COMPSCI 4Z03: Directed Readings in Distance Geometry

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Topic

Distance geometry concerns the estimation or characterization of points in an abstract space using only pairwise measurements of the distance between these points. For example, the classic problem of calculating the area of a triangle in $\mathbb{R}^2$ given only its side lengths was solved over 2000 years ago by Heron of Alexandria [1]. This ancient problem captures the essence of modern research in distance geometry, which typically involves a large number of points, may consider non-Euclidean spaces (e.g., smooth manifolds like the surface of a hypersphere), and is concerned with efficient computation. Some practical applications include the synchronization of clocks, determination of molecular structure, sensor network localization, data analysis, robot control, and computer vision [2].

The proposed directed readings course will begin with the fundamentals of distance geometry as covered by survey papers [3] and elementary textbooks [1]. The student will then identify an application domain or family of computational methods of interest and conduct a literature review on the state of the art. Possible topics include convex analysis and optimization for distance geometry [2], methods from algebraic geometry [4], or applications of distance geometry in robotics.

Learning Goals and Objectives

By the end of the course, the successful student will be able to:

1. summarize the scope of distance geometry;
2. formulate a variety of problems in the language of distance geometry;
3. describe and categorize different problems in distance geometry;
4. recognize similar distance-geometric patterns in problems from a variety of application domains;
5. list and explain open computational problems in distance geometry; and
6. implement software basic algorithms for solving distance geometry problems.

The overall goal, which requires that the student achieve these learning objectives, is to prepare the student for graduate level research in distance geometry or a related field.

Resources Required

Due to the availability of free academic literature, datasets, and scientific computing software, the student only requires a commodity laptop computer or its equivalent.

Deliverables and Evaluation

The course will have three written deliverables:

1. a broad survey of the basics of distance geometry (30% and due Feb. 15, 2024);
2. a short proposal document identifying a state-of-the-art research area of interest (20% and due Mar. 1, 2024); and
3. a detailed literature review of the research area of interest (50% and due at the end of the term).

Depending on the student's progress and interests, the first and third deliverables may also include a computational component or mathematical derivations and proofs.

Planned Contact with Instructor

The student will have weekly hour-long virtual or in-person progress meetings with the instructor, during which they can request direct feedback, present their latest progress, and work through problems together. Additional ad hoc meetings are also an option. Finally, detailed written feedback will be provided in a timely manner for each of the three deliverables.

References

[1] L. Liberti and C. Lavor, *Euclidean Distance Geometry*, ser. Springer Undergraduate Texts in Mathematics and Technology. Springer International Publishing, 2017.

[2] J. Dattorro, *Convex Optimization & Euclidean Distance Geometry.* MeBoo, 2005.

[3] I. Dokmanic, R. Parhizkar, J. Ranieri, and M. Vetterli, "Euclidean Distance Matrices: Essential theory, algorithms, and applications," *IEEE Signal Processing Magazine*, vol. 32, no. 6, pp. 12–30, Nov. 2015.

[4] D. A. Cox, J. Little, and D. O'Shea, *Ideals, Varieties, and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra*, ser. Undergraduate Texts in Mathematics. Cham: Springer International Publishing, 2015.