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SINGLE TOP QUARK PRODUCTION
AND V_{tb} CKM MATRIX ELEMENT MEASUREMENT
IN HIGH ENERGY e^+e^- COLLISIONS

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Abstract

Dokholyan N.V., Jikia G.V. Single Top Quark Production and V_{tb} CKM Matrix Element Measurement in High Energy e^+e^- Colisions: IHEP Preprint 93-111. – Protvino, 1993. – p. 9, figs. 6, tables 1, refs.: 16.

The new method of determination of CKM mixing matrix element V_{tb} has been proposed. It has been shown, that at the future colliders one will measure the tb-mixing element with the accuracy 12 – 28 %.

Аннотация

Дохолян Н.В., Джикия Г.В. Одиночное рождение топ-кварка и измерение элемента матрицы ККМ V_{tb} в высокоэнергетичных e^+e^--столкновениях: Препринт ИФВЭ 93-111. – Протвино, 1993. – 9 с, 6 рис., 1 табл., библиогр.: 16.

Был предложен новый метод определения элемента матрицы смешивания Кабиббо-Кобаяши-Масакава V_{tb}. Показано, что на будущих ускорителях станет возможным измерение $|V_{tb}|^2$ с точностью 12–28%.

INTRODUCTION

The experimental measurements of the CKM matrix element V_{tb} and/or top quark width Γ_t are known not to be an easy problem for heavy top quark $m_t > 108$ GeV [1] due to the very short life time of the top quark which is decaying with almost unit probability to b-jet and a W-boson. It seems that the best possibilities are provided by next generation linear e^+e^- colliders [2].

Several methods have been proposed to measure V_{tb} and Γ_t:

(1) Energy scan in the threshold region of $t\bar{t}$ production can give the accuracy of the top quark width measurement of $\Delta\Gamma_t = {(+10) \atop (-30)}\%$ [2,8,9,11]. The experimental difficulties are due to initial state radiation, beam energy spread and beamstrahlung effects:

- Beam effects strongly influence the threshold shape [9]. The beam energy spread plays the main role in the smearing of the peak and the beamstrahlung causes some reduction of the luminosity. So that, the switching on of both these phenomena leads to the strong suppression of the peak and the height of the peak on the threshold of the reaction $e^+e^- \to t\bar{t}$ falls down from approximately 1.32 pb to 0.40 pb, i.e. 70 % ($m_t = 150$ GeV, $|V_{tb}|^2 = 1.$, $\alpha_s = 0.12$) and the peak, factually, disappears [9]. The fact that beamstrahlung influences usable luminosity is clearly seen if we switch off only beamstrahlung. The cross-section of this reaction at the peak decreases by approximately 0.57 pb, i.e. 43 %. This means that we have to know the beam energy spectrum with high resolution.

- The QCD corrections also influence the curve near the threshold [9], [10]. Thus, when $\Lambda^{(5)}_{\overline{MS}}$ changes from 0.22 GeV to 0.12 GeV the height of the peak falls from 680 fb to 560 fb and the peak position shifts to the right by approximately 1 GeV (by $\sqrt{s}$-axis).

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So, this method of determining $|V_{tb}|^2$ depends on many different parameters, and each of them we are supposed to know with high precision. That's why this way is characterized by a low precision of $|V_{tb}|^2$ measuring.

(2) The measurement of the soft gluon or photon radiation pattern in the reaction of $t\bar{t}$ production above the threshold can give a rough estimate of the Γ_t up to a factor of two [3,4,5].

(3) The sensitivity to the value of V_{tb} of the top quark momentum distribution and forward-backward asymmetry, which measures the degree of overlap of S- and P-waves due to a finite Γ_t, has been analyzed recently [6] including full $O(\alpha_S)$ corrections. Both methods require high luminosity. The first one can give a statistical error of $\Delta|V_{tb}|^2 = \pm0.05$ for the integrated luminosity of 100 fb^{-1}, while the second one can give a statistical error of $\Delta|V_{tb}|^2 = \pm0.07$ for 40 k events of $t\bar{t}$ production.

Here we propose to measure the V_{tb} matrix element in the reaction of single t-quark production well above the threshold. It seems this process does not suffer from theoretical uncertainties due to higher order QCD corrections and can be experimentally studied at moderate luminosity.

This process was discussed recently by S. Ambrosanio and B. Mele, who considered single top quark production at e^+e^- collisions with $\sqrt{s}$ below the $t\bar{t}$-pair threshold. We found complete agreement with [15]. We studied this region either, however, for our purposes the number of events of single top quark production below the $t\bar{t}$ threshold is too small for accurate measurement of $|V_{tb}|^2$.

1. PROCESS DISCUSSION

Let's consider the process:

$$e^-(p1) \; e^+(p2) \; \longrightarrow \; t(p3) \; \bar{b}(p4) \; W^-(p5).$$

Our calculations for this reaction are described by seven diagrams in fig.1. We have also calculated this process in 't Hooft-Feynman gauge. Although the latter gauge appends two additional diagrams, the propagator of the W^- - boson contains only one term which is proportionate to $g_{\mu\nu}$.
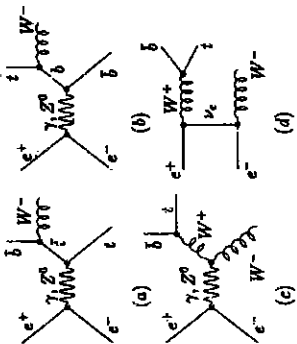

Figure 1. Feynman diagrams in unitary gauge.

• **Matrix element** [3] includes the vertex of the decay $\bar{t} \to \bar{b}W^-$, proportional to V_{tb} (the element of CKM mixing matrix) and Breit-Wigner propagator of the virtual top-quark (see fig.1(a)):

$$D^t \propto \frac{1}{p_t^2 - m_t^2 + i\cdot m_t\Gamma}, \qquad (1)$$

where Γ is top width decay, directly proportionate to $|V_{tb}|^2$. Therefore, diagram (a) gives the main contribution nearby the peak (where the intermediate top-quark lies on the mass shell ($\sqrt{s_{bW}} \simeq m_t$)), and its matrix element squared is proportional to

$$|M_{if}^{(a)}| \propto \frac{|V_{tb}|^2}{(p_t^2 - m_t^2)^2 + \xi\cdot|V_{tb}|^4} = \frac{|V_{tb}|^2}{(s_{bW} - m_t^2)^2 + \xi\cdot|V_{tb}|^4}, \qquad (2)$$

where $\xi = \frac{\Gamma^2 m_t^2}{|V_{tb}|^4}$ does not depend on $|V_{tb}|^2$, $s_{bW} = (p_4 + p_5)^2$.

• **Total Cross-Section**

$$\sigma_{tot} = \int \frac{|M_{if}|^2}{J} \cdot d\Phi, \qquad (3)$$

where J means flux and $d\Phi$ is phase space. After the integration over full two-particles phase space, any information about $|V_{tb}|^2$ is lost (see diagram (a) as the example). Hence, we need to step aside from the peak ($\sqrt{s_{bW}} \simeq m_t$). But a new problem arises: cross - section lessens strongly. It is clearly seen for different $\sqrt{s}$ from fig.2 and fig.3. Consequently, we have to find the 'golden mean', i.e. optimal composition of the cross-section and information about V_{tb}.

• **It is worth mentioning** that there is no broad selection as to how one cuts the peak, owing to experimental restriction on the resolution of hadronic calorimeter [8],[9]:

$$\frac{\sigma_E}{E} = \frac{\Delta}{E} = \sqrt{\left(\frac{0.1}{\sqrt{E(GeV)}}\right)^2 + (0.02)^2}. \tag{4}$$

As it is obviously seen from formula (4), Δ should not be less than $3 \div 5$ GeV near the peak. After cutting we will investigate the value:

$$\sigma_\Delta = \int_{E_{bW}^{min}}^{m_t-\Delta} \frac{\partial \sigma}{\partial E_{bW}} dE_{bW} + \int_{m_t+\Delta}^{E_{bW}^{max}} \frac{\partial \sigma}{\partial E_{bW}} dE_{bW}, \tag{5}$$

where $E_{bW}^{min} = m_W + m_b$ and $E_{bW}^{max} = \sqrt{s} - m_t$. This dependence of σ_Δ on Δ for different $|V_{tb}|^2 = 1.15, 1.00, 0.85$ is represented in fig.4 and fig.5. All the curves gather at the point $\Delta = 0$ because of lack of information about $|V_{tb}|^2$ at the peak (where $\sqrt{s_{bW}} \simeq m_t$). Further, for $\sqrt{s} = 500$ GeV and for integrated luminosity $L = 10fb^{-1}$ we have obtained about 400 events when we back out for $\Delta = 5$ GeV, what is in general sufficient for statistics.
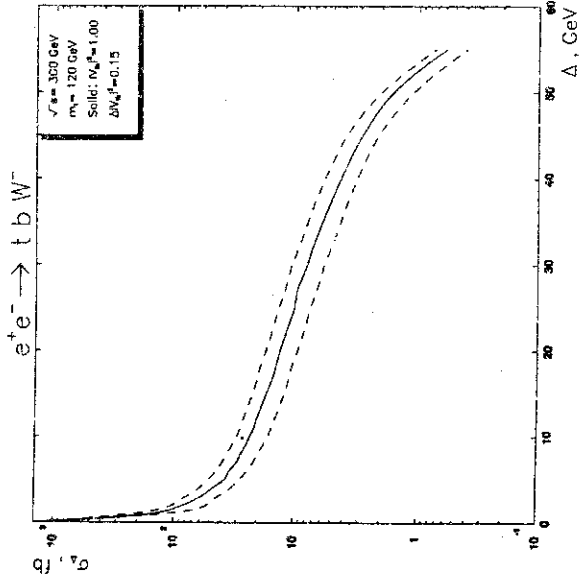

Figure 4. Cross-section σ_Δ depending on cutting parameter Δ for $\sqrt{s}=300$ GeV. $|V_{tb}|^2$ is the same as in fig.2.
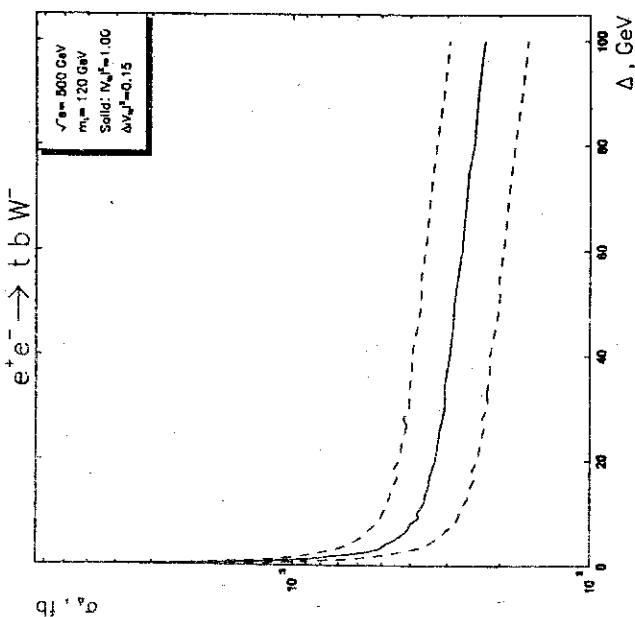

Figure 2. Differential cross-section depending on $\Xi = E_{bW} - m_t$ for $\sqrt{s}=300$ GeV and $m_t=120$ GeV. Solid line is for $|V_{tb}|^2=1.00$, upper line is for $|V_{tb}|^2=1.15$ and lower line is for $|V_{tb}|^2=0.85$.
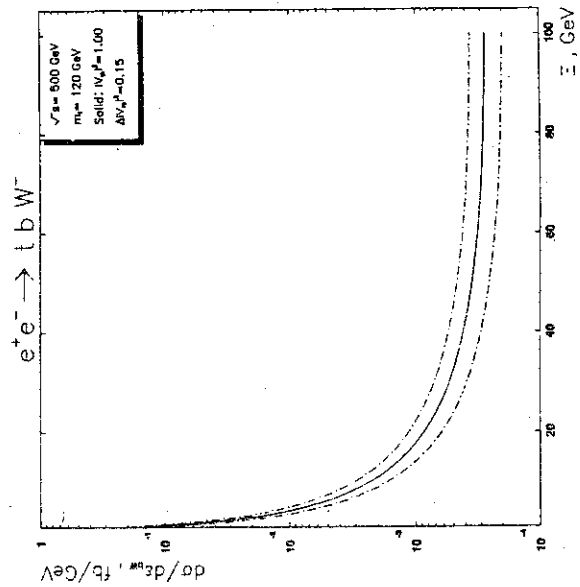
Figure 3. The same as in fig.2, but for $\sqrt{s}=500$ GeV.

• Event Selection. Minimizing [14]

Figure 5. See caption to fig.4, but for $\sqrt{s}$=500 GeV.

we find such a value of $|V_{tb}|^2_{min}$ that χ^2 from (6) becomes minimal – χ^2_{min}.

$$\chi^2 = \sigma_{tot} \cdot L \cdot \epsilon_{eff} \cdot \sum_{k=1}^{N_{bin}} \frac{(\tilde{n}_k - n_k)^2}{n_k} \quad (6)$$

Here σ_{tot} is the total cross-section, L is the luminosity, ϵ_{eff} is the efficiency of $t\bar{t}$-pair registration [4], N_{bin}-number of bins of $E_{bW} = \sqrt{s_{bW}}$-axis broken according to (4). It is worse taking into account, that while we can measure the energy of the beam with accuracy (4), the minimal bins length is about 5 GeV. The frequencies $n_k = N_k/N$ and $\tilde{n}_k = \tilde{N}_k/N$ do not depend on the total number of events [5] N, and correspond to the $|V_{tb}|^2 \neq 1$ and $|V_{tb}|^2 = 1$, respectively. Further, basing on the following system:

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[4] $\epsilon_{eff} \simeq 0.30$ [2]

[5] All the cross-sections were calculated with the help of integrating package VEGAS[12]. The events were generated with the aid of program BASES/SPRING[13]

$$\begin{cases} \chi^2 = \chi^2_{min} + \Delta\chi^2 \\ |V_{tb}|^2 = |V_{tb}|^2_{min} \pm \Delta|V_{tb}|^2 \end{cases} \quad (7)$$

and according to the value of $\Delta\chi^2$ we can already judge with what precision $\Delta|V_{tb}|^2$ we measure $|V_{tb}|^2$.

2. RESULTS AND DISCUSSION

We computed quantity $\Delta|V_{tb}|^2$ for $\Delta = 0$ and $5\ GeV$, $\sqrt{s} = 500\ GeV$ ($m_t = 120, 150, 180\ GeV$) and $\sqrt{s} = 300\ GeV$ ($m_t = 120, 130$ and $140\ GeV$). All the results are collected in Table 1. Evidently, the higher cross-section is, the better we measure $|V_{tb}|^2$. The cross-section falls down while the top-quark mass is rising, because of the fact, that when m_t increases we approach the reaction threshold. However the picture changes for $\sqrt{s} = 500\ GeV$ and $\Delta = 5\ GeV$. σ_Δ is growing, when m_t is increasing. It follows from the fact, that the main contribution gains from the peak (where $\sqrt{s_{bW}} = m_t$) and the peak width for $m_t = 120\ GeV$ is less than the width for $m_t = 150\ GeV$, etc. There such phenomenon is not observed for $\sqrt{s} = 500\ GeV$, because the decrease of the cross-section due to m_t dominante the phenomenon discussed above.

It is easy to see, that the results for $\Delta = 0\ GeV$ and $\Delta = 5\ GeV$ are almost not different. One should have expected it because of $|V_{tb}|^2$ information absence at the peak (see fig.4–6) discussed above. The small difference of results is due to lack of accuracy of cutting peak (5), and it is dictated by stipulation (4).

Table 1.

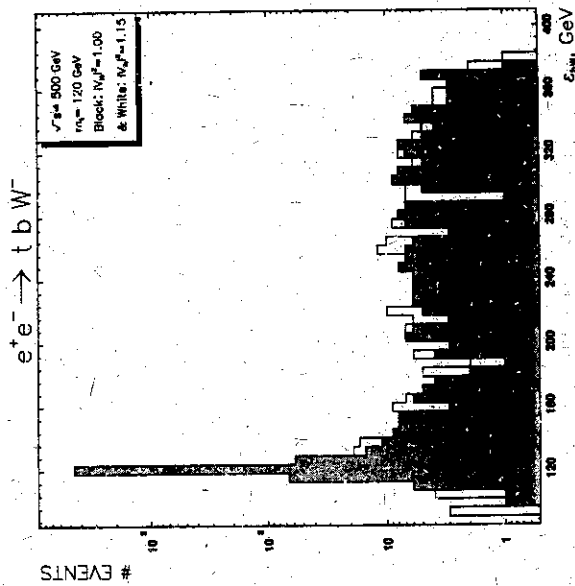| m_t, GeV	$\sqrt{s}=500GeV, \Delta=0GeV$ $\Delta\|V_{tb}\|^2$ 99%CL	95%CL	σ_{tot}, fb	m_t, GeV	$\sqrt{s}=500GeV, \Delta=5GeV$ $\Delta\|V_{tb}\|^2$ 99%CL	95%CL	σ_Δ, fb
120	0.197	0.175	707.3	120	0.199	0.179	45.4
150	0.231	0.205	651.8	150	0.190	0.168	54.8
180	0.283	0.252	571.4	180	0.182	0.156	69.0
	$\sqrt{s}=300GeV, \Delta=0GeV$				$\sqrt{s}=300GeV, \Delta=5GeV$		
120	0.152	0.132	1410.1	120	0.145	0.124	39.4
130	0.203	0.176	1181.1	130	0.193	0.168	37.8
140	0.286	0.241	854.3	140	0.265	0.219	29.0

Figure 6. The histogram for process $e^+e^- \rightarrow t\bar{b}W^-$ is generated for $\sqrt{s}=500$ GeV, $m_t=120$ GeV. Black color corresponds to $|V_{tb}|^2 = 1.00$. Black and white colors correspond to $|V_{tb}|^2 = 1.15$.

3. CONCLUSIONS

Summing it up one might say, that it is possible to measure $|V_{tb}|^2$ at the reaction $e^+e^- \rightarrow t\bar{b}W^-$ with the accuracy $12 \div 28$ %. Even though did not we investigate simulation of decay of top quark, we can say that the efficiency of $t\bar{t}$-pair registration was counted with the help of the parameter ϵ_{eff}. Full Monte Carlo generation including t-quark decay was done by K. Fujii [9] for the reactions of $t\bar{t}$-pair production. For this reaction, $e^-e^+ \rightarrow t\bar{t} \rightarrow 6jet$ he obtained accuracy of $|V_{tb}|^2$ order to $\Delta |V_{tb}|^2 \approx 35 \div 40\%$.

Acknowledgements

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