The following code is adapted from http://www.sagargv.com/blog/meta-learning-in-pytorch/ and illustrates how to use pytorch to differentiate through an unrolled iterative optimization a common technique in "meta-learning".

Specifically consider f_s which will define a randomly sampled sinusoid with a different frequency. We want to have a function $f_w(x)$ that fits the sinusoid using just 4 samples of $(x, f_s(x))$ and a few iterations of gradient descent.

To do this we will formulate the problem as finding a good initialization w_0 to rapidly obtain the best fit, which also generalizes to the unseen points. Assume we have some way to sample various sinusoid to learn this intiliazation and we split these samples for each one into $(X_{train}, f_s(X_{train}))$ and a validation set $(X_{val}, f_s(X_{val}))$

We now solve the following nested optimzation

$\min_{w_0} \mathcal{L}(f_{w_T}(X_{val}), f_t(X_{val}))$ where w_T is obtained by iterating T times $w_t = w_{t-1} - \alpha \nabla \mathcal{L}(f_{w_{t-1}}(X_{train}), f_s(X_{train}))$.

Observe that to take a gradient descent step on this objective we must differentiate through the T gradient steps of the inner problem, including the gradient operation! Fortunately pytorch is able to do this in almost the same way we are used to. Your take away from this lab should be that we can differentiate through multiple iterations of gradient descent allowing us to construct these kind of nested objective functions.

Run the code cell below and try to get an understanding of what it is doing

```
# From http://www.sagargv.com/blog/meta-learning-in-pytorch/

import math
import random
import torch
from torch import nn
from torch.nn import functional as F
import matplotlib.pyplot as plt

def net(x, params):
    x = F.linear(x, params[0], params[1])
    x = F.tanh(x)

    x = F.linear(x, params[2], params[3])
    x = F.tanh(x)

    x = F.linear(x, params[4], params[5])
    return x

params = [
    torch.Tensor(32, 1).uniform_(-1., 1.).requires_grad_(),
    torch.Tensor(32).zero_().requires_grad_(),
```

```
    torch.Tensor(32, 32).uniform_(-1./math.sqrt(32), 1./math.sqrt(32)).requires_grad_(),
    torch.Tensor(32).zero_().requires_grad_(),

    torch.Tensor(1, 32).uniform_(-1./math.sqrt(32), 1./math.sqrt(32)).requires_grad_(),
    torch.Tensor(1).zero_().requires_grad_(),
]

opt = torch.optim.SGD(params, lr=1e-2)
n_inner_loop = 3
alpha = 3e-2

for it in range(5000):
    # sample frequency
    a = torch.rand(1).item()*0.5+0.5

    #Data for the inner loop
    x = torch.rand(4, 1)*4*math.pi - 2*math.pi
    y = torch.sin(a*x)

    #Data for the outer loop
    v_x = torch.rand(4, 1)*4*math.pi - 2*math.pi
    v_y = torch.sin(a*v_x)

    opt.zero_grad()

    new_params = params
    for k in range(n_inner_loop):
        f = net(x, new_params)
        loss = F.l1_loss(f, y)

        # create_graph=True because computing grads here is part of the forward pass.
        # We want to differentiate through the SGD update steps and get higher order
        # derivatives in the backward pass.
        grads = torch.autograd.grad(loss, new_params, create_graph=True)
        new_params = [(new_params[i] - alpha*grads[i]) for i in range(len(params))]

        if it % 500 == 0: print('Iteration %d -- Inner loop %d -- Loss: %.4f' % (it, k, lo

    v_f = net(v_x, new_params)
    loss2 = F.l1_loss(v_f, v_y)
    loss2.backward()

    opt.step()

    if it % 500 == 0: print('Iteration %d -- Outer Loss: %.4f' % (it, loss2))
```

```
/usr/local/lib/python3.7/dist-packages/torch/nn/functional.py:1698: UserWarning: nn.f
  warnings.warn("nn.functional.tanh is deprecated. Use torch.tanh instead.")
Iteration 0 -- Inner loop 0 -- Loss: 0.5677
Iteration 0 -- Inner loop 1 -- Loss: 0.5127
Iteration 0 -- Inner loop 2 -- Loss: 0.4579
Iteration 0 -- Outer Loss: 0.6380
Iteration 500 -- Inner loop 0 -- Loss: 0.8761
Iteration 500 -- Inner loop 1 -- Loss: 0.4141
Iteration 500 -- Inner loop 2 -- Loss: 0.1089
```

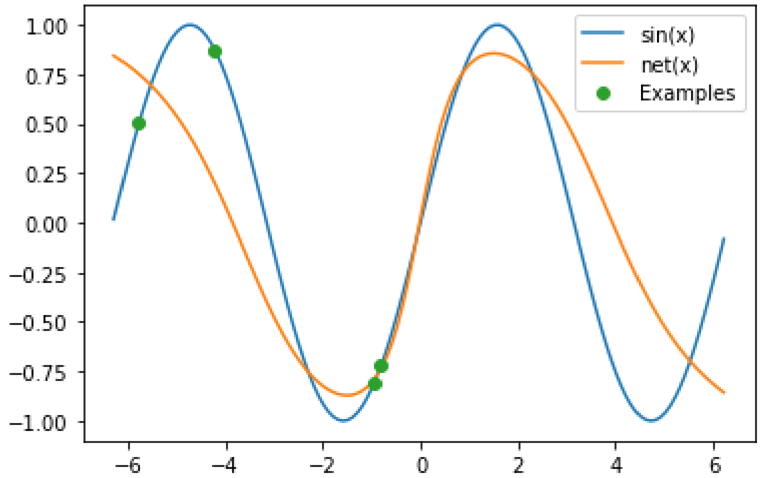
```
Iteration 500 -- Outer Loss: 0.2063
Iteration 1000 -- Inner loop 0 -- Loss: 0.2679
Iteration 1000 -- Inner loop 1 -- Loss: 0.1904
Iteration 1000 -- Inner loop 2 -- Loss: 0.2458
Iteration 1000 -- Outer Loss: 0.2600
Iteration 1500 -- Inner loop 0 -- Loss: 0.1043
Iteration 1500 -- Inner loop 1 -- Loss: 0.0618
Iteration 1500 -- Inner loop 2 -- Loss: 0.0338
Iteration 1500 -- Outer Loss: 0.3144
Iteration 2000 -- Inner loop 0 -- Loss: 0.3098
Iteration 2000 -- Inner loop 1 -- Loss: 0.1406
Iteration 2000 -- Inner loop 2 -- Loss: 0.0990
Iteration 2000 -- Outer Loss: 0.0918
Iteration 2500 -- Inner loop 0 -- Loss: 0.6753
Iteration 2500 -- Inner loop 1 -- Loss: 0.4958
Iteration 2500 -- Inner loop 2 -- Loss: 0.3182
Iteration 2500 -- Outer Loss: 0.3013
Iteration 3000 -- Inner loop 0 -- Loss: 0.3219
Iteration 3000 -- Inner loop 1 -- Loss: 0.1564
Iteration 3000 -- Inner loop 2 -- Loss: 0.0807
Iteration 3000 -- Outer Loss: 0.1614
Iteration 3500 -- Inner loop 0 -- Loss: 0.1660
Iteration 3500 -- Inner loop 1 -- Loss: 0.1149
Iteration 3500 -- Inner loop 2 -- Loss: 0.0854
Iteration 3500 -- Outer Loss: 0.3453
Iteration 4000 -- Inner loop 0 -- Loss: 0.3837
Iteration 4000 -- Inner loop 1 -- Loss: 0.1397
Iteration 4000 -- Inner loop 2 -- Loss: 0.1158
Iteration 4000 -- Outer Loss: 0.2105
Iteration 4500 -- Inner loop 0 -- Loss: 0.2148
Iteration 4500 -- Inner loop 1 -- Loss: 0.1220
Iteration 4500 -- Inner loop 2 -- Loss: 0.1011
Iteration 4500 -- Outer Loss: 0.1242
```

1) We can now "evaluate" the initialization we learned. Run the cell below a few times to get an idea of how well it fits a new sinusoid. You don't need to add any new code here.

```
t_b = 0 #math.pi
a = torch.rand(1).item()*0.5+0.5
t_x = torch.rand(4, 1)*4*math.pi - 2*math.pi
# print(t_x)
t_y = torch.sin(a*t_x + t_b)
# print(t_y)
opt.zero_grad()
t_params = params
for k in range(n_inner_loop):
    t_f = net(t_x, t_params)
    t_loss = F.l1_loss(t_f, t_y)
    # print(t_loss)
    grads = torch.autograd.grad(t_loss, t_params, create_graph=True)
    t_params = [(t_params[i] - alpha*grads[i]) for i in range(len(params))]

test_x = torch.arange(-2*math.pi, 2*math.pi, step=0.1).unsqueeze(1)
test_y = torch.sin(a*test_x + t_b)

test_f = net(test_x, t_params)
```

```
# print(test_f)
plt.plot(test_x.data.numpy(), test_y.data.numpy(), label='sin(x)')
plt.plot(test_x.data.numpy(), test_f.data.numpy(), label='net(x)')
plt.plot(t_x.data.numpy(), t_y.data.numpy(), 'o', label='Examples')
plt.legend()
plt.show()
```

```
/usr/local/lib/python3.7/dist-packages/torch/nn/functional.py:1698: UserWarning: nn.f
  warnings.warn("nn.functional.tanh is deprecated. Use torch.tanh instead.")
```

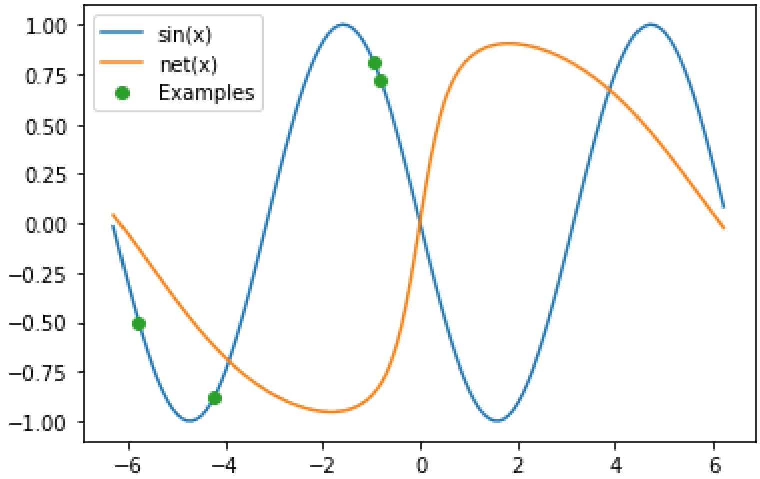


We are going to try to understand what the above code does.

2) Modify the above evaluation cell to set t_b = math.pi. Qualitatively how does the solution compare to t_b = 0. Explain the behavior.

```
t_b = math.pi #math.pi
t_y = torch.sin(a*t_x + t_b)

opt.zero_grad()

t_params = params
for k in range(n_inner_loop):
    t_f = net(t_x, t_params)
    t_loss = F.l1_loss(t_f, t_y)

    grads = torch.autograd.grad(t_loss, t_params, create_graph=True)
    t_params = [(t_params[i] - alpha*grads[i]) for i in range(len(params))]



test_y = torch.sin(a*test_x + t_b)
test_f = net(test_x, t_params)

plt.plot(test_x.data.numpy(), test_y.data.numpy(), label='sin(x)')
plt.plot(test_x.data.numpy(), test_f.data.numpy(), label='net(x)')
plt.plot(t_x.data.numpy(), t_y.data.numpy(), 'o', label='Examples')
plt.legend()
plt.show()
```

```
/usr/local/lib/python3.7/dist-packages/torch/nn/functional.py:1698: UserWarning: nn.f
  warnings.warn("nn.functional.tanh is deprecated. Use torch.tanh instead.")
```



It's really bad , the predicted sin wave looks like shifted by and flatten

3) Rerun the cell from (1) and then write a code to compare the result on the same sinusoid to just initializing the network with a random w_0 instead of one learnt with the nested objective. Fit the same function with the same number of steps (n_inner_loop).

```
def net(x, params):
    x = F.linear(x, params[0], params[1])
    x = F.tanh(x)

    x = F.linear(x, params[2], params[3])
    x = F.tanh(x)

    x = F.linear(x, params[4], params[5])
    return x

params = [
    # torch.Tensor(32, 1).rand().requires_grad_(),
    torch.rand(32, 1).requires_grad_(),
    torch.Tensor(32).zero_().requires_grad_(),

    torch.Tensor(32, 32).uniform_(-1./math.sqrt(32), 1./math.sqrt(32)).requires_grad_(),
    torch.Tensor(32).zero_().requires_grad_(),

    torch.Tensor(1, 32).uniform_(-1./math.sqrt(32), 1./math.sqrt(32)).requires_grad_(),
    torch.Tensor(1).zero_().requires_grad_(),
]

opt = torch.optim.SGD(params, lr=1e-2)
n_inner_loop = 3
alpha = 3e-2

for it in range(5000):
    # # sample frequency
    # a = torch.rand(1).item()*0.5+0.5
```

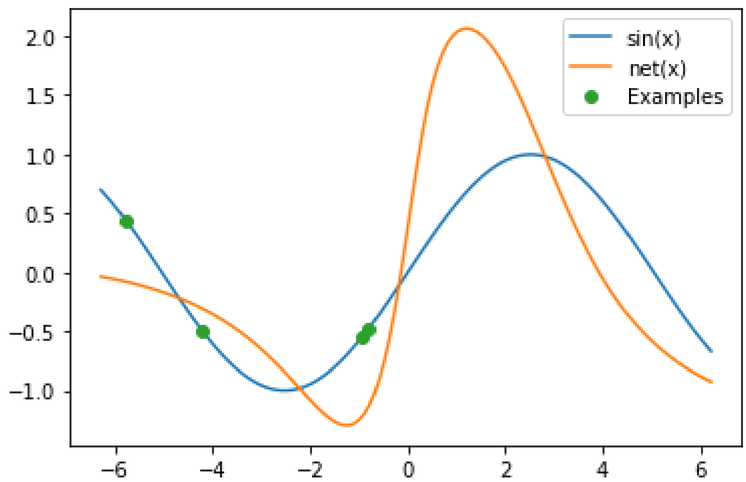
```
    # #Data for the inner loop
    # x = torch.rand(4, 1)*4*math.pi - 2*math.pi
    # y = torch.sin(a*x)

    # #Data for the outer loop
    # v_x = torch.rand(4, 1)*4*math.pi - 2*math.pi
    # v_y = torch.sin(a*v_x)

    opt.zero_grad()

    new_params = params
    for k in range(n_inner_loop):
        f = net(x, new_params)
        loss = F.l1_loss(f, y)

        # create_graph=True because computing grads here is part of the forward pass.
        # We want to differentiate through the SGD update steps and get higher order
        # derivatives in the backward pass.
        grads = torch.autograd.grad(loss, new_params, create_graph=True)
        new_params = [(new_params[i] - alpha*grads[i]) for i in range(len(params))]

        if it % 500 == 0: print('Iteration %d -- Inner loop %d -- Loss: %.4f' % (it, k, lo

    v_f = net(v_x, new_params)
    loss2 = F.l1_loss(v_f, v_y)
    loss2.backward()

    opt.step()

    if it % 500 == 0: print('Iteration %d -- Outer Loss: %.4f' % (it, loss2))
```

```
/usr/local/lib/python3.7/dist-packages/torch/nn/functional.py:1698: UserWarning: nn.f
  warnings.warn("nn.functional.tanh is deprecated. Use torch.tanh instead.")
Iteration 0 -- Inner loop 0 -- Loss: 0.5309
Iteration 0 -- Inner loop 1 -- Loss: 0.5176
Iteration 0 -- Inner loop 2 -- Loss: 0.5042
Iteration 0 -- Outer Loss: 0.9492
Iteration 500 -- Inner loop 0 -- Loss: 2.8151
Iteration 500 -- Inner loop 1 -- Loss: 2.0769
Iteration 500 -- Inner loop 2 -- Loss: 1.3137
Iteration 500 -- Outer Loss: 0.2826
Iteration 1000 -- Inner loop 0 -- Loss: 2.4529
Iteration 1000 -- Inner loop 1 -- Loss: 1.7242
Iteration 1000 -- Inner loop 2 -- Loss: 1.0397
Iteration 1000 -- Outer Loss: 0.1562
Iteration 1500 -- Inner loop 0 -- Loss: 2.1900
Iteration 1500 -- Inner loop 1 -- Loss: 1.5213
Iteration 1500 -- Inner loop 2 -- Loss: 0.9341
Iteration 1500 -- Outer Loss: 0.0898
Iteration 2000 -- Inner loop 0 -- Loss: 1.9486
Iteration 2000 -- Inner loop 1 -- Loss: 1.2906
Iteration 2000 -- Inner loop 2 -- Loss: 0.7262
Iteration 2000 -- Outer Loss: 0.1189
Iteration 2500 -- Inner loop 0 -- Loss: 2.0215
Iteration 2500 -- Inner loop 1 -- Loss: 1.4217
Iteration 2500 -- Inner loop 2 -- Loss: 0.9087
Iteration 2500 -- Outer Loss: 0.0728
Iteration 3000 -- Inner loop 0 -- Loss: 1.8790
```

```
Iteration 3000 -- Inner loop 1 -- Loss: 1.2834
Iteration 3000 -- Inner loop 2 -- Loss: 0.7779
Iteration 3000 -- Outer Loss: 0.0489
Iteration 3500 -- Inner loop 0 -- Loss: 1.9366
Iteration 3500 -- Inner loop 1 -- Loss: 1.3580
Iteration 3500 -- Inner loop 2 -- Loss: 0.8670
Iteration 3500 -- Outer Loss: 0.0654
Iteration 4000 -- Inner loop 0 -- Loss: 1.7385
Iteration 4000 -- Inner loop 1 -- Loss: 1.1860
Iteration 4000 -- Inner loop 2 -- Loss: 0.7207
Iteration 4000 -- Outer Loss: 0.0644
Iteration 4500 -- Inner loop 0 -- Loss: 1.7378
Iteration 4500 -- Inner loop 1 -- Loss: 1.1902
Iteration 4500 -- Inner loop 2 -- Loss: 0.7286
Iteration 4500 -- Outer Loss: 0.0431
```

```
t_b = 0 #math.pi
a = torch.rand(1).item()*0.5+0.5
t_y = torch.sin(a*t_x + t_b)
# print(t_y)
opt.zero_grad()
t_params = params
for k in range(n_inner_loop):
    t_f = net(t_x, t_params)
    t_loss = F.l1_loss(t_f, t_y)
    # print(t_loss)
    grads = torch.autograd.grad(t_loss, t_params, create_graph=True)
    t_params = [(t_params[i] - alpha*grads[i]) for i in range(len(params))]

test_y = torch.sin(a*test_x + t_b)

test_f = net(test_x, t_params)
# print(test_f)
plt.plot(test_x.data.numpy(), test_y.data.numpy(), label='sin(x)')
plt.plot(test_x.data.numpy(), test_f.data.numpy(), label='net(x)')
plt.plot(t_x.data.numpy(), t_y.data.numpy(), 'o', label='Examples')
plt.legend()
plt.show()
```

```
/usr/local/lib/python3.7/dist-packages/torch/nn/functional.py:1698: UserWarning: nn.f
  warnings.warn("nn.functional.tanh is deprecated. Use torch.tanh instead.")
```



4) Now let's imagine we also want to find the best learning rate to use in the inner loop using gradient descent. We can now add the learning rate as a parameter to optimize in the outer objective. Thus we have

$\min_{w,\alpha} \mathcal{L}(f_{w_T}(X_{val}), f_t(X_{val}))$ where w is obtained by iterating T times $w_t = w_{t-1} - \alpha \nabla \mathcal{L}(f_{w_{t-1}}(X_{train}), f_s(X_{train}))$. Note w_T is dependent on w_0 and α.

The code below implements this idea. Modify it to print the gradient of α in the first iteration of the outer loop and run the code.

```
# From http://www.sagargv.com/blog/meta-learning-in-pytorch/

import math
import random
import torch
from torch import nn
from torch.nn import functional as F
import matplotlib.pyplot as plt

def net(x, params):
    x = F.linear(x, params[0], params[1])
    x = F.tanh(x)

    x = F.linear(x, params[2], params[3])
    x = F.tanh(x)

    x = F.linear(x, params[4], params[5])
    return x

params = [
    torch.Tensor(32, 1).uniform_(-1., 1.).requires_grad_(),
    torch.Tensor(32).zero_().requires_grad_(),

    torch.Tensor(32, 32).uniform_(-1./math.sqrt(32), 1./math.sqrt(32)).requires_grad_(),
    torch.Tensor(32).zero_().requires_grad_(),

    torch.Tensor(1, 32).uniform_(-1./math.sqrt(32), 1./math.sqrt(32)).requires_grad_(),
    torch.Tensor(1).zero_().requires_grad_(),
]

alpha = torch.FloatTensor([0.1]).requires_grad_()

opt = torch.optim.SGD(params, lr=1e-2)
opt2 = torch.optim.Adam([alpha], lr=1e-4)
n_inner_loop = 5

for it in range(5000):
    # sample frequency
    a = torch.rand(1).item()*0.5+0.5

    #Data for the inner loop
    x = torch.rand(4, 1)*4*math.pi - 2*math.pi
```

```
    y = torch.sin(a*x)

    #Data for the outer loop
    v_x = torch.rand(4, 1)*4*math.pi - 2*math.pi
    v_y = torch.sin(a*v_x)

    opt.zero_grad()
    opt2.zero_grad()

    new_params = params
    for k in range(n_inner_loop):
        f = net(x, new_params)
        loss = F.l1_loss(f, y)

        # create_graph=True because computing grads here is part of the forward pass.
        # We want to differentiate through the SGD update steps and get higher order
        # derivatives in the backward pass.
        grads = torch.autograd.grad(loss, new_params, create_graph=True)
        new_params = [(new_params[i] - alpha*grads[i]) for i in range(len(params))]

        if it % 500 == 0: print('Iteration %d -- Inner loop %d -- Loss: %.4f' % (it, k, lo

    v_f = net(v_x, new_params)
    loss2 = F.l1_loss(v_f, v_y)
    loss2.backward()

    #TODO print the gradient of alpha

    opt.step()
    opt2.step()

    if it % 500 == 0:
      print('Iteration %d -- Outer Loss: %.4f, learning rate: %.5f' % (it, loss2, alpha))
      print("alpha grad=",alpha.grad)
```

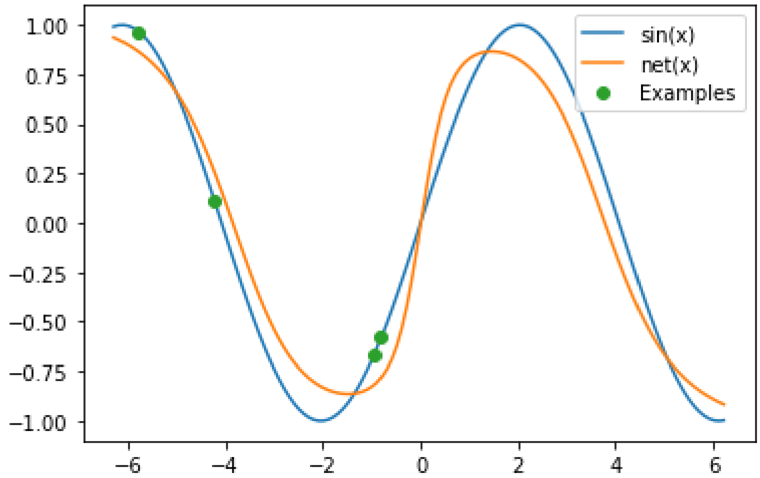
```
Iteration 500 -- Inner loop 4 -- Loss: 0.4832
Iteration 500 -- Outer Loss: 0.6964, learning rate: 0.08678
alpha grad= tensor([-0.7778])
Iteration 1000 -- Inner loop 0 -- Loss: 0.1057
Iteration 1000 -- Inner loop 1 -- Loss: 0.0918
Iteration 1000 -- Inner loop 2 -- Loss: 0.1059
Iteration 1000 -- Inner loop 3 -- Loss: 0.0912
Iteration 1000 -- Inner loop 4 -- Loss: 0.1060

Iteration 1000 -- Outer Loss: 0.2944, learning rate: 0.07558
alpha grad= tensor([0.3883])
Iteration 1500 -- Inner loop 0 -- Loss: 0.4531
Iteration 1500 -- Inner loop 1 -- Loss: 0.3680
Iteration 1500 -- Inner loop 2 -- Loss: 0.2939
Iteration 1500 -- Inner loop 3 -- Loss: 0.2238
Iteration 1500 -- Inner loop 4 -- Loss: 0.2096
Iteration 1500 -- Outer Loss: 0.1250, learning rate: 0.06116
alpha grad= tensor([-1.4812])
Iteration 2000 -- Inner loop 0 -- Loss: 0.4415
Iteration 2000 -- Inner loop 1 -- Loss: 0.3259
Iteration 2000 -- Inner loop 2 -- Loss: 0.2418
Iteration 2000 -- Inner loop 3 -- Loss: 0.1760
```

```
Iteration 2000 -- Inner loop 4 -- Loss: 0.1558
Iteration 2000 -- Outer Loss: 0.1600, learning rate: 0.04408
alpha grad= tensor([-4.2657])
Iteration 2500 -- Inner loop 0 -- Loss: 0.1985
Iteration 2500 -- Inner loop 1 -- Loss: 0.1940
Iteration 2500 -- Inner loop 2 -- Loss: 0.1783
Iteration 2500 -- Inner loop 3 -- Loss: 0.1657
Iteration 2500 -- Inner loop 4 -- Loss: 0.1795
Iteration 2500 -- Outer Loss: 0.3859, learning rate: 0.02753
alpha grad= tensor([2.3833])
Iteration 3000 -- Inner loop 0 -- Loss: 0.2127
Iteration 3000 -- Inner loop 1 -- Loss: 0.1333
Iteration 3000 -- Inner loop 2 -- Loss: 0.0936
Iteration 3000 -- Inner loop 3 -- Loss: 0.0751
Iteration 3000 -- Inner loop 4 -- Loss: 0.0843
Iteration 3000 -- Outer Loss: 0.2477, learning rate: 0.01924
alpha grad= tensor([6.8957])
Iteration 3500 -- Inner loop 0 -- Loss: 0.3324
Iteration 3500 -- Inner loop 1 -- Loss: 0.2805
Iteration 3500 -- Inner loop 2 -- Loss: 0.2381
Iteration 3500 -- Inner loop 3 -- Loss: 0.2039
Iteration 3500 -- Inner loop 4 -- Loss: 0.1761
Iteration 3500 -- Outer Loss: 0.1262, learning rate: 0.01573
alpha grad= tensor([8.7041])
Iteration 4000 -- Inner loop 0 -- Loss: 0.2974
Iteration 4000 -- Inner loop 1 -- Loss: 0.1121
Iteration 4000 -- Inner loop 2 -- Loss: 0.0917
Iteration 4000 -- Inner loop 3 -- Loss: 0.1448
Iteration 4000 -- Inner loop 4 -- Loss: 0.1320
Iteration 4000 -- Outer Loss: 0.0975, learning rate: 0.01521
alpha grad= tensor([-9.9040])
Iteration 4500 -- Inner loop 0 -- Loss: 0.3073
Iteration 4500 -- Inner loop 1 -- Loss: 0.2831
Iteration 4500 -- Inner loop 2 -- Loss: 0.2632
Iteration 4500 -- Inner loop 3 -- Loss: 0.2466
Iteration 4500 -- Inner loop 4 -- Loss: 0.2323
Iteration 4500 -- Outer Loss: 0.5125, learning rate: 0.01537
```

```python
t_b = 0 #math.pi
a = torch.rand(1).item()*0.5+0.5
t_y = torch.sin(a*t_x + t_b)
# print(t_y)
opt.zero_grad()
t_params = params
for k in range(n_inner_loop):
    t_f = net(t_x, t_params)
    t_loss = F.l1_loss(t_f, t_y)
    # print(t_loss)
    grads = torch.autograd.grad(t_loss, t_params, create_graph=True)
    t_params = [(t_params[i] - alpha*grads[i]) for i in range(len(params))]

test_y = torch.sin(a*test_x + t_b)

test_f = net(test_x, t_params)
# print(test_f)
plt.plot(test_x.data.numpy(), test_y.data.numpy(), label='sin(x)')
plt.plot(test_x.data.numpy(), test_f.data.numpy(), label='net(x)')
plt.plot(t_x.data.numpy(), t_y.data.numpy(), 'o', label='Examples')
```

```
plt.legend()
plt.show()
```

```
/usr/local/lib/python3.7/dist-packages/torch/nn/functional.py:1698: UserWarning: nn.f
  warnings.warn("nn.functional.tanh is deprecated. Use torch.tanh instead.")
```



Apparently, this is the highest fit