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Single top quark production and V_{tb} CKM matrix element measurement in high energy e^+e^- collisions

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Abstract

A new method for the determination of the CKM mixing matrix element V_{tb} has been proposed It has been shown, that at future colliders one will measure the tb-mixing element with an accuracy 12–28 %

1. Introduction

The experimental measurements of the CKM matrix element V_{tb} and/or top quark width Γ_t are known to be an uneasy problem for a heavy top quark $m_t >$ 129 GeV [1] due to the very short life time of the top quark which is decaying with almost unit probability to a b-jet and a W-boson It seems that the best possibilities are provided by next generation linear e^+e^- colliders [2]

Several methods have been proposed to measure V_{tb} and Γ_t

(1) An energy scan in the threshold region of $t\bar{t}$ production can give the accuracy of the top quark width measurement of $\Delta\Gamma_t = \left({}^{+10}_{-30}\right)\%$ [2,8,9,11]. The experimental difficulties are due to initial state radiation, beam energy spread and beamstrahlung effects·

- Beam effects strongly influence the threshold shape [9]. The energy spread of the beam plays the main role in the smearing of the peak and the beamstrahlung causes some reduction of the luminosity. So the switching on of both of these phenomena leads to a strong suppression of the peak and the height of the peak on the threshold of the reaction $e^+e^- \to t\bar{t}$ falls down from approximately 1 32 pb to 0 40 pb, i e 70 % (m_t = 150 GeV, $|V_{tb}|^2$ = 1., α_s = 0 12) and the peak, factually, disappears [9]. The fact that beamstrahlung influences usable luminosity is clearly seen if we switch off only beamstrahlung The cross-section of this reaction at the peak decreases by approximately 0.57 pb, i.e. 43 %. This means that we have to know the beam energy spectrum with high resolution.
- The QCD corrections also influence the curve near the threshold [9,10] Thus, when $\Lambda^{(5)}_{\overline{\rm MS}}$ changes from 0.22 GeV to 0.12 GeV the height of the peak falls from 680 fb to 560 fb and the peak position shifts to the right by approximately 1 GeV (by $\sqrt{s}$-axis).

So, this method of determining $|V_{tb}|^2$ depends on many different parameters, and we are supposed to know each of them with a high precision. That is why this method is characterized by a low precision of the

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mesurement of $|V_{tb}|^2$

(2) The measurement of the soft gluon or photon radiation pattern in the reaction of $t\bar{t}$ production above the threshold can give a rough estimate of the Γ_t up to a factor of two [3–5]

(3) The sensitivity to the value of V_{tb} of the top quark momentum distribution and forward–backward asymmetry, which measures the degree of overlap of S- and P-waves due to a finite Γ_t, has been analyzed recently [6] including full $\mathcal{O}(\alpha_S)$ corrections Both methods require a high luminosity The first one can give a statistical error of $\Delta|V_{tb}|^2 = \pm 0\ 05$ for the integrated luminosity of 100 fb^{-1}, while the second one can give a statistical error of $\Delta|V_{tb}|^2 = \pm 0.07$ for 40 k events of $t\bar{t}$ production

Here we propose to measure the V_{tb} matrix element in the reaction of single t-quark production well above the threshold It seems that this process does not suffer from theoretical uncertainties due to higher order QCD corrections and can be experimentally studied at moderate luminosity

This process was discussed recently by S Ambrosanio and B Mele, who considered single top quark production at e^+e^- collisions with $\sqrt{s}$ below the $t\bar{t}$-pair threshold. We found complete agreement with [17] We studied this region either, however, for our purposes the number of events of single top quark production below the $t\bar{t}$ threshold is too small for an accurate measurement of $|V_{tb}|^2$

2. Process discussion

Let us consider the process.

$$e^-(p1)e^+(p2) \longrightarrow t(p3)\bar{b}(p4)W^-(p5)$$

Our calculations for this reaction are described by seven diagrams in unitary gauge shown in Fig 1. We have also calculated this process in 't Hooft-Feynman gauge Although the latter gauge appends two additional diagrams, the propagator of the W^--boson contains only one term which is proportional to $g_{\mu\nu}$

- Matrix element [3] includes the vertex of the decay $\bar{t} \to \bar{b}W^-$, proportional to V_{tb} (the element of CKM mixing matrix) and Breit-Wigner propagator of the virtual top-quark (see Fig. 1a)·

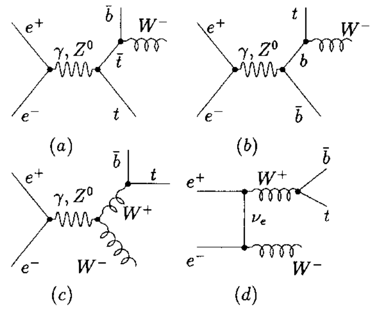

Fig 1 Feynman diagrams in unitary gauge

$$D^t \propto \frac{1}{p_t^2 - m_t^2 + i\, m_t \Gamma}, \tag{1}$$

where Γ – top width decay – is proportional to $|V_{tb}|^2$ Therefore, diagram (a) gives the main contribution nearby the peak (where the intermediate top-quark lies on the mass shell ($\sqrt{s_{bW}} \simeq m_t$)), and its matrix element squared is proportional to

$$\left|M_{if}^{(a)}\right|^2 \propto \frac{|V_{tb}|^2}{(p_t^2 - m_t^2)^2 + \xi \cdot |V_{tb}|^4} = \frac{|V_{tb}|^2}{(s_{bW} - m_t^2)^2 + \xi \cdot |V_{tb}|^4}, \tag{2}$$

where $\xi = \Gamma^2 m_t^2 / |V_{tb}|^4$ does not depend on $|V_{tb}|^2$, $s_{bW} = (p_4 + p_5)^2$.

- Total cross-section

$$\sigma_{\text{tot}} = \int \frac{|M_{if}|^2}{J}\ d\Phi, \tag{3}$$

where J means flux and $d\Phi$ is phase space

After the integration over full two-particles phase space, any information about $|V_{tb}|^2$ is lost (see diagram (a) as the example) Hence, we need to step aside from the peak ($\sqrt{s_{bW}} \simeq m_t$). But a new problem arises: cross-section lessens strongly It is clearly seen for different $\sqrt{s}$ from Fig 2 and Fig 3 Consequently, we have to find the "golden mean", i e optimal composition of the cross-section and information about V_{tb}.

- It is worth mentioning, that there is no broad selection as to how one cuts the peak, owing to ex-

[3] We used the program FORM in order to calculate squared matrix element $|M_{if}|^2$ in a symbolic level [18]

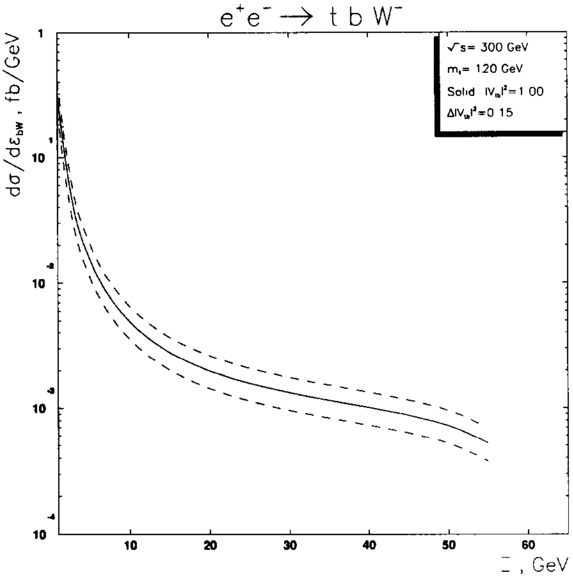

Fig 2 Differential cross-section depending on $\Xi = E_{bW} - m_t$ for $\sqrt{s} = 300$ GeV and $m_t = 120$ GeV Solid line is for $|V_{tb}|^2 = 1\ 00$, upper line is for $|V_{tb}|^2 = 1\ 15$ and lower line is for $|V_{tb}|^2 = 0\ 85$

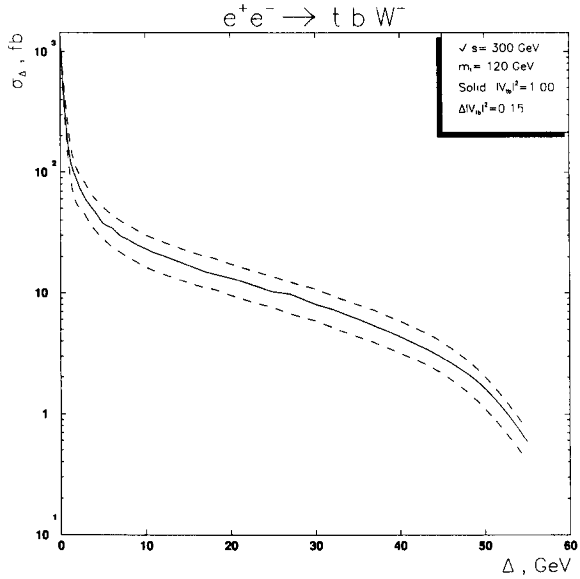

Fig 4 Cross-section σ_Δ depending on cutting parameter Δ for $\sqrt{s} = 300$ GeV $|V_{tb}|^2$ is the same as in Fig 2

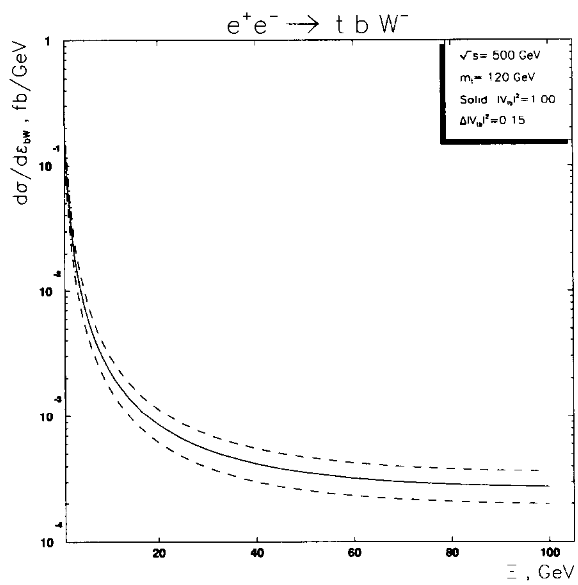

Fig 3 The same as in Fig 2, but for $\sqrt{s} = 500$ GeV

perimental restriction on the resolution of hadronic calorimeter [8,9].

$$\frac{\sigma_E}{E} = \frac{\Delta}{E} = \sqrt{\left(\frac{0.1}{\sqrt{E(\mathrm{GeV})}}\right)^2 + (0\ 02)^2}. \qquad (4)$$

As it is obviously seen from formula (4), Δ should not be less than 3–5 GeV near the peak. After cutting we will investigate the value

$$\sigma_\Delta = \int_{E_{bW}^{\min}}^{m_t-\Delta} \frac{\partial\sigma}{\partial E_{bW}} dE_{bW} + \int_{m_t+\Delta}^{E_{bW}^{\max}} \frac{\partial\sigma}{\partial E_{bW}} dE_{bW}, \qquad (5)$$

where $E_{bW}^{\min} = m_W + m_b$ and $E_{bW}^{\max} = \sqrt{s} - m_t$.

This dependence of σ_Δ on Δ for different $|V_{tb}|^2 = 1.15, 1\ 00, 0.85$ is represented in Fig. 4 and Fig 5 All the curves gather at the point $\Delta = 0$ because of lack of information about $|V_{tb}|^2$ at the peak (where $\sqrt{s_{bW}} \simeq m_t$). Further, for $\sqrt{s} = 500$ GeV and for an integrated luminosity $L = 10\ \mathrm{fb}^{-1}$ [14] we have obtained about 400 events when we back out for $\Delta = 5$ GeV, what is in general sufficient for statistics.

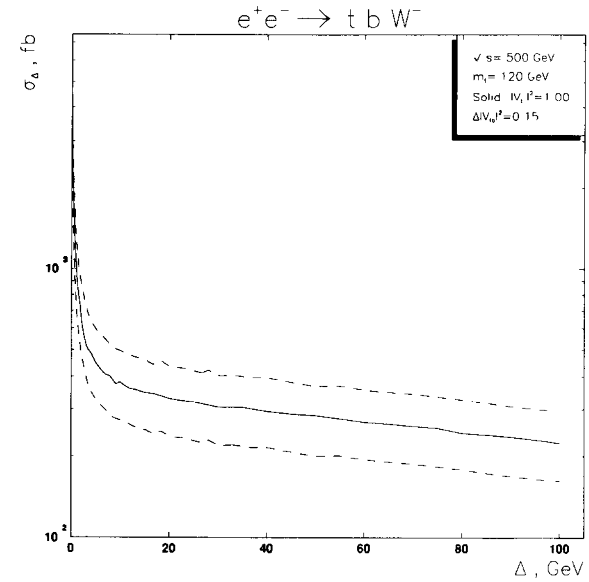

Fig 5 See caption to Fig 4, but for $\sqrt{s} = 500$ GeV

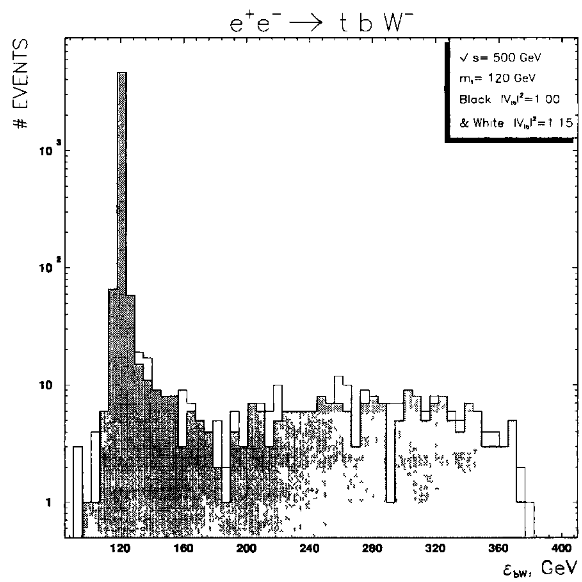

Fig 6 The histogram for process $e^+e^- \longrightarrow t\bar{b}W^-$ is generated for $\sqrt{s}$ = 500 GeV, m_t = 120 GeV Black color corresponds to $|V_{tb}|^2$ = 1 00 Black and white colors correspond to $|V_{tb}|^2$ = 1 15

3. Event selection

This process $e^+e^- \longrightarrow t\bar{b}W^-$ is just a single stage of the reaction. Then the t-quark whose energy is above threshold is decaying to a b-quark and W^+ The W^+ and W^--bosons are decaying to

1) $q\bar{q}q\bar{q}$,
2) $q\bar{q}l\nu_l$,
3) $l\nu_l l\nu_l$

Therefore, the final states in our reaction could be

1) 6 jets,
2) 4 jets and $l\nu_l$,
3) 2 jets and $l\nu_l l\nu_l$

Here we took into account only the pure hadronic mode of decaying, applying to the fact that the efficiency of the registration of the t- and $\bar{t}$-quarks is lying in the range 26 2–30% [15] Taking into account the other decay modes 2) and 3), we would have improved our results by 8 7% + 22.7% = 31.4% with respect to our results To select final $t\bar{b}W^-$ states the experimental procedure [15] can be used The authors of [15] proposed the method of selecting the $t\bar{t}$-pairs and calculated efficiency of the registration of $t\bar{t}$ in the process $e^+e^- \longrightarrow t\bar{t} \longrightarrow$ 6 jets, 4 jets and $l\nu_l$, 2 jets and $l\nu_l l\nu_l$ The method for pure hadronic final states is based on the folowing criteria

- Particle multiplicity (charged and neutral) more than 40,
- Thrust < 0 85,
- Thrust angle $|\cos\theta_{\rm th}| < 0$ 95,
- At least 5 jets in the event

However, there is still a problem here. There is a combinatorial background coming from the contribution of t and $\bar{t}$ production when both of them are on shell It seems, that it should be possible to exclude such events by the following additional criteria

- b-jets tagging,
- requirement that the invariant mass of any 2 jets + 1 b-jet combination far from m_t, e g $|M_{2j+b} - m_t| >$ 10 GeV

Minimizing [16]

$$\chi^2 = \sigma_{\rm tot}\ L\ \epsilon_{\rm eff} \sum_{k=1}^{N_{\rm bin}} \frac{(\tilde{n}_k - n_k)^2}{n_k}\,, \qquad (6)$$

we find such a value of $|V_{tb}|^2_{\rm min}$ that χ^2 from (6) becomes minimal – $\chi^2_{\rm min}$ Here $\sigma_{\rm tot}$ is the total cross-section, L is the luminosity, $\epsilon_{\rm eff}$ is the efficiency of

$t\bar{t}$-pair registration[4], $N_{\rm bin}$ is the number of bins of $E_{bW} = \sqrt{s_{bW}}$ – the axis broken according to (4). It is worth taking into account, that while we can measure the energy of the beam with accuracy (4), the minimal bin length is about 5 GeV. The frequencies $n_k = N_k/N$ and $\tilde{n}_k = \tilde{N}_k/N$ do not depend on the total number of events[5] N, and correspond to the $|V_{tb}|^2 \neq 1$ and $|V_{tb}|^2 = 1$, respectively

Further, based on the following system:

$$\begin{cases} \chi^2 = \chi^2_{\rm min} + \Delta\chi^2 , \\ |V_{tb}|^2 = |V_{tb}|^2_{\rm min} \pm \Delta\, |V_{tb}|^2 , \end{cases} \qquad (7)$$

and according to the value of $\Delta\chi^2$ we can already judge with what precision $\Delta\,|V_{tb}|^2$ we measure $|V_{tb}|^2$

4. Results and discussion

We computed quantity $\Delta\,|V_{tb}|^2$ for $\Delta = 0$ and 5 GeV, $\sqrt{s} = 500$ GeV (m_t = 120, 150, 180 GeV) and $\sqrt{s} =$ 300 GeV (m_t = 120, 130 and 140 GeV) All the results are collected in Table 1 Evidently, the higher cross-section is, the better we measure $|V_{tb}|^2$ The cross-section falls down while the top-quark mass is rising, because of the fact that when m_t increases we approach the reaction threshold However the picture changes for $\sqrt{s}$ = 500 GeV and Δ = 5 GeV: σ_Δ is growing, when m_t is increasing It follows from the fact, that the main contribution comes from the peak (where $\sqrt{s_{bW}} = m_t$) and the peak width for m_t = 120 GeV is less than the width for m_t = 150 GeV, etc There, such a phenomenon is not observed for $\sqrt{s}$ = 500 GeV, because the decrease of the cross-section due to m_t dominates the phenomenon discussed above

It is easy to see that the results for Δ = 0 GeV and for Δ = 5 GeV are almost the same One should have expected this because of the lack of $|V_{tb}|^2$ information at the peak (see Figs 4–6) discussed above The small difference of results is due to lack of accuracy of the cutting peak (5), and it is dictated by stipulation (4)

[4] $\epsilon_{\rm eff} \simeq 0$ 30 [2]

[5] All the cross-sections were calculated with the help of integrating package VEGAS [12] The events were generated with the aid of program BASES/SPRING [13]

Table 1
Cross-sections and accuracy of $|V_{tb}|^2$ measurement for 95% CL and 99% CL for different m_t, $\sqrt{s}$ and Δ

$\sqrt{s}$ (GeV)	Δ (GeV)	m_t (GeV)	$\Delta\|V_{tb}\|^2$		$\sigma_{\rm tot}$ (fb)
			99% CL	95% CL	
500	0	120	0 197	0 175	707 3
		150	0 231	0 205	651 8
		180	0 283	0 252	571 4
300	0	120	0 152	0 132	1410 1
		130	0 203	0 176	1181 1
		140	0 286	0 241	854 3
$\sqrt{s}$ (GeV)	Δ (GeV)	m_t (GeV)	$\Delta\|V_{tb}\|^2$		σ_Δ (fb)
			99% CL	95% CL	
500	5	120	0 199	0 179	45 4
		150	0 190	0 168	54 8
		180	0 182	0 156	69 0
300	5	120	0 145	0 124	39 4
		130	0 193	0 168	37 8
		140	0 265	0 219	29 0

5. Conclusions

Summing it up one might say that it should be possible to measure $|V_{tb}|^2$ at the reaction $e^+e^- \to t\bar{b}W^-$ with an accuracy of 12–28 % Even though we did not investigate simulation of the decay of top quark, the efficiency of $t\bar{t}$-pair registration was counted with the help of the parameter $\epsilon_{\rm eff}$ Full Monte Carlo generation including t-quark decay was done by K Fujii [9] for the reactions of $t\bar{t}$-pair production. For this reaction, $e^-e^+ \to t\bar{t} \to 6$ jet, he obtained an accuracy for $|V_{tb}|^2$ of the order of $\Delta\,|V_{tb}|^2 \approx$ 35–40%. Of course, a detailed Monte Carlo simulation is also needed to come to final conclusions concerning V_{tb} measurements in the process of single top-quark production. However, at the first stage of this study we conclude that this method is quite promising, especially taking into account recent CDF data in favor of the existance of a heavy top quark $m_t = 174 \pm 10^{+13}_{-12}$ GeV/c^2 [19]

6. Acknowledgements

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References

[1] P Grannis, Presented at the 9th Topical Workshop on $p\bar{p}$ Colloder Physics, Tsukuba, Japan, October 17-22 1993, FERMILAB-PUB-94/097-E

[2] P Igi-Kemenes, Plenary talk at the 2nd International Workshop on Physics and Experiments at Linear e^+e^- Colliders, Waikoloa, Hawaii April 26–30, 1993

[3] G Jikia, Phys Lett B 257 (1991) 196

[4] VA Khoze, L H Orr and WJ Stirling, Nucl Phys B 378 (1992) 413,
L H Ore, Talk at the 2nd International Workshop on Physics and Experiments at Linear e^+e^- Colliders, Waikoloa, Hawaii April 26–30, 1993

[5] G F Abu Leil, preprint DTP/92/84, 1992

[6] H Murayama, Y Sumino, Phys Rev D 47 (1993) 82,
Y Sumino, Talk at the 2nd International Workshop on Physics and Experiments at Linear e^+e^- Colliders, Waikoloa, Hawaii April 26–30, 1993

[7] V Fadin and V Khoze, Sov J Nucl Phys 48 (1988) 309

[8] K Fujii, preprint KEK 90-187 (1991)

[9] K Fujii, preprint KEK 92-31 (1992)

[10] M Strassler and M Peskin, Phys Rev D 43 (1991) 1500,
W Kwong, Phys Rev D 43 (1991) 1488

[11] S Komamaiya, SLAC–PUB–5324 (1991)

[12] G P Lepage, J Comput Phys 27 (1978) 192

[13] S Kawabata, Comput Phys Commun 41 (1986) 127

[14] B Wiik, Proceedings of the 2nd International Workshop on Physics and Experiments at Linear e^+e^- Colliders, Waikoloa, Hawaii April 26–30, 1993, volume I, p 46

[15] G Bagliesi et al, Proceedings of the Workshop e^+e^--collisions at 500 GeV The Physics Potential, DESY 92-123A, ed PM Zerwas, Part A, p 327

[16] D J Hudson STATISTICS Lectures on Elementary Statistics and Probability, Geneva, 1964

[17] S Ambrosanio, B Mele, Dipartimento di Fisica, Universitá di Roma "La Sapienza", I N F N - Sezione di Roma, preprint n 979 (1993)

[18] J A M Vermaseren, Symbolic Manipulation with FORM, published by CAN, Kruislaan 413, 1098 SJ Amsterdam, 1991, ISBN 90-74116-01-9

[19] F Abe et al, FERMILAB-PUB-94/097-E, April 1994