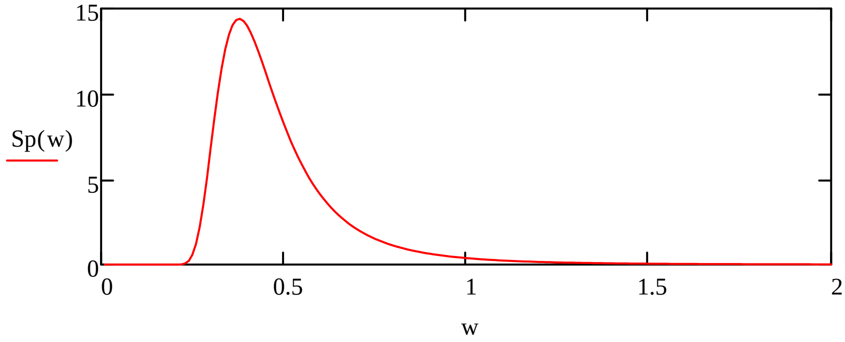
Geração de série temporal das elevações do mar: Espectro Pierson-Moskovitz

$Hs := 7.8$ Altura significativa de onda

$Tz := 11.8$ Período cruzamento zero

$$B := \frac{\left(\frac{2\cdot\pi}{Tz}\right)^4}{\pi} \qquad A := B\cdot\frac{Hs^2}{4} \qquad Sp(w) := \frac{A}{w^5}\cdot\exp\left(\frac{-B}{w^4}\right)$$

$w := 0, 0.01 .. 2$



Intervalo de Frequências	Intervalo de Tempo	Tempo de Simulação
$\omega i := 0.2$	$\Delta t := 0.125$	$T := 10800$
$\omega f := 2.0$		$Ts := T \qquad T = 1.08 \times 10^4$

Momentos Espectrais

$$mo := \int_{\omega i}^{\omega f} \omega^0\cdot Sp(\omega)\, d\omega \qquad m2 := \int_{\omega i}^{\omega f} \omega^2\cdot Sp(\omega)\, d\omega \qquad m4 := \int_{\omega i}^{\omega f} \omega^4\cdot Sp(\omega)\, d\omega$$

$mo = 3.796 \qquad m2 = 1.029 \qquad m4 = 0.57$

$\sigma := \sqrt{mo} \qquad \sigma = 1.948 \qquad \varepsilon := \sqrt{1 - \frac{m2^2}{mo\cdot m4}}$

$\nu_m := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{m4}{m2}} \quad \nu_m = 0.118 \quad \nu_o := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{m2}{mo}} \quad \nu_o = 0.083 \quad \varepsilon = 0.715$ Largura de banda

Relação entre Hs e m_0

$$\frac{Hs}{\sqrt{mo}} = 4.003$$

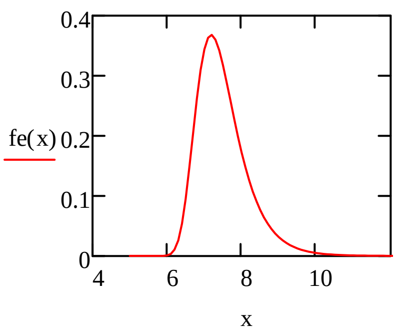
Estatística do Valor Extremo (amplitude da onda individual extrema)

$u := \sqrt{mo}\cdot\sqrt{2\cdot\ln(\nu_o\cdot T)}$ $\quad u = 7.184 \quad \nu_o\cdot T = 895.092$

$\alpha := \sqrt{2\cdot\ln(\nu_o\cdot T)}\cdot\frac{1}{\sqrt{mo}}$ $\quad \alpha = 1.892 \quad \mu_e := u + \frac{0.5772}{\alpha} \quad \mu_e = 7.489$

$\sigma_e := \frac{\pi}{\sqrt{6}\cdot\alpha}$ $\quad \sigma_e = 0.678$

$fe(x) := \exp[-\alpha\cdot(x-u) - \exp[-\alpha\cdot(x-u)]]$ $\quad x := 5, 5.1 .. 14$



$Hmax := 2\cdot u \quad Hmax = 14.368$

$\frac{Hmax}{Hs} = 1.842$ $\quad N := \nu_o\cdot T = 895.092$ $\quad$ Se fosse 1000 a relação seria 1.86!

Geração de uma série temporal

No de componentes

$N\omega := 500$

Intervalo de frequência

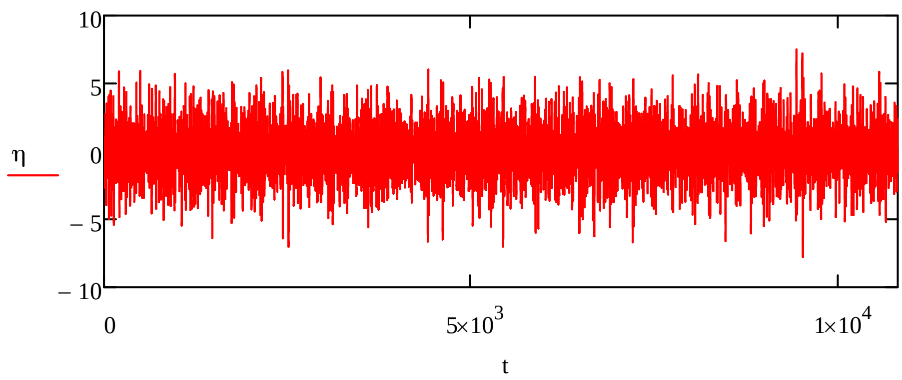
$\Delta\omega := \dfrac{\omega f - \omega i}{N\omega}$ $\quad \Delta\omega = 3.6 \times 10^{-3}$

Simulação no domínio do tempo

$NP := \dfrac{T}{\Delta t} + 1$ $\quad NP = 8.64 \times 10^{4}$ $\quad i := 1, 2 .. NP$ $\quad t_i := (i - 1)\cdot\Delta t$

$$
\begin{aligned}
\mathrm{Serie}(N\omega, \Delta\omega, NP, \Delta t) := \;& \phi \leftarrow \mathrm{runif}(N\omega, 0, 2\cdot\pi) \\
& \phi\omega \leftarrow \mathrm{runif}(N\omega, 0, 1) \\
& \text{for } i \in 1, 2 .. N\omega \\
& \quad \omega m_i \leftarrow \frac{\Delta\omega\cdot(i-1) + \Delta\omega\cdot i + \omega i\cdot 2}{2} \\
& \quad \omega_i \leftarrow \omega i + \Delta\omega\cdot(i-1) + \Delta\omega\cdot\phi\omega_i \\
& \quad A_i \leftarrow \sqrt{2\cdot Sp(\omega m_i)\,\Delta\omega} \\
& \text{for } k \in 1, 2 .. NP \\
& \quad t_k \leftarrow (k-1)\cdot\Delta t \\
& \quad y_k \leftarrow \sum_{j=1}^{N\omega} \left(A_j\cdot\cos\left(\omega_j\cdot t_k + \phi_j\right)\right) \\
& y
\end{aligned}
$$

$\eta := \mathrm{Serie}(N\omega, \Delta\omega, NP, \Delta t)$

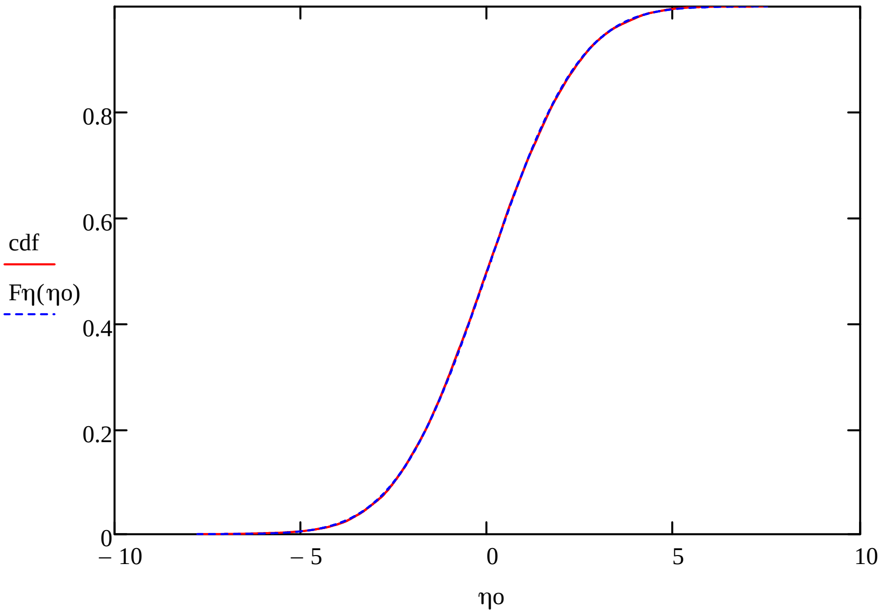


Distribuição de probabilidades do processo aleatório (comparação com a NORMAL)

$\eta o := \mathrm{sort}(\eta)$ $\mu_\eta := \mathrm{mean}(\eta)$ $\sigma_\eta := \mathrm{stdev}(\eta)$

$cdf_i := \dfrac{i}{NP + 1}$ $F\eta(\eta) := \mathrm{cnorm}\left(\dfrac{\eta - \mu_\eta}{\sigma_\eta}\right)$

$\mu_\eta = 6.822 \times 10^{-4}$ $\sigma_\eta = 1.95$ $\sqrt{mo} = 1.948$



Distribuição dos máximos - Distribuição de Rice

$\nu_o = 0.083$ $\varepsilon = 0.715$

$$\mathrm{fmax}(\eta) := \frac{\varepsilon}{\sqrt{mo}\cdot\sqrt{2\cdot\pi}}\cdot\exp\left(\frac{-1}{2}\cdot\frac{\eta^2}{mo}\right) + \frac{\eta}{mo}\cdot\sqrt{1-\varepsilon^2}\cdot\exp\left(\frac{-\eta^2}{2\cdot mo}\right)\cdot\mathrm{cnorm}\left(\frac{\eta}{mo\cdot\varepsilon}\cdot\sqrt{1-\varepsilon^2}\right) \qquad \mathrm{Fmax}(\eta) := \int_{-6}^{\eta}\mathrm{fmax}(x)\,dx$$

Distribuição a partir da amostra

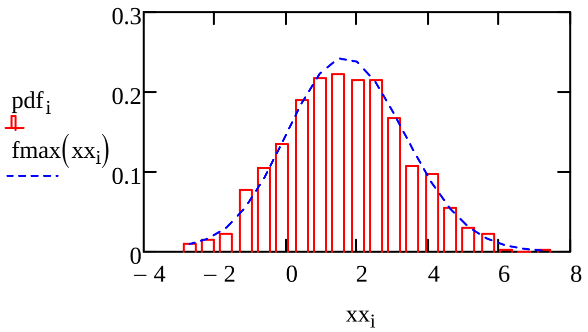
Rotina para separar os máximos

```
pm(x) := | N ← rows(x)
         | v_1 ← 0
         | v_N ← 0
         | for j ∈ 2,3 .. N − 1
         |    | i ← 0
         |    | i ← 1 if (x_{j−1} < x_j) ∧ (x_j > x_{j+1})
         |    | v_j ← i
         | m ← 0
         | for k ∈ 1,2 .. N
         |    | aux ← m
         |    | m ← v_k + aux
         |    | p_m ← x_k if v_k > 0
         | p
```

Histograma dos máximos

$xm := pm(\eta)$ $Nint := 20$ $v := \mathrm{histogram}(Nint, xm)$ $xx := v^{\langle 1\rangle}$ $\Delta := xx_2 - xx_1$

$Nm := \mathrm{rows}(xm)$ $pdf := \dfrac{v^{\langle 2\rangle}}{Nm\cdot\Delta}$ $\Delta = 0.524$ $i := 1,2 .. Nint$



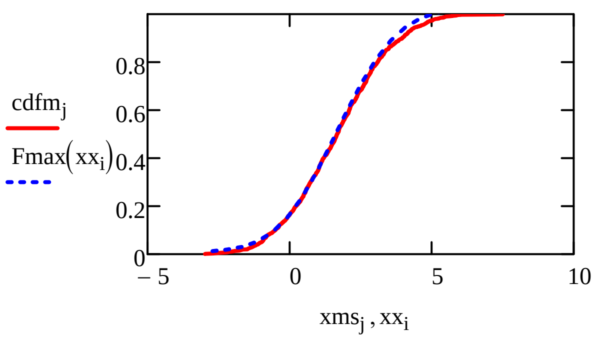
PDF

$\max(xm) = 7.502$ $xms := \mathrm{sort}(xm)$

$\min(xm) = -2.975$ $Nm := \mathrm{rows}(xm)$

$Nm = 1.279\times 10^3$ $j := 1,2 .. Nm$

$cdfm_j := \dfrac{j}{Nm+1}$



CDF

Geração de uma amostra de valores extremos (processo demorado)

$$Sample\_m(N) := \left| \begin{array}{l} \text{for } j \in 1, 2 .. N \\ \quad xm_j \leftarrow \max((Serie(N\omega, \Delta\omega, NP, \Delta t))) \\ xm \end{array} \right.$$

$Nm := 20$ $\qquad xm := Sample\_m(Nm)$

xm =

	1
1	6.719
2	7.342
3	6.83
4	9.3
5	7.086
6	8.526
7	6.56
8	7.167
9	7.459
10	7.242
11	6.957
12	8.268
13	7.672
14	7.904
15	6.499
16	...

$mm := mean(xm)$ $\qquad sm := stdev(xm)$

$mm = 7.377$ $\qquad sm = 0.698$ $\qquad \alpha m := \frac{\pi}{\sqrt{6} \cdot sm}$ $\qquad \alpha m = 1.837$

$um := mm - \frac{0.5722}{\alpha m}$ $\qquad um = 7.065$

Valores Teóricos

$\mu_e = 7.489$ $\qquad \sigma_e = 0.678$ $\qquad u = 7.184$ $\qquad \alpha = 1.892$ $\qquad i := 1, 2 .. Nm$

$xms := sort(xm)$ $\qquad Fms_i := \frac{i}{Nm + 1}$ $\qquad Fm(x) := \exp[-\exp[-\alpha \cdot (x - u)]]$

Cálculo da densidade espectral: FFT

$NP = 8.64 \times 10^{4}$ $\qquad NC := 2^{16}$ Maior número de pontos que pode ser escrito como potência de 2

$NC = 6.554 \times 10^{4}$ $\qquad i := 1, 2 .. NC$ $\qquad \eta a_i := \eta_i$

$u := FFT(\eta a)$ $\qquad ncoef := length(u) - 1$ $\qquad TC := (NC - 1) \cdot \Delta t$ $\qquad \Delta w := \dfrac{2 \cdot \pi}{TC}$

$k := 1, 2 .. ncoef$ $\qquad ncoef = 3.277 \times 10^{4}$

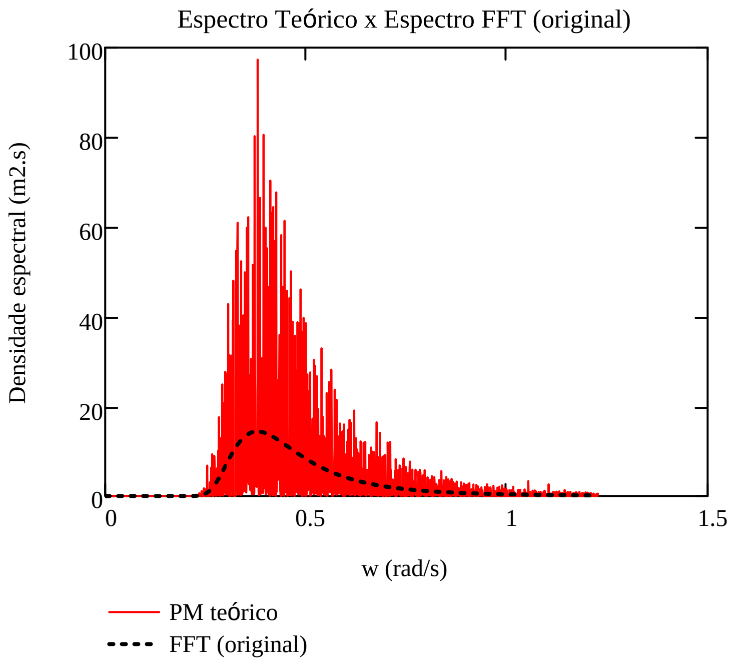
$w_k := (k - 1) \cdot \Delta w$

$an_k := 2 \cdot Re(u_k)$ $\qquad bn_k := -2 \cdot Im(u_k)$

$an_1 := 0.0$ $\qquad bn_1 := 0.0$

$$SS_k := \frac{(an_k)^2 + (bn_k)^2}{2 \cdot \Delta w} \qquad \sum_{k=1}^{ncoef} \frac{(an_k)^2 + (bn_k)^2}{2} = 3.807 \qquad mo = 3.796$$

$j := 1, 2 .. 1600$ $\qquad S_j := SS_j$

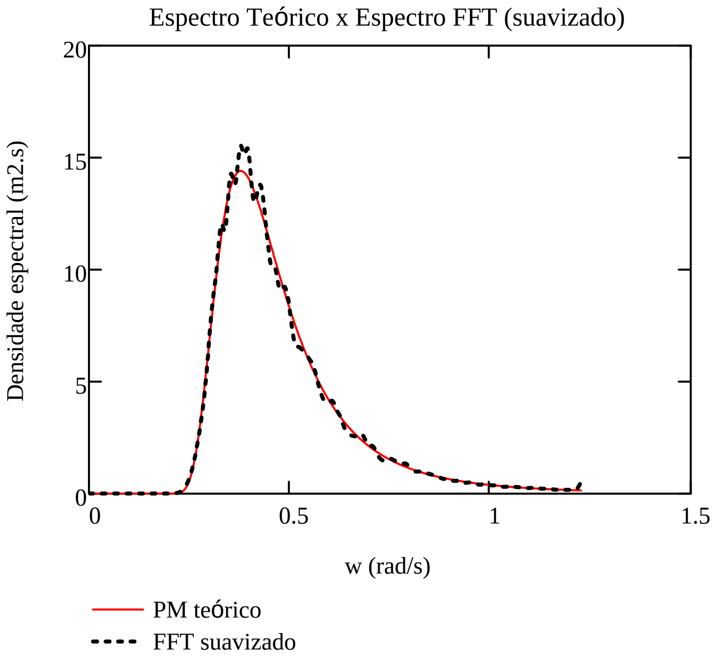


Suavização do Espectro

$$\text{Hann}(S, \text{nvezes}) := \left| \begin{array}{l} \text{Nw} \leftarrow \text{rows}(S) \\ \text{for } i \in 1, 2 .. \text{nvezes} \\ \quad \left| \begin{array}{l} SS \leftarrow S \\ \text{for } j \in 2, 3 .. \text{Nw} - 1 \\ \quad S_j \leftarrow 0.5 \cdot SS_j + 0.25 \cdot \left(SS_{j-1} + SS_{j+1} \right) \end{array} \right. \\ S \end{array} \right.$$

$\text{nvezes} := 200$

$\text{Snew} := \text{Hann}(S, \text{nvezes})$



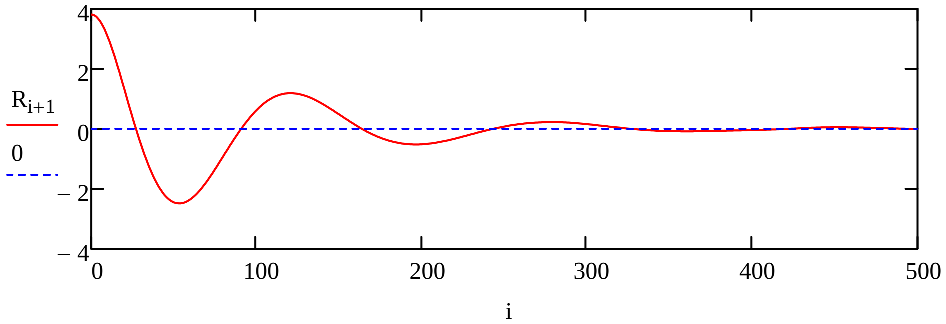
Função de Auto-correlação

$i := 0, 1 .. NPP$

$NPP := 500$

$tt_{i+1} := i \cdot \Delta t$

$$R_{i+1} := \sum_{j=1}^{NP-NPP} \frac{\eta_j \eta_{j+i}}{NP - NPP}$$



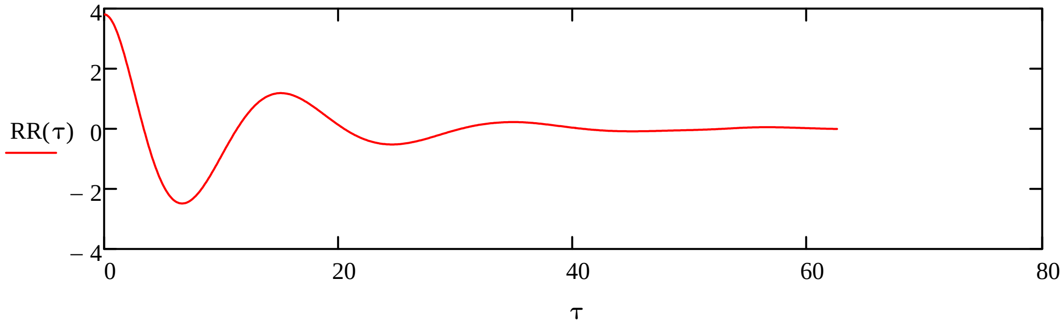
$vs := lspline(tt, R)$

$Tm := tt_{rows(tt)}$

$Tm = 62.5$

$RR(\tau) := interp(vs, tt, R, \tau)$

$\tau := 0, 0.1 .. Tm$



$$S(w) := 4 \cdot \frac{\int_0^{Tm} RR(\tau) \cdot \cos(w \cdot \tau) \, d\tau}{2 \cdot \pi}$$

$w := 0.1, 0.1125 .. 1.5$