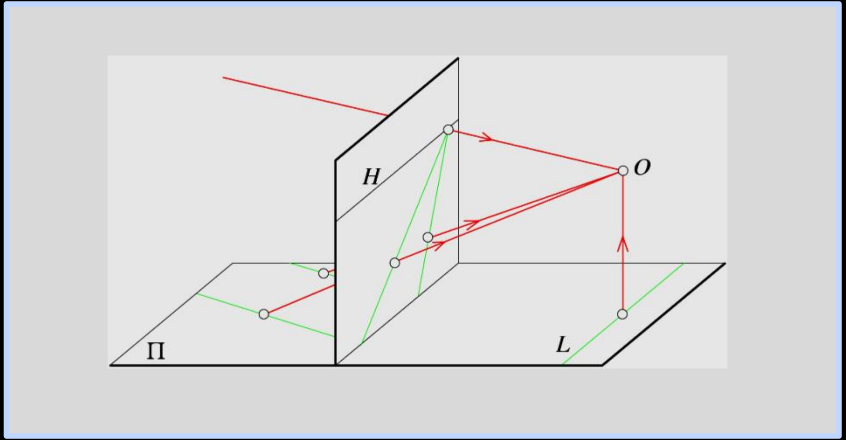
Perspective Projection

- How does scaling the projection matrix change the transformation?

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z} \right)$$

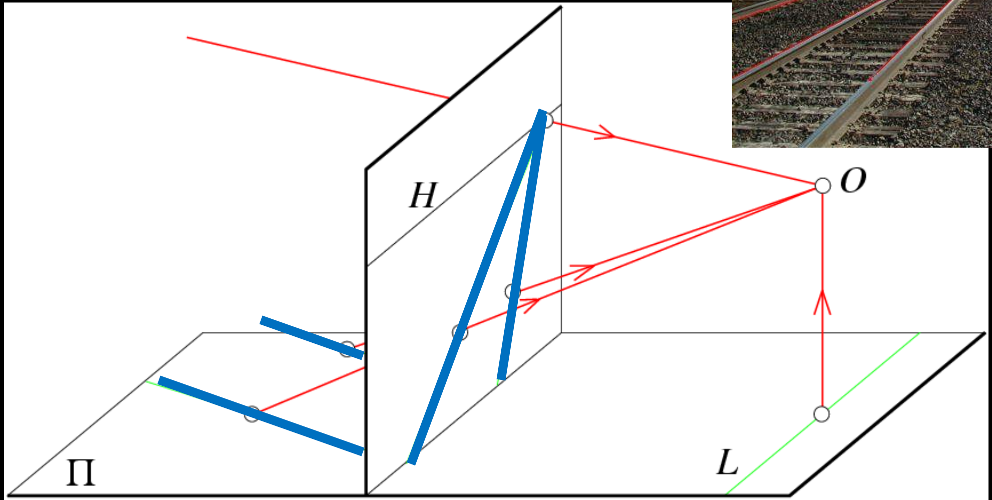
Geometric properties of projection

- Points go to **points**
- Lines go to **lines**
- Polygons go to **polygons**



Parallel lines in the world meet in the image

"**Vanishing**" point



Parallel lines converge in math too…

Line in 3-space

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$

$$z(t) = z_0 + ct$$

Perspective projection of the line

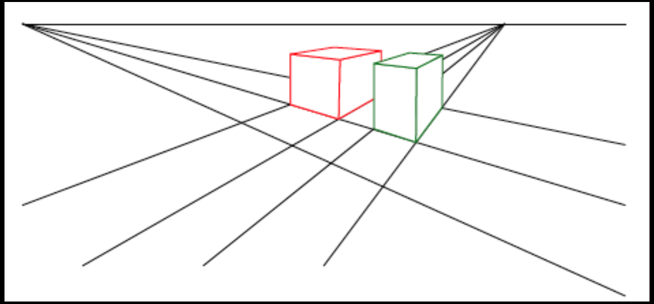
$$x'(t) = \frac{fx}{z} = \frac{f(x_0 + at)}{z_0 + ct}$$

$$y'(t) = \frac{fy}{z} = \frac{f(y_0 + bt)}{z_0 + ct}$$

In the limit as $t \to \pm\infty$ we have (for $c \neq 0$):

$$x'(t) \to \frac{fa}{c}, \quad y'(t) \to \frac{fb}{c}$$

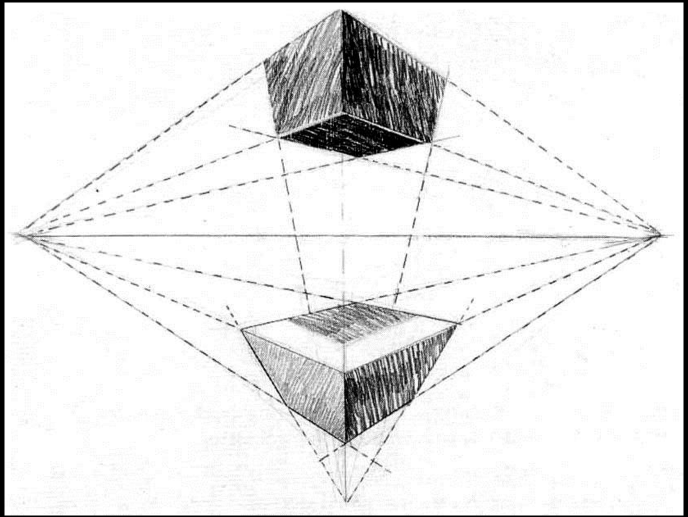
Vanishing points



- Each set of parallel lines (=direction) meets at a different point
- Sets of parallel lines on the same plane lead to collinear vanishing points.
  - The line is called the horizon for that plane
- Good ways to spot faked images
  - scale and perspective don't work
  - vanishing points behave badly

More vanishing points

3-point perspective: Different directions correspond to different vanishing points

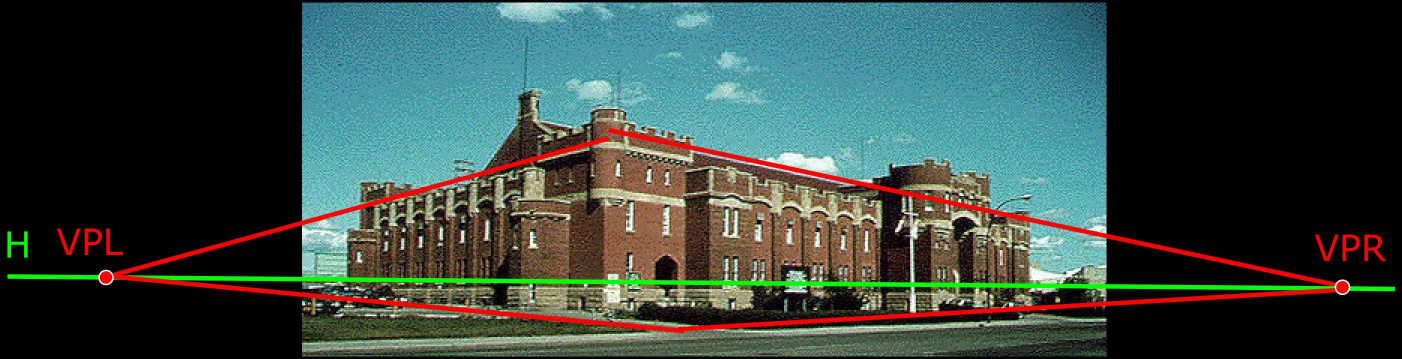


VP_1

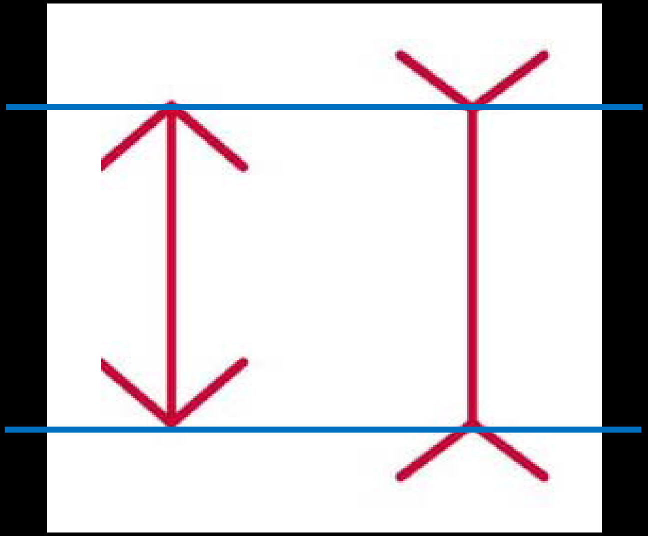
VP_2

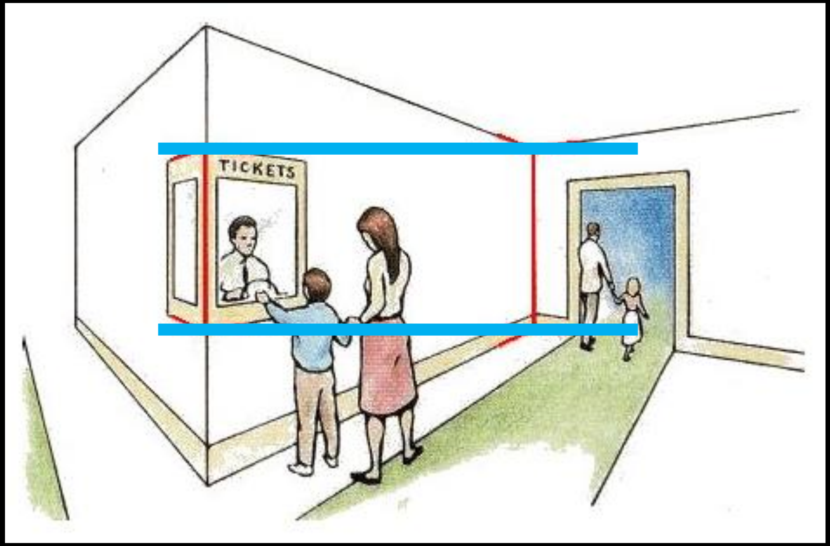
VP_3

Vanishing points



Human vision: Müller-Lyer Illusion





Which line is longer?

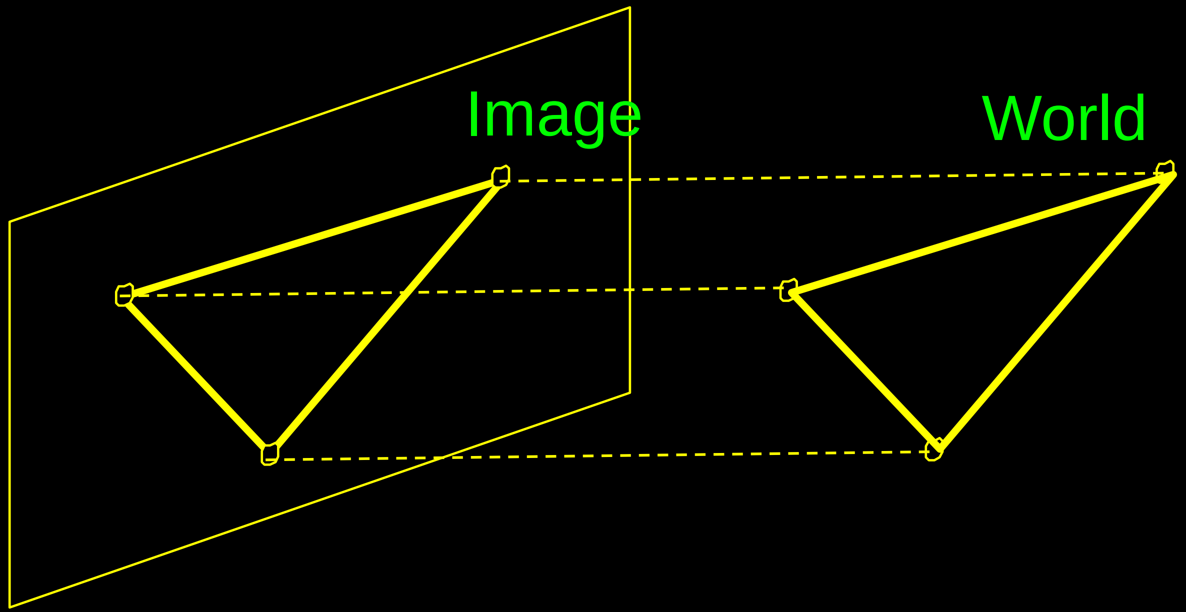
http://www.michaelbach.de/ot/sze_muelue/index.html

Quiz

What determines at what point in the image parallel lines intersect?

a) The direction the lines have in the world.
b) Whether the world lines are on the ground plane?
c) The orientation of the camera.
d) (a) and (c)

Other models: Orthographic projection

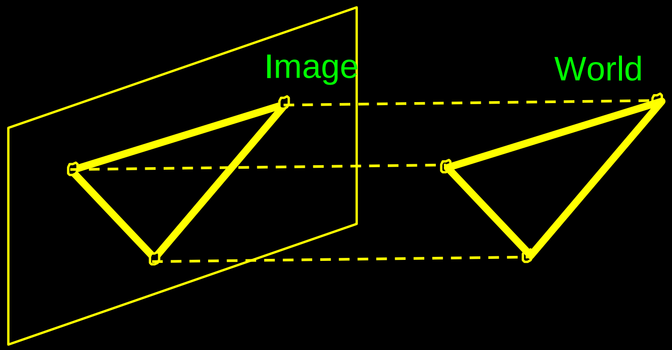


Other models: Orthographic projection

- Special case of perspective projection
  - Distance from the COP to the image plane is infinite

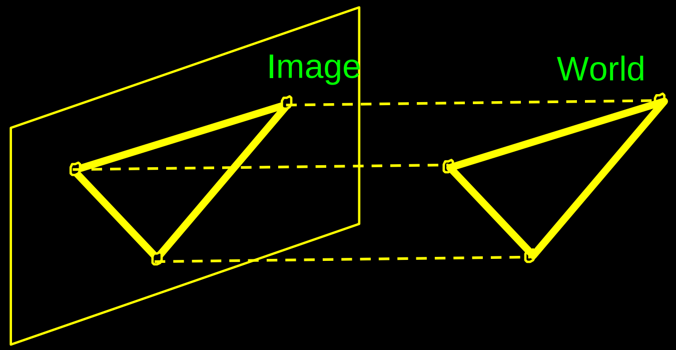
    => Both f and Z are very large
  - Good approximation for telephoto optics
  - Also called "parallel projection":

    $(x, y, z) \rightarrow (x, y)$



Other models: Orthographic projection

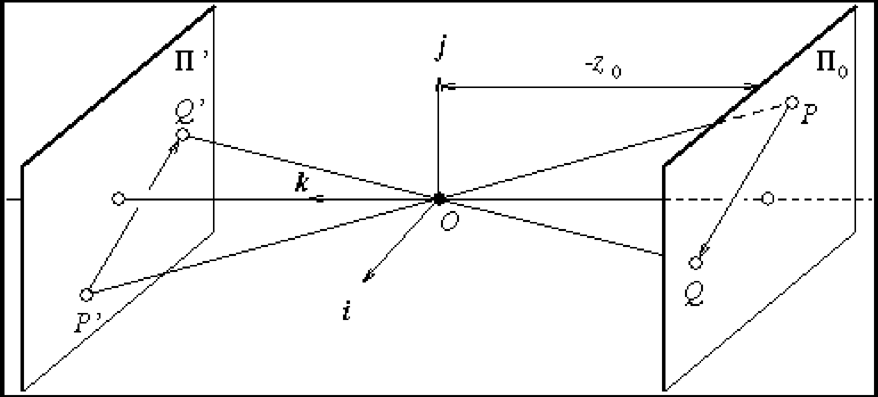
- Special case of perspective projection
  - What's the projection matrix?



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Other projection models: Weak perspective

- *Perspective* effects, but not over the scale of *individual* objects
- Collect points into a group at about the same depth, then divide each point by the depth of its group
- Advantage: easy
- Disadvantage : only approximate



$$(x, y, z) \rightarrow \left(\frac{fx}{z_0}, \frac{fy}{z_0} \right)$$

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- Disadvantage : only approximate

$$(x, y, z) \to \left(\frac{fx}{z_0}, \frac{fy}{z_0} \right)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/s \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1/s \end{bmatrix} \Rightarrow (sx, sy)$$

Three camera projections

	3-d point	2-d image position
(1) Perspective:	(x,y,z)	$\rightarrow \left(\dfrac{fx}{z}, \dfrac{fy}{z}\right)$
(2) Weak perspective:	(x,y,z)	$\rightarrow \left(\dfrac{fx}{z_0}, \dfrac{fy}{z_0}\right)$
(3) Orthographic:	(x,y,z)	$\rightarrow (x,y)$