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A STORAGE-SIZE SELECTION PROBLEM

D.K. FRIESEN

Computer Science Department, Texas A&M University, College Station, TX 77843, U.S.A.

M.A. LANGSTON

Computer Science Department, Washington State University, Pullman, WA 99164-1210, U.S.A.

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In the usual bin packing problem [1,2], one is given a set of pieces and a bin size, and the objective is to minimize the number of bins used to pack the pieces. This is equivalent to finding a packing with the least wasted space. Herein we investigate the question of what can be done if, in addition, we are allowed to choose the bin size (which must be the same for all bins). How should one compute the best bin size, α, and the best packing into bins of this size to minimize wasted space? If there is no bound on the maximum bin size, the problem is trivial (use only one bin). We thus assume the existence of a maximum bin size, which we normalize for convenience to 1.

This formulation reflects a memory allocation problem where, perhaps for simplicity of addressing, memory is to be divided into equal sized blocks. The problem then is to determine the block size and a packing which minimizes the total memory allocated. Also this problem models a situation where a manufacturer must buy containers for distributing his products. If the price he (or she) pays for containers or their handling is considerably less if he orders all one size, he might wish to try to choose the best single size for his product mix.

The problem is clearly NP-hard, reducing to the usual bin packing problem if the maximum bin size equals the maximum piece size. If there is only a small fixed number of sizes available, one can simply use a bin packing algorithm for each size and select the best packing produced. We are interested here in the case where *any* bin size less than or equal to 1 may be chosen.

Thus we are given a set of pieces $P = \{p_1, p_2, \ldots, p_n\}$ with sizes $s(p_i) \leqslant 1$ for all i. We wish to choose a number $\alpha \leqslant 1$, and produce a packing of P into N bins of size α with $N\alpha - \sum_{i=1}^{n} s(p_i)$ as small as possible. In analyzing worst-case performance, we shall compare our result to $OPT_{\alpha_0}(P)$, the minimal space required using a fixed bin size $\alpha_0 \leqslant 1$.

The purpose of this article is to observe that if a bin packing algorithm A can guarantee a worst-case ratio R for the usual bin packing problem, then an iterated version, A*, can be used to achieve a bound as close to R as desired for this size selection problem. (The bound for A* may in fact be better than R, but a proof of this would presumably depend on the algorithm used and may, we fear, be quite difficult.)

Suppose bound $R + \epsilon$ is desired, where R is algorithm A's worst-case bound for the usual bin packing problem. We will generate a sequence of

sizes $\alpha_1, \alpha_2, \ldots, \alpha_k$ such that if $\alpha_{i+1} < \alpha_0 \leqslant \alpha_i$, then running algorithm A on bins of size α_i will insure a space usage $A_{\alpha_i}(P)$ less than $R + \epsilon$ times that of $OPT_{\alpha_0}(P)$.

Claim. *If*

$$\alpha_{i+1} = \alpha_i R/(R+\epsilon) \quad \textit{and} \quad \alpha_{i+1} < \alpha_0 \leqslant \alpha_i,$$

then

$$A_{\alpha_i}(P) < (R+\epsilon)OPT_{\alpha_0}(P).$$

Proof. Consider an optimal packing of P into bins of size α_i. No fewer bins can suffice if packing P into bins of size α_0. Thus

$$\begin{aligned} A_{\alpha_i}(P) &\leqslant R \cdot OPT_{\alpha_i}(P) \\ &\leqslant R \cdot OPT_{\alpha_0}(P)(\alpha_i/\alpha_0) \\ &< R \cdot OPT_{\alpha_0}(P)(\alpha_i/\alpha_{i+1}) \\ &= (R+\epsilon)OPT_{\alpha_0}(P). \quad \square \end{aligned}$$

Using the sequence $1 = \alpha_1, \alpha_2, \ldots, \alpha_k$ will guarantee a bound of $R + \epsilon$, regardless of the value of α_0. The last value, α_k, may be determined in either of two ways. Since it is not possible to use a bin size smaller than the maximum piece size, this provides one lower bound. If in packing with a bin size α_i, all bins are filled to at least level $\alpha_i/(R+\epsilon)$, then certainly the algorithm has produced a packing within the bound. This must happen if the maximum piece size is no greater than $\alpha_i - \alpha_i/(R+\epsilon)$. Hence, if the maximum piece size is $\leqslant 1 - 1/(R+\epsilon)$, the first packing is guaranteed to be within the required bound, and no further packings are needed. Thus one would never need an $\alpha_i \leqslant 1 - 1/(R+\epsilon)$.

To illustrate the implementation of this scheme we look at a simple example. The FFD packing algorithm provides a worst-case ratio of 11/9 [2]. (This is an asymptotic bound, so the best we can expect is an asymptotic bound for FFD*.) If we wish to achieve a bound of 3/2 for our problem, we would use

$$\alpha_1 = 1,$$

$$\alpha_2 = (11/9)/(3/2) = 22/27 \cong 0.81,$$

$$\alpha_3 = (22/27)^2 \cong 0.66,$$

$$\alpha_4 = (22/27)^3 \cong 0.54,$$

$$\alpha_5 \cong 0.44,$$

$$\alpha_6 \cong 0.36,$$

$$\alpha_7 \cong 0.29 \leqslant 1/3.$$

Thus in this case it is sufficient to select the best packing from among the six produced. In general, the precise number of iterations, k, is $\lceil \log(1 - 1/(R+\epsilon)) \rceil$ where the base of the logarithm is $R/(R+\epsilon)$.

Thus, known, previously analyzed bin packing algorithms can be employed to yield near-optimal solutions to this storage-size selection problem. We conclude this article by briefly suggesting a couple of other approaches one might take. The fully polynomial time approximation scheme of Karmarkar and Karp [3] should be adaptable, though it is based on complicated mathematical programming techniques and may not be very attractive in practice. One can also imagine the possibility of reasonable one-pass algorithms that modify the bin size as the packing proceeds.

References

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