Imperial College London
Department of Earth Science and Engineering
MSc in Applied Computational Science and Engineering

Independent Research Project
Project Plan

# Ice Fracture Modelling on Spherical Volumetric Domains

by

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# 1 Introduction

The outer solar system has been the focus of a number of missions towards the end of the 20th century. The Pioneer and Voyager missions provided us a first glimpse into the numerous satellite worlds of Jupyter and Saturn; but it wouldn't be until later missions when enough data would be collected to begin unravelling the mysteries of those moons (Nimmo and Pappalardo 2016). The Galileo spacecraft reached the Jovian system in 1995. Part of its mission was to collect data from the Galilean satellites. Among them, Europa, stands out as an icy ocean body with striking surface features and the prospect of harboring life in its subsurface ocean (Nimmo and Pappalardo 2016; Vance et al. 2018).

Numerous studies have been conducted throughout the years in attempts to distinguish the moon's internal makeup (Golombek and Banerdt 1990; Billings and Kattenhorn 2005; Bray et al. 2014;), map and model the formation and evolution of its lineament fractures (Helfenstein and Parmentier 1985; Greeley et al. 2000; Kattenhorn 2002; Figueredo and Greeley 2004; Marshall and Kattenhorn 2005; Aydin 2006; Preblich et al. 2007; Dombard et al. 2013; Leonard, Pappalardo, and Yin 2018; Leonard, Yin, and Pappalardo 2020) and understand its geological features and processes (Kattenhorn and Marshall 2006; Goldreich and Mitchell 2010; Han and Showman 2010; Johnston and Montési 2014; Craft et al. 2016; Vance et al. 2018). Europa has a radius of 1561km with a density of about $2989kg/m^3$ (Vance et al. 2018). Gravity field measurements, magnetic induction and surface features, places the estimate for the outer water and trace minerals layers to be about 120km; a liquid ocean underneath an outer ice shell (Nimmo and Pappalardo 2016; Billings and Kattenhorn 2005). The water layer can be further divided into 3 sub-layers: "the outer brittle/elastic ice layer, an underlying ductile layer of, potentially, convecting ice and the lower liquid layer" (Billings and Kattenhorn 2005). An accurate estimation of the brittle layer thickness and its history is desirable, as many geological and biological processes depend on it (Vance et al. 2018). Thermal analyses using "convection models, calculations of thermal equilibrium and temperature gradients through the ice shell" give an estimation of a thick icy layer (ranging from 10-30km); "impact studies infer a thickness of 2.4-19km"; and mechanical studies such as: "modeling the flexure of the ice shell due to loading at the surface, buoyancy of floating ice rafts and applications of fracture mechanics" place the ice layer thickness at 0.1-10km (Billings and Kattenhorn 2005). The discrepancy between the estimates can be attributed to the methods implemented, as "thermodynamic models estimate the entire ice shell thickness" while "mechanical models only estimate the thickness of the elastic portion of the shell" (Billings and Kattenhorn 2005). There is evidence (Greeley et al. 2000; Leonard, Pappalardo, and Yin 2018; Leonard, Yin, and Pappalardo 2020) that points to a change in the thickness of the ice shell over time. However, further work is required to test this hypothesis.

# 2 Project Description

The goal of this project is to model fracture formation and growth of Europa's entire ice shell. This task is divided into two sub-projects: the first investigating the stresses that spawn and drive the expansion of fracture units and the second exploring the means to model fracture growth on spherical volumetric shell of such scale and resolution; the end result will be be a simulation that resembles the surface lineaments present on Europa.The former part of the project will be tackled by another ACSE student, while the latter will be my undertaking. Code will be written in C++. The project will make use of the Imperial College Geomechanics Toolkit (ICGT) for fracture growth and adaptive remeshing during the simulation runtime.

## 2.1 Domain Generation

The first task at hand is to construct a spherical volumetric shell domain or mesh that will be used for the modeling. There are a number of techniques one can implement to generate a spherical volumetric domain, however there are a number of constraints that need to be considered before selection. The estimated surface area of Europa is $3.06 \times 10^7\ km^2$ (Vance et al. 2018). To accurately represent a sphere of such a scale with a mesh, would require a forbidding number of elements. Remeshing of the domain to account for fracture growth would also increase the number of elements, so research on the types of fracture units and their history was conducted in order to estimate the level of resolution required to properly model such features.

### 2.1.1 Types of fracture units and domain resolution

A number of geological mappings (Helfenstein and Parmentier 1985; Greeley et al. 2000; Figueredo and Greeley 2004; Leonard, Pappalardo, and Yin 2018) have been done throughout the years. Here I will present a summary of the classification done by Leonard, Pappalardo, and Yin 2018, as they enrich the existing classification efforts by taking into account the highest resolution images available in their mappings. The main categories of surface units are: ridge units, plain units, chaos units and band material. These are further subdivided into smaller groups according to their unique characteristics (eg. texture, morphology), but for now we are primarily concerned with the range of width or aperture size of the fractures, the determining factor for the desired resolution of the domain and size of the fracture elements. Ridge units can have an aperture size that ranges from 400m to more than 10km, plain units are characterised by "a series of small-scale (200-500m) high-albedo ridges", chaos units range from "tens of kilometers to tens of meters" in aperture size and band material can extend to more than 10 km in aperture. Thus, the generated domain will need to account for fractures with 0.1 to a few tens of kilometers in width.

These constraints indicate that using the standard Finite Element Method (FEM) would be computationally and memory expensive, as the mesh to be created would need to have a considerable number of elements in order to both model the domain and fractures. Thus, I will now propose another approach that scales better for domains of such geometry and desired features.

### 2.1.2 NURBS and Isogeometric Analysis

A construct that allows us to accurately model spherical volumes and enables straightforward mesh refinement and augmentation of a non uniform mesh, are Non-Uniform Rational B-Splines (NURBSs). NURBS are a generalization of B-Splines. "The B-spline parametric is local to 'patches' rather than elements" and those "patches play the role of subdomains within which element types and material models are assumed to be uniform" (Hughes, Cottrell, and Bazilevs 2005). NURBSs are constructed using a basis function, knot vectors and control points that define the geometry of the patch. Generating NURBS bases is done with a simple recursion algorithm and the resulting coarse mesh encapsulates the desired geometry exactly (Hughes, Cottrell, and Bazilevs 2005). Also, there are a number of operations (h-refinement/know insertion, p-refinement, k-refinement) that can be applied to a patch in order to make refinement more robust, since there is no need for interaction with the CAD system (Hughes, Cottrell, and Bazilevs 2005). The power of NURBS to accurately model geometries with such few elements, makes them (and the techniques that use them) a desirable choice for this project.

This approach to analysis has been named Isogeometric Analysis (IGA) and there is much research that explores its capabilities in modeling problems. Work that is relevant to this project and will be studied to reach the goal has been done by: Gu et al. 2015; Ge et al. 2016; Rasool, Corbett, and Sauer 2016; Coox et al. 2017; Greco et al. 2017; Perduta and Putanowicz 2019; Gao, Guo, and Zheng 2019; Shafei, Faroughi, and Rabczuk 2020.

### 2.1.3 Prototyping and future plans

Code has been written to compute the NURBS basis function in Python and it was applied to curves and surfaces. Curves have been constructed using pure Python and surfaces using the NURBS-Python library (as it provided a straightforward way to visualize the control points and surface) (Bingol and Krishnamurthy 2019). Early results for a circular curve and spherical surface are presented in Figure 1.

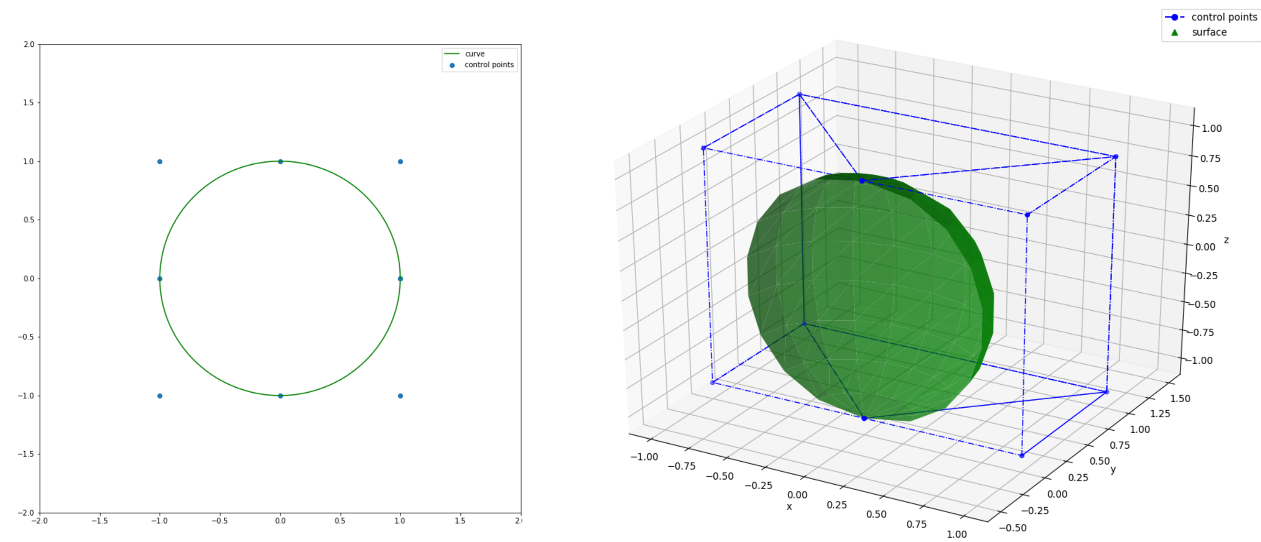


Figure 1: Left: NURBS Curve for unit circle. Right: NURBS Surface of half sphere.

The next steps would be to properly define a spherical shell volume using NURBS (in C++), explore means of visualizing the domain and the simulation output (I will make an effort to implement this using Python in Blender) and begin applying concepts of IGA and fracture formation/growth in the constructed domain.

## 3 Objectives

- Produce rigorous code that generates high quality spherical meshes.
- Implement a number of mesh refinement techniques to properly simulate fracture creation and growth.
- Run the simulation and iterate on mesh generation and refinement.

# 4 Schedule

## 4.1 June

- Formulate project description and theoretical background:
  - Gather and read sources on Europa, specifically:
    - Ice crust evolutionary models,
    - Types of fractures and their formation history,
- Study approaches for mesh generation and refinement, specifically:
  - Types of mesh,
  - Spherical volumetric mesh generation,
  - Basis functions for surfaces and volumes,
  - Fracture growth,
- Prototype code to generate a volumetric spherical cell mesh in true scale using a NURBS patch.
- Finalize literature review and theoretical background for Europa's ice crust evolutionary models and mesh generation for the final report.

## 4.2 July

- Produce, document and iterate over mesh generation code/theory (IGA/FEM) and prototype fracture "injection" and growth algorithms.
- Implement scripts to assist visualization of mesh states.
- Finalize literature review and theoretical background for: Europa's fracture types, formation and growth; and mesh augmentation and fracture growth for the final report.
- Produce, document and iterate over fracture creation and growth code/theory; begin applying those algorithms in the generated mesh.
- Produce simulations and iterate on the existing code for optimization and correctness.
- Report on the outcomes of the simulations and any improvements implemented.

## 4.3 August

- Finalize simulation: report, analyze and visualize the final output.
- Finalize documentation.
- Prepare for report submission.

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