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Applications of statistical physics to the oil industry: predicting oil recovery using percolation theory

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Abstract

In this paper we apply scaling laws from percolation theory to the problem of estimating the time for a fluid injected into an oil field (for the purposes of recovering the oil) to breakthrough into a production well. The main contribution is to show that percolation theory, when applied to a realistic model, can be used to obtain the same results as calculated in a more conventional way but significantly more quickly. Specifically, we found that a previously proposed scaling form for the breakthrough time distribution when applied to a real oil field is in good agreement with more time consuming simulation results. Consequently these methods can be used in practical engineering circumstances to aid decision making for real field problems.

1. Introduction

There are many practical ways in which statistical physics is being used by the oil industry. To list just a few, real space renormalisation ideas are used to derive large-scale flow parameters given a detailed geological description of an oil reservoir [1,2], concepts similar to Potts models are used to represent detailed geological structure in reservoirs [3–5], understanding pattern formation in granular materials helps understand geological processes [6], non-linear dynamics of drill strings is being used to reduce drilling costs [7], simulated annealing is used to optimise business decisions adding hundreds of millions of dollars to the value of projects [8]. And so the list goes

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on. There are too many applications to go into any detail in this presentation. Instead we shall concentrate on the problem of how to predict the uncertainty in recovery given statistical information of the underlying disorder in an oil reservoir.

Oil reservoirs are extremely complex, containing geological heterogeneities on all length scales from centimetres to kilometres. These heterogeneities have a significant impact on hydrocarbon recovery by perturbing the displacement front, leading to a reduction in recovery by around 25% (a further 25% being lost by trapping at the pore scale by interfacial tension).

The most common method of oil recovery is by displacement. Either water or a miscible gas is injected into wells, to push the oil to production wells. Ultimately, the injected fluid breaks through at the production wells reducing the amount of oil that can be produced. These fluids must be separated from the produced oil and suitably disposed of, which can be very costly. For economic purposes it is important to know when the injected fluid will break through and what the rate of decline in oil production will be. In this paper we concentrate on the first of these problems.

The reservoir rock is highly heterogeneous because of the sedimentary processes that deposited it. In many cases it is possible to distinguish between "good" rock (high permeability) and "poor" rock (low or zero permeability). For all practical purposes, the flow takes place just in the good rock. For example, the good rock may be ancient river channels producing sandbodies of length tens of kilometres, width tens to hundreds of metres and thickness up to tens of metres. It is the interconnectivity of these channels that controls the flow. The spatial distribution of the sandbodies is also governed by the geological process but can frequently be considered to be statistically independent or of a short-range correlation. Hence, the problem of the connectivity of the sandbodies is precisely a continuum percolation problem. The place of the occupancy probability p of percolation theory is taken by the volume fraction of good sand (known as the net-to-gross ratio in the oil industry literature).

We have very little direct knowledge about the distribution of rock properties in the reservoir. Direct measurements are limited to samples that represent around 10^{-13} of the total reservoir volume. Away from the wells, rock properties must be inferred from statistical models, based on knowledge of the general geological environment and measurements made from modern day examples or from surface outcrops of a similar type. Hence, there is a great deal of uncertainty in the prediction of the rock properties. This leads to a large uncertainty in our prediction of time to breakthrough of the injected fluid. We need to assess this uncertainty for economic risk evaluation.

The conventional approach is to build detailed numerical models of the reservoir and then perform flow simulations to predict breakthrough. This is repeated with many realisations of the subsurface models to build up an estimate of the uncertainty. Unfortunately this process is very computer intensive with each flow simulation taking typically many hours of CPU time. In order to get good statistics one must repeat the calculation many hundreds or thousands of times. Additional calculations are necessary in order to estimate the uncertainty associated with an alternative pattern of injection and production wells, which is required to determine the optimal well pattern. The purpose of this study is to use the methods of percolation theory to make the estimation of uncertainty much more efficient. Previously percolation theory has been used to estimate static connectivity [9], and we now apply similar concepts to the dynamic displacement problem.

2. Flow model

To simplify the model, we shall assume that the permeability is either zero (shale) or one (sand). The sandbodies are cuboidal (and in the first instance isotropic in shape). They are distributed independently and randomly in space to a volume fraction of p. Further we shall assume that the displacing fluid has the same viscosity and density as the displaced fluid. This has the advantage that as the injected fluid displaces the oil the pressure field is unchanged. This pressure field is determined by the solution of the single-phase flow equations. That is, the local flow rate is given by Darcy's law $v = -K\nabla P$, which coupled with the conservation condition for an incompressible fluid $(\nabla \cdot v = 0)$ gives the equation for pressure as $\nabla \cdot (K\nabla P) = 0$. The injected flow then just follows the streamlines (normals to the isobars) of this flow. The permeability, K, is either zero or one as assumed above. The boundary conditions are fixed pressure of $+1$ at the injection well and 0 at the production well. In this work we shall only consider a single well pair separated by a Euclidean distance r. The breakthrough time then corresponds to the first passage time for transport between the injector and the producer.

For a given geometry of the reservoir we can then sample for different locations of the wells (or equivalently for the same well locations for different models of the reservoir with the same underlying statistics) and plot the distribution of breakthrough times. This is the conditional probability for the breakthrough time, t_{br}, given that the reservoir size (measured in dimensionless units of sandbody length) is L, the distance between wells is r, and the net-to-gross ratio is p, i.e., $P(t_{\mathrm{br}}|r, L, p)$. In previous studies [10–14] we have demonstrated by extensive simulations that this distribution obeys the following scaling Ansatz:

$$P(t_{\mathrm{br}}|r, L, p) \sim \frac{1}{r^{d_t}} \left(\frac{t_{\mathrm{br}}}{r^{d_t}}\right)^{-g_t} f_1\left(\frac{t_{\mathrm{br}}}{r^{d_t}}\right) f_2\left(\frac{t_{\mathrm{br}}}{L^{d_t}}\right) f_3\left(\frac{t_{\mathrm{br}}}{|p - p_c|^{-\nu d_t}}\right), \tag{1}$$

where

$$f_1(x) = \exp(-ax^{-\phi}), \tag{2}$$

$$f_2(x) = \exp(-bx^{\psi}), \tag{3}$$

$$f_3(x) = \exp(-cx^{\theta}). \tag{4}$$

Hence d_t is the exponent characterizing how $\langle t_{\mathrm{br}} \rangle$ scales with r, and ν is the correlation length exponent. In this paper we will not discuss the background to this scaling relationship, but concentrate on how well it succeeds in predicting the breakthrough time for a realistic permeability field.

Fig. 1. (a) Permeability map for North Sea field. Note the vertical exaggeration; the true dimensions are 700 m $\times$ 700 m $\times$ 170 m vertically. Colors are chosen from a rainbow scale, from red (high permeability of $\approx$900 mD) to blue (low permeability of $\approx$100 mD) (b) Subset of Fig. 1a. showing only the region with permeability larger than a cutoff of 273 mD.

3. Application to a real oil field

We take as an example a model of a North Sea oil field. The permeability map is shown in Fig. 1a. This is a turbidite reservoir. That is, the sands were deposited by submarine turbidity currents ("avalanches") in a deep sea environment. The resulting sands consist of channels (from the avalanche scour) and lobe-shaped sandbodies from the resulting avalanche plumes. The permeability is bi-modal (Fig. 2). We apply a cutoff in permeability at 273 mD to separate the sand into good and bad. The resulting sandbodies are shown in Fig. 1b. This permeability cutoff was chosen as it corresponds to the threshold value for the system at which the incipient infinite cluster appears. Thus for this cutoff value the sandbodies are just connected. We then carried out simulations of the flow to determine the distribution of breakthrough times for well pairs at different locations and a variety of separations (Fig. 3a, solid lines).

From the scaling result, Eq. (1), and using the fact that

$$d_t = 1.33 \,, \tag{5}$$

we would expect to get data collapse if we plot $r^{1.33}P(t_{\rm br}|r, L, p)$ against $t_{\rm br}/r^{1.33}$ [11]. Fig. 3b shows this reasonably well but there is a lot of noise which somewhat obscures it. However, we can determine what we would expect the breakthrough time distribution to be from the scaling relationship. We plot this in Fig. 3a, using dashed lines. The agreement with the Monte Carlo predictions is certainly good enough for engineering purposes. The main point is that the scaling predictions took a fraction of a second of CPU time compared with the hours required for the conventional Monte Carlo approach, making this a practical tool to be used for engineering and management decisions.

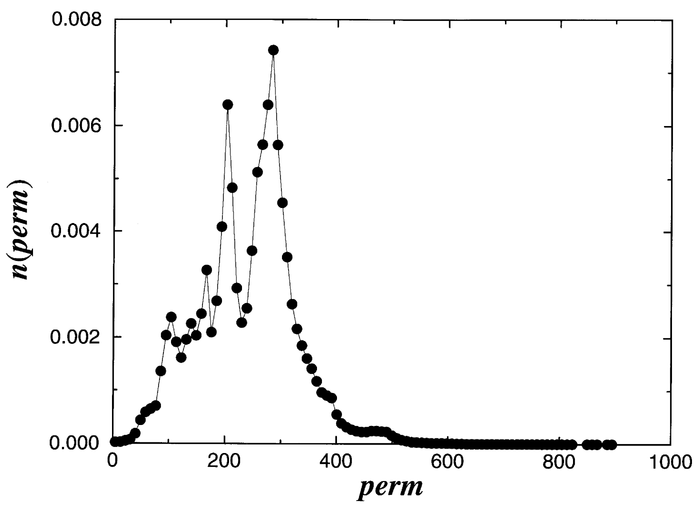

Fig. 2. Permeability distribution for North Sea Field. The approximate bimodal form is often found.

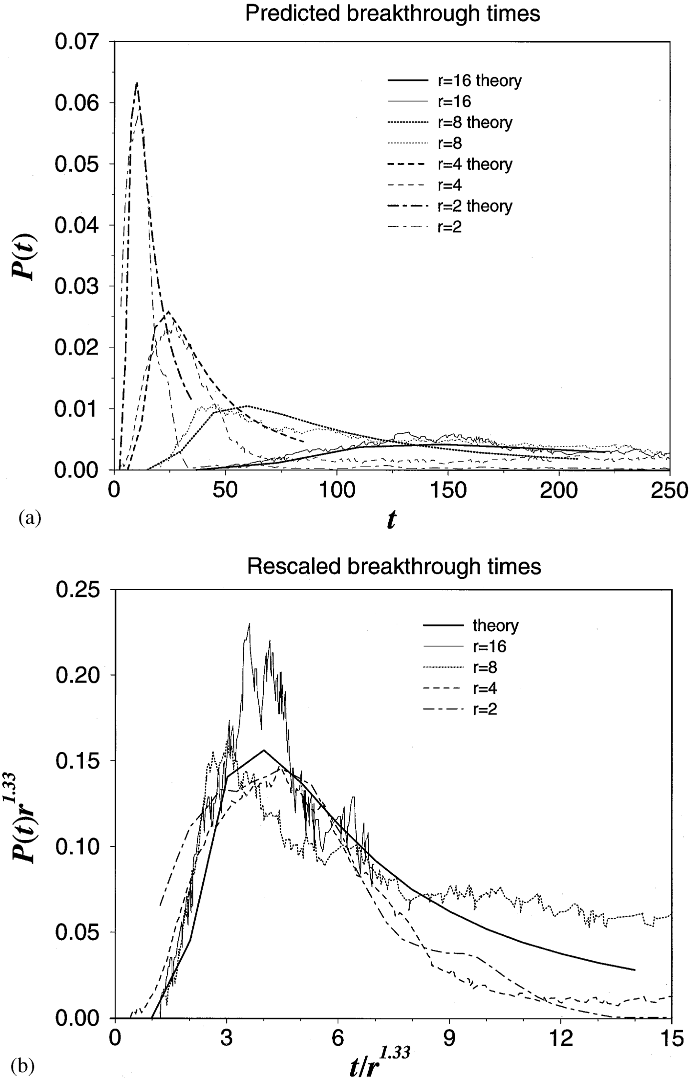

Fig. 3. (a) Distributions of breakthrough times for four different values of the well spacing r. Lines are determined by explicit Monte Carlo calculations, and from the theoretical scaling law of Eq. (1). (b) Rescaled distributions of breakthrough time for the same data, with the exponent value discussed in Section 3.

4. Conclusions

We have applied results obtained earlier for the scaling law [10,11] for breakthrough time distributions for oil field recovery to realistic data from a North Sea field. We have shown that agreement between the theory and the conventional Monte Carlo approach is accurate for engineering purposes and therefore makes it a practical tool for decision making.

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