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# 1.A

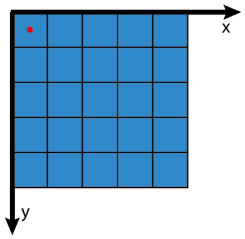
https://medium.com/mlreview/a-guide-to-receptive-field-arithmetic-for-convolutional-neural-networks-e0f514068807

$n$ : number of features
$r$ : receptive field size
$j$ : jump (distance between two consecutive features)
$start$ : center coordinate of the first feature

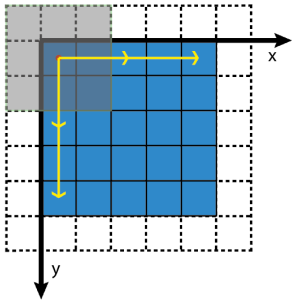
$k$ : convolution kernel size
$p$ : convolution padding size
$s$ : convolution stride size

$$n_{out} = \left\lfloor \frac{n_{in} + 2p - k}{s} \right\rfloor + 1$$
$$j_{out} = j_{in} * s$$
$$r_{out} = r_{in} + (k-1) * j_{in}$$
$$start_{out} = start_{in} + \left(\frac{k-1}{2} - p\right) * j_{in}$$

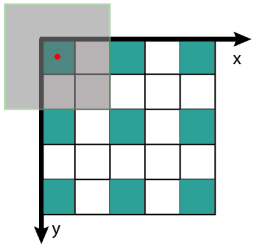
Layer 0: $n_0 = 5;\ r_0 = 1;\ j_0 = 1;\ start_0 = 0.5$



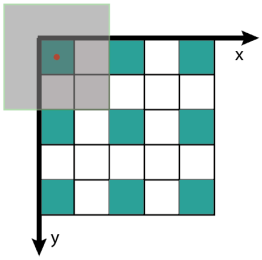

Conv1: $k_1 = 3;\ p_1 = 1;\ s_1 = 2$



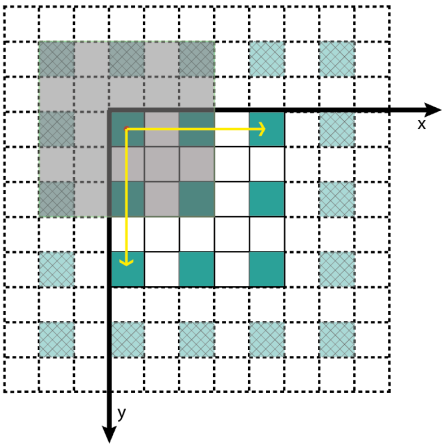

Layer 1: $n_1 = 3;\ r_1 = 3;\ j_1 = 2;\ start_1 = 0.5$



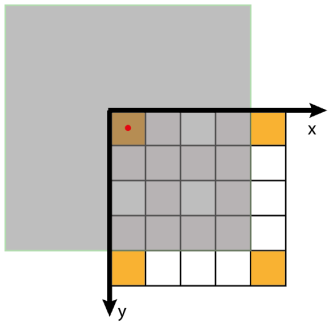

Layer 1: $n_1 = 3;\ r_1 = 3; j_1 = 2;\ start_1 = 0.5$




Conv2: $k_2 = 3;\ p_2 = 1;\ s_2 = 2$




Layer 2: $n_3 = 2;\ r_3 = 7; j_3 = 4;\ start_1 = 0.5$




In [11]:

```
# [filter size, stride, padding]
#Assume the two dimensions are the same
#Each kernel requires the following parameters:
# - k_i: kernel size
# - s_i: stride
# - p_i: padding (if padding is uneven, right padding will higher than left padding; "S
#
#Each layer i requires the following parameters to be fully represented:
# - n_i: number of feature (data layer has n_1 = imagesize )
# - j_i: distance (projected to image pixel distance) between center of two adjacent fe
# - r_i: receptive field of a feature in layer i
```

```
# - start_i: position of the first feature's receptive field in layer i (idx start from

import math
convnet =   [[11,4,0],[3,2,0],[5,1,2],[3,2,0],[3,1,1],[3,1,1],[3,1,1],[3,2,0],[6,1,0],
layer_names = ['conv1','pool1','conv2','pool2','conv3','conv4','conv5','pool5','fc6-con
imsize = 227

def outFromIn(conv, layerIn):
    n_in = layerIn[0]
    j_in = layerIn[1]
    r_in = layerIn[2]
    start_in = layerIn[3]
    k = conv[0]
    s = conv[1]
    p = conv[2]

    n_out = math.floor((n_in - k + 2*p)/s) + 1
    actualP = (n_out-1)*s - n_in + k
    pR = math.ceil(actualP/2)
    pL = math.floor(actualP/2)

    j_out = j_in * s
    r_out = r_in + (k - 1)*j_in
    start_out = start_in + ((k-1)/2 - pL)*j_in
    return n_out, j_out, r_out, start_out

def printLayer(layer, layer_name):
    print(layer_name + ":")
    print("\t n features: %s \n \t jump: %s \n \t receptive size: %s \t start: %s " % (

layerInfos = []
#first layer is the data layer (image) with n_0 = image size; j_0 = 1; r_0 = 1; and sta
# print ("-------Net summary------")
currentLayer = [imsize, 1, 1, 1]
printLayer(currentLayer, "input image")

for i in range(len(convnet)):
    currentLayer = outFromIn(convnet[i], currentLayer)
    layerInfos.append(currentLayer)
    printLayer(currentLayer, layer_names[i])
print ("-------------------------")
layer_name ="conv1"
print_layer_names = ['conv1','pool1','conv2','pool2']
for layer_name in  print_layer_names:
    layer_idx = layer_names.index(layer_name)
    idx_x = 1
    idx_y = 1
    n = layerInfos[layer_idx][0]
    j = layerInfos[layer_idx][1]
    r = layerInfos[layer_idx][2]
    start = layerInfos[layer_idx][3]
    if(idx_x < n):
#     assert(idx_y < n)
        print(layer_name,':')
        print('number of features=',n)
        print ("receptive field: (%s, %s)" % (r, r))
        print ("start: (%s, %s)" % ((start+idx_x*j)-r/2, (start+idx_y*j)-r/2))
        print ("center: (%s, %s)" % (start+idx_x*j, start+idx_y*j))
        print ("end: (%s, %s)" % ((start+idx_x*j)+r/2, (start+idx_y*j)+r/2))
        print('-------------------------')
```

```
input image:
         n features: 227
         jump: 1
         receptive size: 1       start: 1
conv1:
         n features: 55
         jump: 4
         receptive size: 11      start: 6.0
pool1:
         n features: 27
         jump: 8
         receptive size: 19      start: 10.0
conv2:
         n features: 27
         jump: 8
         receptive size: 51      start: 10.0
pool2:
         n features: 13
         jump: 16
         receptive size: 67      start: 18.0
conv3:
         n features: 13
         jump: 16
         receptive size: 99      start: 18.0
conv4:
         n features: 13
         jump: 16
         receptive size: 131     start: 18.0
conv5:
         n features: 13
         jump: 16
         receptive size: 163     start: 18.0
pool5:
         n features: 6
         jump: 32
         receptive size: 195     start: 34.0
fc6-conv:
         n features: 1
         jump: 32
         receptive size: 355     start: 114.0
fc7-conv:
         n features: 1
         jump: 32
         receptive size: 355     start: 114.0
------------------------
conv1 :
number of features= 55
receptive field: (11, 11)
start: (4.5, 4.5)
center: (10.0, 10.0)
end: (15.5, 15.5)
------------------------
pool1 :
number of features= 27
receptive field: (19, 19)
start: (8.5, 8.5)
center: (18.0, 18.0)
end: (27.5, 27.5)
------------------------
conv2 :
number of features= 27
receptive field: (51, 51)
start: (-7.5, -7.5)
center: (18.0, 18.0)
end: (43.5, 43.5)
```

```
-------------------------
pool2 :
number of features= 13
receptive field: (67, 67)
start: (0.5, 0.5)
center: (34.0, 34.0)
end: (67.5, 67.5)
-------------------------
```

# 1.B

In [14]:
```
import torch.nn as nn
import numpy as np
import torch as torch
import cv2
from torch.autograd import Variable
import torch.nn.functional as F
from torch.optim import Adam
model = torch.hub.load('pytorch/vision:v0.9.0', 'alexnet', pretrained=True)
model.eval()
```

```
Using cache found in C:\Users\Administrator/.cache\torch\hub\pytorch_vision_v0.9.0
```

Out[14]:
```
AlexNet(
  (features): Sequential(
    (0): Conv2d(3, 64, kernel_size=(11, 11), stride=(4, 4), padding=(2, 2))
    (1): ReLU(inplace=True)
    (2): MaxPool2d(kernel_size=3, stride=2, padding=0, dilation=1, ceil_mode=False)
    (3): Conv2d(64, 192, kernel_size=(5, 5), stride=(1, 1), padding=(2, 2))
    (4): ReLU(inplace=True)
    (5): MaxPool2d(kernel_size=3, stride=2, padding=0, dilation=1, ceil_mode=False)
    (6): Conv2d(192, 384, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
    (7): ReLU(inplace=True)
    (8): Conv2d(384, 256, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
    (9): ReLU(inplace=True)
    (10): Conv2d(256, 256, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
    (11): ReLU(inplace=True)
    (12): MaxPool2d(kernel_size=3, stride=2, padding=0, dilation=1, ceil_mode=False)
  )
  (avgpool): AdaptiveAvgPool2d(output_size=(6, 6))
  (classifier): Sequential(
    (0): Dropout(p=0.5, inplace=False)
    (1): Linear(in_features=9216, out_features=4096, bias=True)
    (2): ReLU(inplace=True)
    (3): Dropout(p=0.5, inplace=False)
    (4): Linear(in_features=4096, out_features=4096, bias=True)
    (5): ReLU(inplace=True)
    (6): Linear(in_features=4096, out_features=1000, bias=True)
  )
)
```

In [15]:
```
# def get_img_tensor(img_file_name):
#     input_image = Image.open(img_file_name)
# #     print(input_image)
#     preprocess = transforms.Compose([
#         transforms.Resize(224),
#         transforms.CenterCrop(224),
#         transforms.ToTensor(),
#         transforms.Normalize(mean=[0.485, 0.456, 0.406], std=[0.229, 0.224, 0.225]),
#     ])
#     return preprocess(input_image)
```

In [16]:
```
def preprocess(image_path):
    ALEX_MEAN = [0.485, 0.456, 0.406]
    ALEX_STD=[0.229, 0.224, 0.225]
    original_image = cv2.imread(image_path)
    image = np.copy(original_image)

    out = cv2.resize(image, (224, 224))
    out = out.astype(np.float32) / 256.
    image = np.clip(image, 0, 1)
    out = out[:, :, [2, 1, 0]]  # swap channel from BGR to RGB
    out[:, :, 0] = (out[:, :, 0] - ALEX_MEAN[0]) / ALEX_STD[0]
    out[:, :, 1] = (out[:, :, 1] - ALEX_MEAN[1]) / ALEX_STD[1]
    out[:, :, 2] = (out[:, :, 2] - ALEX_MEAN[2]) / ALEX_STD[2]
    out = np.transpose(out, (2, 0, 1))
    return out
```

In [17]:
```
def deprocess(tensor):
    ALEX_MEAN = [0.485, 0.456, 0.406]
    ALEX_STD=[0.229, 0.224, 0.225]

    out = tensor.data.cpu().numpy()
    out = np.reshape(out,[3, 224, 224])
    out = np.transpose(out, (1, 2, 0))

    out[:, :, 0] = (out[:, :, 0] * ALEX_STD[0]) + ALEX_MEAN[0]
    out[:, :, 1] = (out[:, :, 1] * ALEX_STD[1]) + ALEX_MEAN[1]
    out[:, :, 2] = (out[:, :, 2] * ALEX_STD[2]) + ALEX_MEAN[2]
    out = out[:, :, [2, 1, 0]]  # swap channel from RGB to BGR
    out = np.clip(out, 0, 1)
    out = out * 256

    out = out.astype(np.uint8)
    return out
```

In [30]:
```
# sample execution (requires torchvision)
from PIL import Image
from torchvision import transforms
def get_image_label(img, is_tensor=False):
    if is_tensor ==False:
        input_tensor =torch.from_numpy(preprocess(img))
    else:
        input_tensor=img
    input_batch = input_tensor.unsqueeze(0) # create a mini-batch as expected by the mo

    # move the input and model to GPU for speed if available
    if torch.cuda.is_available():
        input_batch = input_batch.to('cuda')
        model.to('cuda')

    with torch.no_grad():
        output = model(input_batch)
    # Tensor of shape 1000, with confidence scores over Imagenet's 1000 classes
    # print(output[0])
    # The output has unnormalized scores. To get probabilities, you can run a softmax o
    probabilities = torch.nn.functional.softmax(output[0], dim=0)
    # print(probabilities)
```

```
    # Read the categories
    with open("imagenet_classes.txt", "r") as f:
        categories = [s.strip() for s in f.readlines()]
    # Show top categories per image
    # top5_prob, top5_catid = torch.topk(probabilities, 5)
    # for i in range(top5_prob.size(0)):
    #     print(categories[top5_catid[i]], top5_prob[i].item())

    top_prob, top_catid = torch.topk(probabilities, 1)
#     print(top_prob)
    return top_catid[0],categories[top_catid[0]]
```

In [62]:

```
from imageio import imread
import matplotlib.pyplot as plt
urls=[["https://raw.githubusercontent.com/ajschumacher/imagen/master/imagen/n01726692_4
      ["https://raw.githubusercontent.com/ajschumacher/imagen/master/imagen/n01674464_3
      ["https://raw.githubusercontent.com/ajschumacher/imagen/master/imagen/n01770393_1
      ["https://raw.githubusercontent.com/ajschumacher/imagen/master/imagen/n02129165_2
      ["https://raw.githubusercontent.com/ajschumacher/imagen/master/imagen/n02129604_2

for url in urls:
#     image = imread(url[0])
#     plt.imshow(image)
    # url, filename = ("https://github.com/pytorch/hub/raw/master/images/dog.jpg", "dog
    # !wget("https://github.com/ajschumacher/imagen/blob/master/imagen/n00007846_147031_
    http_url, filename = (url[0], url[1])
    label_code,label= get_image_label(filename)
    url.append(label_code.item())
    print("ground truth=",filename," predicted=",label)
```

```
ground truth= snake.jpg  predicted= water snake
ground truth= lizard.jpg  predicted= agama
ground truth= scorpion.jpg  predicted= scorpion
ground truth= lion.jpg  predicted= lion
ground truth= tiger.jpg  predicted= tiger
```

In [64]:

```
print(urls)
```

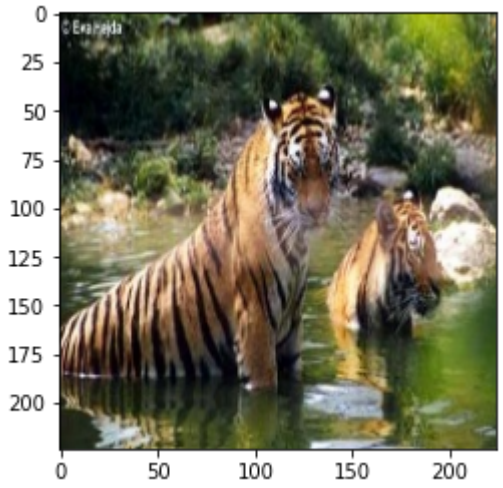
```
[['https://raw.githubusercontent.com/ajschumacher/imagen/master/imagen/n01726692_4802_sn
ake.jpg', 'snake.jpg', 58], ['https://raw.githubusercontent.com/ajschumacher/imagen/mast
er/imagen/n01674464_3490_lizard.jpg', 'lizard.jpg', 42], ['https://raw.githubuserconten
t.com/ajschumacher/imagen/master/imagen/n01770393_12410_scorpion.jpg', 'scorpion.jpg', 7
1], ['https://raw.githubusercontent.com/ajschumacher/imagen/master/imagen/n02129165_2762
_lion.jpg', 'lion.jpg', 291], ['https://raw.githubusercontent.com/ajschumacher/imagen/ma
ster/imagen/n02129604_20374_tiger.jpg', 'tiger.jpg', 292]]
```

# 1.C

In [34]:

```
img = preprocess(urls[4][1])
img_tensor = torch.from_numpy(img)
img_tensor = img_tensor.unsqueeze_(0)
img_variable = Variable(img_tensor, requires_grad=True)

pre_image = deprocess(img_tensor)
cv2.imwrite("pre_processed.jpg", pre_image)
plt.figure()
plt.imshow(cv2.cvtColor(cv2.imread("pre_processed.jpg"),cv2.COLOR_BGR2RGB))
get_image_label("pre_processed.jpg")
```

Out[34]: (tensor(292), 'tiger')



In [75]:
```
# def generate_adverserial(input_image,ground_truth):
#     print('target_variable=',target_variable)
#     optimizer = Adam([img_variable], lr=0.005)
#     criteron = nn.CrossEntropyLoss()
#     print("\nStarting Optimization...")
#     for iteration in range(1,100):
#         final_image = deprocess(img_variable)
#         cv2.imwrite("temp.jpg", final_image)
#         adv_label=get_image_label("temp.jpg")[0].item()
#         print('adv_label',adv_label)
#         if adv_label==ground_truth:
#             fc_out = model(input_image)
#             fc_out = F.softmax(fc_out, dim=1)

#             optimizer.zero_grad()
#             loss = criteron(fc_out, target_variable)
#             print(img_variable.grad.shape)

#             loss.backward(retain_graph = True)
#         #     grads = torch.sign(img_variable.grad.data)
#         #     print(img_variable.grad.data)
#         #     optimizer.zero_grad()
#             optimizer.step()
#         #     epsilon = [0.0001, 0.001, 0.0013, 0.009, 0.015, 0.120, 0.25, 0.302]
#         #     for i in epsilon:
#         #         img_variable = img_variable.data + i

#             if iteration == 1 or iteration%50 == 0:
#                 print("Current Iteration: ",iteration,"\tCurrent Cost: ", loss)

#         else:
#             break
#     return input_image
```

In [77]:
```
def generate_adverserial(input_image,target_variable,iterations):
    optimizer = Adam([img_variable], lr=0.005)
    criteron = nn.CrossEntropyLoss()
    print("\nStarting Optimization...")
    for iteration in range(1,iterations):
```

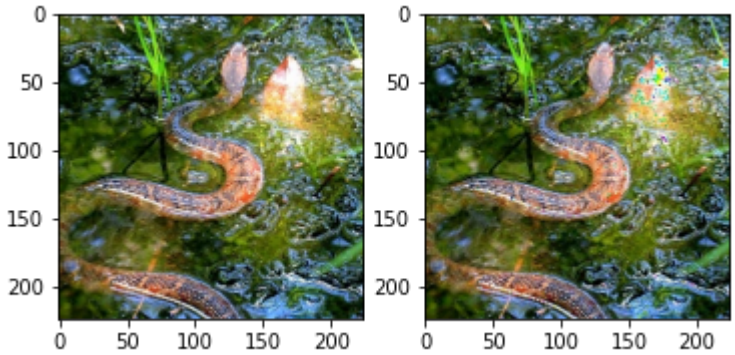
```
        fc_out = model(input_image)
        fc_out = F.softmax(fc_out, dim=1)

        optimizer.zero_grad()
        loss = criteron(fc_out, target_variable)

        loss.backward(retain_graph = True)
    #     grads = torch.sign(img_variable.grad.data)
    #     print(img_variable.grad.data)
    #     optimizer.zero_grad()
        optimizer.step()
    #     epsilon = [0.0001, 0.001, 0.0013, 0.009, 0.015, 0.120, 0.25, 0.302]
    #     for i in epsilon:
    #         img_variable = img_variable.data + i

        if iteration == 1 or iteration%50 == 0:
            print("Current Iteration: ",iteration,"\tCurrent Cost: ", loss)
    #         final_image = deprocess(img_variable)
    #         cv2.imwrite("temp.jpg", final_image)
    #         print(get_image_label("temp.jpg"))
    #         print(fc_out,target_variable)
    return input_image
```

In [79]:

```
targets=[5,10,15]
for url in urls:
    img = preprocess(url[1])
    img_tensor = torch.from_numpy(img)
    img_tensor = img_tensor.unsqueeze_(0)
    pre_image = deprocess(img_tensor)
    for target in targets:
        pre_processed="pre_processed"+url[1]+"-"+str(target)+".jpg"
        cv2.imwrite(pre_processed, pre_image)
        img = preprocess(url[1])
        img_tensor = torch.from_numpy(img)
        img_tensor = img_tensor.unsqueeze_(0)
        img_variable = Variable(img_tensor, requires_grad=True)
        target_vector = np.array([target], dtype=np.int64)
        target_tensor = torch.from_numpy(target_vector)
        target_variable = Variable(target_tensor)
#         adv_image=generate_adverserial(img_variable,target_variable)
#         adv_image=generate_adverserial(img_variable,url[2])
        adv_image=generate_adverserial(img_variable,target_variable,30)
        final_image = deprocess(adv_image)
        advout="adverseial_"+url[1]+"-"+str(target)+".jpg"
        cv2.imwrite(advout, final_image)
        print("original:",get_image_label(pre_processed))
        print("adverserial:",get_image_label(advout))
        f = plt.figure()
        f.add_subplot(1,2, 1)
        plt.imshow(cv2.cvtColor(cv2.imread(pre_processed),cv2.COLOR_BGR2RGB))
        f.add_subplot(1,2, 2)
        plt.imshow(cv2.cvtColor(cv2.imread(advout),cv2.COLOR_BGR2RGB))
        plt.show(block=True)
```

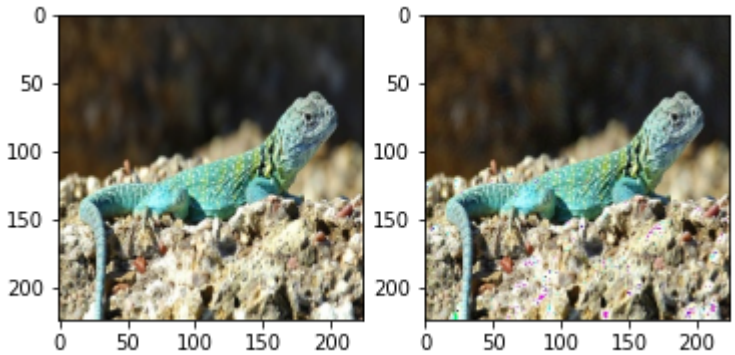
```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9094, grad_fn=<NllLossBackward>)
original: (tensor(58), 'water snake')
adverserial: (tensor(30), 'bullfrog')
```

```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9094, grad_fn=<NllLossBackward>)
original: (tensor(58), 'water snake')
adverserial: (tensor(58), 'water snake')
```



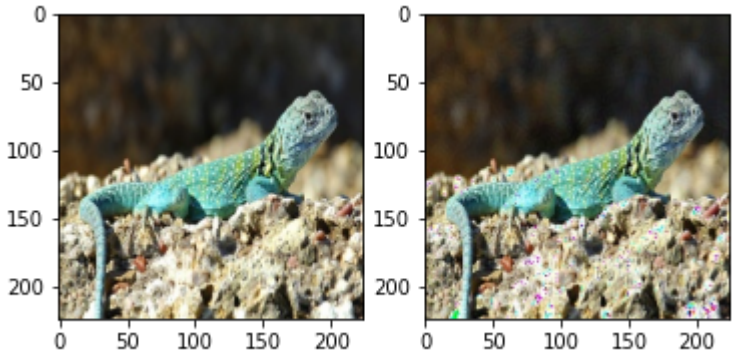
```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9094, grad_fn=<NllLossBackward>)
original: (tensor(58), 'water snake')
adverserial: (tensor(15), 'robin')
```

```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9090, grad_fn=<NllLossBackward>)
original: (tensor(42), 'agama')
adverserial: (tensor(300), 'tiger beetle')
```

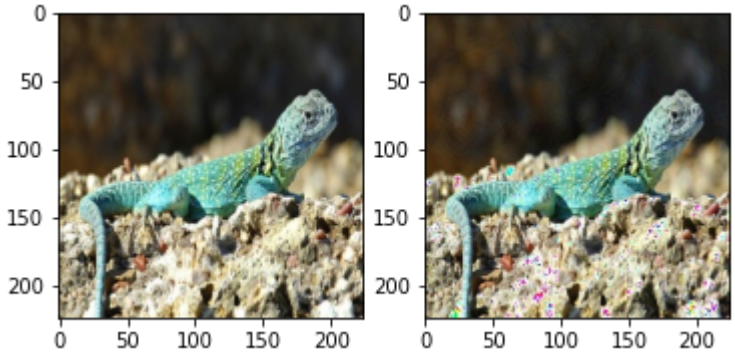


```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9090, grad_fn=<NllLossBackward>)
```

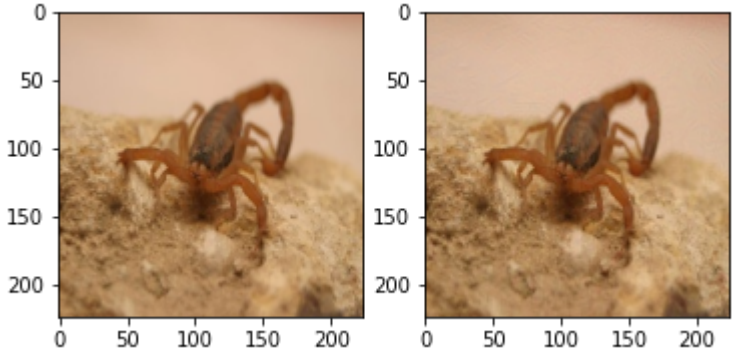
```
original: (tensor(42), 'agama')
adverserial: (tensor(10), 'brambling')
```



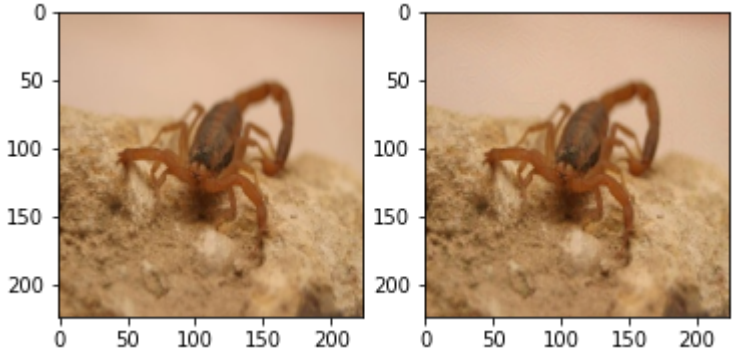
```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9090, grad_fn=<NllLossBackward>)
original: (tensor(42), 'agama')
adverserial: (tensor(15), 'robin')
```



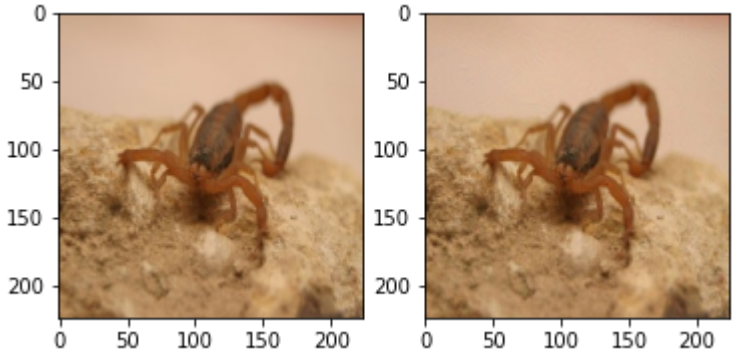
```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9093, grad_fn=<NllLossBackward>)
original: (tensor(71), 'scorpion')
adverserial: (tensor(5), 'electric ray')
```



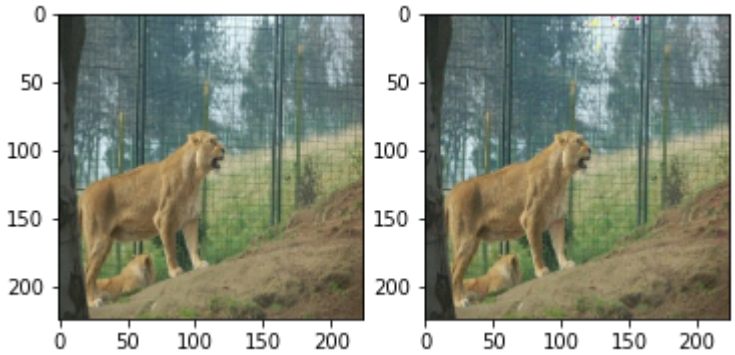
```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9093, grad_fn=<NllLossBackward>)
original: (tensor(71), 'scorpion')
adverserial: (tensor(73), 'barn spider')
```



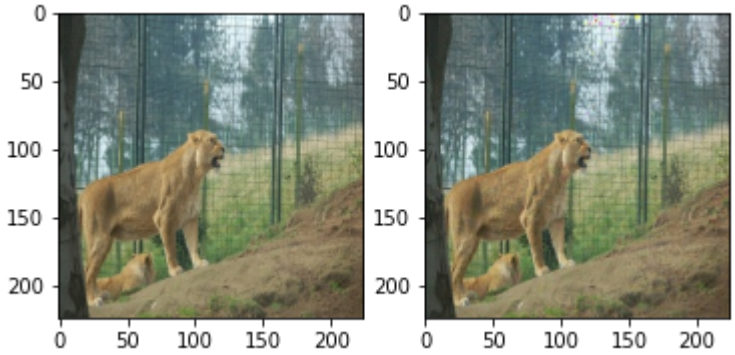
```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9093, grad_fn=<NllLossBackward>)
original: (tensor(71), 'scorpion')
adverserial: (tensor(73), 'barn spider')
```



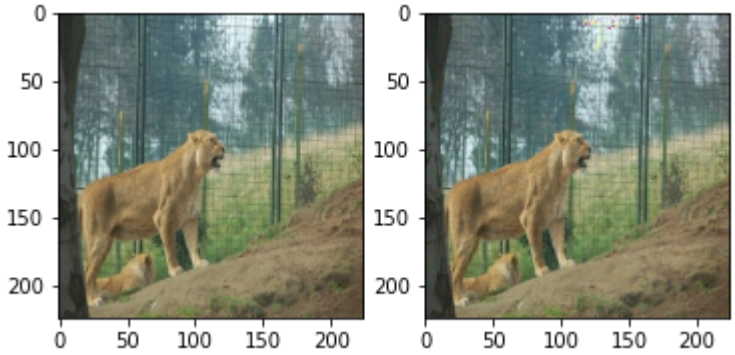
```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9089, grad_fn=<NllLossBackward>)
original: (tensor(291), 'lion')
adverserial: (tensor(48), 'Komodo dragon')
```



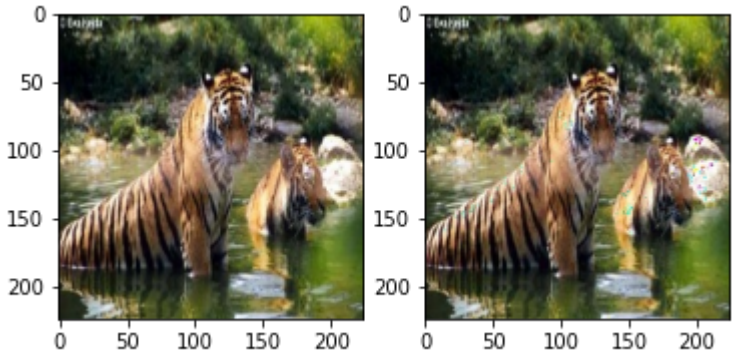
```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9089, grad_fn=<NllLossBackward>)
original: (tensor(291), 'lion')
adverserial: (tensor(10), 'brambling')
```



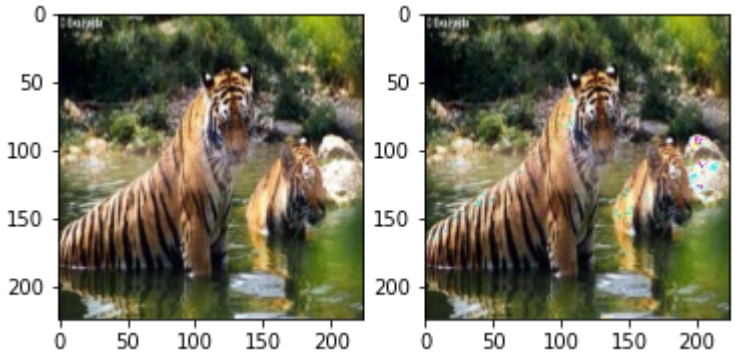
```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9089, grad_fn=<NllLossBackward>)
original: (tensor(291), 'lion')
adverserial: (tensor(15), 'robin')
```



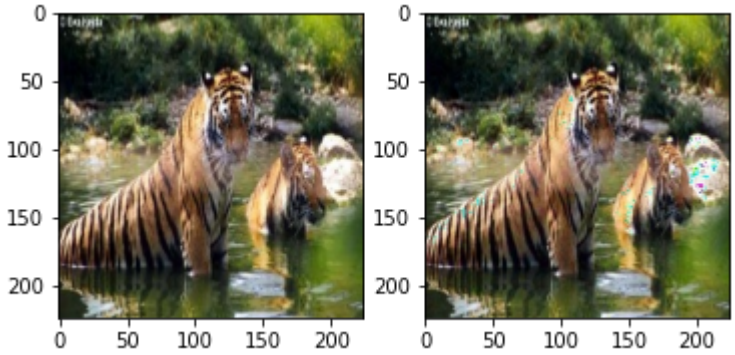
```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9093, grad_fn=<NllLossBackward>)
original: (tensor(292), 'tiger')
adverserial: (tensor(340), 'zebra')
```



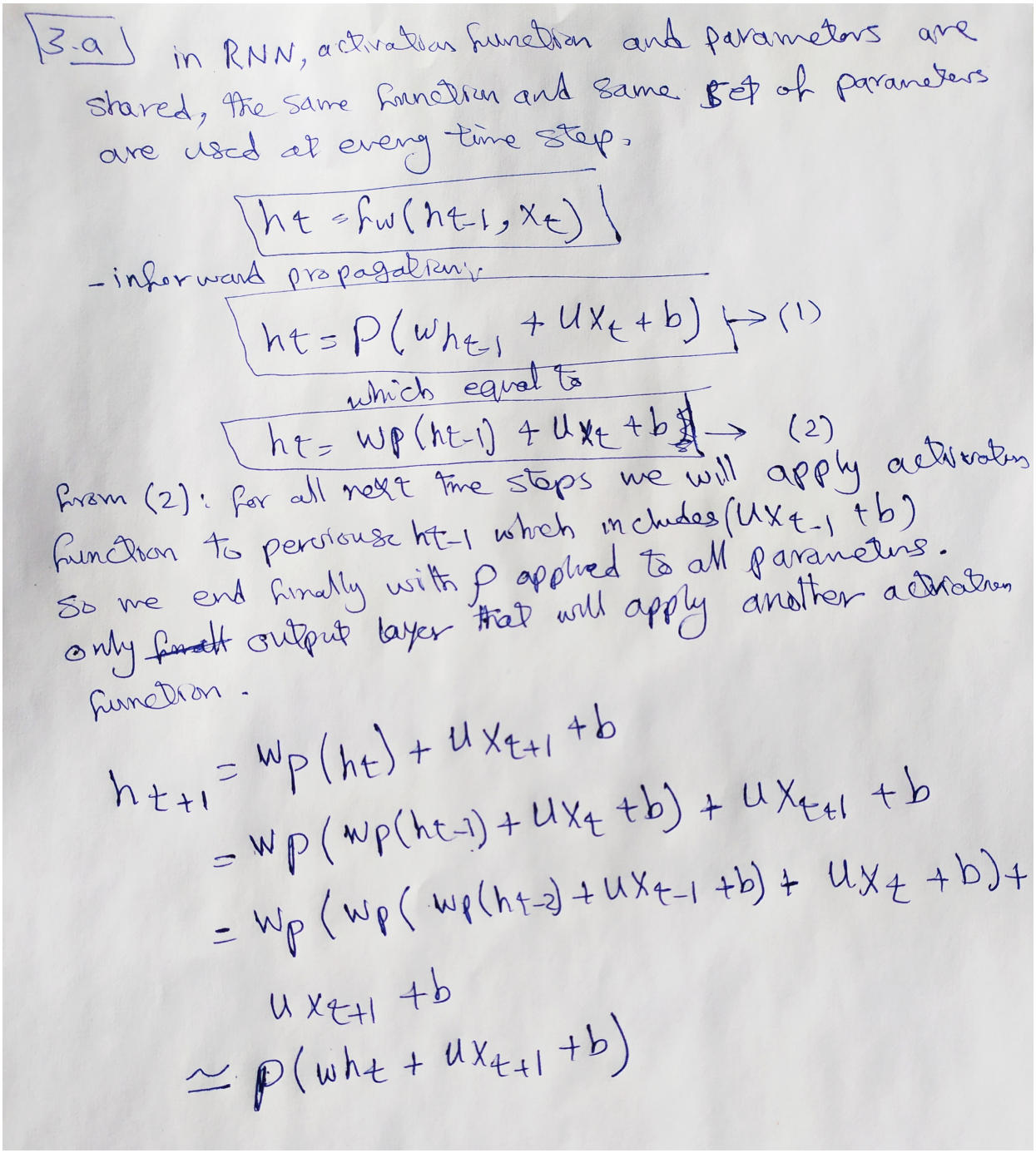
```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9093, grad_fn=<NllLossBackward>)
original: (tensor(292), 'tiger')
adverserial: (tensor(396), 'lionfish')
```



```
Starting Optimization...
Current Iteration:  1   Current Cost:  tensor(6.9093, grad_fn=<NllLossBackward>)
original: (tensor(292), 'tiger')
adverserial: (tensor(15), 'robin')
```



# 3.B




In [190…

```
import numpy as np
np.random.seed(0)
class RecurrentNetwork(object):
    """W_hh means a weight matrix that accepts a hidden state and produce a new hidden
    Similarly, W_xh represents a weight matrix that accepts an input vector and produce
    """
    def __init__(self):
        self.hidden_state = np.zeros((3, 3))
        self.hidden_state2 = np.zeros((3, 3))
        self.W_hh = np.random.randn(3, 3)
        self.W_xh = np.random.randn(3, 3)
        self.W_hy = np.random.randn(3, 3)
```

```
        self.Bh = np.random.randn(3,)
        self.By = np.random.rand(3,)

    def forward_prop(self, x):
        # The order of which you do dot product is entirely up to you. The gradient upd
        # as long as the matrix dimension matches up.
        self.hidden_state = np.tanh(np.dot(self.hidden_state, self.W_hh) + np.dot(x, se
#         print(self.W_hy.dot(self.hidden_state) + self.By)
        self.hidden_state2= np.tanh(np.dot(self.hidden_state2, self.W_hh)) + np.dot(x,
#         print(self.W_hy.dot(self.hidden_state2) + self.By)
        return self.W_hy.dot(self.hidden_state) + self.By,self.W_hy.dot(self.hidden_sta

input_vector = np.ones((3, 3))
RNN = RecurrentNetwork()

# Notice that same input, but leads to different ouptut at every single time step.
print(RNN.forward_prop(input_vector))
print(RNN.forward_prop(input_vector))
print(RNN.forward_prop(input_vector))
```

```
(array([[-1.73665315, -2.40366542, -2.72344361],
       [ 1.61591482,  1.45557046,  1.13262256],
       [ 1.68977504,  1.54059305,  1.21757531]]), array([[-3.13455948, -9.50547134, -9.4
2482   ],
       [ 1.96646911,  3.23649703,  2.81313303],
       [ 2.07884955,  3.51721453,  3.08274661]]))
(array([[-2.15023381, -2.41205828, -2.71701457],
       [ 1.71962883,  1.45767515,  1.13101034],
       [ 1.80488553,  1.542929  ,  1.21578594]]), array([[ -6.22857733, -12.5994636 ,  -
6.39824394],
       [  2.74235888,   4.01238038,   2.0541557 ],
       [  2.93999702,   4.37835488,   2.24036999]]))
(array([[-2.15024751, -2.41207375, -2.720968  ],
       [ 1.71963227,  1.45767903,  1.13200175],
       [ 1.80488935,  1.54293331,  1.21688628]]), array([[ -6.22857733, -12.59948899,  -
6.41337362],
       [  2.74235888,   4.01238675,   2.05794978],
       [  2.93999702,   4.37836194,   2.24458098]]))
```

# 3.C

https://mmuratarat.github.io/2019-02-07/bptt-of-rnn

In vanilla RNNs, vanishing/exploding gradient comes from the repeated application of the recurrent connections. which happen because of recursive derivative we need to compute

$$\prod_{j=k}^{t} \frac{\partial h_{j+1}}{\partial h_j} = \frac{\partial h_{t+1}}{\partial h_k} = \frac{\partial h_{t+1}}{\partial h_t}\frac{\partial h_t}{\partial h_{t-1}} \cdots \frac{\partial h_{k+1}}{\partial h_k}$$

if we want to backpropagate through t-k timesteps, this gradient will be:

$$\prod_{j=k}^{t} \frac{\partial h_{j+1}}{\partial h_j} = \prod_{j=k}^{t} diag(\phi_h'(W_{xh}^T \cdot X_{j+1} + W_{hh}^T \cdot h_j + b_h)W_{hh}$$

if the dominant eigenvalue of the matrix Whh is greater than 1, the gradient explodes. If it is less than 1, the gradient vanishes.

The gradient $\frac{\partial h_{t+1}}{\partial h_k}$ is a product of Jacobian matrices that are multiplied many times, $t-k$ times in our case:

$$\left\| \frac{\partial h_{t+1}}{\partial h_k} \right\| = \left\| \prod_{j=k}^{t} \frac{\partial h_{j+1}}{\partial h_j} \right\| \leq (\gamma_W \gamma_h)^{t-k}$$

This can become very small or very large quickly, and the locality assumption of gradient descent breaks down as the sequence gets longer (i.e the distance between $t$ and $k$ increases). Then the value of $\gamma$ will determine if the gradient either gets very large (explodes) or gets very small (vanishes).

Since $\gamma$ is associated with the leading eigenvalues of $\frac{\partial h_{j+1}}{\partial h_j}$, the recursive product of $t-k$ Jacobian matrices makes it possible to influence the overall gradient in such a way that for $\gamma < 1$ the gradient tends to vanish while for $\gamma > 1$ the gradient tends to explode.

# 3.D

gradient tends to vanish when Y<1 and tends to explode when Y>1.

# 4.B

Self attention sees its input as a set, not a sequence. If we permute the input sequence, the output sequence will be exactly the same, except permuted also (i.e. self-attention is permutation equivariant). We will mitigate this somewhat when we build the full transformer, but the self-attention by itself actually ignores the sequential nature of the input.
http://peterbloem.nl/blog/transformers

In [79]:
```
from scipy.special import softmax
import itertools
import numpy as np
from copy import copy, deepcopy
T=5
D=5
p0=range(0,T)
permutations=list(itertools.permutations(p0))
X = np.random.randint(10,size=(T, D))
Wq=np.random.rand(D, D)
Wk=np.random.rand(D, D)
Wv=np.random.rand(D, D)
# print(X)
def s_x(x):
    Q=x@Wq
    # print(Wq)
    K=x@Wk
    # print(Wk)
    V=x@Wv
    # print(X)
    alpha=2
    A=(Q@K.transpose())/alpha
    s=softmax(A)@V
```

```
    return s

for p in permutations:
    psx=deepcopy(s_x(X))
    psx[p0]=psx[list(p)]
    px=deepcopy(X)
    px[p0]=px[list(p)]
    spx=s_x(px)
    print('p:',p,'-> psx==spx:',np.allclose(psx,spx))
```

```
p: (0, 1, 2, 3, 4) -> psx==spx: True
p: (0, 1, 2, 4, 3) -> psx==spx: True
p: (0, 1, 3, 2, 4) -> psx==spx: True
p: (0, 1, 3, 4, 2) -> psx==spx: True
p: (0, 1, 4, 2, 3) -> psx==spx: True
p: (0, 1, 4, 3, 2) -> psx==spx: True
p: (0, 2, 1, 3, 4) -> psx==spx: True
p: (0, 2, 1, 4, 3) -> psx==spx: True
p: (0, 2, 3, 1, 4) -> psx==spx: True
p: (0, 2, 3, 4, 1) -> psx==spx: True
p: (0, 2, 4, 1, 3) -> psx==spx: True
p: (0, 2, 4, 3, 1) -> psx==spx: True
p: (0, 3, 1, 2, 4) -> psx==spx: True
p: (0, 3, 1, 4, 2) -> psx==spx: True
p: (0, 3, 2, 1, 4) -> psx==spx: True
p: (0, 3, 2, 4, 1) -> psx==spx: True
p: (0, 3, 4, 1, 2) -> psx==spx: True
p: (0, 3, 4, 2, 1) -> psx==spx: True
p: (0, 4, 1, 2, 3) -> psx==spx: True
p: (0, 4, 1, 3, 2) -> psx==spx: True
p: (0, 4, 2, 1, 3) -> psx==spx: True
p: (0, 4, 2, 3, 1) -> psx==spx: True
p: (0, 4, 3, 1, 2) -> psx==spx: True
p: (0, 4, 3, 2, 1) -> psx==spx: True
p: (1, 0, 2, 3, 4) -> psx==spx: True
p: (1, 0, 2, 4, 3) -> psx==spx: True
p: (1, 0, 3, 2, 4) -> psx==spx: True
p: (1, 0, 3, 4, 2) -> psx==spx: True
p: (1, 0, 4, 2, 3) -> psx==spx: True
p: (1, 0, 4, 3, 2) -> psx==spx: True
p: (1, 2, 0, 3, 4) -> psx==spx: True
p: (1, 2, 0, 4, 3) -> psx==spx: True
p: (1, 2, 3, 0, 4) -> psx==spx: True
p: (1, 2, 3, 4, 0) -> psx==spx: True
p: (1, 2, 4, 0, 3) -> psx==spx: True
p: (1, 2, 4, 3, 0) -> psx==spx: True
p: (1, 3, 0, 2, 4) -> psx==spx: True
p: (1, 3, 0, 4, 2) -> psx==spx: True
p: (1, 3, 2, 0, 4) -> psx==spx: True
p: (1, 3, 2, 4, 0) -> psx==spx: True
p: (1, 3, 4, 0, 2) -> psx==spx: True
p: (1, 3, 4, 2, 0) -> psx==spx: True
p: (1, 4, 0, 2, 3) -> psx==spx: True
p: (1, 4, 0, 3, 2) -> psx==spx: True
p: (1, 4, 2, 0, 3) -> psx==spx: True
p: (1, 4, 2, 3, 0) -> psx==spx: True
p: (1, 4, 3, 0, 2) -> psx==spx: True
p: (1, 4, 3, 2, 0) -> psx==spx: True
p: (2, 0, 1, 3, 4) -> psx==spx: True
p: (2, 0, 1, 4, 3) -> psx==spx: True
p: (2, 0, 3, 1, 4) -> psx==spx: True
p: (2, 0, 3, 4, 1) -> psx==spx: True
p: (2, 0, 4, 1, 3) -> psx==spx: True
```

```
p: (2, 0, 4, 3, 1) -> psx==spx: True
p: (2, 1, 0, 3, 4) -> psx==spx: True
p: (2, 1, 0, 4, 3) -> psx==spx: True
p: (2, 1, 3, 0, 4) -> psx==spx: True
p: (2, 1, 3, 4, 0) -> psx==spx: True
p: (2, 1, 4, 0, 3) -> psx==spx: True
p: (2, 1, 4, 3, 0) -> psx==spx: True
p: (2, 3, 0, 1, 4) -> psx==spx: True
p: (2, 3, 0, 4, 1) -> psx==spx: True
p: (2, 3, 1, 0, 4) -> psx==spx: True
p: (2, 3, 1, 4, 0) -> psx==spx: True
p: (2, 3, 4, 0, 1) -> psx==spx: True
p: (2, 3, 4, 1, 0) -> psx==spx: True
p: (2, 4, 0, 1, 3) -> psx==spx: True
p: (2, 4, 0, 3, 1) -> psx==spx: True
p: (2, 4, 1, 0, 3) -> psx==spx: True
p: (2, 4, 1, 3, 0) -> psx==spx: True
p: (2, 4, 3, 0, 1) -> psx==spx: True
p: (2, 4, 3, 1, 0) -> psx==spx: True
p: (3, 0, 1, 2, 4) -> psx==spx: True
p: (3, 0, 1, 4, 2) -> psx==spx: True
p: (3, 0, 2, 1, 4) -> psx==spx: True
p: (3, 0, 2, 4, 1) -> psx==spx: True
p: (3, 0, 4, 1, 2) -> psx==spx: True
p: (3, 0, 4, 2, 1) -> psx==spx: True
p: (3, 1, 0, 2, 4) -> psx==spx: True
p: (3, 1, 0, 4, 2) -> psx==spx: True
p: (3, 1, 2, 0, 4) -> psx==spx: True
p: (3, 1, 2, 4, 0) -> psx==spx: True
p: (3, 1, 4, 0, 2) -> psx==spx: True
p: (3, 1, 4, 2, 0) -> psx==spx: True
p: (3, 2, 0, 1, 4) -> psx==spx: True
p: (3, 2, 0, 4, 1) -> psx==spx: True
p: (3, 2, 1, 0, 4) -> psx==spx: True
p: (3, 2, 1, 4, 0) -> psx==spx: True
p: (3, 2, 4, 0, 1) -> psx==spx: True
p: (3, 2, 4, 1, 0) -> psx==spx: True
p: (3, 4, 0, 1, 2) -> psx==spx: True
p: (3, 4, 0, 2, 1) -> psx==spx: True
p: (3, 4, 1, 0, 2) -> psx==spx: True
p: (3, 4, 1, 2, 0) -> psx==spx: True
p: (3, 4, 2, 0, 1) -> psx==spx: True
p: (3, 4, 2, 1, 0) -> psx==spx: True
p: (4, 0, 1, 2, 3) -> psx==spx: True
p: (4, 0, 1, 3, 2) -> psx==spx: True
p: (4, 0, 2, 1, 3) -> psx==spx: True
p: (4, 0, 2, 3, 1) -> psx==spx: True
p: (4, 0, 3, 1, 2) -> psx==spx: True
p: (4, 0, 3, 2, 1) -> psx==spx: True
p: (4, 1, 0, 2, 3) -> psx==spx: True
p: (4, 1, 0, 3, 2) -> psx==spx: True
p: (4, 1, 2, 0, 3) -> psx==spx: True
p: (4, 1, 2, 3, 0) -> psx==spx: True
p: (4, 1, 3, 0, 2) -> psx==spx: True
p: (4, 1, 3, 2, 0) -> psx==spx: True
p: (4, 2, 0, 1, 3) -> psx==spx: True
p: (4, 2, 0, 3, 1) -> psx==spx: True
p: (4, 2, 1, 0, 3) -> psx==spx: True
p: (4, 2, 1, 3, 0) -> psx==spx: True
p: (4, 2, 3, 0, 1) -> psx==spx: True
p: (4, 2, 3, 1, 0) -> psx==spx: True
p: (4, 3, 0, 1, 2) -> psx==spx: True
p: (4, 3, 0, 2, 1) -> psx==spx: True
p: (4, 3, 1, 0, 2) -> psx==spx: True
p: (4, 3, 1, 2, 0) -> psx==spx: True
```

```
p: (4, 3, 2, 0, 1) -> psx==spx: True
p: (4, 3, 2, 1, 0) -> psx==spx: True
```

# 4.C apply average function

In [194…

```
from skimage.measure import block_reduce
print(X)
for p in permutations:
# p= permutations[2]
    px=deepcopy(X)
    px[p0]=px[list(p)]
    spx=s_x(px)
    sx=s_x(X)

    sx_mean=np.average(sx)
#     print(sx_mean)
#     spx_mean=block_reduce(spx, (3,3), np.mean)
    spx_mean=np.average(spx)
#     print(spx_mean)
    print('p:',p,'-> sx_avg==spx_avg:',np.allclose(sx_mean,spx_mean))
```

```
[[6 7 7 0 0]
 [1 4 5 1 0]
 [8 1 0 0 1]
 [6 6 9 5 6]
 [6 3 2 2 3]]
p: (0, 1, 2, 3, 4) -> sx_avg==spx_avg: True
p: (0, 1, 2, 4, 3) -> sx_avg==spx_avg: True
p: (0, 1, 3, 2, 4) -> sx_avg==spx_avg: True
p: (0, 1, 3, 4, 2) -> sx_avg==spx_avg: True
p: (0, 1, 4, 2, 3) -> sx_avg==spx_avg: True
p: (0, 1, 4, 3, 2) -> sx_avg==spx_avg: True
p: (0, 2, 1, 3, 4) -> sx_avg==spx_avg: True
p: (0, 2, 1, 4, 3) -> sx_avg==spx_avg: True
p: (0, 2, 3, 1, 4) -> sx_avg==spx_avg: True
p: (0, 2, 3, 4, 1) -> sx_avg==spx_avg: True
p: (0, 2, 4, 1, 3) -> sx_avg==spx_avg: True
p: (0, 2, 4, 3, 1) -> sx_avg==spx_avg: True
p: (0, 3, 1, 2, 4) -> sx_avg==spx_avg: True
p: (0, 3, 1, 4, 2) -> sx_avg==spx_avg: True
p: (0, 3, 2, 1, 4) -> sx_avg==spx_avg: True
p: (0, 3, 2, 4, 1) -> sx_avg==spx_avg: True
p: (0, 3, 4, 1, 2) -> sx_avg==spx_avg: True
p: (0, 3, 4, 2, 1) -> sx_avg==spx_avg: True
p: (0, 4, 1, 2, 3) -> sx_avg==spx_avg: True
p: (0, 4, 1, 3, 2) -> sx_avg==spx_avg: True
p: (0, 4, 2, 1, 3) -> sx_avg==spx_avg: True
p: (0, 4, 2, 3, 1) -> sx_avg==spx_avg: True
p: (0, 4, 3, 1, 2) -> sx_avg==spx_avg: True
p: (0, 4, 3, 2, 1) -> sx_avg==spx_avg: True
p: (1, 0, 2, 3, 4) -> sx_avg==spx_avg: True
p: (1, 0, 2, 4, 3) -> sx_avg==spx_avg: True
p: (1, 0, 3, 2, 4) -> sx_avg==spx_avg: True
p: (1, 0, 3, 4, 2) -> sx_avg==spx_avg: True
p: (1, 0, 4, 2, 3) -> sx_avg==spx_avg: True
p: (1, 0, 4, 3, 2) -> sx_avg==spx_avg: True
p: (1, 2, 0, 3, 4) -> sx_avg==spx_avg: True
p: (1, 2, 0, 4, 3) -> sx_avg==spx_avg: True
p: (1, 2, 3, 0, 4) -> sx_avg==spx_avg: True
p: (1, 2, 3, 4, 0) -> sx_avg==spx_avg: True
p: (1, 2, 4, 0, 3) -> sx_avg==spx_avg: True
p: (1, 2, 4, 3, 0) -> sx_avg==spx_avg: True
```

```
p: (1, 3, 0, 2, 4) -> sx_avg==spx_avg: True
p: (1, 3, 0, 4, 2) -> sx_avg==spx_avg: True
p: (1, 3, 2, 0, 4) -> sx_avg==spx_avg: True
p: (1, 3, 2, 4, 0) -> sx_avg==spx_avg: True
p: (1, 3, 4, 0, 2) -> sx_avg==spx_avg: True
p: (1, 3, 4, 2, 0) -> sx_avg==spx_avg: True
p: (1, 4, 0, 2, 3) -> sx_avg==spx_avg: True
p: (1, 4, 0, 3, 2) -> sx_avg==spx_avg: True
p: (1, 4, 2, 0, 3) -> sx_avg==spx_avg: True
p: (1, 4, 2, 3, 0) -> sx_avg==spx_avg: True
p: (1, 4, 3, 0, 2) -> sx_avg==spx_avg: True
p: (1, 4, 3, 2, 0) -> sx_avg==spx_avg: True
p: (2, 0, 1, 3, 4) -> sx_avg==spx_avg: True
p: (2, 0, 1, 4, 3) -> sx_avg==spx_avg: True
p: (2, 0, 3, 1, 4) -> sx_avg==spx_avg: True
p: (2, 0, 3, 4, 1) -> sx_avg==spx_avg: True
p: (2, 0, 4, 1, 3) -> sx_avg==spx_avg: True
p: (2, 0, 4, 3, 1) -> sx_avg==spx_avg: True
p: (2, 1, 0, 3, 4) -> sx_avg==spx_avg: True
p: (2, 1, 0, 4, 3) -> sx_avg==spx_avg: True
p: (2, 1, 3, 0, 4) -> sx_avg==spx_avg: True
p: (2, 1, 3, 4, 0) -> sx_avg==spx_avg: True
p: (2, 1, 4, 0, 3) -> sx_avg==spx_avg: True
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p: (2, 3, 0, 1, 4) -> sx_avg==spx_avg: True
p: (2, 3, 0, 4, 1) -> sx_avg==spx_avg: True
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p: (2, 3, 1, 4, 0) -> sx_avg==spx_avg: True
p: (2, 3, 4, 0, 1) -> sx_avg==spx_avg: True
p: (2, 3, 4, 1, 0) -> sx_avg==spx_avg: True
p: (2, 4, 0, 1, 3) -> sx_avg==spx_avg: True
p: (2, 4, 0, 3, 1) -> sx_avg==spx_avg: True
p: (2, 4, 1, 0, 3) -> sx_avg==spx_avg: True
p: (2, 4, 1, 3, 0) -> sx_avg==spx_avg: True
p: (2, 4, 3, 0, 1) -> sx_avg==spx_avg: True
p: (2, 4, 3, 1, 0) -> sx_avg==spx_avg: True
p: (3, 0, 1, 2, 4) -> sx_avg==spx_avg: True
p: (3, 0, 1, 4, 2) -> sx_avg==spx_avg: True
p: (3, 0, 2, 1, 4) -> sx_avg==spx_avg: True
p: (3, 0, 2, 4, 1) -> sx_avg==spx_avg: True
p: (3, 0, 4, 1, 2) -> sx_avg==spx_avg: True
p: (3, 0, 4, 2, 1) -> sx_avg==spx_avg: True
p: (3, 1, 0, 2, 4) -> sx_avg==spx_avg: True
p: (3, 1, 0, 4, 2) -> sx_avg==spx_avg: True
p: (3, 1, 2, 0, 4) -> sx_avg==spx_avg: True
p: (3, 1, 2, 4, 0) -> sx_avg==spx_avg: True
p: (3, 1, 4, 0, 2) -> sx_avg==spx_avg: True
p: (3, 1, 4, 2, 0) -> sx_avg==spx_avg: True
p: (3, 2, 0, 1, 4) -> sx_avg==spx_avg: True
p: (3, 2, 0, 4, 1) -> sx_avg==spx_avg: True
p: (3, 2, 1, 0, 4) -> sx_avg==spx_avg: True
p: (3, 2, 1, 4, 0) -> sx_avg==spx_avg: True
p: (3, 2, 4, 0, 1) -> sx_avg==spx_avg: True
p: (3, 2, 4, 1, 0) -> sx_avg==spx_avg: True
p: (3, 4, 0, 1, 2) -> sx_avg==spx_avg: True
p: (3, 4, 0, 2, 1) -> sx_avg==spx_avg: True
p: (3, 4, 1, 0, 2) -> sx_avg==spx_avg: True
p: (3, 4, 1, 2, 0) -> sx_avg==spx_avg: True
p: (3, 4, 2, 0, 1) -> sx_avg==spx_avg: True
p: (3, 4, 2, 1, 0) -> sx_avg==spx_avg: True
p: (4, 0, 1, 2, 3) -> sx_avg==spx_avg: True
p: (4, 0, 1, 3, 2) -> sx_avg==spx_avg: True
p: (4, 0, 2, 1, 3) -> sx_avg==spx_avg: True
p: (4, 0, 2, 3, 1) -> sx_avg==spx_avg: True
p: (4, 0, 3, 1, 2) -> sx_avg==spx_avg: True
```

```
p: (4, 0, 3, 2, 1) -> sx_avg==spx_avg: True
p: (4, 1, 0, 2, 3) -> sx_avg==spx_avg: True
p: (4, 1, 0, 3, 2) -> sx_avg==spx_avg: True
p: (4, 1, 2, 0, 3) -> sx_avg==spx_avg: True
p: (4, 1, 2, 3, 0) -> sx_avg==spx_avg: True
p: (4, 1, 3, 0, 2) -> sx_avg==spx_avg: True
p: (4, 1, 3, 2, 0) -> sx_avg==spx_avg: True
p: (4, 2, 0, 1, 3) -> sx_avg==spx_avg: True
p: (4, 2, 0, 3, 1) -> sx_avg==spx_avg: True
p: (4, 2, 1, 0, 3) -> sx_avg==spx_avg: True
p: (4, 2, 1, 3, 0) -> sx_avg==spx_avg: True
p: (4, 2, 3, 0, 1) -> sx_avg==spx_avg: True
p: (4, 2, 3, 1, 0) -> sx_avg==spx_avg: True
p: (4, 3, 0, 1, 2) -> sx_avg==spx_avg: True
p: (4, 3, 0, 2, 1) -> sx_avg==spx_avg: True
p: (4, 3, 1, 0, 2) -> sx_avg==spx_avg: True
p: (4, 3, 1, 2, 0) -> sx_avg==spx_avg: True
p: (4, 3, 2, 0, 1) -> sx_avg==spx_avg: True
p: (4, 3, 2, 1, 0) -> sx_avg==spx_avg: True
```

# 4.D

The Positionwise Feedforward network brings in some non-linear to MultiHead Attension

In [171…

```
class PositionwiseFeedForward(nn.Module):
    """
    Does a Linear + RELU + Linear on each of the timesteps
    """
    def __init__(self,B,T,D):
        """
        Parameters:
            input_depth: Size of last dimension of input
            filter_size:Hidden size of the middle layer
            output_depth: Size last dimension of the final output
        """
        input_depth, filter_size, output_depth=B,T,D
        super(PositionwiseFeedForward, self).__init__()


        self.layers = nn.ModuleList([
                      torch.nn.Linear(input_depth, filter_size),
                      torch.nn.Linear(filter_size, filter_size),
                      torch.nn.Linear(filter_size, output_depth)
                    ])
        self.relu = nn.ReLU()
    def forward(self, inputs):
        x = inputs
        for i, layer in enumerate(self.layers):
            x = layer(x)
            if i < len(self.layers):
                x = self.relu(x)
        return x
```

In [172…

```
class PositionwiseFeedForward_Conv(nn.Module):
    """
    Does a Linear + RELU + Linear on each of the timesteps
    """
    def __init__(self,B,T,D):
        """
```

```
        Parameters:
            input_depth: Size of last dimension of input
            filter_size:Hidden size of the middle layer
            output_depth: Size last dimension of the final output
        """
        input_depth, filter_size, output_depth=B,T,D
        super(PositionwiseFeedForward_Conv, self).__init__()


        self.layers = nn.ModuleList([
                      torch.nn.Conv1d(input_depth, filter_size,1),
                      torch.nn.Conv1d(filter_size, filter_size,1),
                      torch.nn.Conv1d(filter_size, output_depth,1)
                    ])
        self.relu = nn.ReLU()
    def forward(self, inputs):
        x = inputs
        for i, layer in enumerate(self.layers):
            x = layer(x)
            if i < len(self.layers):
                x = self.relu(x)
        return x
```

In [177…

```
Z=torch.randn([4,3,4])
permuted_tensor = Z.permute(0,2,1).clone().contiguous()
pwff_conv=PositionwiseFeedForward_Conv(Z.shape[0],Z.shape[1],Z.shape[2])
tensor_pwff_conv=pwff_conv.forward(permuted_tensor)

pwff=PositionwiseFeedForward(Z.shape[0],Z.shape[1],Z.shape[2])
for i in range(len(pwff_conv.layers)):
    pwff.layers[i].weight= torch.nn.Parameter(pwff_conv.layers[i].weight.squeeze(2))
    pwff.layers[i].bias = torch.nn.Parameter(pwff_conv.layers[i].bias)
tensor_pwff=pwff.forward(Z).permute(0,2,1).clone().contiguous()

print('pwff Linear out=',tensor_pwff.detach())
print('pwff_Conv1D Out=',tensor_pwff_conv.detach())
print( 'pwff_Linear==spwff_Conv1D:',np.allclose(tensor_pwff.detach(),tensor_pwff_conv.d
```

```
pwff Linear out= tensor([[[0.1803, 0.2522, 0.1838],
         [0.0000, 0.0421, 0.0439],
         [0.0000, 0.0000, 0.0000],
         [0.0000, 0.0612, 0.0000]],

        [[0.1809, 0.1785, 0.2299],
         [0.0054, 0.0037, 0.0000],
         [0.0000, 0.0000, 0.0000],
         [0.0000, 0.0000, 0.0312]],

        [[0.2116, 0.2455, 0.1926],
         [0.0278, 0.0335, 0.1105],
         [0.0000, 0.0000, 0.0000],
         [0.0000, 0.0467, 0.0000]],

        [[0.1785, 0.2554, 0.2053],
         [0.0037, 0.0598, 0.0000],
         [0.0000, 0.0000, 0.0000],
         [0.0000, 0.0598, 0.0000]]])
pwff_Conv1D Out= tensor([[[0.1803, 0.2522, 0.1838],
         [0.0000, 0.0421, 0.0439],
         [0.0000, 0.0000, 0.0000],
         [0.0000, 0.0612, 0.0000]],
```

```
        [[0.1809, 0.1785, 0.2299],
         [0.0054, 0.0037, 0.0000],
         [0.0000, 0.0000, 0.0000],
         [0.0000, 0.0000, 0.0312]],

        [[0.2116, 0.2455, 0.1926],
         [0.0278, 0.0335, 0.1105],
         [0.0000, 0.0000, 0.0000],
         [0.0000, 0.0467, 0.0000]],

        [[0.1785, 0.2554, 0.2053],
         [0.0037, 0.0598, 0.0000],
         [0.0000, 0.0000, 0.0000],
         [0.0000, 0.0598, 0.0000]]])
pwff_Linear==spwff_Conv1D: True
```

In [ ]:

In this excercise we will run a basic RNN based language model and answer some questions about the code. It is advised to use GPU to run this. First run the code then answer the questions below that require modifying it.

In [1]:
```
# Some part of the code was referenced from below.
# https://github.com/pytorch/examples/tree/master/word_language_model
# https://github.com/yunjey/pytorch-tutorial/tree/master/tutorials/02-intermediate/lang

! git clone https://github.com/yunjey/pytorch-tutorial/
%cd pytorch-tutorial/tutorials/02-intermediate/language_model/

import torch
import torch.nn as nn
import numpy as np
from torch.nn.utils import clip_grad_norm_

# Device configuration
device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')

# Hyper-parameters
embed_size = 128
hidden_size = 1024
num_layers = 1
num_epochs = 5
num_samples = 50     # number of words to be sampled
batch_size = 20
seq_length = 30
learning_rate = 0.002
```

```
fatal: destination path 'pytorch-tutorial' already exists and is not an empty directory.
/content/pytorch-tutorial/tutorials/02-intermediate/language_model
```

In [44]:
```
from data_utils import Dictionary, Corpus

# Load "Penn Treebank" dataset
corpus = Corpus()
ids = corpus.get_data('data/train.txt', batch_size)
print(ids)
vocab_size = len(corpus.dictionary)
print(vocab_size)
num_batches = ids.size(1) // seq_length
print(num_batches)
```

```
tensor([[   0,    1,    2,  ...,  152, 4955, 4150],
        [  93,  718,  590,  ...,  170, 6784,  133],
        [  27,  930,   42,  ...,  392, 4864,   26],
        ...,
        [ 997,   42,  507,  ...,  682, 6849, 6344],
        [ 392, 5518, 3034,  ..., 2264,   42, 3401],
        [4210,  467, 1496,  ..., 9999,  119, 1143]])
10000
1549
```

Model definition

In [3]:
```
# RNN based language model
class RNNLM(nn.Module):
    def __init__(self, vocab_size, embed_size, hidden_size, num_layers):
```

```
        super(RNNLM, self).__init__()
        self.embed = nn.Embedding(vocab_size, embed_size)
        self.lstm = nn.LSTM(embed_size, hidden_size, num_layers, batch_first=True)
        self.linear = nn.Linear(hidden_size, vocab_size)

    def forward(self, x, h):
        # Embed word ids to vectors
        x = self.embed(x)

        # Forward propagate LSTM
        out, (h, c) = self.lstm(x, h)

        # Reshape output to (batch_size*sequence_length, hidden_size)
        out = out.reshape(out.size(0)*out.size(1), out.size(2))

        # Decode hidden states of all time steps
        out = self.linear(out)
        return out, (h, c)
```

Training .. should take a few minutes with GPU

In [4]:

```
model = RNNLM(vocab_size, embed_size, hidden_size, num_layers).to(device)

# Loss and optimizer
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.Adam(model.parameters(), lr=learning_rate)

# Truncated backpropagation
def detach(states):
    return [state.detach() for state in states]


# Train the model
for epoch in range(num_epochs):
    # Set initial hidden and cell states
    states = (torch.zeros(num_layers, batch_size, hidden_size).to(device),
              torch.zeros(num_layers, batch_size, hidden_size).to(device))

    for i in range(0, ids.size(1) - seq_length, seq_length):
        # Get mini-batch inputs and targets
        inputs = ids[:, i:i+seq_length].to(device)
        targets = ids[:, (i+1):(i+1)+seq_length].to(device)

        # Forward pass
        states = detach(states)
        outputs, states = model(inputs, states)
        loss = criterion(outputs, targets.reshape(-1))

        # Backward and optimize
        optimizer.zero_grad()
        loss.backward()
        clip_grad_norm_(model.parameters(), 0.5)
        optimizer.step()

        step = (i+1) // seq_length
        if step % 100 == 0:
            print ('Epoch [{}/{}], Step[{}/{}], Loss: {:.4f}, Perplexity: {:5.2f}'
                   .format(epoch+1, num_epochs, step, num_batches, loss.item(), np.exp(
```

```
Epoch [1/5], Step[0/1549], Loss: 9.2154, Perplexity: 10050.66
Epoch [1/5], Step[100/1549], Loss: 6.0428, Perplexity: 421.07
Epoch [1/5], Step[200/1549], Loss: 5.9603, Perplexity: 387.74
Epoch [1/5], Step[300/1549], Loss: 5.7509, Perplexity: 314.47
Epoch [1/5], Step[400/1549], Loss: 5.6951, Perplexity: 297.40
Epoch [1/5], Step[500/1549], Loss: 5.1242, Perplexity: 168.05
Epoch [1/5], Step[600/1549], Loss: 5.1864, Perplexity: 178.83
Epoch [1/5], Step[700/1549], Loss: 5.3664, Perplexity: 214.10
Epoch [1/5], Step[800/1549], Loss: 5.2182, Perplexity: 184.60
Epoch [1/5], Step[900/1549], Loss: 5.0592, Perplexity: 157.46
Epoch [1/5], Step[1000/1549], Loss: 5.0750, Perplexity: 159.97
Epoch [1/5], Step[1100/1549], Loss: 5.3712, Perplexity: 215.11
Epoch [1/5], Step[1200/1549], Loss: 5.1832, Perplexity: 178.25
Epoch [1/5], Step[1300/1549], Loss: 5.1171, Perplexity: 166.84
Epoch [1/5], Step[1400/1549], Loss: 4.8107, Perplexity: 122.82
Epoch [1/5], Step[1500/1549], Loss: 5.1486, Perplexity: 172.20
Epoch [2/5], Step[0/1549], Loss: 5.4245, Perplexity: 226.89
Epoch [2/5], Step[100/1549], Loss: 4.5731, Perplexity: 96.85
Epoch [2/5], Step[200/1549], Loss: 4.6927, Perplexity: 109.14
Epoch [2/5], Step[300/1549], Loss: 4.6652, Perplexity: 106.19
Epoch [2/5], Step[400/1549], Loss: 4.5912, Perplexity: 98.61
Epoch [2/5], Step[500/1549], Loss: 4.0974, Perplexity: 60.18
Epoch [2/5], Step[600/1549], Loss: 4.4198, Perplexity: 83.08
Epoch [2/5], Step[700/1549], Loss: 4.4268, Perplexity: 83.66
Epoch [2/5], Step[800/1549], Loss: 4.4558, Perplexity: 86.12
Epoch [2/5], Step[900/1549], Loss: 4.2247, Perplexity: 68.36
Epoch [2/5], Step[1000/1549], Loss: 4.3176, Perplexity: 75.00
Epoch [2/5], Step[1100/1549], Loss: 4.5346, Perplexity: 93.18
Epoch [2/5], Step[1200/1549], Loss: 4.4663, Perplexity: 87.04
Epoch [2/5], Step[1300/1549], Loss: 4.2586, Perplexity: 70.71
Epoch [2/5], Step[1400/1549], Loss: 3.9051, Perplexity: 49.66
Epoch [2/5], Step[1500/1549], Loss: 4.3613, Perplexity: 78.36
Epoch [3/5], Step[0/1549], Loss: 4.4603, Perplexity: 86.51
Epoch [3/5], Step[100/1549], Loss: 3.8699, Perplexity: 47.94
Epoch [3/5], Step[200/1549], Loss: 4.0718, Perplexity: 58.66
Epoch [3/5], Step[300/1549], Loss: 3.9270, Perplexity: 50.76
Epoch [3/5], Step[400/1549], Loss: 3.9246, Perplexity: 50.64
Epoch [3/5], Step[500/1549], Loss: 3.3873, Perplexity: 29.59
Epoch [3/5], Step[600/1549], Loss: 3.7703, Perplexity: 43.40
Epoch [3/5], Step[700/1549], Loss: 3.8043, Perplexity: 44.89
Epoch [3/5], Step[800/1549], Loss: 3.8573, Perplexity: 47.34
Epoch [3/5], Step[900/1549], Loss: 3.5485, Perplexity: 34.76
Epoch [3/5], Step[1000/1549], Loss: 3.6283, Perplexity: 37.65
Epoch [3/5], Step[1100/1549], Loss: 3.7443, Perplexity: 42.28
Epoch [3/5], Step[1200/1549], Loss: 3.8194, Perplexity: 45.58
Epoch [3/5], Step[1300/1549], Loss: 3.5264, Perplexity: 34.00
Epoch [3/5], Step[1400/1549], Loss: 3.1919, Perplexity: 24.34
Epoch [3/5], Step[1500/1549], Loss: 3.6495, Perplexity: 38.46
Epoch [4/5], Step[0/1549], Loss: 3.5889, Perplexity: 36.19
Epoch [4/5], Step[100/1549], Loss: 3.2794, Perplexity: 26.56
Epoch [4/5], Step[200/1549], Loss: 3.4924, Perplexity: 32.86
Epoch [4/5], Step[300/1549], Loss: 3.3518, Perplexity: 28.55
Epoch [4/5], Step[400/1549], Loss: 3.4131, Perplexity: 30.36
Epoch [4/5], Step[500/1549], Loss: 2.8256, Perplexity: 16.87
Epoch [4/5], Step[600/1549], Loss: 3.3928, Perplexity: 29.75
Epoch [4/5], Step[700/1549], Loss: 3.2491, Perplexity: 25.77
Epoch [4/5], Step[800/1549], Loss: 3.4220, Perplexity: 30.63
Epoch [4/5], Step[900/1549], Loss: 3.0265, Perplexity: 20.63
Epoch [4/5], Step[1000/1549], Loss: 3.2141, Perplexity: 24.88
Epoch [4/5], Step[1100/1549], Loss: 3.2332, Perplexity: 25.36
Epoch [4/5], Step[1200/1549], Loss: 3.3068, Perplexity: 27.30
Epoch [4/5], Step[1300/1549], Loss: 3.0439, Perplexity: 20.99
Epoch [4/5], Step[1400/1549], Loss: 2.6748, Perplexity: 14.51
Epoch [4/5], Step[1500/1549], Loss: 3.1453, Perplexity: 23.23
Epoch [5/5], Step[0/1549], Loss: 3.0393, Perplexity: 20.89
```

```
Epoch [5/5], Step[100/1549], Loss: 2.8959, Perplexity: 18.10
Epoch [5/5], Step[200/1549], Loss: 3.1254, Perplexity: 22.77
Epoch [5/5], Step[300/1549], Loss: 2.9773, Perplexity: 19.63
Epoch [5/5], Step[400/1549], Loss: 3.0717, Perplexity: 21.58
Epoch [5/5], Step[500/1549], Loss: 2.5135, Perplexity: 12.35
Epoch [5/5], Step[600/1549], Loss: 3.0250, Perplexity: 20.59
Epoch [5/5], Step[700/1549], Loss: 2.9458, Perplexity: 19.03
Epoch [5/5], Step[800/1549], Loss: 3.0710, Perplexity: 21.56
Epoch [5/5], Step[900/1549], Loss: 2.7675, Perplexity: 15.92
Epoch [5/5], Step[1000/1549], Loss: 2.8360, Perplexity: 17.05
Epoch [5/5], Step[1100/1549], Loss: 2.8814, Perplexity: 17.84
Epoch [5/5], Step[1200/1549], Loss: 2.9925, Perplexity: 19.94
Epoch [5/5], Step[1300/1549], Loss: 2.7103, Perplexity: 15.03
Epoch [5/5], Step[1400/1549], Loss: 2.3566, Perplexity: 10.56
Epoch [5/5], Step[1500/1549], Loss: 2.8948, Perplexity: 18.08
```

Q2 (a) (10 points) The above code implements a version of truncated backpropagation through time. The implementation only requires the detach() function (L7-9 of the cell) defined above the loop and used once inside the training loop. Explain the implementation (compared to not using truncated backprop through time). What does the detach() call here achieve? Draw a computational graph. You may choose to answer this question outside the notebook. When using using line 7-9 we will typically observe less GPU memory being used during training, explain why in your answer.

# Q2.a Answer

## Not using truncated backprop through time

- Conventional backpropagation will be performed which will consume alot of time and does not consider the dependencies between model variables and shared parameter during the recursive iterations

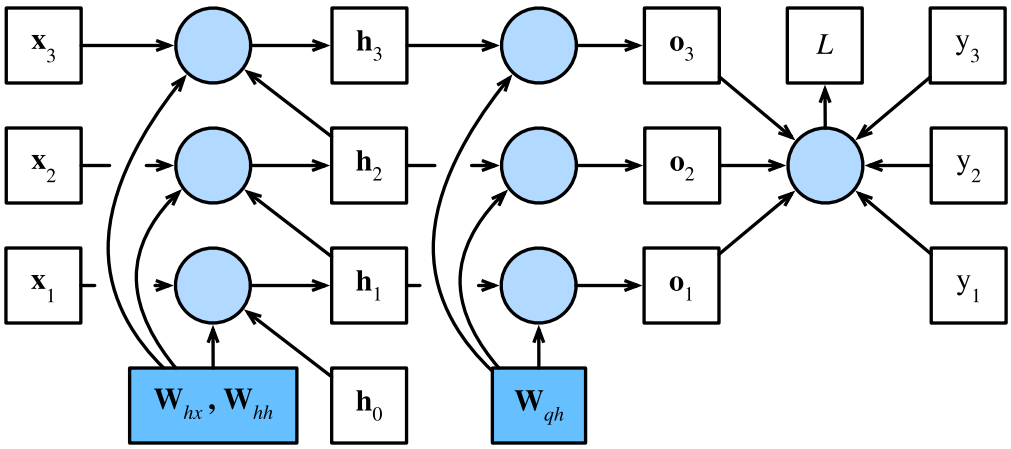
## main problems of BPTT

- high cost of a single parameter update, which makes it impossible to use a large number of iterations.
- the gradient becomes too small with long sequances. if the number of time steps is long then far past sequance information will effectively be discarded.
- large memoery and train time reqired to maintain the large sequence gradient updates through online backpropagation

https://mmuratarat.github.io/2019-02-07/bptt-of-rnn

https://d2l.ai/chapter_recurrent-neural-networks/bptt.html#equation-eq-bptt-partial-ht-wh-gen

# gradient computation for RNN sequance




$$L(\hat{y}, y) = \sum_{t=1}^{T} L_t(\hat{y}_t, y_t) = -\sum_{t}^{T} y_t \log \hat{y}_t = -\sum_{t=1}^{T} y_t log\left[softmax(o_t)\right]$$

$$\frac{\partial L}{\partial W_{yh}} = \sum_{t}^{T} \frac{\partial L_t}{\partial W_{yh}} = \sum_{t}^{T} \frac{\partial L_t}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial o_t} \frac{\partial o_t}{\partial W_{yh}} = \sum_{t}^{T} (\hat{y}_t - y_t) \otimes h_t$$

we can take the derivative with respect to $W_{xh}$ over the whole sequence as :

$$\frac{\partial L}{\partial W_{xh}} = \sum_{t}^{T} \sum_{k=1}^{t+1} \frac{\partial L_{t+1}}{\partial \hat{y}_{t+1}} \frac{\partial \hat{y}_{t+1}}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial h_k} \frac{\partial h_k}{\partial W_{xh}}$$

## Observes less GPU memory being used during training

Detaching the gradients in detach() function (L7-9 of the cell). will help to avoid too long RNN outputs are kept in memory before doing backprop on a batch. thus, leads to an approximation of the true gradient with limited memory needs

Now we will sample from the model

In [60]:

```
# Sample from the model
with torch.no_grad():
    with open('sample.txt', 'w') as f:
        # Set intial hidden ane cell states
        state = (torch.zeros(num_layers, 1, hidden_size).to(device),
                 torch.zeros(num_layers, 1, hidden_size).to(device))

        # Select one word id randomly
        prob = torch.ones(vocab_size)
        # input = torch.multinomial(prob, num_samples=1).unsqueeze(1).to(device)
        input=torch.tensor([[100]]).to(device)
        print(input)
        for i in range(num_samples):
            # Forward propagate RNN
            output, state = model(input, state)
```

```
            # Sample a word id
            prob = output.exp()
            word_id = torch.multinomial(prob, num_samples=1).item()
            # print(word_id)
            # Fill input with sampled word id for the next time step
            input.fill_(word_id)

            # File write
            word = corpus.dictionary.idx2word[word_id]
            # print(word)
            word = '\n' if word == '<eos>' else word + ' '
            f.write(word)

            if (i+1) % 100 == 0:
                print('Sampled [{}/{}] words and save to {}'.format(i+1, num_samples, '
! cat sample.txt
```

```
tensor([[100]], device='cuda:0')
standard a bureaucrats result
we dinkins countries confidence the buy-out but prediction on europe as anything of acti
on 's impact compatible at least being foods inflation the $ N billion of gm officials c
o banking
but always foreign is an sisulu big letter france gold an <unk> 's
```

Q2 (b) (5 points) Consider the sampling procedure. The current code samples the word to feed the model from the softmax at each output step feeding those to the next timestep. Copy below the above cell and modify this sampling to use a greedy sampling which selects the highest probability word at each time step to feed as the next input.

In [71]:

```
# Sample greedily from the model

# Sample from the model
with torch.no_grad():
    with open('sample.txt', 'w') as f:
        # Set intial hidden ane cell states
        state = (torch.zeros(num_layers, 1, hidden_size).to(device),
                 torch.zeros(num_layers, 1, hidden_size).to(device))

        # Select one word id randomly
        prob = torch.ones(vocab_size)
        # input = torch.multinomial(prob, num_samples=1).unsqueeze(1).to(device)
        input=torch.tensor([[100]]).to(device)
        print(input)

        for i in range(num_samples):
            # Forward propagate RNN
            output, state = model(input, state)

            # Sample a word id
            prob = output.exp()
            # print(prob)
            # print(torch.argmax(torch.flatten(prob, start_dim=0)))
            # word_id = torch.multinomial(prob, num_samples=1).item()
            # print(torch.flatten(prob, start_dim=0))
            word_id=torch.argmax(torch.flatten(prob, start_dim=1)).item()
            # print(word_id)
            # Fill input with sampled word id for the next time step
            input.fill_(word_id)
            # print(input)
            # File write
```

```
            word = corpus.dictionary.idx2word[word_id]
            # print(word)
            word = '\n' if word == '<eos>' else word + ' '
            f.write(word)

            if (i+1) % 100 == 0:
                print('Sampled [{}/{}] words and save to {}'.format(i+1, num_samples, '
! cat sample.txt
```

```
tensor([[100]], device='cuda:0')

the <unk> of the <unk> of the <unk> of the <unk> of the <unk> of the <unk> of the <unk>
of the <unk> of the <unk> of the <unk> of the <unk>
the <unk> <unk> <unk> <unk> <unk> <unk> <unk> <unk> <unk> <unk> <unk> <unk> <unk> <unk>
<unk>
```

In [72]:

```
print(26,corpus.dictionary.idx2word[26])
```

```
26 <unk>
```

Q2 (c) (5 points) The model above has learned a specific set of word embeddings. Write a function that takes in 2 words and prints the euclidean distance between their embeddings using the word embeddings from the above model. Use it to print the euclidean distance of the word "army" and the word "taxpayer". Refer to the sampling code for how to output the words corresponding to each index. To get the index you can use the function corpus.dictionary.word2idx.

In [73]:

```
def calc_euclidean_dist(emba,emb2):
  return torch.norm(emb2 - emb1)
```

In [92]:

```
idx1=corpus.dictionary.word2idx['army']
idx2=corpus.dictionary.word2idx['taxpayer']
input=torch.tensor([[idx1],[idx2]]).to(device)
embedings=model.embed(input)
emb1=embedings[0][0]
print('emb1=',emb1)
emb2=embedings[1][0]
print('emb2=',emb2)
print('euclidean_dist=',calc_euclidean_dist(emb1,emb2))
```

```
emb1= tensor([-2.2605,  1.8347,  0.3290,  1.2438,  0.5797, -0.8029, -1.7835, -1.7043,
        -1.9972,  0.8936,  0.4586,  0.9676,  0.6952, -0.4859,  0.3806,  0.0866,
         1.0970, -0.0312, -1.0015, -1.3252, -1.2698,  0.4583, -0.1687,  0.4805,
         0.9958, -0.8913,  0.3243,  0.0583,  0.1725,  1.1637,  0.2934,  1.7897,
         0.6411, -0.1767, -0.1446,  1.2658,  2.5794, -1.3246, -0.8371,  0.2599,
         1.9956,  0.8422,  1.2673, -0.0601, -0.7049,  1.8812,  0.5234, -1.7571,
        -0.2755,  0.1172,  0.0358, -1.3997, -0.7592,  0.3979,  0.6130,  0.4447,
        -1.0921, -0.8210,  0.7058,  1.2346, -0.6560, -1.9122, -2.8719,  1.5943,
         0.0417,  0.1118,  0.3425, -0.1908, -0.2210, -0.9062, -0.6764, -2.1598,
         0.4907,  0.1816, -1.0560, -0.2304, -0.8227, -1.6182, -0.4912, -1.7678,
        -0.2781,  0.1765, -0.4360,  0.1429,  0.8224, -0.3750,  0.8260, -1.5832,
         1.8599, -1.1047, -0.6410, -0.3978,  0.0879, -0.4928, -1.1259,  0.1938,
        -0.1726, -0.5636, -2.0747, -0.2870,  0.4424,  1.6880, -0.6350, -1.3068,
         0.4195, -0.6620, -0.2466,  0.8768,  0.7050,  1.2635, -0.4820,  0.8348,
        -0.0529, -0.0718,  0.6743,  0.7258, -1.8350,  0.8995,  1.1814, -0.8911,
        -1.2975, -0.2758, -0.4911,  1.1866, -0.6611,  0.3857, -1.5217,  0.2267],
       device='cuda:0', grad_fn=<SelectBackward>)
emb2= tensor([ 0.2332, -0.3169,  0.1712,  0.2031,  1.1109,  0.2677,  0.8109,  0.5747,
        -0.5917,  1.1332, -0.2610,  0.2653,  2.0403,  1.2999, -1.2561, -0.9575,
```

```
        -1.6528,  0.7856, -0.6016, -0.7846,  0.5881,  0.9525,  0.7121, -0.0404,
        -0.0587,  0.5197, -0.6857, -0.4102, -0.5130,  1.6504, -0.5783, -1.2738,
         0.9794,  0.4101, -0.0657,  0.7646, -0.6332,  0.3777,  1.4272, -0.6644,
        -1.4570,  1.0993,  0.1187, -0.7335,  0.3964,  1.0111,  0.8567,  1.3628,
         0.9063, -2.2283,  0.1024, -0.1236,  0.0568, -1.3278,  2.6409,  0.3308,
        -0.4690,  0.9344,  0.0150, -1.5370, -0.1914,  0.4623, -0.6456,  1.0381,
        -0.0633,  1.6121,  1.2612, -1.3139, -0.8432, -0.2772,  0.0647,  0.5816,
        -2.1104, -0.7644, -0.2722, -1.7793, -0.8148,  1.6828,  0.0145,  1.4473,
         0.5370,  0.8873, -0.4203, -1.2486,  0.5086,  0.7202,  0.1146,  1.2036,
        -0.8572,  1.0427,  0.1403,  0.3393, -0.2534, -0.9276,  0.9246, -0.1006,
         0.0594,  0.7682, -0.2759,  3.1614, -0.9177,  0.3148,  0.5130, -0.4845,
         1.0830, -0.5981, -0.7253, -1.8724, -0.6145, -0.6529,  1.6785, -1.7053,
        -1.3276, -0.4010,  0.0984,  0.1668, -1.0333, -0.0244, -2.2978,  0.5253,
        -0.3659,  1.7339, -0.3771, -0.3961,  0.6169, -0.2665, -0.7288, -0.3646],
       device='cuda:0', grad_fn=<SelectBackward>)
euclidean_dist= tensor(17.3774, device='cuda:0', grad_fn=<CopyBackwards>)
```

In [ ]: