Python for control purposes

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February 7, 2017

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Chapter 1

Introduction

1.1 Install the packages

1.2 The simplest way

I prepared a VirtualBox disk image [1] with a Debian distribution and all the required packages. VirtualBox is available for Windows, Linux, OS X and Solaris. All the features described in this document are available.
Please contact me via email to receive the link to the zipped file.

1.3 Linux

The required modules can be simply installed using the usual package manager of the Linux distribution. It is also possible to install the Anaconda distribution [2] for Linux to get the basic Python modules.
It is important to check the versions of the Python modules, in particular numpy, scipy and sympy. Old versions of these packages don't allows to perform all the tasks described in this document. In case of old versions, it is possible to download the last versions of these packages from the SciPy download page [3], and install them from a Linux shell.
Under Debian jessie we can use the apt manager to install the following packages:

- python-numpy (Vers. $\geq$ 1.8.2)
- python-scipy (Vers. $\geq$ 0.14)
- python-matplotlib
- python-sympy (Vers. $\geq$ 0.7.5)
- python-setuptools
- python-psutils
- ipython
- ipython-qtconsole

Under Debian and Ubuntu it is possible to check if all the required development packages are correctly installed using the shell command

```
sudo apt-get build-dep python-scipy
```

The following packages are not available as distribution packages and should be installed separately.

- The Python Control toolbox [4]
- The Slycot libraries [5]
- The SupsiCtrl package [6]

For the second part of the project (code generation etc.) the following packages are required

- python-pyqt4
- python-pyqt4-dev
- python-qwt5-qt4

This features presented in the second part of this document are at present only interesting under the Linux OS, because the real-time code is generated for a Linux PREEMPT-RT machine..

1.4 Windows

Under Windows it is sufficient to install the "Anaconda" package [2], to have all the python and ipython modules installed. The Slycot libraries for Windows can be downloaded from here [7]. At present it is not possible to perform hybrid simulation and code generation under the Windows OS.

1.5 Mac OSX

The Anaconda package [2] is available for Mac OSX. The Slycot libraries can be downloaded from here [7].

Chapter 2

Python - Some hints for Matlab users

2.1 Basics

There are important differences between Matlab and Python. In particular, the Python approach to matrices and to indexed objects is quite different compared to Matlab.
More information about a comparison between Python and Matlab is available online at [8].
The web contains a lot of documentation about Python and its packages. In particular, the book of David Pine [9] gives a good introduction about the features of Python for scientific applications.
Other links present tutorials for **numpy** [10], **scipy** [11], **matplotlib** [12] and **sympy** [13].

2.2 The python shell

A Python script can run within a Python shell, but can also be launched as executable.
The basic python shell is similar to the Matlab shell without the java improvements (**matlab -nojvm**).
A better shell is for example **ipython**. In this interactive form, when started with the extension **–pylab**, ipython already loads at startup a set of functions and modules.
Another interesting environment, more similar to the Matlab shell, is represented by the **Spyder** application. In this application it is possible to debug scripts and functions like in the Matlab environment.
In this document we are mostly working with **ipython** launched with the shell commands

```
ipython qtconsole --pylab inline
```

or

```
ipython --pylab
```

Sometimes not all the functions and modules are explicitly loaded at the beginning of the examples. In addition, **ipython** implements some useful commands like for example **whos** and **run** (for launching scripts).
In the ipython shell it is possible to start single commands, paste a set of commands or launch a ".py" program using **run**.

```
In [1]: # single command

In [2]: a = 5

In [3]: # paste a set of commands

In [4]: a=5
   ...: b=7
   ...: c=a*b
   ...: print c
   ...:
35

In [5]: # run a .py file

In [6]: run DCmotorKane.py
Matrix([[-Dm*w(t) + kt*I(t)]])
Matrix([[-J*Derivative(w(t), t)]])
[[0 1]
 [0 -Dm/J]]
[[0]
 [kt/J]]
```

2.3 Python vs. Matlab

Differently from Matlab, Python implements more types of variables

```
In [1]: a=5

In [2]: b=2.7

In [3]: c=[[1,2,3],[4,5,6]]

In [4]: d='Ciao'

In [5]: whos
Variable   Type     Data/Info
-----------------------------
a          int      5
b          float    2.7
c          list     n=2
d          str      Ciao
```

2.4 List, array and matrix

Python implements three kind of multidimensional objects: **list**, **array** and **matrix**. These objects are handled differently than in Matlab.

2.5 List

A Python **list** implements the Matlab **cell**. It represents the simplest and default indexed object.

```
In [1]: a=[[[1,2],[3,4]],'abcd',2]

In [2]: b=[[1,2,3],[4,5,6],[7,8,9]]

In [3]: whos
Variable   Type    Data/Info
-----------------------------
a          list    n=3
b          list    n=3
```

2.6 Arrays

In Python the **array** is a multidimensional variable that implements sets of values of the same type. Usually the elements of an array are numbers, but can also be booleans, strings, or other objects. An array is the basic instance for most scientific applications.

Operations like *, /, ** etc. implement the **dot** operations of the Matlab environment (**.***, **./** and **.^**). For example, the multiplication of two arrays $a * a$ represents the value-by-value multiplication implemented in Matlab with the operation $a. * a$.

```
In [1]: from numpy import mat, matrix, array

In [2]: a=array([[1,2,3],[4,5,6]])

In [3]: b=array([[1],[2]])

In [4]: print a*a
[[ 1  4  9]
 [16 25 36]]

In [5]: print a*b
[[ 1  2  3]
 [ 8 10 12]]
```

2.7 Matrices

The **matrix** object is useful in case of linear algebra operations. In this case the variables are instanced using the **mat** or the **matrix** function.

```
In [1]: from numpy import mat, matrix, array

In [2]: a=mat(a)

In [3]: b=array([[1],[2],[3]])

In [4]: a*b
Out[5]:
matrix([[14],
        [32]])

In [6]: a=array(a)
```

```
In [7]: a*b
---------------------------------------------------------------------------
ValueError                                Traceback (most recent
    call last)
<ipython-input-9-8201c27d19b7> in <module>()
----> 1 a*b

ValueError: operands could not be broadcast together with
    shapes (2,3) (3,1)

In [8]: b=mat(b)

In [9]: a*b
Out[10]:
matrix([[14],
        [32]])
```

2.8 Indexing

Indexing in Python is quite different compared with the syntax used in Matlab. Indices start from **0** (and not **1** as in Matlab). In addition, the syntax is different for lists, arrays and matrices.

2.9 Lists

1-dimension lists can be accessed using one index (ex. $a[2]$). Multidimensional lists require multiple indices in the form $[i][j]\ldots$

```
In [1]: a=[1,2,3,4,5]

In [2]: %whos
Variable   Type    Data/Info
----------------------------
a          list    n=5

In [3]: a[3]
Out[3]: 4

In [4]: b=[[1,2,3],[4,5,6]]

In [5]: %whos
Variable   Type    Data/Info
----------------------------
a          list    n=5
b          list    n=2

In [6]: b[1][2]
Out[6]: 6

In [7]: b[0]
Out[7]: [1, 2, 3]
```

2.10 Arrays

Multidimensional arrays allow the use of indices in the forms $[i, j]$ and $[i][j]$.

```
In [1]: from numpy import array

In [2]: a=array([1,2,3,4,5])

In [3]: b=array([[1,2,3],[4,5,6]])

In [4]: %whos
Variable   Type       Data/Info
-------------------------------
a          ndarray     5: 5 elems, type `int64`, 40 bytes
b          ndarray    2x3: 6 elems, type `int64`, 48 bytes
```

```
In [5]: a.shape
Out[5]: (5,)

In [6]: b.shape
Out[6]: (2, 3)

In [7]: a[3]
Out[7]: 4

In [8]: b[0,2]
Out[8]: 3

In [9]: b[0][2]
Out[9]: 3

In [10]: b[:,0]
Out[10]: array([1, 4])

In [11]: b[0,:]
Out[11]: array([1, 2, 3])

In [12]: b[0]
Out[12]: array([1, 2, 3])
```

2.11 Matrices

Matrices can be only indexed using the $[i, j]$ syntax. A matrix has always a minimum of 2 dimensions.

```
In [1]: from numpy import mat

In [2]: a=array([1,2,3,4,5])

In [3]: b=array([[1,2,3],[4,5,6]])

In [4]: %whos
Variable   Type      Data/Info
------------------------------
a          matrix    [[1 2 3 4 5]]
b          matrix    [[1 2 3]\n [4 5 6]]

In [5]: a.shape
Out[5]: (1, 5)

In [6]: b.shape
Out[6]: (2, 3)
```

```
In [7]: a[0,2]
Out[7]: 3

In [8]: b[1,1]
Out[8]: 5

In [9]: b[:,0]
Out[9]:
matrix([[1],
        [4]])

In [10]: b[0,:]
Out[10]: matrix([[1, 2, 3]])
```

2.12 Multidimensional arrays and matrices

Matrices and arrays can be defined with more than 2 dimensions.

```
In [1]: from numpy import array, mat

In [2]: a=zeros((3,3,3),int8)

In [3]: a.shape
Out[3]: (3, 3, 3)

In [4]: %whos
Variable   Type       Data/Info
-------------------------------
a          ndarray    3x3x3: 27 elems, type `int8`, 27
    bytes

In [5]: a[1,1,1]
Out[5]: 0
In [6]: a[1,1,1]=5

In [7]: a
Out[7]:
array([[[0, 0, 0],
        [0, 0, 0],
        [0, 0, 0]],

       [[0, 0, 0],
        [0, 5, 0],
        [0, 0, 0]],

       [[0, 0, 0],
        [0, 0, 0],
        [0, 0, 0]]], dtype=int8)
```

Chapter 3

The Python Control System toolbox

3.1 Basics

The Python Control Systems Library, is a package initially developed by Richard Murray at Caltech. This toolbox contains a set of python classes and functions that implement common operations for the analysis and design of feedback control systems. In addition, a MATLAB compatibility package (control.matlab) has been integrated in order to provide functions equivalent to the commands available in the MATLAB Control Systems Toolbox.

3.2 Models

LTI systems can be described in state-space form or as transfer functions.

3.3 Continuous systems

3.4 State-space representation

```
In [1]: from control import *

In [2]: a=[[0,1],[-1,-1]]

In [3]: b=[[0],[1]]

In [4]: c=[1,0]

In [5]: d=0

In [6]: sys = ss(a,b,c,d)

In [7]: print sys
A = [[ 0   1]
 [-1  -1]]

B = [[0]
  [1]]

C = [[1  0]]

D = [[0]]
```

3.5 Transfer function

```
 In [1]: from control import *

In [2]: g=tf(1,[1,1,1])

In [3]: print g

      1
-----------
s^2 + s + 1
```

3.6 Zeros-Poles-Gain

This method is not implemented in control toolbox yet. It is available in the package **scipy.signal** but it is not completely compatible with the class of LTI objects defined in the Python control toolbox.

3.7 Discrete time systems

An additional fields (**dt**) in the **StateSpace** and **TransferFunction** classes is used to differentiate continuous-time and discrete-time systems.

3.8 State-space representation

```
In [4]: a=[[0,1],[-1,1]]

In [5]: b=[[0],[1]]

In [6]: c=[1,-1]

In [7]: d=0

In [8]: sysd = ss(a,b,c,d,0.01)

In [9]: print sysd
A = [[ 0  1]
 [-1  1]]

B = [[0]
 [1]]

C = [[ 1 -1]]

D = [[0]]

dt = 0.01
```

3.9 Transfer function

```
In [1]: from control import *

In [2]: g=tf([1,-1],[1,-1,1],0.01)

In [3]: print g

    z - 1
-----------
z^2 - z + 1

dt = 0.01
```

3.10 Conversions

The Python control system toolbox only implements conversion from continuous time systems to discrete-time systems (**c2d**) with the methods "zoh", "tustin" and "matched". No conversion from discrete to continuous has been implemented yet.

The supsictrl.yottalab package implements both functions **c2d** and **d2c** with the methods "zoh', "foh", "tustin" and "matched" ("matched" is only implemented in c2d).

```
In [1]: from control import *

In [2]: from control.Matlab import *

In [3]: g=tf(1,[1,1,1])

In [4]  # Matlab compatibility

In [5]: gd = c2d(g,0.01)

In [6]  # control toolbox

In [7]: gd2 = sample_system(g,0.01)

In [8]: print g

      1
-----------
s^2 + s + 1


In [9]: print gd

4.983e-05 z + 4.967e-05
-----------------------
  z^2 - 1.99 z + 0.99

dt = 0.01
```

```
In [1]: from control import *

In [2]: from control.Matlab import c2d

In [3]: from supsictrl.yottalab import d2c

In [4]: g=tf(1,[1,1,1])

In [5]: gd =c2d(g,0.01)

In [6]: g2=d2c(gd)

In [7]: print g

      1
-----------
s^2 + s + 1


In [8]: print g2

1.729e-14 s + 1
---------------
  s^2 + s + 1
```

3.11 Casting

The control.matlab module implements the casting functions to transform LTI systems to a transfer function (**tf**) or to a state-space form (**ss**).

```
In [8]: g = tf(sys)

In [9]: print g

      1
-----------
s^2 + s + 1
```

and transfer functions into one of the state-space representation

```
In [10]: sys = ss(g)

In [11]: print sys
A = [[ 0.  -1.]
  [ 1.  -1.]]

B = [[-1.]
  [ 0.]]

C = [[ 0.  -1.]]

D = [[ 0.]]
```

3.12 Models interconnection

Commands like **parallel** and **series** are available in order to interconnect systems. The operators + and * have been overloaded for the LTI class to perform the same operations. In addition the command **feedback** is implemented exactly as in Matlab.

```
In [1]: from control import *

In [2]: g1=tf(1,[1,1])

In [3]: g2=tf(1,[1,2])

In [4]: print parallel(g1,g2)

   2 s + 3
-------------
s^2 + 3 s + 2


In [5]: print g1+g2

   2 s + 3
-------------
s^2 + 3 s + 2
```

```
In [6]: print series(g1,g2)

      1
-------------
s^2 + 3 s + 2

In [7]: print g1*g2

      1
-------------
s^2 + 3 s + 2
```

```
In [8]: print feedback(g1,g2)

    s + 2
-------------
s^2 + 3 s + 3
```

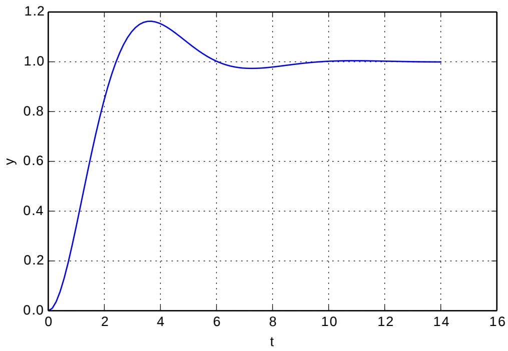
Chapter 4

System analysis

4.1 Time response

The Python Control toolbox offers own functions to simulate the time response of systems. For Matlab users, the control.matlab module gives the possibility to work with the same syntax as in Matlab. Please take care about the order of the return values!
Examples of time responses are shown in the figures 4.1, 4.2, 4.3, 4.4 and 4.5.

```
In [1]: from control import *

In [2]: import matplotlib.pyplot as plt

In [3]: g = tf(1,[1,1,1])

In [4]: t,y = step_response(g)

In [5]: plt.plot(t,y)
   ...: plt.grid()
   ...: plt.xlabel('t')
   ...: plt.ylabel('y')
```

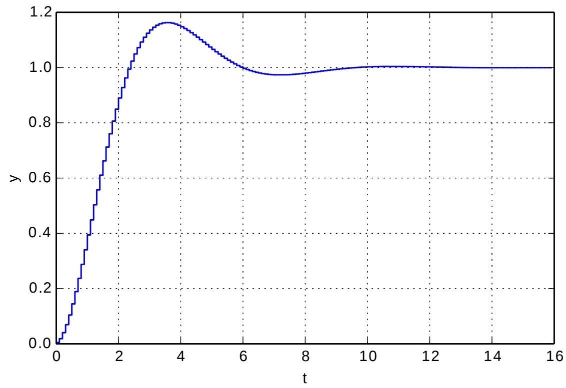


or alternatively

```
In [1]: from control import *

In [2]: from control.matlab import *

In [3]: import matplotlib.pyplot as plt

In [4]: g = tf(1,[1,1,1])

In [5]: y,t = step(g)

In [6]: plt.plot(t,y)
   ...: plt.xlabel('t')
   ...: plt.ylabel('y')
   ...: plt.grid()
```

Figure 4.1: Step response for continuous-time systems

```
In [1]: from control import *

In [2]: from control.matlab import c2d

In [3]: import matplotlib.pyplot as plt

In [4]: g = tf(1,[1,1,1])

In [5]: gz=c2d(g,0.1)

In [6]: t=arange(0,16,0.1)

In [7]: t1,y = step_response(gz,t)

In [8]: plt.step(t,y.T[0]) # transpose
   col matrix y[0]
   ...: plt.grid()
   ...: plt.xlabel('t')
   ...: plt.ylabel('y')
```

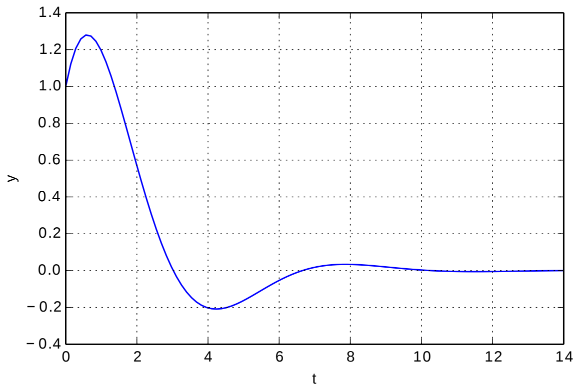


or alternatively

```
In [1]: from control import *

In [2]: from control.matlab import *

In [3]: import matplotlib.pyplot as plt

In [4]: g = tf(1,[1,1,1])

In [5]: gz=c2d(g,0.1)

In [6]: t=arange(0,16,0.1)

In [7]: y,t1 = step(gz,t)

In [8]: plt.step(t,y[0])  # get first
   row from y matrix
   ...: plt.grid()
   ...: plt.xlabel('t')
   ...: plt.ylabel('y')
```

Figure 4.2: Step response for discrete-time systems

```
In [1]: from control import *

In [2]: import matplotlib.pyplot as plt

In [3]: a=[[0,1],[-1,-1]]

In [4]: b=[[0],[1]]

In [5]: c=[1,0]

In [6]: d=[0]

In [7]: sys=ss(a,b,c,d)

In [8]: t,y=initial_response(sys,
                                        X0=[1,1])

In [9]: plt.plot(t,y)
   ...: plt.grid()
   ...: plt.xlabel('t')
   ...: plt.ylabel('y')
```



or alternatively

```
In [1]: from control import *

In [2]: from control.matlab import *

In [3]: import matplotlib.pyplot as plt

In [4]: a=[[0,1],[-1,-1]]

In [5]: b=[[0],[1]]

In [6]: c=[1,0]

In [7]: d=[0]

In [8]: sys=ss(a,b,c,d)

In [9]: y,t=initial(sys,X0=[1,1])

In [10]: plt.plot(t,y)
   ...: plt.xlabel('t')
   ...: plt.ylabel('y')
   ...: plt.grid()
```

Figure 4.3: Continuous time systems - Initial condition response

```
In [1]: from control import *

In [2]: import matplotlib.pyplot as plt

In [3]: g = tf(1,[1,1,1])

In [4]: t,y = impulse_response(g)

In [5]: plt.plot(t,y)
   ...: plt.grid()
   ...: plt.xlabel('t')
   ...: plt.ylabel('y')
```

or alternatively

```
In [1]: from control import *

In [2]: from control.matlab import *

In [3]: import matplotlib.pyplot as plt

In [4]: g = tf(1,[1,1,1])

In [5]: y,t = impulse(g)

In [6]: plt.plot(t,y)
   ...: plt.grid()
   ...: plt.xlabel('t')
   ...: plt.ylabel('y')
```

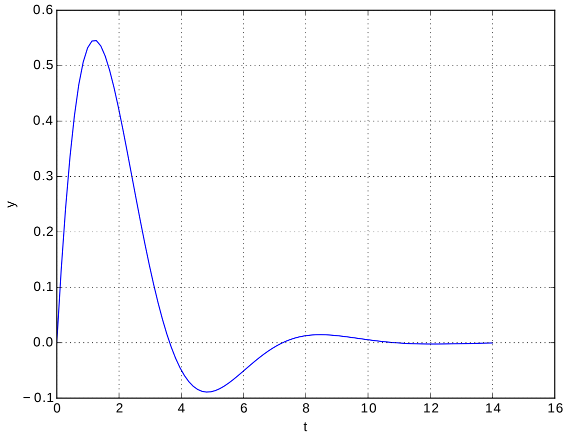
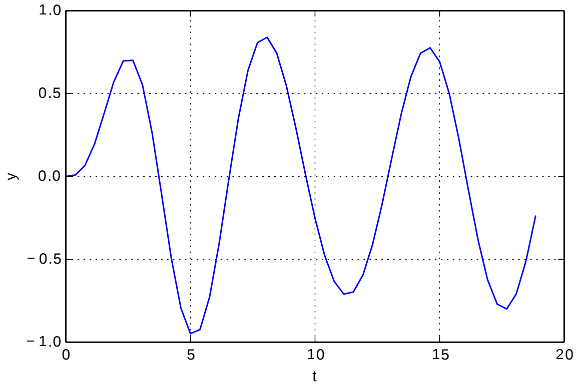


Figure 4.4: Continuous time systems - Impulse response

```
In [1]: from control import *

In [2]: import matplotlib.pyplot as plt

In [3]: g=tf([1,2],[1,2,3,4])

In [4]: t=linspace(0,6*pi)

In [5]: u=sin(t)

In [6]: t,y,x = forced_response(g,t,u)

In [7]: plt.plot(t,y)
   ...: plt.xlabel('t')
   ...: plt.ylabel('y')
   ...: plt.grid()
```



or alternatively

```
In [1]: from control import *

In [2]: from control.matlab import *

In [3]: import matplotlib.pyplot as plt

In [4]: g=tf([1,2],[1,2,3,4])

In [5]: t=linspace(0,6*pi)

In [6]: u=sin(t)

In [7]: y,t,x = lsim(g,u,t)

In [8]: plt.plot(t,y)
   ...: plt.xlabel('t')
   ...: plt.ylabel('y')
   ...: plt.grid()
```

Figure 4.5: Continuous time systems - Generic input

4.2 Frequency analysis

The frequency analysis includes some commands like **bode_response**, **nyquist_response**, **nichols_response** and the corresponding Matlab versions **bode**, **nyquist** and **nichols**. (See figures 4.6, 4.7 and 4.8)

```
In [1]: from control import *

In [2]: g=tf([1],[1,0.5,1])

In [3]: bode_plot(g, dB=True);
```

or alternatively

```
In [1]: from control import *

In [2]: from control.matlab import *

In [3]: g=tf([1],[1,0.5,1])

In [4]: bode(g, dB=True);
```

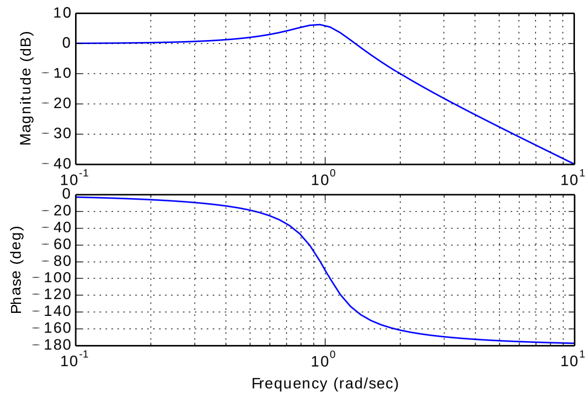


Figure 4.6: Bode plot

The command **margins** returns the gain margin, the phase margin and the corresponding crossover frequencies.

```
In [1]: from control import *

In [2]: g=tf(2,[1,2,3,1])

In [3]: gm, pm, wg, wp = margin(g)

In [4]: gm                          # Gain, not dB!
Out[4]: 2.5000000000000013

In [5]: pm
Out[5]: 76.274075256921392          # deg

In [6]: wg
Out[6]: 0.85864877610167201         # rad/s

In [7]: wp
Out[7]: 1.7320508075688776          # rad/s
```

In addition, the command **stability_margins** returns the stability margin and the corresponding frequency. The stability margin values w_s and s_m, which correspond to the shortest distance from the Nyquist curve to the critical point -1, are useful for the sensitivity analysis.

```
In [1]: from control import *
In [2]: import matplotlib.pyplot as plt
In [3]: g=tf([1],[1,2,1])
In [3]: nyquist_plot(g), plt.grid()
```

or alternatively

```
In [1]: from control import *
In [2]: import matplotlib.pyplot as plt
In [3]: from control.matlab import *
In [4]: g=tf(1,[1,2,1])
In [5]: nyquist(g), plt.grid()
```

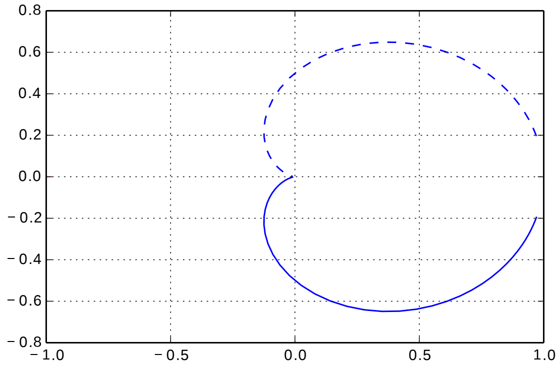


Figure 4.7: Nyquist plot

```
In [1]: from control import *
In [2]: g=tf(1,[1,2,3,4,0])
In [3]: nichols_plot(g)
```

or alternatively

```
In [1]: from control import *
In [2]: g=tf(1,[1,2,3,4,0])
In [3]: nichols(g)
```

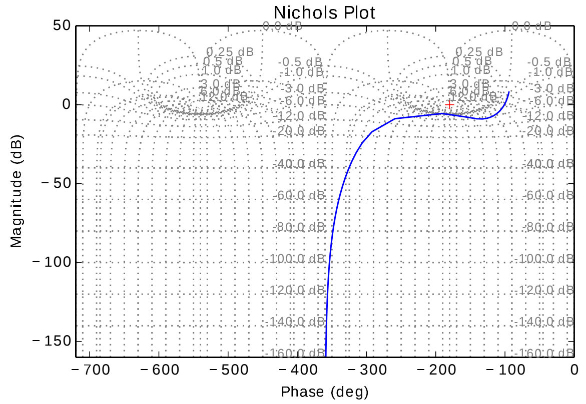


Figure 4.8: Nichols plot

```
In [1]: from control import *

In [2]: g=tf(2,[1,2,3,1])

In [3]: gm, pm, sm, wg, wp, ws = stability_margins(g)

In [4]: gm
Out[4]: 2.5000000000000013        # Gain not dB'

In [5]: pm
Out[5]: 76.274075256921392        # deg

In [6]: wg
Out[6]: 1.7320508075688776        # rad/s

In [7]: wp
Out[7]: 0.85864877610167201       # rad/s

In [8]: sm
Out[8]: 0.54497577553096421       #

In [9]: ws
Out[9]: 1.3669371206538097        # rad/s
```

4.3 Poles, zeros and root locus analysis

Poles and zeros of an open loop system can be calculated with the commands **pole**, **zero** or plotted and calculated with **pzmap**.
In addition there are two functions that implement the root locus command: **rlocus** and **root_locus**. At present no algorithm to automatically choose the values of K has been implemented: if not provided, the K vector is calculated in rlocus with log values between 10^{-3} and 10^3. For the **root_locus** function the K values should be provided.
If the ipython shell is not launched with the **–inline** flag, the root locus is plotted on an external window and it is possible to get the values of gain and damp by clicking with the mouse on the curves.

```
Clicked at    -0.5724    +1.293j gain     1.722 damp
    0.4048
Clicked at     -1.119  +0.01874j gain     2.252 damp
    0.9999
Clicked at    -0.7545    +1.293j gain     1.114 damp
         0.504
```

```
In [1]: from control import *

In [2]: from control.pzmap import pzmap

In [3]: g=tf([1,1],[1,2,3,4,0])

In [4]: g.pole()
Out[4]:
array([-1.65062919+0.j        ,
       -0.17468540+1.54686889j,
       -0.17468540-1.54686889j,
        0.00000000+0.j        ])

In [5]: g.zero()
Out[5]: array([-1.])

In [6]: poles, zeros = pzmap(g), grid()

In [7]: poles

Out[7]:
array([-1.65062919+0.j        ,
       -0.17468540+1.54686889j,
       -0.17468540-1.54686889j,
        0.00000000+0.j        ])

In [8]: zeros
Out[8]: array([-1.])
```

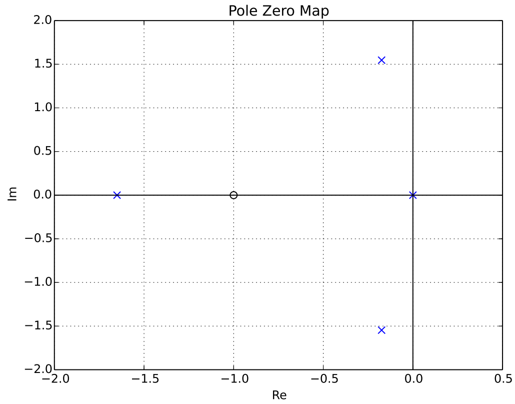


Figure 4.9: Poles and zeros

```
In [1]: from control import *

In [2]: g=tf(1,[1,2,3,0])

In [3]: rlocus(g); grid()
```

or alternatively

```
In [1]: from control import *

In [2]: g=tf(1,[1,2,3,0])

In [3]: k=logspace(-3,3,100)

In [4]: root_locus(g,k); grid()
```

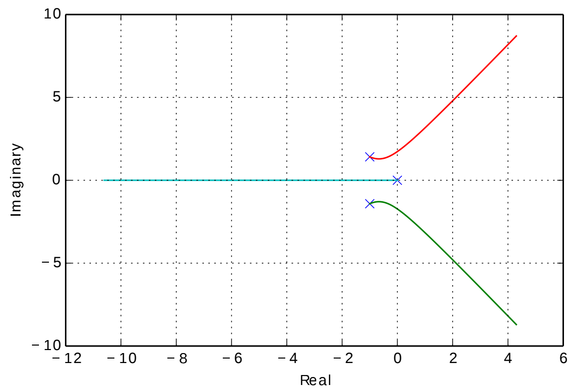


Figure 4.10: Root locus plot

Chapter 5

Modeling

The **sympy** module (symbolic python) contains a full set of operations to manage physical systems. In particular, it is possible to find the linearized model of a mechanical system using the Lagrange's method or the Kane's method. More details about the Kane's method are available at [14], [15], [16], [17], [18] and [19].

In the next sections we present the modelling of 3 plants that we can find in our laboratories and that are quite familiar to us.

5.1 Model of a DC motor

5.1.1 Plant

In this first example we model a DC servo motor with a current input in order to find its state-space representation. The motor is characterized by a torque constant k_t, an inertia (motor+load) J and a friction constant D_m.

The input of the plant is the current I and the output is the position φ. The rotation center is the point **O**, the main coordinates system is **N** and we add a local reference frame **Nr** which rotates with the load (angle φ and speed ω).

5.1.2 Modules and constants

```
n [1]: from sympy import symbols, Matrix,pi
   ...: from sympy.physics.mechanics import *
   ...: import numpy as np
   ...:
   ...: # Modeling the system with Kane method
   ...:
   ...: # Signals
   ...: ph = dynamicsymbols('ph')     # motor angle
   ...: w  = dynamicsymbols('w')      # motor rot. speed
   ...: I  = dynamicsymbols('I')      # input current
   ...:
   ...: # Constants
   ...: Dm = symbols('Dm')            # friction
   ...: M, J = symbols('M_J')         # Mass and inertia
   ...: t = symbols('t')              # time
   ...: kt  = symbols('kt')           # torque constant
   ...:
```

5.1.3 Reference frames

```
In [2]: # Reference frame for the motor and Load
   ...: N = ReferenceFrame('N')
   ...:
   ...: O = Point('O')           # center of rotation
   ...: O.set_vel(N,0)
   ...:
   ...: # Reference frames for the rotating disk
   ...: Nr = N.orientnew('Nr', 'Axis',[ph, N.x])      #
      rotating reference (load)
   ...:
   ...: Nr.set_ang_vel(N, w*N.x)
   ...:
```

5.1.4 Body and inertia of the load

```
In [3]: # Mechanics
   ...: Io = J*outer(Nr.x, Nr.x)
   ...:
   ...: InT = (Io, O)
   ...:
   ...: B = RigidBody('B', O, Nr, M, InT)
   ...:
```

5.1.5 Forces and torques

In order to find the dynamic model of the plant we need some other definitions, in particular the relation between angle φ and angular velocity ω, the forces and torques applied to the system and a vector that contains the rigid bodies of the system.

```
In [4]: # Forces and torques
   ...: forces = [(Nr,(kt*I-Dm*w)*N.x)]
   ...:
   ...: kindiffs = [(ph.diff(t)-w)]
   ...:
   ...: bodies=[B]
   ...:
```

5.1.6 Model

Using the Kane's method is now possible to find the dynamic matrices related to the plant.

```
In [5]: # Model
   ...: KM = KanesMethod(N,q_ind=[ph],u_ind=[w],kd_eqs=
        kindiffs)
   ...: fr, frstar = KM.kanes_equations(forces,bodies)
   ...:
   ...: print fr
   ...: print frstar
   ...:
Matrix([[-Dm*w(t) + kt*I(t)]])
Matrix([[-J*Derivative(w(t), t)]])
```

5.1.7 State-space matrices

From the results of the Kane's model identification, we can now extract the matrices A and B of the state-space representation.

```
In [6]: # symbolically linearize about arbitrary
     equilibrium
   ...: linear_state_matrix, linear_input_matrix, inputs =
KM.linearize(new_method=True)
   ...:
   ...: # set the the equilibrium point
   ...: eq_pt = [0, 0]
   ...: eq_dict = dict(zip([ph,w], eq_pt))
   ...:
   ...: f_A_lin = linear_state_matrix.subs(eq_dict)
   ...: f_B_lin = linear_input_matrix.subs(eq_dict)
   ...: m_mat = KM.mass_matrix_full.subs(eq_dict)
   ...:
   ...: # compute A and B matrices
   ...: A = np.matrix(m_mat.inv() * f_A_lin)
   ...: B = np.matrix(m_mat.inv() * f_B_lin)
```

```
In [6]: print A
   ...: print B
   ...:
[[0  1]
 [0 -Dm/J]]
[[0]
 [kt/J]]
```

5.2 Model of the inverted pendulum

The second example is represented by the classical inverted pendulum as shown in figure 5.1.

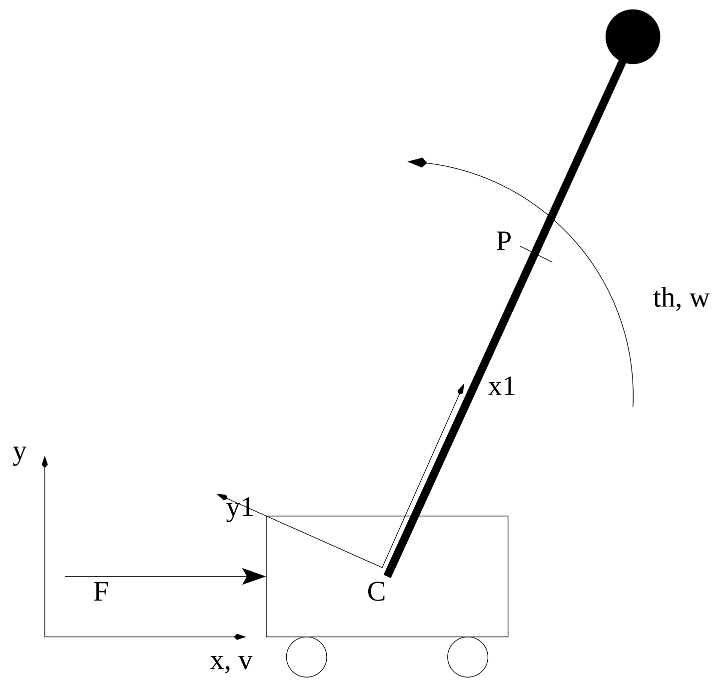


Figure 5.1: Inverted pendulum

The global reference frame is **Nf** (x, y) The point **P** is the center of mass of the pendulum. The car is moving with speed **v** and position **C**. The pole is rotating with the angle **th** and angular velocity **w**, In addition to the main coordinate frame **Nf** (x, y), we define a local body-fixed frame to the pendulum **Npend** (x_1, y_1).

Figure 5.2: Inverted pendulum - Real plant

5.2.1 Modules and constants

```
In [1]: from sympy import symbols, Matrix, pi
   ...: from sympy.physics.mechanics import *
   ...: import numpy as np
   ...:
   ...: # Modeling the system with Kane method
   ...:
   ...: # Signals
   ...: x, th  = dynamicsymbols('x th')
   ...: v, w = dynamicsymbols('v w')
   ...: F = dynamicsymbols('F')
  ...:
   ...: # Constants
   ...: d = symbols('d') # friction
   ...: m, r = symbols('m r')
   ...: M = symbols('M')
   ...: g, t = symbols('g t')
   ...: J = symbols('J')
   ...:
```

5.2.2 Frames - Car and pendulum

```
In [2]: # Frames and Coord. system
   ...:
   ...: # Car - reference x,y
   ...: Nf = ReferenceFrame('Nf')
   ...: C = Point('C')
   ...: C.set_vel(Nf, v*Nf.x)
   ...: Car = Particle('Car',C,M)
   ...:
   ...: # Pendulum - reference x1, y1
   ...: Npend = Nf.orientnew('Npend','Axis',[th,Nf.z])
   ...: Npend.set_ang_vel(Nf,w*Nf.z)
   ...:
   ...: P = C.locatenew('P',r*Npend.x)
   ...: P.v2pt_theory(C,Nf,Npend)
   ...: Pa = Particle('Pa', P, m)
   ...:
```

5.2.3 Points, bodies, masses and inertias

```
In [3]: I = outer (Nf.z, Nf.z)
   ...: Inertia_tuple = (J*I, P)
   ...: Bp = RigidBody('Bp', P, Npend, m, Inertia_tuple)
   ...:
```

5.2.4 Forces, frictions and gravity

```
In [4]: # Forces and torques
   ...: forces = [(C,F*Nf.x-d*v*Nf.x),(P,-m*g*Nf.y)]
   ...: frames = [Nf,Npend]
   ...: points = [C,P]
   ...:
   ...: kindiffs = [x.diff(t)-v, th.diff(t) - w]
   ...: particles = [Car,Bp]
   ...:
```

5.2.5 Final model and linearized state-space matrices

```
n [5]: # Model
   ...: KM = KanesMethod(Nf, q_ind=[x,th], u_ind=[v,w],
        kd_eqs=kindiffs)
   ...: fr, frstar = KM.kanes_equations(forces, particles)
   ...:
   ...: # Equilibrium point
   ...: eq_pt = [0, pi/2,0,0]
   ...: eq_dict = dict(zip([x,th,v,w], eq_pt))
   ...:
   ...: # symbolically linearize about arbitrary
        equilibrium
   ...: linear_state_matrix, linear_input_matrix, inputs =
KM.linearize(new_method=True)
   ...:
   ...: # sub in the equilibrium point and the parameters
   ...: f_A_lin = linear_state_matrix.subs(eq_dict)
   ...: f_B_lin = linear_input_matrix.subs(eq_dict)
   ...: m_mat = KM.mass_matrix_full.subs(eq_dict)
   ...:
   ...: # compute A and B
   ...: A = m_mat.inv() * f_A_lin
   ...: B = m_mat.inv() * f_B_lin
   ...:
```

```
In [6]: A
Out[6]:
Matrix([
[0,   0,   1,  0],
[0,   0,   0,  1],
[0,    g*m**2*r**2/(J*M + J*m + M*m*r**2), -d*(m**2*r**2/((
      M + m)*(J*M + J*m
+ M*m*r**2)) + 1/(M + m)),  0],
[0,  g*m*r*(M + m)/(J*M + J*m + M*m*r**2),
-d*m*r/(J*M + J*m + M*m*r**2),  0]])
```

```
In [7]: B
Out[7]:
Matrix([
[
      0],
[
      0],
[m**2*r**2/((M + m)*(J*M + J*m + M*m*r**2)) + 1/(M + m)],
[                                m*r/(J*M + J*m + M*m*r**2)]])
```

And the results can be written in a better form as

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{gm^2r^2}{JcM+Jcm+Mmr^2} & -\frac{d\left(Jc+mr^2\right)}{JcM+Jcm+Mmr^2} & 0 \\ 0 & \frac{gmr(M+m)}{JcM+Jcm+Mmr^2} & -\frac{dmr}{JcM+Jcm+Mmr^2} & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{Jc+mr^2}{JcM+Jcm+Mmr^2} \\ \frac{mr}{JcM+Jcm+Mmr^2} \end{bmatrix}$$

5.3 Model of the Ball-on-Wheel plant

A more complex plant is represented by the Ball-on-Wheel system of figure 5.3, where a ball must be maintened in the unstable equilibrium point on the top of a bike wheel.

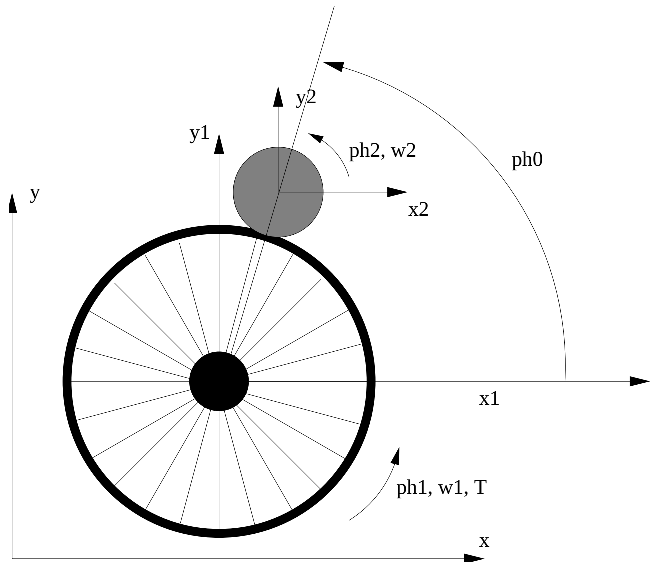


Figure 5.3: Ball-On-Wheel plant

In this system we have 4 reference frames. The frame **N** is the main reference frame, **N0** rotates with the line connecting the centers of mass of the wheel (**O**) and of the ball (**CM2**), **N1** (x_1, y_1) rotates with the wheel and **N2** (x_2, y_2) is body-fixed to the ball.

The radius of the wheel and of the ball are respectively R_1 and R_2. The non sliding condition is given by

$$(R_1 + R_2) \cdot ph_0 = R_1 \cdot ph_1 + R_2 \cdot ph_2$$

The input of the system is represented by the torque T applied to the wheel.

5.3.1 Modules and constants

```
In [1]: from sympy import symbols, Matrix, pi
   ...: from sympy.physics.mechanics import *
   ...: import numpy as np
   ...:
   ...: ph0, ph1, ph2 = dynamicsymbols('ph0 ph1 ph2')
   ...: w1, w2 = dynamicsymbols('w1 w2')
   ...:
   ...: T = dynamicsymbols('T')
   ...:
   ...: J1, J2 = symbols('J1 J2')
   ...: M1, M2 = symbols('M1 M2')
   ...: R1, R2 = symbols('R1 R2')
   ...: d1     = symbols('d1')
   ...: g      = symbols('g')
   ...: t      = symbols('t')
   ...:
```

5.3.2 Reference frames

```
In [2]: N = ReferenceFrame('N')
   ...:
   ...: O = Point('O')
   ...: O.set_vel(N,0)
   ...:
   ...: ph0 = (R1*ph1+R2*ph2)/R1
   ...:
   ...: N0 = N.orientnew('N0','Axis',[ph0,N.z])
   ...: N1 = N.orientnew('N1','Axis',[ph1,N.z])
   ...: N2 = N.orientnew('N2','Axis',[ph2,N.z])
   ...: N1.set_ang_vel(N,w1*N.z)
   ...: N2.set_ang_vel(N,w2*N.z)
   ...:
```

5.3.3 Centers of mass of the ball

```
In [3]: CM2 = O.locatenew('CM2',(R1+R2)*N0.y)
   ...: CM2.v2pt_theory(O,N,N0)
   ...:
Out[3]: (-R1*ph1' - R2*ph2')*N0.x
```

5.3.4 Masses and inertias

```
In [4]: Iz = outer(N.z,N.z)
   ...: In1T = (J1*Iz, O)
   ...: In2T = (J2*Iz, CM2)
   ...:
   ...: B1 = RigidBody('B1', O, N1, M1, In1T)
   ...: B2 = RigidBody('B2', CM2, N2, M2, In2T)
   ...:
```

5.3.5 Forces and torques

```
In [5]: #forces = [(N1, (T-d1*w1)*N.z), (CM2,-M2*g*N.y)]
   ...: forces = [(N1, T*N.z), (CM2,-M2*g*N.y)]
   ...:
   ...: kindiffs = [ph1.diff(t)-w1,ph2.diff(t)-w2]
   ...:
```

5.3.6 Kane's model and linearized state-space matrices

```
In [6]: KM = KanesMethod(N,q_ind=[ph1, ph2],u_ind=[w1, w2
    ],kd_eqs=kindiffs)
   ...: fr, frstar = KM.kanes_equations(forces,[B1, B2])
   ...:

In [7]: # Equilibrium point
   ...: eq_pt = [0, 0, 0, 0, 0]
   ...: eq_dict = dict(zip([ph1,ph2,w1,w2, T], eq_pt))
   ...:

In [8]: # symbolically linearize about arbitrary
    equilibrium
   ...: linear_state_matrix, linear_input_matrix, inputs =
KM.linearize(new_method=True)
   ...:
   ...: # sub in the equilibrium point and the parameters
   ...: f_A_lin = linear_state_matrix.subs(eq_dict)
   ...: f_B_lin = linear_input_matrix.subs(eq_dict)
   ...: m_mat = KM.mass_matrix_full.subs(eq_dict)
   ...:
   ...: # compute A and B
   ...: A = m_mat.inv() * f_A_lin
   ...: B = m_mat.inv() * f_B_lin
```

```
In [9]: A
Out[9]:
Matrix([
[0, 0, 1, 0],
[0, 0, 0, 1],
[-M2**2*R1**2*R2**2*g/((R1 + R2)*(J1*J2 + J1*M2*R2**2 + J2
    *M2*R1**2)) +
M2*R1**2*g*(M2**2*R1**2*R2**2/((J1 + M2*R1**2)*(J1*J2 + J1
    *M2*R2**2 +
J2*M2*R1**2)) + 1/(J1 + M2*R1**2))/(R1 + R2), -M2**2*R1*R2
    **3*g/((R1 +
R2)*(J1*J2 + J1*M2*R2**2 + J2*M2*R1**2)) + M2*R1*R2*g*(M2
    **2*R1**2*R2**2/((J1 +
M2*R1**2)*(J1*J2 + J1*M2*R2**2 + J2*M2*R1**2)) + 1/(J1 +
    M2*R1**2))/(R1 + R2),
0, 0],
[                                              -M2**2*R1**3*
    R2*g/((R1 + R2)*(J1*J2
+ J1*M2*R2**2 + J2*M2*R1**2)) + M2*R1*R2*g*(J1 + M2*R1**2)
    /((R1 + R2)*(J1*J2 +
J1*M2*R2**2 + J2*M2*R1**2)),
-M2**2*R1**2*R2**2*g/((R1 + R2)*(J1*J2 + J1*M2*R2**2 + J2*
    M2*R1**2)) +
M2*R2**2*g*(J1 + M2*R1**2)/((R1 + R2)*(J1*J2 + J1*M2*R2**2
    + J2*M2*R1**2)), 0,
0]])
In [10]: B
Out[10]:
Matrix([
[
           0],
[
           0],
[M2**2*R1**2*R2**2/((J1 + M2*R1**2)*(J1*J2 + J1*M2*R2**2 +
    J2*M2*R1**2)) +
1/(J1 + M2*R1**2)],
[                                                 -M2*R1*R2/(
    J1*J2 + J1*M2*R2**2 +
J2*M2*R1**2)]])
```

or as formula

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{J_2 M_2 R_1^2 g}{J_1 J_2 R_1 + J_1 J_2 R_2 + J_1 M_2 R_1 R_2^2 + J_1 M_2 R_2^3 + J_2 M_2 R_1^3 + J_2 M_2 R_1^2 R_2} & \frac{J_2 M_2 R_1 R_2 g}{J_1 J_2 R_1 + J_1 J_2 R_2 + J_1 M_2 R_1 R_2^2 + J_1 M_2 R_2^3 + J_2 M_2 R_1^3 + J_2 M_2 R_1^2 R_2} & 0 & 0 \\ \frac{J_1 M_2 R_1 R_2 g}{(R_1 + R_2)\left(J_1 J_2 + J_1 M_2 R_2^2 + J_2 M_2 R_1^2\right)} & \frac{J_1 M_2 R_2^2 g}{(R_1 + R_2)\left(J_1 J_2 + J_1 M_2 R_2^2 + J_2 M_2 R_1^2\right)} & 0 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{M_2^2 R_1^2 R_2^2}{\left(J_1 + M_2 R_1^2\right)\left(J_1 J_2 + J_1 M_2 R_2^2 + J_2 M_2 R_1^2\right)} + \frac{1}{J_1 + M_2 R_1^2} \\ -\frac{M_2 R_1 R_2}{J_1 J_2 + J_1 M_2 R_2^2 + J_2 M_2 R_1^2} \end{bmatrix}$$

Chapter 6

Control design

6.1 PI+Lead design example

6.1.1 Define the system and the project specifications

In this first example we design a controller for a plant with the transfer function

$$G(s) = \frac{1}{s^2 + 6 \cdot s + 5}$$

The requirements for the control are

$$e_\infty = 0$$

for a step input

$$PM \geq 60^o$$

and

$$\omega_{gc} = 10 rad/s$$

The controller can be written in the form

$$C(s) = K \cdot \frac{1 + s \cdot T_i}{s \cdot T_i} \cdot \frac{1 + \alpha \cdot T_D \cdot s}{1 + s \cdot T_D}$$

with a PI and a lead part.
We have to design the controller and find the values of $\mathbf{T_i}$, α, $\mathbf{T_D}$ and $\mathbf{K}$. The full design is performed using the bode diagram.
After installing the required modules, we can define the plant transfer function and the requirements of the project.

```
In [1]: # Modules

In [2]: from matplotlib.pyplot import *

In [3]: from control import *

In [4]: from numpy import pi, linspace

In [5]: from scipy import sin, sqrt

In [6]: from supsictrl.yottalab import *

In [7]: g=tf([1],[1,6,5])

In [8]: bode(g,dB=True);

In [9]: subplot(211), legend(['G(s)'],prop={'size':10})
Out[9]:
(<matplotlib.axes.AxesSubplot at 0x7f85b5193550>,
 <matplotlib.legend.Legend at 0x7f85b47e6950>)

In [10]: wgc = 10          # Desired Bandwidth

In [11]: desiredPM = 60    # Desired Phase margin
```

Figure 6.1 shows the bode diagram of the plant.

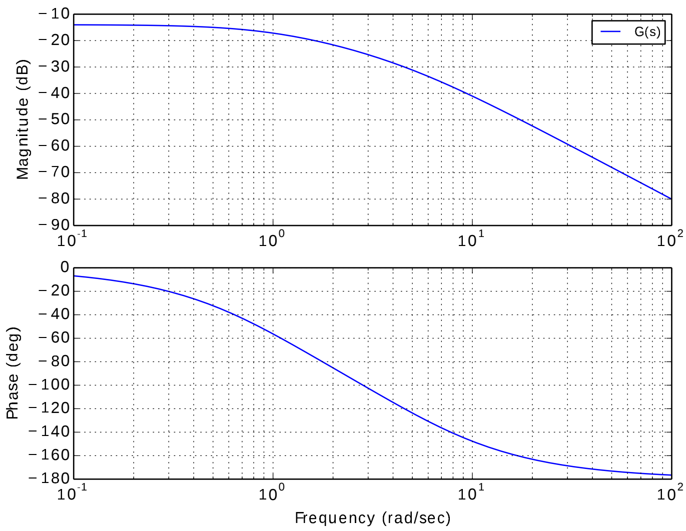


Figure 6.1: Bode diagram of the plant

6.1.2 PI part

Now we choose the integration time for the PI part of the controller. In this example we set

$$T_i = 0.15s$$

```
In [12]: # PI part

In [13]: Ti=0.15

In [14]: Gpi=tf([Ti,1],[Ti,0])

In [15]: print "PI part is: ", Gpi
PI part is:
0.15 s + 1
----------
  0.15 s


In [16]: figure()
Out[16]: <matplotlib.figure.Figure at 0x7f85b47eaa10>

In [17]: bode(g,dB=True,linestyle='dashed');

In [18]: hold
Out[18]: <function matplotlib.pyplot.hold>

In [19]: bode(Gpi*g,dB=True);

In [20]: subplot(211), legend(('G(s)','Gpi(s)*G(s)'),
    prop={'size':10})
Out[20]:
(<matplotlib.axes.AxesSubplot at 0x7f85b4806250>,
 <matplotlib.legend.Legend at 0x7f85b4303850>)
```

Figure 6.2 shows the bode plot of the plant with and without the PI controller part.

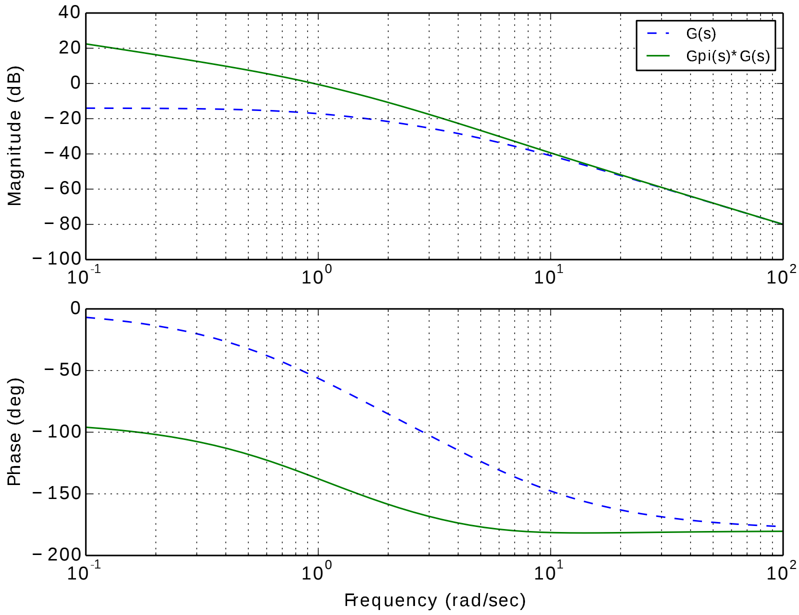


Figure 6.2: Bode diagram: G (dashed) and Gpi*G

6.1.3 Lead part

Now we can get the PM at the frequency ω_{gc} in order to calculate the additional phase contribution of the lead part of the controller.

```
In [21]: mag,phase,omega = bode(Gpi*g,[wgc],Plot=False)

In [22]: ph = phase[0]

In [23]: if ph>=0:
   ...:       ph = phase[0]-360;
   ...:

In [24]: Phase = -180+desiredPM

In [25]: dPM = Phase-ph

In [26]: print "Additional phase from Lead part: ", dPM
Additional phase from Lead part:  61.4144232114
```

Now it is possible to calculate the lead controller by finding the values of α and T_D.

```
In [27]: # Lead part

In [28]: dPMrad = dPM/180*pi

In [29]: alfa = (1+sin(dPMrad))/(1-sin(dPMrad));

In [30]: print "Alpha is: ", alfa
Alpha is:  15.4073552425

In [31]: TD = 1/(sqrt(alfa)*wgc);

In [32]: Glead = tf([alfa*TD,1],[TD,1])

In [33]: print "Lead part is: ", Glead
Lead part is:
 0.3925 s + 1
-------------
0.02548 s + 1


In [34]: figure()
Out[34]: <matplotlib.figure.Figure at 0x7f85b43462d0>

In [35]: bode(g,dB=True,linestyle='dashed');

In [36]: hold
Out[36]: <function matplotlib.pyplot.hold>

In [37]: bode(Gpi*Glead*g, dB=True);

In [38]: subplot(211),
legend(('G(s)','Gpi(s)*G(s)','Gpi(s)*GLead(s)*G(s)'),
    prop={'size':10})
Out[38]:
(<matplotlib.axes.AxesSubplot at 0x7f85b43736d0>,
 <matplotlib.legend.Legend at 0x7f85b3b1f450>)
```

Figure 6.3 shows now the bode plot of the plant, the plant with the PI part and the plant with PI and Lead part

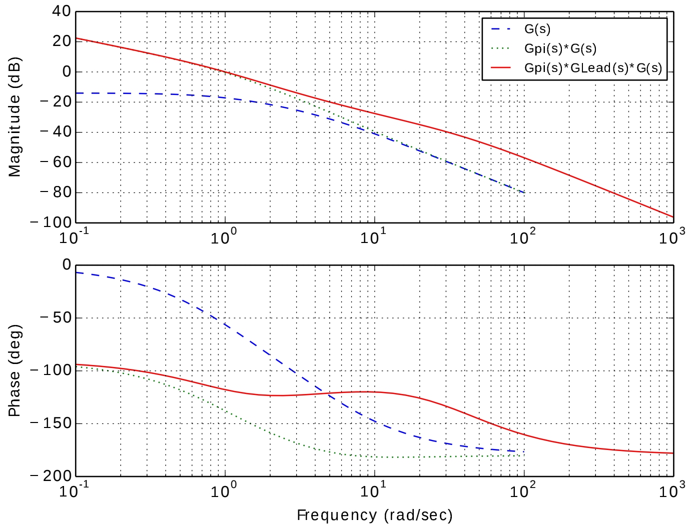


Figure 6.3: Bode diagram - G (dashed), Gpi*G (dotted) and Gpi*GLead*G

6.1.4 Controller Gain

The last step is to find the amplification K of the controller which move up the bode gain plot in order to obtain the required crossover frequency ω_{gc}.

```
In [39]: mag,phase,omega = bode(Gpi*Glead*g,[wgc],Plot=
     False)

In [40]: print "Phase at wgc is: ", phase[0]
Phase at wgc is:   -120.0

In [41]: K=1/mag[0]

In [42]: print "Gain to have MAG at gwc 0dB: ", K
Gain to have MAG at gwc 0dB:   23.8177769548

In [43]: figure()
Out[43]: <matplotlib.figure.Figure at 0x7f85b3a703d0>

In [44]: bode(g,dB=True,linestyle='dashed');

In [45]: hold
Out[45]: <function matplotlib.pyplot.hold>

In [46]: bode(Gpi*Glead*g, dB=True,linestyle='-.');

In [47]: bode(K*Gpi*Glead*g, dB=True);

In [48]:
subplot(211),legend(([ 'G(s)','Gpi(s)*G(s)','Gpi(s)*GLead(s
     )*G(s)',
'K*Gpi(s)*GLead(s)*G(s)']),prop={'size':10})
Out[48]:
(<matplotlib.axes.AxesSubplot at 0x7f85b3a76690>,
 <matplotlib.legend.Legend at 0x7f85b33e6f90>)
```

In the figure 6.4 we see now that the gain plot has been translated up to get $0dB$ at the gain crossover frequency ω_{gc}.

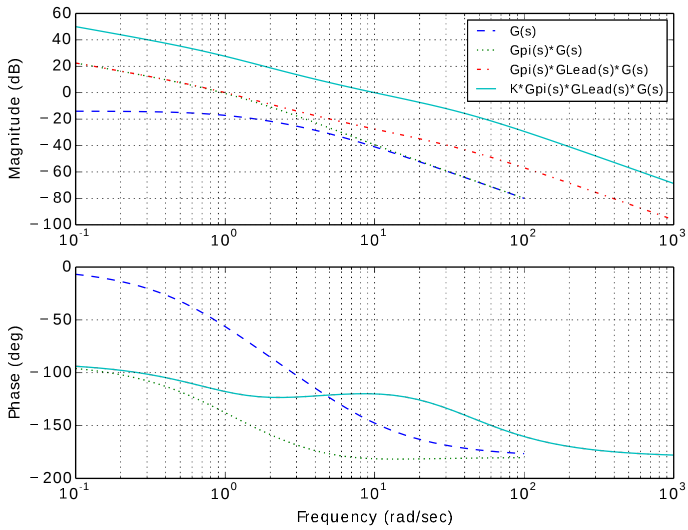


Figure 6.4: Bode diagram - G (dashed), Gpi*G (dotted), Gpi*GLead*G (dot-dashed) and K*Gpi*GLead*G

6.1.5 Simulation of the controlled system

Now it is possible to simulate the controlled system after closing the loop.

```
In [49]: Contr = K*Gpi*Glead

In [50]: print "Full controller: ", Contr
Full controller:
1.402 s^2 + 12.92 s + 23.82
-----------------------------
    0.003821 s^2 + 0.15 s

In [51]: mag,phase,omega = bode(K*Gpi*Glead*g,[wgc],Plot=
     False)

In [52]: print "Data at wgc - wgc: ", omega[0], "Magnitude
     : ",mag[0], "Phase:
",phase[0]
Data at wgc - wgc:  10 Magnitude:  1.0 Phase:  -120.0

In [53]: gt=feedback(K*Gpi*Glead*g,1)

In [54]: t=linspace(0,1.5,300)

In [55]: y,t = step(gt,t)

In [56]: figure()
Out[56]: <matplotlib.figure.Figure at 0x7f85b3514290>

In [57]: plot(t,y), xlabel('t'), ylabel('y'), title('Step 
     response of the
controlled plant')
Out[57]:
([<matplotlib.lines.Line2D at 0x7f85b34252d0>],

In [58]: grid()
```

The simulation of the controlled plant with a step input is shown in figure 6.5.

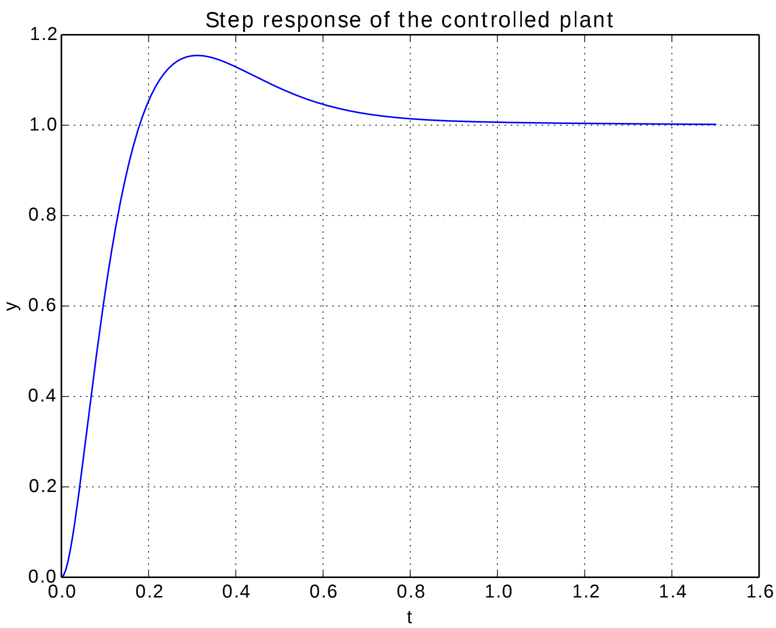


Figure 6.5: Step response of the controlled plant

6.2 Discrete-state feedback controller design

6.2.1 Plant and project specifications

In this example we design a discrete-state feedback controller for a DC servo motor.

We want to have a controlled system with a maximum of 4% overshooting and an error $e_\infty = 0$ with a step input. In addition we desire a bandwidth of the controlled system of at least 6 rad/s.

The step response of the motor with the current input of $I_{in} = 500mA$) has been saved into the file "MOT".

6.2.2 Motor parameters identification

We try to find the parameters of the srvo motor using a least square identification from the collected data.

The transfer function of the DC motor from input current $I(s)$ to output angle $\Phi(s)$ can be represented as

$$G(s) = \frac{\Phi(s)}{I_{in}(s)} = \frac{K_t/J}{s^2 + s \cdot D/J}$$

6.2.3 Required modules

```
In [1]: from scipy.optimize import leastsq

In [2]: from scipy.signal import step2

In [3]: import numpy as np

In [4]: import scipy as sp

In [5]: from control import *

In [6]: from control.Matlab import *

In [7]: from supsictrl.yottalab import *
```

6.2.4 Function for least square identification

We define now the function **residuals** which returns the error between the collected and the simulated data. Using this function we can try to minimize the error using a least square approach.

```
In [8]: # Motor response for least square identification

In [9]: def residuals(p, y, t):
   ...:     [k,alpha] = p
   ...:     g = tf(k,[1,alpha,0])
   ...:     Y,T = step(g,t)
   ...:     err=y-Y
   ...:     return err
   ...:
```

6.2.5 Parameter identification

We load the collected data to perform the parameter identification of the numerator $K = K_t/J$ and the denominator value $\alpha = D/J$.

```
In [10]: # Identify motor

In [11]: x = np.loadtxt('MOT');

In [12]: t = x[:,0]

In [13]: y = x[:,2]

In [14]: Io = 1000

In [15]: y1 = y/Io

In [16]: p0 = [1,4]

In [17]: plsq = leastsq(residuals, p0, args=(y1, t))

In [18]: kt = 0.0000382            # Motor torque constant

In [19]: Jm=kt/plsq[0][0]          # Motor Inertia

In [20]: Dm=plsq[0][1]*Jm          # Motor friction

In [21]: g=tf([kt/Jm],[1,Dm/Jm,0])  # Transfer function
```

6.2.6 Check of the identified parameters

The next step is to check how good our parameters have been identified by comparing the simulated function with the measured data (see figure 6.6)

```
In  [22]:  Y,T = step(g,t)

In  [23]:  plot(T,Y,t,y1), legend(('Identified␣transfer␣
     function','Collected
data'),prop={'size':10},loc=2), xlabel('t'), ylabel('y'),
     title('Step
response'), grid()
Out[23]:
([<matplotlib.lines.Line2D at 0x7fb9a1b6b590>,
  <matplotlib.lines.Line2D at 0x7fb9a1b6b710>],
 <matplotlib.legend.Legend at 0x7fb9a1b6bb10>,
 <matplotlib.text.Text at 0x7fb9a3cec310>,
 <matplotlib.text.Text at 0x7fb9a1b8b910>,
 <matplotlib.text.Text at 0x7fb9a1b3cbd0>,
 None)
```

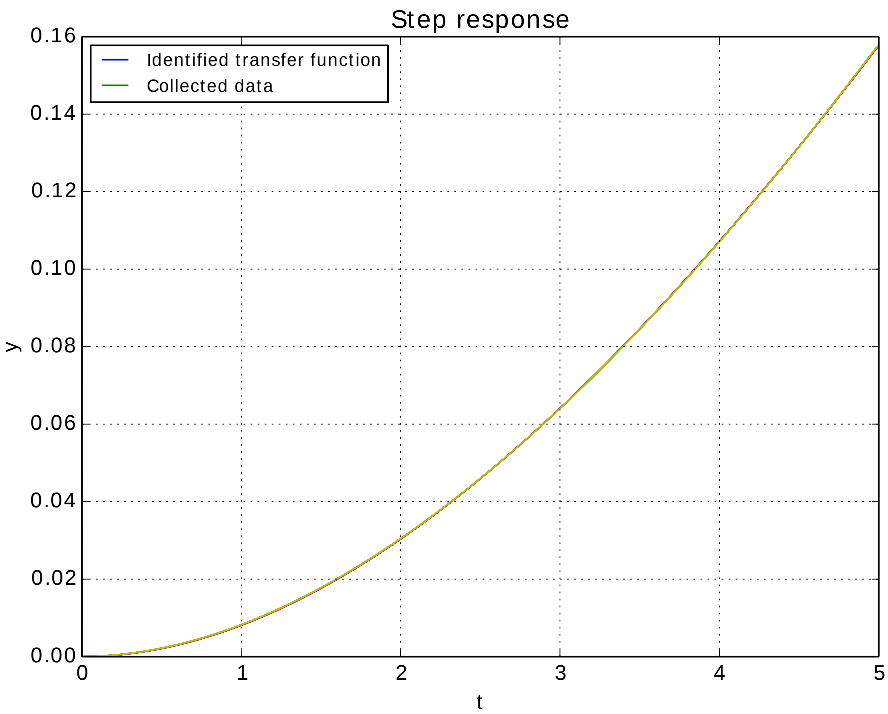


Figure 6.6: Step response and collected data

6.2.7 Continuous and discrete model

For the state controller design we need to model our motor in the state-space form. We define the continuous-state and the discrete-state space model

```
In [24]: # Design Controller Motor 1

In [25]: a=[[0,1],[0,-Dm/Jm]]

In [26]: b=[[0],[1]]

In [27]: c=[[kt/Jm,0]];

In [28]: d=[0];

In [29]: sysc=ss(a,b,c,d)                    # Continuous
     state-space form

In [30]: Ts=0.01                             # Sampling time

In [31]: sys = c2d(sysc,Ts,'zoh')            # Discrete ss
     form
```

6.2.8 Controller design

For the controller we set a bandwidth to 6 rad/s with a damping factor of $\xi = \sqrt{2}/2$.

```
In [32]: # Control system design

In [33]: print rank(ctrb(sys.A,sys.B))==2     #
     Controllability check
True

In [34]: # State feedback with integral part

In [35]: wn=6

In [36]: xi=np.sqrt(2)/2

In [37]: angle = np.arccos(xi)
```

We add a discrete integral part to eliminate the steady state error and we obtain an additional state for the error between reference and output signal. The two matrices Φ and Γ required by the pole placement routine must be extended with the additional state.

```
In [38]: cl_poles = -wn*array([1, exp(1j*angle), exp(-1j*
    angle)] ) # three poles

In [39]: cl_polesd=sp.exp(cl_poles*Ts)    # Desired
    discrete poles

In [40]: sz1=sp.shape(sys.A);

In [41]: sz2=sp.shape(sys.B);

In [42]: # Add discrete integrator for steady state zero
    error

In [43]: Phi_f=np.vstack((sys.A,-sys.C*Ts))

In [44]: Phi_f=np.hstack((Phi_f,[[0],[0],[1]]))

In [45]: G_f=np.vstack((sys.B,zeros((1,1))))

In [46]: k=place(Phi_f,G_f,cl_polesd)
```

6.2.9 Observer design

Now we can implement the observer: in this example we choose a reduced-order observer and we can use the function provided by the yottalab module to obtain it.

```
In [47]: #Reduced order observer

In [48]: print rank(obsv(sys.A,sys.C))==2     #
    Observability check
True

In [49]: p_oc=-10*max(abs(cl_poles))

In [50]: p_od=sp.exp(p_oc*Ts);

In [51]: T=[0,1]

In [52]: r_obs=red_obs(sys,T,[p_od])
```

6.2.10 Controller in compact form

The yottalab function **comp_form_i** allows to integrate the controller gains and the observer into an unique block.

```
In [53]: # Controller + integral + observer in compact
    form

In [54]: contr_I=comp_form_i(sys,r_obs,k)
```

6.2.11 Anti windup

The last operation consists in dividing the controller into an input part and a feedback part in order to realize the anti-windup mechanism and considering the saturation block.

```
In [55]: # Anti windup

In [56]: [gss_in , gss_out]=set_aw(contr_I ,[0 ,0])
```

6.2.12 Simulation of the controlled plant

The block diagram of the final controlled system is represented in figure 6.7.

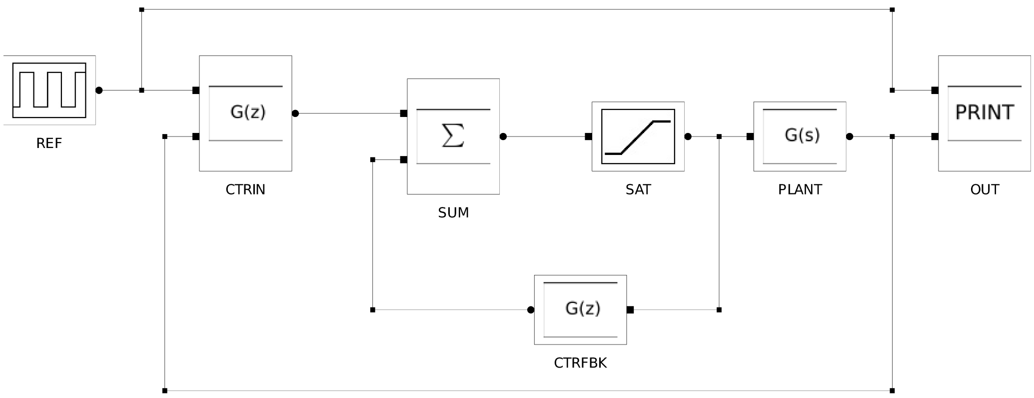


Figure 6.7: Block diagram of the controlled system

It is not possible to simulate the resulting system in Python because of:

- The controller is discrete and the plant is continuous. At present it is not possible to perform hybrid simulation. In some cases we can substitute the plant with the discrete-time system and perform a discrete simulation.
- The block “CTRIN” has two inputs. The step function can only find the output from a single input.
- The control toolbox can handle only linear system (and there is a saturation in the final system).

Chapter 7

Hybrid simulation and code generation

7.1 Basics

CACSD environments usually offer a graphical editor to perform the hybrid simulation (Matlab↔Simuli Scioslab↔Scicos, Scilab↔xCos etc.).
The “pyEdit.py” application should cover this task for the Python Control environment.
In the following we'll explain how it is possible, from the pyEdit schematics, to generate code for the hybrid simulation. Code for the RT controller can be generated in the same way: users should only replace the mathematical model of the plant with the blocks interfacing the sensors and the actuators of the real system.

7.2 pyEdit

The application “pyEdit“ is a block diagram editor to design schematics for simulation and code generation.
Starting points for the pyEdit application were the PySimEd project ([20]) and the qtnodes-develop project ([21]).
PyEdit offers the most used blocks in control design. A little set of these blocks is shown in figure 7.1.
The application offers set of operations in the toolbar as shown in the figure 7.2 and other operations are available as popup menu by clicking on a block.
By clicking with the right mouse button on a block, a popup menu is shown, offering the following operations:

Block I/Os to modify (if possible) the number of input and output ports of the block

Flip block Flip left/right the block

Change name Each block in the diagram must have **a unique name**

Block parameters to modify the parameters: this operation is available with a double click tool

Clone block to get a copy of the selected block

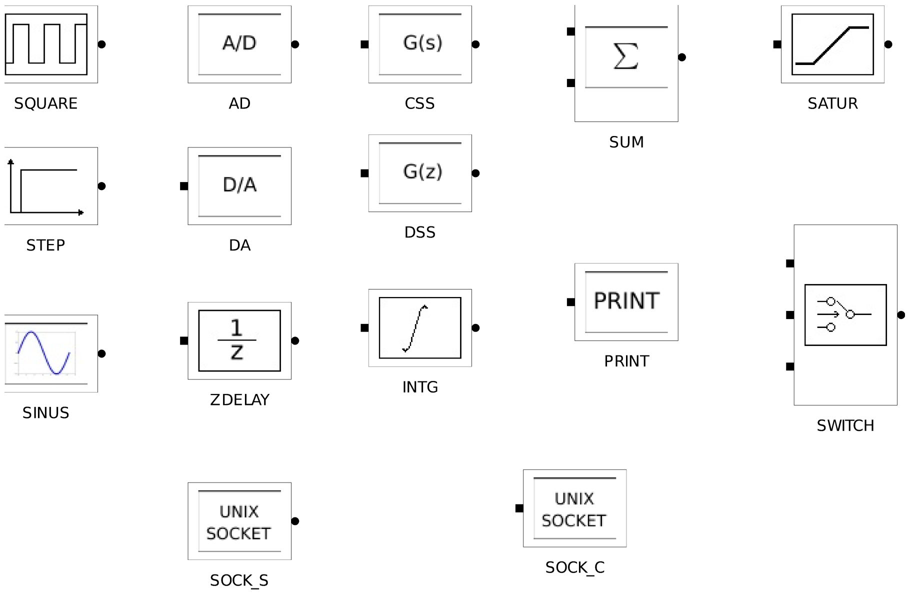


Figure 7.1: Some pyEdit blocks for control design

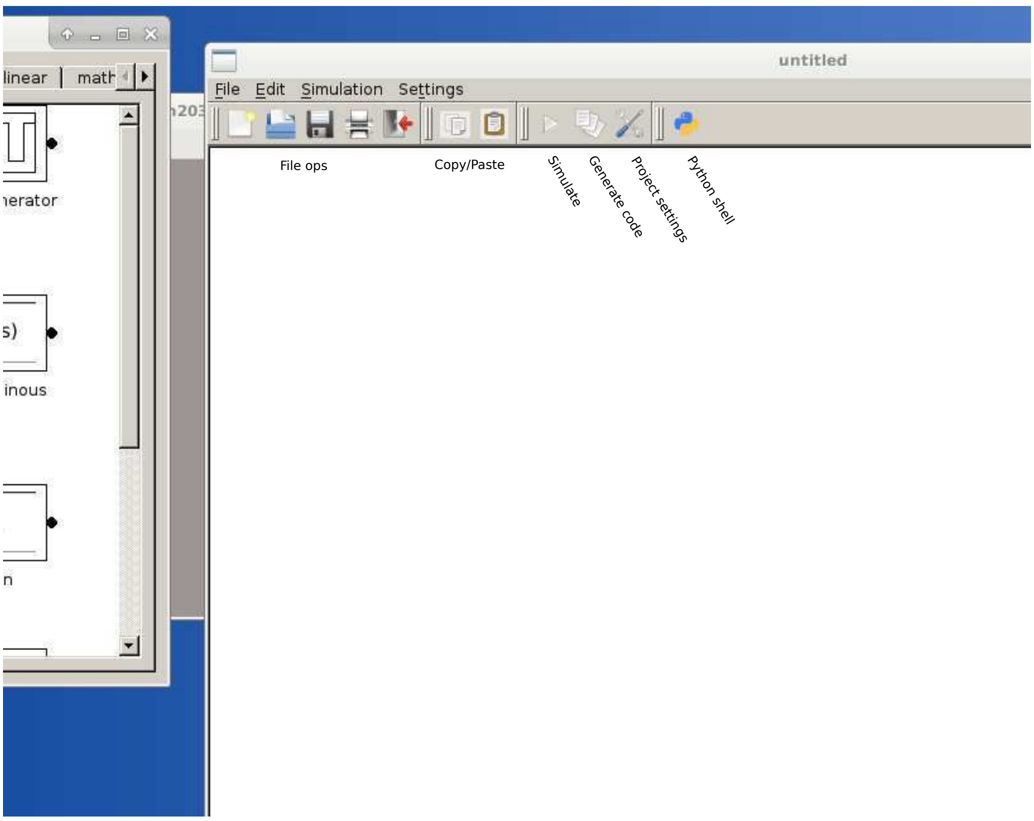


Figure 7.2: The pyEdit application

7.3 Basic operations

7.4 Configuring some parameters

The file const.py in the pyeditor folder contains some definitions required ba he applications. In particular it is possible to change the variables ”path“ and ”pycmd“.

7.4.1 Inserting a block

Get a block from a library and drag it into the main window.

7.4.2 Connecting blocks

- Move to the output port of a block or to a node.
- Click and release the left button of the mouse.
- Move the mouse to draw the connection.
- Click again the left mouse on an input port of a block to finish the connection or click the mouse to obtain a ”node“ and to continue to draw the connection.

7.4.3 Inserting a node

- Move to a connection and click the right mouse button
- Select the ”insert node“ menu.

If a new ”node“ is needed into a connection simply click on it with the right mouse button.

7.4.4 Deleting a block or a node

- Move to a block or node and click with the right mouse button.
- Choose the submenu ”delete“

7.5 Remove a node

- Move to the node.
- Click with the right mouse button on the node.
- Choose the submenu ”Bind node“ The connection is maintained but the node is cleared.

Chapter 8

Simulation and Code generation

Each element of a block diagram is defined with three or four functions:

The interface function that describes how the block must be drawn in the block diagram

The Implementation function that contains the code to be executed to perform the tasks related with this block.

The translation of the block into the RCPblk class described in the RCPblk.py module

A dlg function to implement a special dialog box for the block parameters (only if required)

In addition we need to know all the nodes connected to the inputs and to the outputs of each block.

8.1 Interface functions

Each block is defined into a library file with extension ".blk", stored in the "blocks" folder. The library file is defined using the XML syntax. The blocks are defined with the following fields:

```
<blockdata>
  <blockname>STEP</blockname>
  <inputs>0</inputs>
  <outputs>1</outputs>
  <settable>0</settable>
  <icon>STEP</icon>
  <params>stepBlk|Step Time: 1|Step Value: 1</params>
</blockdata>
```

blockname is the name of the block which can be changed in the block diagram

iputs give the number of the input ports of the block

outputs give the number of the output ports of the block

settable indicates if the number of input or output ports can be modified in the block diagram

icon is the name of the PNG file of the block icon

params is a list containing the blocks parameters

The block libraries are loaded after launching the pyEdit application as shown in figure 8.1

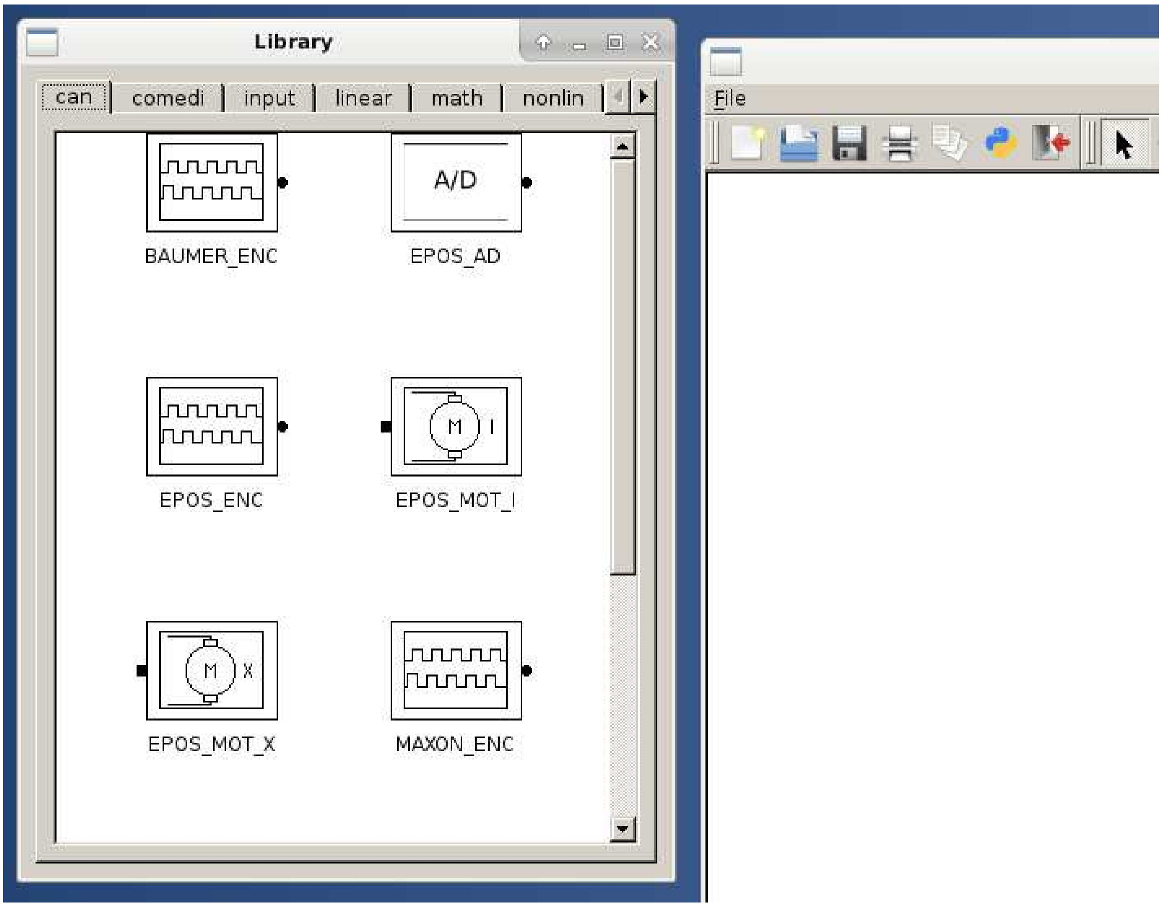


Figure 8.1: Window with the block libraries

Each block must be renamed with a unique name (popup menu "Change name"), and its parameters can be modified directly in the pyEdit application with a double click.

8.2 The implementation functions

In a schematic, each block can be described with the functions (8.1) for continuous-time systems or (8.2) for discrete-time systems.

$$\begin{aligned} \mathbf{y} &= \mathbf{g}(\mathbf{x}, \mathbf{u}, t) \\ \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \end{aligned} \tag{8.1}$$

$$\begin{aligned} \mathbf{y}_k &= \mathbf{g}(\mathbf{x}_k, \mathbf{u}_k, k) \\ \mathbf{x}_{k+1} &= \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k, k) \end{aligned} \tag{8.2}$$

The $\mathbf{g}(\ldots)$ function represents the static part of the block. This function is used to read inputs, read sensors, write actuators or update the outputs of the block.

The second function ($\mathbf{f}(\ldots)$) is only required if the block has internal states, and it is only used by dynamic systems. In addition, each block implements two other functions, one for the block initialization and one to cleanly terminate it.
All these functions are programmed as C-files, compiled and archived into a library.

8.3 Translating the block into the RCPblk class

Before generating the C-Code, each block in the diagram must be translated into an element of the RCPblk class (see section 8.6 for more details). For each block, the corresponding function (the name is given by the 1. string in the parameters line) must exists and should be declared with the required parameters. This function is responsible to fill all the class fields.

8.4 Special dialog box for the block parameters

Usually, the graphic editor build a simple dialog box to enter the block parameters.
In special cases, it is possible to write a special function to enter the parameters.. In this case, the user should provide this function in the RCPDlg.py file. The name of this function is built using the first string of the parameter line, by subsistuting the las 3 letters "Blk" with "Dlg". This new function must receive as input:

- Numper of inputs
- Number of outputs
- The parameters line

This function returns a modified parameters line.

8.5 Example

We can show with an example what happens with a block in the different phases from block to RCPblk class.
The "Pulse generator" input block is stored in the "input.blk" file with the following infos

```
    <blockdata>
      <blockname>PulseGenerator</blockname>
      <inputs>0</inputs>
      <outputs>1</outputs>
      <settable>0</settable>
      <icon>SQUARE</icon>
      <params>squareBlk|Amplitude: 1|Period: 4|Width: 2|Bias: 0|Delay:
0</params>
    </blockdata>
```

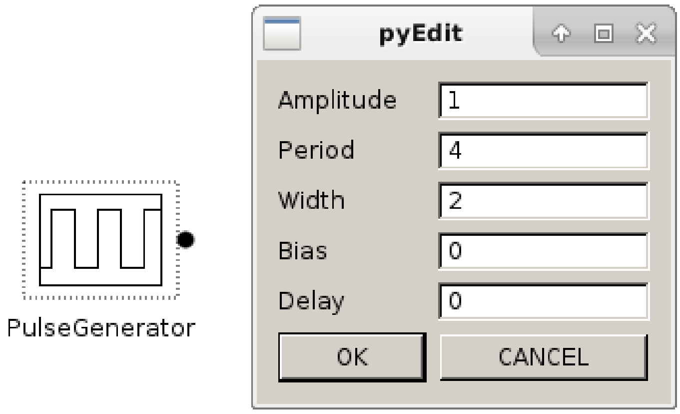


Figure 8.2: Dialog box for the Pulse generator block

The block has no inputs, 1 ouput, the I/O are not modifiable (settable=0).
The "params" line is parsed and translated into the the dialog box shown in figure 8.2.
By generating the element of the class RCPblk, the function "squareBlk" is called with the following parameters:

```
SQUARE = squareBlk(pout, Amp, Period, Width, Bias, Delay)
```

where

pout is the matrix with the id of the inputs (connections)

Amp is the signal amplitude

Period is the period of the signal

width is the duration where the signal has value "Amp-bias"

bias is an offset for the signal

delay represent the time wenn the signal start

The function translate the block into the following object of the RCPblk class

```
 Function            : square
Input ports         : []
Output ports       : [2]
Nr. of states       : [0 0]
Relation u->y       : 0
Real parameters     : [ 4  8  3  0 12]
Integer parameters : []
```

8.6 I/O connections

After clicking on the "code generation" tool on the toolbar, the user is asked to introduce some parameters in a dialog box (see figure 8.3).

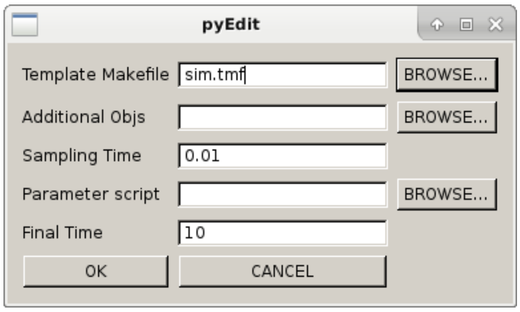


Figure 8.3: Dialog for code generation

In this dialog it is possible to choose the "template makefile" for simulation or real-time execution, the sampling time of the system and some additional libraries, reuired for the simulation with FMU packages.
After this first setup it is possible to translate the block diagram into a list of elements of the class **RCPblk** provided by the **suspictrl** package. This class contains all the information required for the code generation and can be expanded in the future to handle additional fields (ex. the type of the I/O signals: int, double etc.).
This class contains the following fields:

fcn: the name of the C-Function to be used to handle this block

pin: an array containing the id of the input nodes

pout: an array containing the id of the output nodes

nx: the number of internal states (continuous or discrete)

uy: a flag which indicates a direct dependency between input and output signals (feed-through flag).

realPar: an array containing the real parameters of the block

intPar: an array containing the integer parameters of the block

str: a string related to the block

For example, the diagram in figure 8.4 is translated into the following code

```
from supsictrl.RCPblk import *

STEP = stepBlk([1],  1,  1)
PM = sumBlk([1,3],[2],  [1,-1])
CSS = cssBlk([2],[3],  sys,  0)
PRINT = printBlk([1,3])

blks = [STEP,PM,CSS,PRINT,]
fname = 'step'
genCode(fname, 0.01, blks)
genMake(fname, 'sim.tmf', addObj = '')
```

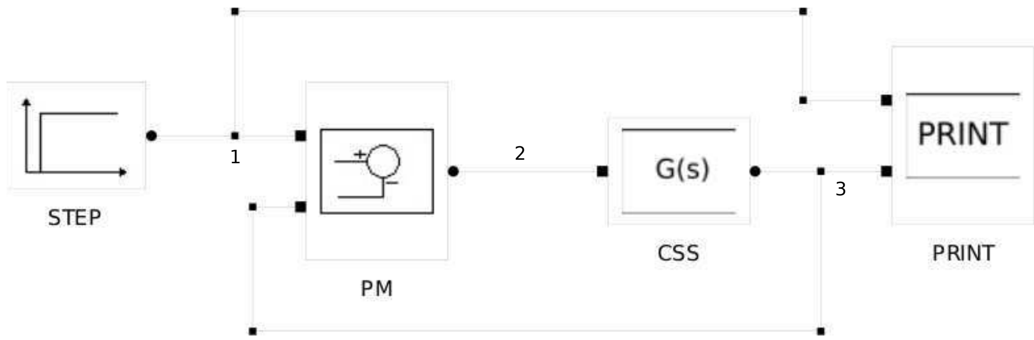


Figure 8.4: Simple block diagram

The block **CSS** has one input connected to node ② and one output connected to node ③, it is a continuous transfer function (cssBlk, $1/(s+1)$) with zero initial conditions. The **PM** block has 2 inputs connected to node ① and ③, one output connected to node ② and performs a subtraction of the output from the input signals.

8.7 Translating the block list into C-code

8.7.1 Finding the right execution sequence

Before starting with the translation of the block diagram into C-code, we need to find the correct sequence of execution of the blocks. This task can be performed by analizing the *uy* flag of the block object. When in a block the *uy* flag is set to 1, we need the output of the blocks connected at his input before starting to update his output. This means that we have to generate a dependency tree of all the blocks and then we must rearrange the order of the block list for code generation.

In linear blocks for examples, the *uy* flag is set if the D matrix is not null.

In the blockdiagram of figure 8.4, the **PM** and the **PRINT** blocks require to know their inputs before update their outputs.

```
In [5]: NrOfNodes = 3

In [6]: ordered_list = detBlkSeq(NrOfNodes,blks)

In [7]: for n in ordered_list:
   ...:     print n
   ...:
Function           : css
Input ports        : [2]
Outputs ports      : [3]
Nr. of states      : [2 0]
Relation u->y      : 0
Real parameters    : [[ 0.  0. -1.  1. -1. -1.
                        0.  0. -1.  0.  0.  0.]]
Integer parameters : [ 2  1  1  1  5  7  9 10]
String Parameter   :

Function           : step
Input ports        : []
Outputs ports      : [1]
Nr. of states      : [0 0]
Relation u->y      : 0
Real parameters    : [1 1]
Integer parameters : []
String Parameter   :

Function           : print
Input ports        : [1 3]
Outputs ports      : []
Nr. of states      : [0 0]
Relation u->y      : 1
Real parameters    : []
Integer parameters : []
String Parameter   :

Function           : sum
Input ports        : [1 3]
Outputs ports      : [2]
Nr. of states      : [0 0]
Relation u->y      : 1
Real parameters    : [ 1 -1]
Integer parameters : []
String Parameter   :
```

If the block diagram contains algebraic loops it is not possible to find a solution for the **detBlkSeq** function and an error is raised.

8.7.2 Generating the C-code

Starting from the ordered list of blocks, it is possible to generate C-code.
The code contains 3 functions:

- The initialization function
- The termination function
- The periodic task

8.7.3 The init function

In this function each block is translated into a python_block structure defined as follows:

```
typedef struct {
  int nin;              /* Number of inputs */
  int nout;             /* Number of outputs */
  int *nx;              /* Cont. and Discr states */
  void **u;             /* inputs */
  void **y;             /* outputs */
  double *realPar;      /* Real parameters */
  int *intPar;          /* Int parameters */
  char * str;           /* String */
  void * ptrPar;        /* Generic pointer */
}python_block;
```

The nodes of the block diagram are defined as "double" variables and the inputs and outputs of the blocks are defined as vectors of pointers to them.

```
...
/* Nodes */
static double Node_1[] = {0.0};
static double Node_2[] = {0.0};
static double Node_3[] = {0.0};

/* Input and outputs */
static void *inptr_0[]  = {0};
static void *outptr_0[] = {0};
static void *outptr_1[] = {0};
static void *inptr_2[]  = {0,0};
static void *inptr_3[]  = {0,0};
static void *outptr_3[] = {0};
...
  inptr_0[0]  = (void *) Node_2;
  outptr_0[0] = (void *) Node_3;
..
  block_test[0].nin  = 1;
  block_test[0].nout = 1;
  block_test[0].nx   = nx_0;
  block_test[0].u    = inptr_0;
  block_test[0].y    = outptr_0;
...
```

After this initialization phase, the implementation functions of the blocks are called with the flag **INIT**.

```
  css(INIT, &block_test[0]);
  step(INIT, &block_test[1]);
  print(INIT, &block_test[2]);
  sum(INIT, &block_test[3]);
```

8.7.4 The termination function

This procedure calls the implementation functions of the blocks with the flag **END**.

8.7.5 The ISR function

This procedure represents the periodic task of the RT execution. First of all, the implementation functions are called with the flag **OUT**, in order to perform the output update of each blocks. As a second step, the implementation functions of the block containing internal states ($nx \neq 0$) are called with the flag **STUPD** (state update).

```
...
css(OUT, &block_test[0]);
step(OUT, &block_test[1]);
print(OUT, &block_test[2]);
sum(OUT, &block_test[3]);
...
css(OUT, &block_test[0]);
css(STUPD, &block_test[0]);
...
```

8.8 The main file

The core of the RT execution is represented by the "python_main_rt.c" file. During the RT execution, the main procedure starts a high priority thread for handling the RT behavior of the system. The following main file, for example, is used to launch the executable in a Linux preempt_rt environment.

```
void *rt_task(void *p)
{
  ...
  param.sched_priority = prio;
  if(sched_setscheduler(0, SCHED_FIFO, &param)==-1){
    perror("sched_setscheduler failed");
    exit(-1);
  }

  ...
  double Tsamp = NAME(MODEL,_get_tsamp)();

  ...
  NAME(MODEL,_init)();

  while(!end){
   /* wait untill next shot */
    clock_nanosleep(CLOCK_MONOTONIC,
                    TIMER_ABSTIME, &t, NULL);

    ...
    /* periodic task */
    NAME(MODEL,_isr)(T);
    ...
  }
  NAME(MODEL,_end)();
}
```

Chapter 9

Example

9.1 The plant

One of the educational plants available at the SUPSI laboratory is the system shown in figure 9.1. This example is located in to the "pycontrol/Tests/ControlDesign/DisksAndSpring" folder,

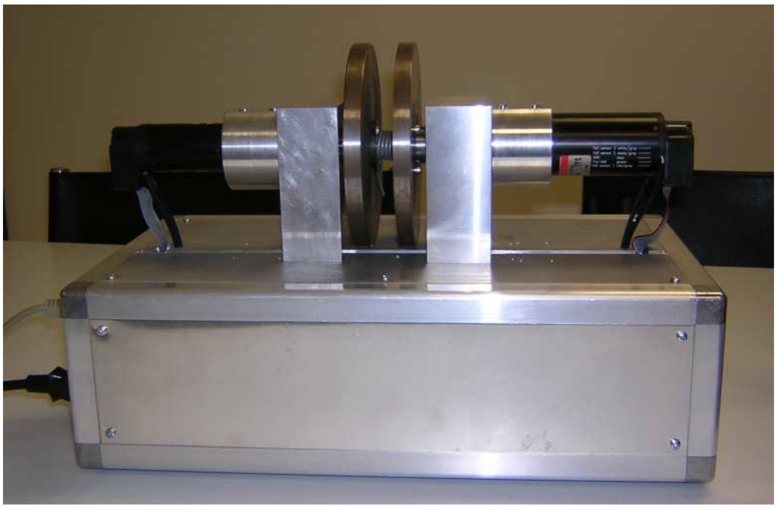


Figure 9.1: The disks and spring plant

Two disks are connected by a spring. The goal for the students is to control the angle of the disk on the right by applying an appropriate torque to the disk on the left.

The physical model of this plant can be directly calculated in python using for example the **sympy** toolbox. Sympy can deliver a symbolic description of the system and through a python *dictionary* it is possible to easily obtain the numerical matrices of the state-space representation of the plant.

```
In [4]: A
Out[4]:
matrix([[0, 0, 1, 0],
        [0, 0, 0, 1],
        [-c/J1, -c/J1, (-d - d1)/J1, -d/J1],
        [-c/J2, -c/J2, -d/J2, (-d - d2)/J2]])

In [5]: B1

Out[5]:
matrix([[0, 0],
        [0, 0],
        [kt1/J1, 0],
        [0, kt2/J2]])

In [6]: B = B1[:,0]

In [7]: C
Out[7]: [[1, 0, 0, 0], [0, 1, 0, 0]]

In [8]: C2
Out[8]: [0, 1, 0, 0]

In [9]: D
Out[9]: [[0], [0]]

In [10]: D2
Out[10]: [0]
```

The control system toolbox and the additional “yottalab.py” package contain all the functions required for the design of the controller. In this case we design a discrete-state feedback controller with integral part for eliminating steady-state errors. The states are estimated with a reduced-order observer. In addition, an anti-windup mechanism has been implemented. The sampling time is set to 10 ms.
The yottalab module offers 3 functions that facilitate the controller design:

- The function **red_obs**(sys, T, poles) which implements the reduced-order observer for the system **sys**, using the submatrix **T** (required to obtain the estimator C-matrix and the desired state-estimator poles **poles**.

$$P = [C; T] \rightarrow C^{*} = C \cdot P^{-1} = [I_q, O_{(n-q)}]$$

- The function **comp_form_i**(sys,obs,K,Cy) that transforms the observer **obs** with the state-feedback gains **K** and the integrator part into a single dynamic block with the reference signal and the two positions φ_1 and φ_2 as inputs and the control current I_1 as output. The vector **Cy** is used to select φ_2 as the output signal that is compared with the reference signal for generating the steady-state error for the integral part of the controller.

- The function **set_aw**(sys,poles) that transforms the previous controller ($Contr(s) = N(s)/D(s)$) in an input state-space system and a feedback state-space system, implementing the anti-windup mechanism. The vector **poles** contains the desired poles of the two new systems ($D_{new}(s)$) (see figure 9.2).

$$sys_{in}(s) = \frac{N(s)}{D_{new}(s)}$$

$$sys_{fbk}(s) = 1 - \frac{D(s)}{D_{new}(s)}$$

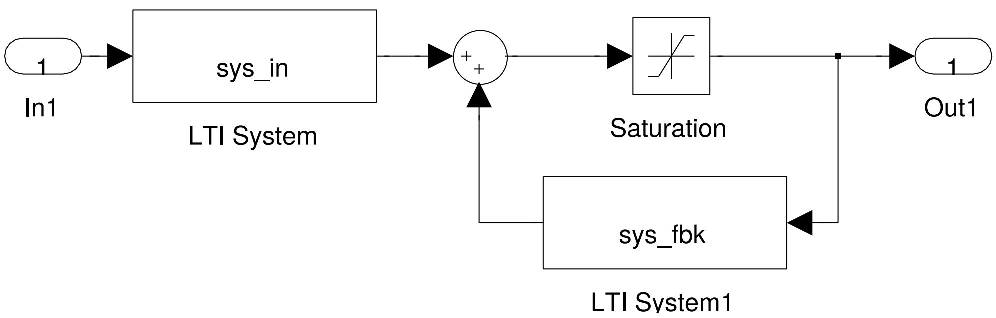


Figure 9.2: Anti windup

9.2 The plant model

```
# Sampling time
ts = 10e-3

gss1 = ss(A,B,C,D)
gss = ss(A,B,C2,D2)
gz = c2d(gss,ts,'zoh')
```

9.3 Controller design

```
# Control design
wn = 10
xi1 = np.sqrt(2)/2
xi2 = 0.85

cl_p1 = [1,2*xi1*wn,wn**2]
cl_p2 = [1,2*xi2*wn,wn**2]
cl_p3 = [1,wn]
cl_poly1 = sp.polymul(cl_p1,cl_p2)
cl_poly = sp.polymul(cl_poly1,cl_p3)
cl_poles = sp.roots(cl_poly)     # Desired continuous
    poles
cl_polesd = sp.exp(cl_poles*ts)  # Desired discrete poles

# Add discrete integrator for steady state zero error
Phi_f = np.vstack((gz.A,-gz.C*ts))
Phi_f = np.hstack((Phi_f,[[0],[0],[0],[0],[1]]))
G_f = np.vstack((gz.B,zeros((1,1))))

# Pole placement
k = placep(Phi_f,G_f,cl_polesd)
```

9.4 Observer design

```
# Observer design - reduced order observer
poli_o = 5*cl_poles[0:2]
poli_oz = sp.exp(poli_o*ts)

disks = ss(A,B,C,D)
disksz = StateSpace(gz.A,gz.B,C,D,ts)
T = [[0,0,1,0],[0,0,0,1]]

# Reduced order observer
r_obs = red_obs(disksz ,T, poli_oz)

# Controller and observer in the same matrix -  Compact
    form
contr_I = comp_form_i(disksz,r_obs,k,[0,1])

# Implement anti windup
[gss_in,gss_out] = set_aw(contr_I,[0.1,0.1,0.1])
```

9.5 Simulation

We can perform the simulation of the discrete-time controller with the continuous-time mathematic plant model using the block diagram of figure 9.3
This diagram is stored as “disks_sim.dgm” in the folder.
The plant is represented by a continuous-time state-space block with 1 input and 2 outputs. The controller implements the state-feedback gains and the state observer and it has been split into a CTRIN block and a CTRFBK block in order to implement the anti-windup mechanism.

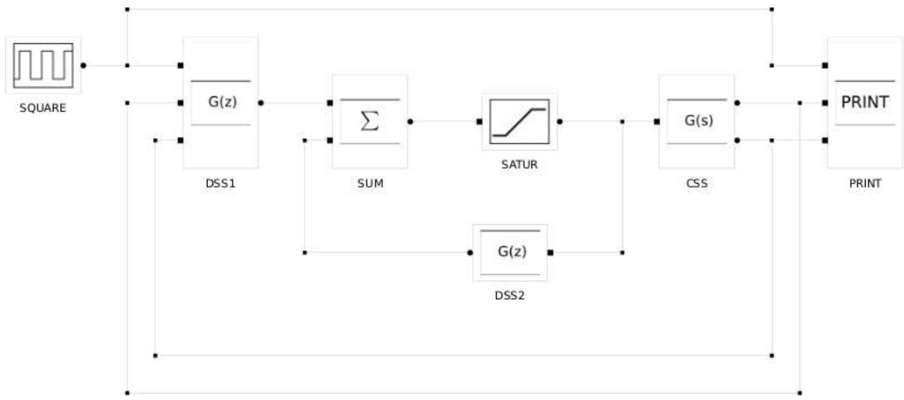


Figure 9.3: Block diagram for the simulation

We can now generate the code for the simulation and launch the generated executable. The template makefile for this executable is **sim.tmf**.
After creating the script for code generation (button in the toolbar) it is possible to proceed with the simulation,
Open a python terminal (for example by clicking on the python button in the toolbar), and give the following commands:

```
run DisksAndSpringKane.py
run -i disks_sim.py
!make
!pySim disks_sim 40
```

The plots resulting from the simulation are shown in figure 9.4.

9.6 Real-time controller

In order to generate the RT controller for the real plant, we first have to substitute the plant with the interfaces for sensors and actuators using blocks that send and receive CAN message using a USB dongle of Peak System. The template makefile for this system is now **rt.tmf**, that allows to generate code with real-time behaviour.
The block diagram for the real-time controller is represented in figure 9.5.
The motor position can be plotted in python at the end of the execution (see figure 9.6).

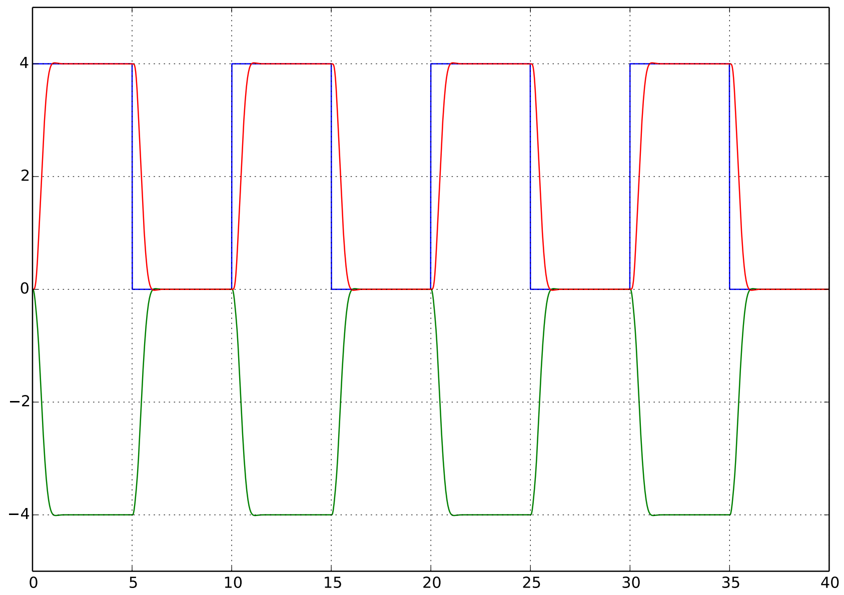


Figure 9.4: Simulation of the plant

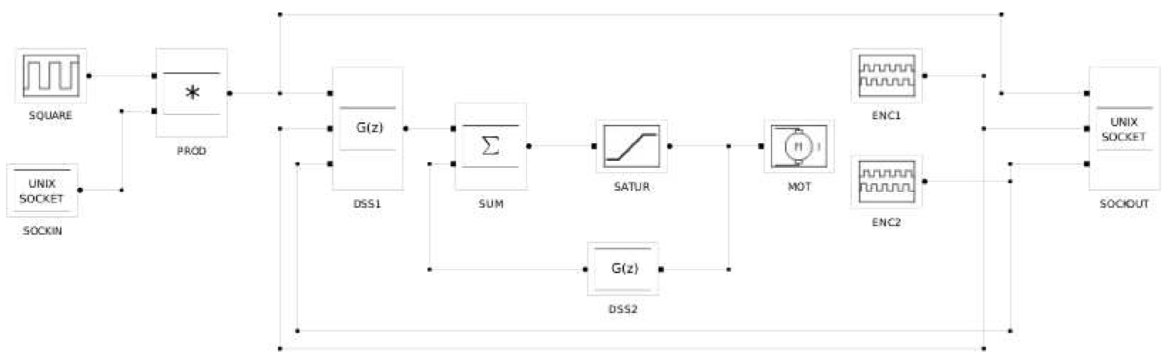


Figure 9.5: Block diagram for the RT implementation

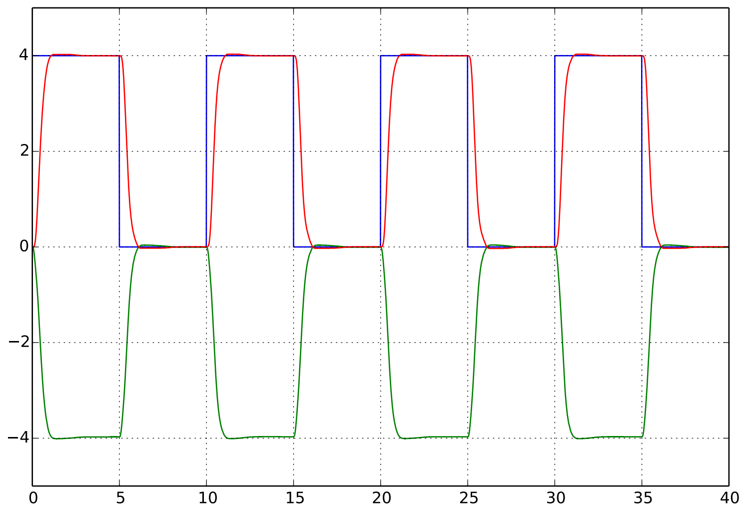


Figure 9.6: RT execution

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