CASCADE SYSTEM CALCULATION

Nomenclature

$\bar{d}_{cav}$	Effective diameter of the cavity, m
A	Area, m^2
A_{ap}	Aperture area of the volumetric receiver, m^2
A_{cav}	Area of the cavity of the volumetric receiver, m^2
$A_{dish,air}$	Heat transfer area of dish receiver between tube and air, m^2
$A_{dishCollector}$	Aperture area of each dish collector, m^2
A_{dish}	Aperture area of all dish collectors, m^2
$A_{stirling,1}$	Heat transfer area of each Stirling engine at air side, m^2
$A_{stirling,2}$	Heat transfer area of each Stirling engine at water side, m^2
$A_{troughCollector}$	Aperture area of each trough collector, m^2
A_{trough}	Aperture area of all trough collectors, m^2
c_p	Heat capacity of Stirling engine working gas at constant pressure, J/(kg·K)
c_r	Heat transfer correction factor of coil tube of volumetric receiver
c_v	Heat capacity of Stirling engine working gas at constant volume, J/(kg·K)
d_i	Inner diameter of trough receiver, m
d_o	Outer diameter of trough receiver, m
d_{ap}	Aperture diameter of volumetric receiver, m
d_{cav}	Diameter of volumetric receiver cavity, m
$d_{i,air}$	Inner diameter of air tube, m

e	Regeneration effectiveness of the Stirling engine
F_e	Soiling factor of the trough collector
Gr	Grashof number
H_C	Height between two adjacent coils of volumetric receiver tube, m
I_{DNI}	Direct normal irradiance, W/m^2
K	Incidence angle modifier of trough collector
k	Specific heat ratio of the working gas in Stirling engine
$k_{air,insu}$	Thermal conductivity of air at the temperature of outside insulating layer, W/(m·K)
L_C	Length of coiled spring tube of volumetric receiver, m
l_{cav}	Depth of volumetric receiver cavity, m
N	Number of coils of the tube in volumetric receiver
n	Amount of working gas in each Stirling engine, mol
n_1	Number of columns of the Stirling engine array
n_2	Number of rows of the Stirling engine array
$n_{dishCollector}$	Number of all dish collectors
n_{se}	Speed of Stirling engine, s^{-1}
$n_{stirlingEngine}$	Number of Stirling engines in the Stirling engine array
$n_{troughCollector}$	Number of all trough collectors
Nu	Nusselt number
p_c	Exhaust pressure of the turbine, Pa
p_s	Main steam pressure of the turbine, Pa
p_{amb}	Ambient pressure, Pa
p_{cp}	Condensate pump outlet pressure, Pa
$p_{deaerator}$	Pressure of deaerator, Pa
p_{dish}	Air pressure in volumetric reciver tube, Pa
$P_{generator}$	Power of generator, We
$P_{stirling}$	Power of the Stirling engine array, We

p_{trough}	Pressure of therminol in the tube of trough receiver, Pa
Pr	Prandtl number
$q^{''}$	Heat flux, W/m^2
$q_{collector,fluid}$	Heat transferred from trough collector to fluid, W
$q_{cond,tot}$	Total conduction loss of volumetric receiver, W
$q_{conv,tot}$	Total convection loss of volumetric receiver, W
$q_{dish,air}$	Energy absorbed by air in the dish collector, W
q_{in}	Solar energy lauched into dish receiver aperture, W
$q_{m,1,r}$	Mass flow rate of air in each row of Stirling engine array, kg/s
$q_{m,1}$	Total mass flow rate of air, kg/s
$q_{m,2,r}$	Mass flow rate of water in each row of Stirling engine array, kg/s
$q_{m,2}$	Total mass flow rate of water, kg/s
$q_{m,3}$	Total mass flow rate of therminol, kg/s
$q_{m,dish}$	Mass flow rate of air in each dish receiver, kg/s
$q_{rad,emit}$	Heat emitted by volumetric receiver cavity, W
$q_{rad,reflect}$	Reflected radiation by volumetric receiver, W
$q_{stirling,cold}$	Heat absorbed by cooling water in the Stirling engine array, W
$q_{stirling}$	Heat provided to the Stirling engine array, W
$q_{sun,collector}$	Solar radiant energy falling on trough collectors, W
Re	Reynolds number
T_H	Highest temperature of expansion space, K
T_L	Lowest temperature of compression space, K
T_R	Regenerator temperature, K
T_s	Main steam temperature of turbine, K
$T_{1,afterstirling}$	Temperature of the air after heating Stirling engine, K
T_{amb}	Ambient temperature, K
$T_{dish,inlet}$	Inlet temperature of air for volumeteric receiver, K

$T_{dish,outlet}$	Outlet temperature of air for volumetric receiver, K
T_{insu}	Temperature of insulating outside layer, K
$T_{s,d}$	Designed mean steam temperature of turbine, K
$T_{trough,outlet}$	Outlet temperature of therminol for trough receiver, K
U	Overall heat transfer coefficient, W/(m^2·K)
$U_{stirling,1}$	Overall heat transfer coefficient of Stirling engine at air side, W/(m^2·K)
$U_{stirling,2}$	Overall heat transfer coefficient of Stirling engine at water side, W/(m^2·K)
v_{wind}	Ambient wind speed, m/s
w_{trough}	Width of trough collector, m
y	Extraction rate of steam turbine

Abbreviations

EES	Engineering Equation Solver
SES	Stirling Energy System

Greek Symbols

α_{abs}	Absorptivity of the absorber selective coating
α_{cav}	Cavity surface absorptance
α_{eff}	Effective absorptance
δ_a	Thickness of air tube in volumetric receiver, m
δ_{insu}	Thickness of receiver insulating layer, m
$\Delta T_{oil,water,min}$	Minimum temperature difference between oil and water in the oil-to-water heat exchagner, K
ϵ_{insu}	Emissivity of reciver insulating layer
η_{dish}	Thermal efficiency of dish
$\eta_{opt,0}$	Optical efficiency with an incidence angle of 0
$\eta_{shading}$	Shading factor
$\eta_{stirling}$	Efficiency of Stirling engine array
γ	Compression ratio

γ	Intercept factor
$\gamma_{stirling}$	Compression ratio of Stirling engine
λ	Thermal conductivity, W/(m·K)
μ	Viscosity, kg/(m·s)
μ_w	Fluid viscosity evaluated at average temperature of tube wall, kg/(m·s)
ϕ	Incidence angle
ρ	Reflectivity
τ	Transmissivity
θ_{dish}	Dish aperture angle (0° is horizental, 90° is vertically down)

Subscripts

i	Inlet
i	Isentropic parameter
$insu$	Insulating layer of the volumetric receiver
o	Outlet
s	Separate system
x	Stirling engine in column x
1	Air
1,1	Air at outlet of dishes and inlet of heat exchanger of Stirling engines
1,2	Air outlet of heat exchanger of Stirling engines and inlet of air-to-water heat exchanger
1,3	Air at outlet of heat exchanger of air-to-water heat exchanger and inlet of dishes
2	Water
2,1	Water at outlet of air-to-water heat exchanger and inlet of turbine
2,10	Water at outlet of evaporator and inlet of superheater
2,11	Water at outlet of superheater and inlet of air-to-water heat exchanger

2,2	Water at outlet of turbine and inlet of condenser
2,3	Water at outlet of turbine bleed point and inlet1 of deaerator
2,4	Water at outlet of condenser and inlet of condenser pump
2,5	Water at outlet of condenser pump and inlet of Stirling engine heat exchanger
2,6	Water at outlet of Stirling engine heat exchanger and inlet2 of deaeretor
2,7	Water at outlet of deaeretor and inlet of feed water pump
2,8	Water at outlet of feed water pump and inlet of preheater
2,9	Water at outlet of preheater and inlet of evaporator
3	Therminol
3,1	Therminol at the outlet of troughes and inlet of superheater
3,2	Therminol at the outlet of superheater and inlet of evaporator
3,3	Therminol at the outlet of evaporator and inlet of preheater
3,4	Therminol at the outlet of preheater and inlet of troughes

1 Introduction

Different types of collectors and different technologies for electricity generation are suitable for different working temperature zones with different costs. An idea of cascade collection and cascade utilization of solar energy with higher efficiency is presented. Parabolic trough collectors are used to collect lower temperature energy with lower cost and dish collectors are used to collect higher temperature energy with higher efficiency. Rankine cycle is used to work in lower temperature zone and Stirling cycle is used to work in higher temperature zone. Furthermore, effective topological structures are considered to take full advantages of thermodynamic characters of different components of the system. The cold chamber of Stirling engine is cooled by condensed fluid of Rankine cycle to use the heat released by Stirling engine.

2 Problem description

To characterize the system, calculation of the system must be applied first. The scale of the system and the dimensions of the parameters must be evaluated.

Before detailed model of the system can be created, several simplifying assumptions are made:

- Steady state at nominal load of the system is analyzed
- Pressure drop due to flow is negligible everywhere
- Same isentropic efficiency of the steam turbine with different loads and in different stages
- Simple models are used of some processes and equipments
- All the Stirling engines in the Stirling engine array has the same speed
- A symmetrical regenerator behavior is assumed so that a single effectiveness can be defined as $e = \dfrac{T_R - T_L}{T_H - T_L}$ [1, 2]
- There is no heat loss to the environment for Stirling engines

For cascade system, water is used as the working fluid of Rankine cycle. Feed water is used to cool the cold chamber of Stirling engine. Figure 1 shows the sketch of cascade system. Air is heated in parabolic dish collectors to be used to heat the hot chamber of Stirling engine and air-to-water heat exchanger successively. Water is heated in the cold chamber of Stirling engine, preheater, evaporator, superheater and air-to-water heat exchanger successively, and then steam expand in the turbine, condense in the condenser. Parabolic trough collectors are used to provide heat for the preheater, evaporator and superheater. Pumps are used to change the pressure of fluids. State numbers are marked on the sketch according to its logical position. Numbers with solid circle means

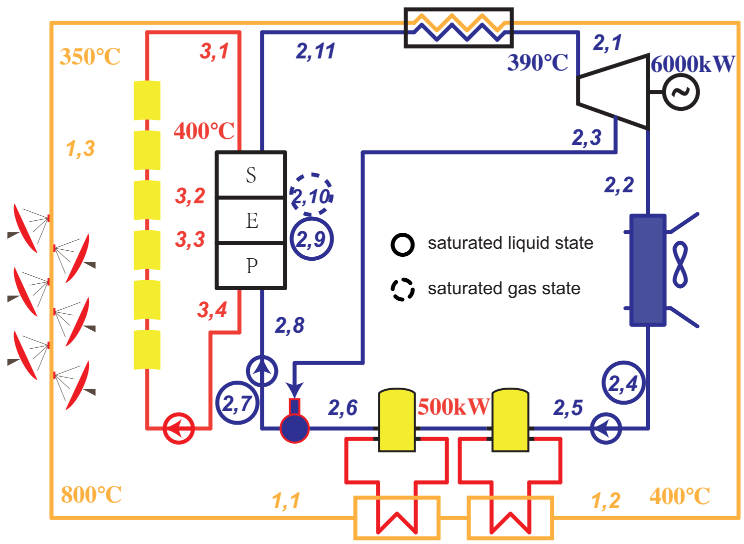

Figure 1: Sketch of cascade system

saturated liquid states ($x = 0$), and with dotted circle means saturated gas states ($x = 1$).

3 Data

Table 1 shows the basic design data of cascade system, such as the nominal electric power of Stirling engine and generator, temperatures of inlet and outlet of dishes, design parameters of turbine, etc. According to these data, some important parameters of the system, such as mass flow rate, fluid temperature, system efficiency, can be obtained by calculation. To calculate the system, the model must be built first. EES is used for the model construction and calculation.

Table 1: System characteristics of cascade system

Nominal electric power		
$P_{generator} = 6 \times 10^6$ We		
Environment		
$I_{DNI} = 700\ \mathrm{W/m^2}$	$T_{amb} = 293\,\mathrm{K}$	$p_{amb} = 1 \times 10^5\,\mathrm{Pa}$
$v_{wind} = 4\,\mathrm{m/s}$		
Dish collector		
$T_{dish,inlet} = 623\,\mathrm{K}$	$T_{dish,outlet} = 1073\,\mathrm{K}$	$p_{dish} = 5 \times 10^5\,\mathrm{Pa}$
$A_{dishCollctor} = 87.7\,\mathrm{m^2}$	Dish type: a product of SES	
Trough collector		
$\Delta T_{oil,water,min} = 15\,\mathrm{K}$ [a]	$T_{trough,outlet} = 623\,\mathrm{K}$	$p_{trough} = 2 \times 10^6\,\mathrm{Pa}$
$A_{troughCollctor} = 545\,\mathrm{m^2}$	Trough type: LS-3	
Stirling engine		
$T_{1,afterstirling} = 673\,\mathrm{K}$	$n_{se} = 10\,\mathrm{s^{-1}}$	$U_{stirling,1} = 30\,\mathrm{W/m^2{\cdot}K}$
$U_{stirling,2} = 150\,\mathrm{W/m^2{\cdot}K}$	$A_{stirling,1} = 6\,\mathrm{m^2}$	$A_{stirling,2} = 6\,\mathrm{m^2}$
$k = 1.4$	$\gamma_{stirling} = 3.375$	$n = 7.73 \times 10^{-2}\,\mathrm{mol}$
$n_{stirlingEngine} = 100$		
Steam turbine		
$T_s = 613\,\mathrm{K}$	$p_s = 2.35 \times 10^6\,\mathrm{Pa}$	$p_c = 1.5 \times 10^4\,\mathrm{Pa}$
$T_{s,d} = 663\,\mathrm{K}$	$p_{cp} = 1 \times 10^6\,\mathrm{Pa}$	Turbine type: a product
Deaerator		
$p_{deaerator} = 1 \times 10^6\,\mathrm{Pa}$		

[a] $\Delta T_{oil,water,min}$ should be chosen carefully, for the pinch temperature of $T_{3,1} - T_{2,11}$ is stable, and $\Delta T_{oil,water,min}$ should be chosen less than this value.

4 Model of the system

The system is built in several blocks. These blocks are made of circuits and efficiency calculations. Two circuits, air circuit and water circuit, are built in some specific states and in some components. Known parameters of the states, we can get the efficiency of the system and the overall efficiency of separated systems.

4.1 Air circuit

Use the function $DishReceiver(A_{dishCollector}, T_{dish,inlet}, T_{dish,outlet} : \eta_{dish})$ to calculate η_{dish}. The explanation of function $DishReceiver$ can be found in Appendix C.

For state [1, 1],

$$\left.\begin{array}{l} p_{1,1} = p_{dish} \\ T_{1,1} = T_{dish,outlet} \end{array}\right\} \Rightarrow h_{1,1} \tag{1}$$

For state [1,2],

$$\left.\begin{array}{l} p_{1,2} = p_{1,1} \\ T_{1,2} = T_{1,afterstirling} \end{array}\right\} \Rightarrow h_{1,2} \tag{2}$$

For Stirling engines, use function $StirlingEngineArray(T_{1,1}, T_{1,2}, q_{m,1}, p_{1,1}, T_{2,5}, (1-y)q_{m,2}, p_{2,5} : \eta_{stirling}, P_{stirling})$ to calculate $\eta_{stirling}$ and $P_{stirling}$. The explanation of function $StirlingEngineArray$ can be found in Appendix B.

$$q_{stirling} = P_{stirling}/\eta_{stirling} \tag{3}$$

$$q_{stirling} = q_{m,1}\left(h_{1,1} - h_{1,2}\right) \tag{4}$$

Since there is no heat loss to the environment for Stirling engines,

$$q_{stirling,cold} = q_{stirling} - P_{stirling} \tag{5}$$

For state [1, 3],

$$\left.\begin{array}{l} p_{1,3} = p_{1,2} \\ T_{1,3} = T_{dish,inlet} \end{array}\right\} \Rightarrow h_{1,3} \tag{6}$$

For dish,

$$q_{dish} = q_{m,1}\left(h_{1,1} - h_{1,3}\right) \tag{7}$$

$$q_{dish} = DNI \cdot A_{dish} \cdot \eta_{dish} \tag{8}$$

$$n_{dishCollector} = ceil(A_{dish}/A_{dishCollector}) \tag{9}$$

4.2 Water circuit

$\eta_{i,turbine}$ is obtained by given design parameters of the turbine.
For state [2, 1],

$$\left.\begin{array}{l} p_{2,1} = p_s \\ T_{2,1} = T_s \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{2,1} \\ s_{2,1} \end{array}\right. \tag{10}$$

For state [2, 2],

$$\left.\begin{array}{l} s_{i,2,2} = s_{2,1} \\ p_{i,2,2} = p_c \end{array}\right\} \Rightarrow h_{i,2,2} \tag{11}$$

$$\eta_{i,turbine} = \frac{h_{2,1} - h_{2,2}}{h_{2,1} - h_{i,2,2}} \tag{12}$$

$$\left.\begin{array}{l} p_{2,2} = p_c \\ h_{2,2} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,2} \\ x_{2,2} \end{array}\right. \tag{13}$$

For state $[2,3]$

$$\left.\begin{array}{l} s_{i,2,3} = s_{2,1} \\ p_{i,2,3} = p_{deaerator} \end{array}\right\} \Rightarrow h_{i,2,3} \tag{14}$$

$$\eta_{i,turbine} = \frac{h_{2,1} - h_{2,3}}{h_{2,1} - h_{i,2,3}} \tag{15}$$

$$\left.\begin{array}{l} p_{2,3} = p_{deaerator} \\ h_{2,3} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,3} \\ x_{2,3} \end{array}\right. \tag{16}$$

$$P_{turbine} = P_{generator}/\eta_{generator} \tag{17}$$

$$P_{turbine} = (1-y)\, q_{m,2} \left(h_{2,1} - h_{2,2}\right) + y q_{m,2} \left(h_{2,1} - h_{2,3}\right) \tag{18}$$

For state $[2,4]$,

$$\left.\begin{array}{l} p_{2,4} = p_{2,2} \\ x_{2,4} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{2,4} \\ s_{2,4} \end{array}\right. \tag{19}$$

For state $[2,5]$,

$$\left.\begin{array}{l} p_{2,5} = p_{2,stirling} \\ s_{2,5} = s_{2,4} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,5} \\ h_{2,5} \end{array}\right. \tag{20}$$

For state $[2,6]$,

$$q_{stirling,cold} = (1-y)\, q_{m,2} \left(h_{2,6} - h_{2,5}\right) \Rightarrow h_{2,6} \tag{21}$$

$$\left.\begin{array}{l} p_{2,6} = p_{2,5} \\ h_{2,6} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,6} \\ s_{2,6} \end{array}\right. \tag{22}$$

For state $[2,7]$,

$$\left.\begin{array}{l} p_{2,7} = p_{deaerator} \\ x_{2,7} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{2,7} \\ s_{2,7} \end{array}\right. \tag{23}$$

$$y h_{2,3} + (1-y)\, h_{2,6} = h_{2,7} \tag{24}$$

For state $[2,8]$,

$$\left.\begin{array}{l} p_{2,8} = p_{s} \\ s_{2,8} = s_{2,7} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,8} \\ h_{2,8} \end{array}\right. \tag{25}$$

For Rankine cycle,

$$\eta_{rankine} = \frac{(1-y)\left(h_{2,1} + h_{2,4} - h_{2,2} - h_{2,5}\right) + y\left(h_{2,1} - h_{2,3}\right) - \left(h_{2,8} - h_{2,7}\right)}{(1-y)\left(h_{2,6} - h_{2,5}\right) + h_{2,1} - h_{2,8}} \tag{26}$$

For state $[2, 9]$,

$$\left.\begin{array}{l} p_{2,9} = p_{2,8} \\ x_{2,9} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,9} \\ h_{2,9} \end{array}\right. \tag{27}$$

For state $[2, 10]$,

$$\left.\begin{array}{l} p_{2,10} = p_{2,9} \\ x_{2,10} = 1 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,10} \\ h_{2,10} \end{array}\right. \tag{28}$$

For state $[2, 11]$,

$$\left.\begin{array}{l} p_{2,11} = p_{2,10} \\ q_{m,1}\left(h_{1,2} - h_{1,3}\right) = q_{m,2}\left(h_{2,1} - h_{2,11}\right) \end{array}\right\} \Rightarrow T_{2,11} \tag{29}$$

$$q_{trough} = q_{m,2}\left(h_{2,11} - h_{2,8}\right) \tag{30}$$

$$q_{trough} = I_p \cdot A_{trough} \cdot \eta_{trough} \tag{31}$$

$$n_{troughCollector} = ceil(A_{trough}/A_{troughCollector}) \tag{32}$$

where $ceil(x)$ is a function that returns the smallest integer larger than x.

4.3 Oil circuit

Use the function $TroughCollector(T_{trough,inlet}, T_{trough,outlet}, q_{m,3} : \eta_{trough})$ to calculate trough collector thermal efficiency. The explanation of function $TroughCollector$ can be found in Appendix A.

For state $[3, 1]$,

$$\left.\begin{array}{l} p_{3,1} = p_{trough} \\ T_{3,1} = T_{trough,outlet} \end{array}\right\} \Rightarrow h_{3,1} \tag{33}$$

For state $[3, 2]$,

$$\left.\begin{array}{l} p_{3,2} = p_{3,1} \\ q_{m,3}\left(h_{3,1} - h_{3,2}\right) = q_{m,2}\left(h_{2,11} - h_{2,10}\right) \\ h_{3,2} = h\left(F\$_3, T_{3,2}, p_{3,2}\right) \end{array}\right\} \Rightarrow T_{3,2} \tag{34}$$

For state $[3, 3]$,

$$\left.\begin{array}{l} p_{3,3} = p_{3,2} \\ q_{m,3}\left(h_{3,2} - h_{3,3}\right) = q_{m,2}\left(h_{2,10} - h_{2,9}\right) \\ h_{3,3} = h\left(F\$_3, T_{3,3}, p_{3,3}\right) \end{array}\right\} \Rightarrow T_{3,3} \tag{35}$$

For state $[3, 4]$,

$$\left.\begin{array}{l} p_{3,4} = p_{3,3} \\ q_{m,3}\left(h_{3,3} - h_{3,4}\right) = q_{m,2}\left(h_{2,9} - h_{2,8}\right) \\ h_{3,4} = h\left(F\$_3, T_{3,4}, p_{3,4}\right) \end{array}\right\} \Rightarrow T_{3,4} \tag{36}$$

$$\Delta T_{oil,water,min} = min(T_{3,4} - T_{2,8}, T_{3,3} - T_{2,9}, T_{3,1} - T_{2,11}) \tag{37}$$

4.4 System efficiency

$$E_{total} = I_{DNI} \cdot (A_{dish} + A_{trough}) \tag{38}$$

$$\eta_{system} = (P_{stirling} + P_{generator}) / E_{total} \tag{39}$$

5 Separate system

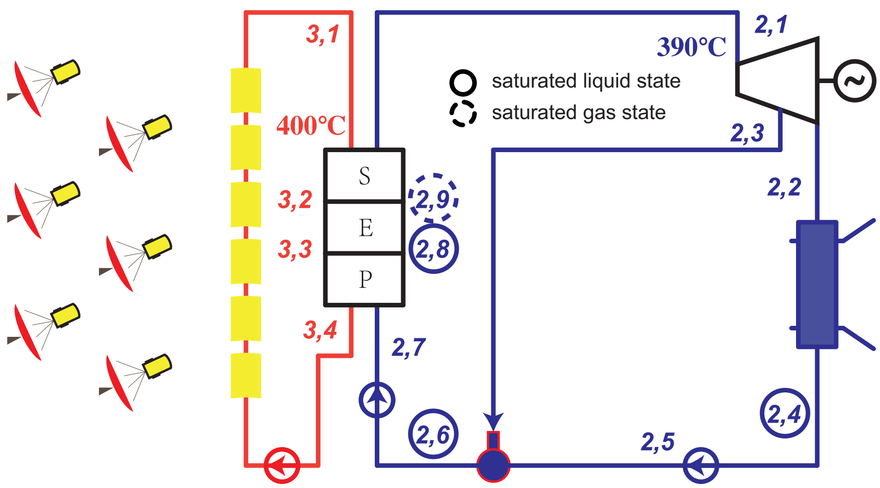

Figure 2: Sketch of separate system

Figure 2 shows the sketch of separate system, which uses the same dish collectors and trough collectors. Stirling engines with the same number of dish collectors are directly put on the focuses of the dish collectors, and water is used for cooling the Stirling engines. Steam turbine has the same main parameters and isentropic efficiency with that in cascade system. Generator has the same efficiency with that in cascade system.

5.1 Separate trough system

$p_{deaerator,s} = p_{deaerator}$, $p_{cp,s} = p_{cp}$.
For state $s[2,1]$,

$$\left.\begin{array}{l} p_{s,2,1} = p_s \\ T_{s,2,1} = T_s \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{s,2,1} \\ s_{s,2,1} \end{array}\right. \tag{40}$$

For state $s[2,2]$,

$$\left.\begin{array}{l} s_{i,s,2,2} = s_{s,2,1} \\ p_{i,s,2,2} = p_c \end{array}\right\} \Rightarrow h_{i,s,2,2} \tag{41}$$

$$\eta_{i,turbine} = \frac{h_{s,2,1} - h_{s,2,2}}{h_{s,2,1} - h_{s,i,2,2}} \tag{42}$$

$$\left.\begin{array}{l} p_{s,2,2} = p_c \\ h_{s,2,2} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{s,2,2} \\ x_{s,2,2} \end{array}\right. \tag{43}$$

and state $s[2,3]$,

$$\left.\begin{array}{l} s_{i,s,2,3} = s_{s,2,1} \\ p_{i,s,2,3} = p_{deaerator,s} \end{array}\right\} \Rightarrow h_{i,s,2,3} \tag{44}$$

$$\eta_{i,turbine} = \frac{h_{s,2,1} - h_{s,2,3}}{h_{s,2,1} - h_{s,i,2,3}} \tag{45}$$

$$\left.\begin{array}{l} p_{s,2,3} = p_{b,s} \\ h_{s,2,3} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{s,2,3} \\ x_{s,2,3} \end{array}\right. \tag{46}$$

$$P_{turbine,s} = (1 - y_s)\, q_{m,2,s} \left(h_{s,2,1} - h_{s,2,2}\right) + y_s q_{m,2,s} \left(h_{s,2,1} - h_{s,2,3}\right) \tag{47}$$

$$P_{generator,s} = P_{turbine,s} \eta_{generator} \tag{48}$$

For state $s[2,4]$,

$$\left.\begin{array}{l} p_{s,2,4} = p_{s,2,2} \\ x_{s,2,4} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{s,2,4} \\ s_{s,2,4} \end{array}\right. \tag{49}$$

For state $s[2,5]$,

$$\left.\begin{array}{l} p_{s,2,5} = p_{cp,s} \\ s_{s,2,5} = s_{s,2,4} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{s,2,5} \\ h_{s,2,5} \end{array}\right. \tag{50}$$

For state $s[2,6]$,

$$\left.\begin{array}{l} p_{s,2,6} = p_{deaerator,s} \\ x_{s,2,6} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{s,2,6} \\ s_{s,2,6} \end{array}\right. \tag{51}$$

$$y_s h_{s,2,3} + (1 - y_s)\, h_{s,2,5} = h_{s,2,6} \tag{52}$$

For state $s[2,7]$,

$$\left.\begin{array}{l} p_{s,2,7} = p_s \\ s_{s,2,7} = s_{s,2,6} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{s,2,7} \\ h_{s,2,7} \end{array}\right. \tag{53}$$

For Rankine cycle,

$$\eta_{rankine,s} = \frac{(1-y_s)\left(h_{s,2,1}+h_{s,2,4}-h_{s,2,2}-h_{s,2,5}\right)+y_s\left(h_{s,2,1}-h_{s,2,3}\right)-\left(h_{s,2,7}-h_{s,2,6}\right)}{h_{s,2,1}-h_{s,2,7}} \tag{54}$$

For state $s[2,8]$,

$$\left.\begin{array}{l} p_{s,2,8} = p_{s,2,7} \\ s_{s,2,8} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{s,2,8} \\ h_{s,2,8} \end{array}\right. \tag{55}$$

For state $s[2,9]$,

$$\left.\begin{array}{l} p_{s,2,9} = p_{s,2,8} \\ x_{s,2,9} = 1 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{s,2,9} \\ h_{s,2,9} \end{array}\right. \tag{56}$$

$$q_{trough} = q_{m,2,s}\left(h_{s,2,1}-h_{s,2,7}\right) \Rightarrow q_{m,2,s} \tag{57}$$

Combine Equation (47), Equation (48) and Equation (57), we can get $P_{generator,s}$.

5.2 Separate dish system

For Stirling turbines, k and γ are chosen as the same value.

$T_{H,s} = 1073$℃, $T_{L,s} = 37$℃[1]

$$T_{R,s} = \frac{T_{H,s}-T_{L,s}}{\ln\dfrac{T_{H,s}}{T_{L,s}}} \tag{58}$$

$$e_s = \frac{T_{R,s}-T_{L,s}}{T_{H,s}-T_{L,s}} \tag{59}$$

$$\eta_{stirling,s} = \frac{T_{H,s}-T_{L,s}}{T_{H,s}+\dfrac{1-e_s}{k-1}\cdot\dfrac{T_{H,s}-T_{L,s}}{\ln\dfrac{T_{H,s}}{T_{L,s}}}} \tag{60}$$

$\eta_{dish,s}$ uses the value 0.8.[3]

$$P_{stirling,s} = q_{dish,s}\eta_{stirling,s} \tag{61}$$

[1] $T_{H,s}$ is chosen to be equal to outlet temperature of air in dish receiver. $T_{L,s}$ is chosen to be 310 K, the default expansion temperature in Fraser's paper for the calculation of 4-95 NKII engine.

5.3 Separate system efficiency

$$\eta_{system,s} = (P_{stirling,s} + P_{generator,s})/E_{total} \tag{62}$$

6 Results

Figure 3 shows some important results of the system. From the results, we can see that the overall of the cascade system $\eta_{system} = 0.1950$ is higher than that of separated system $\eta_{system,s} = 0.1946$. The separated system uses the same collectors of the cascade system and recirculated cooling water with the same temperature of the environment is used to cool cold chamber of Stirling engine.

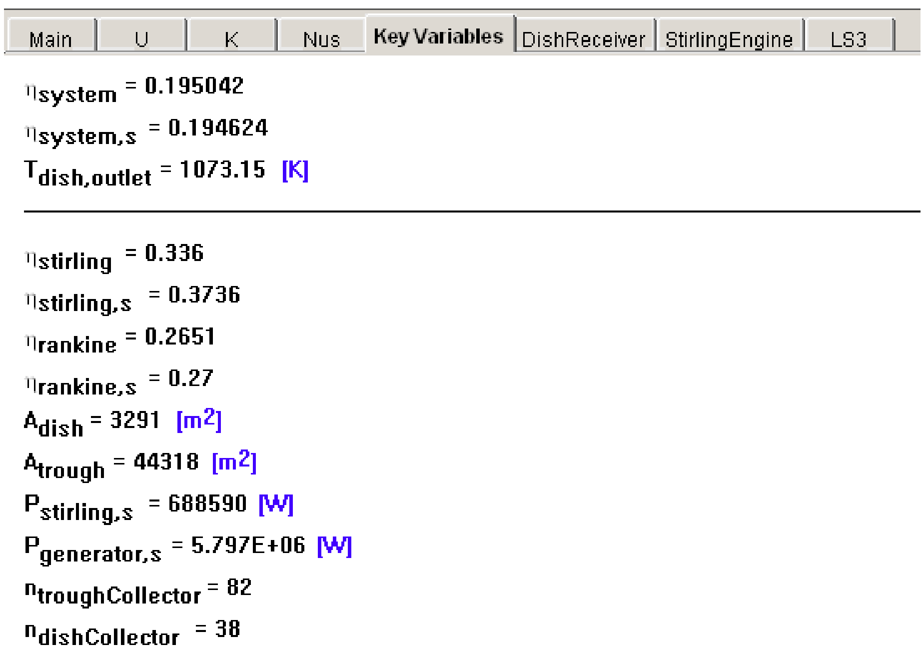

Figure 3: Some important results

Figure 4 shows results of trough collector part, Figure 5 shows results of dish receiver part, Figure 6 shows results of Stirling engine part. The detail explanation of these parts can be found in Appendix.

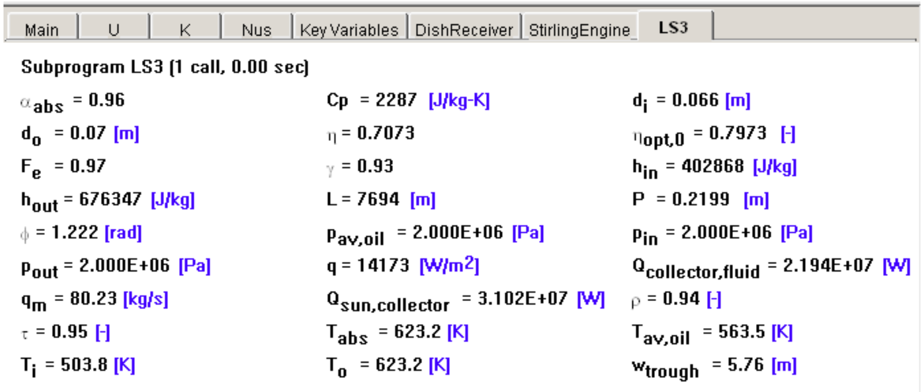

Figure 4: Results of the trough collector part

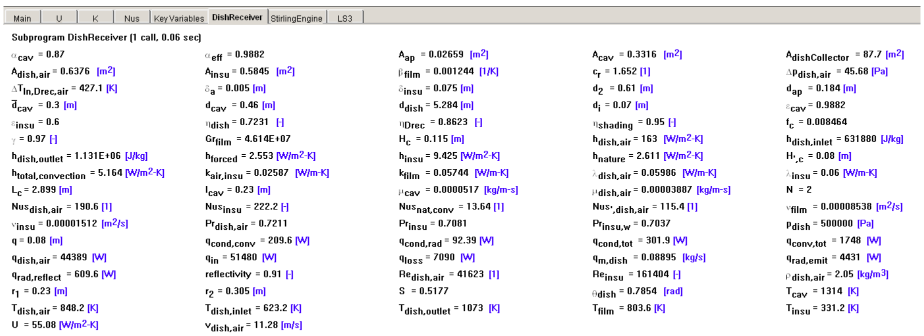

Figure 5: Results of the dish receiver part

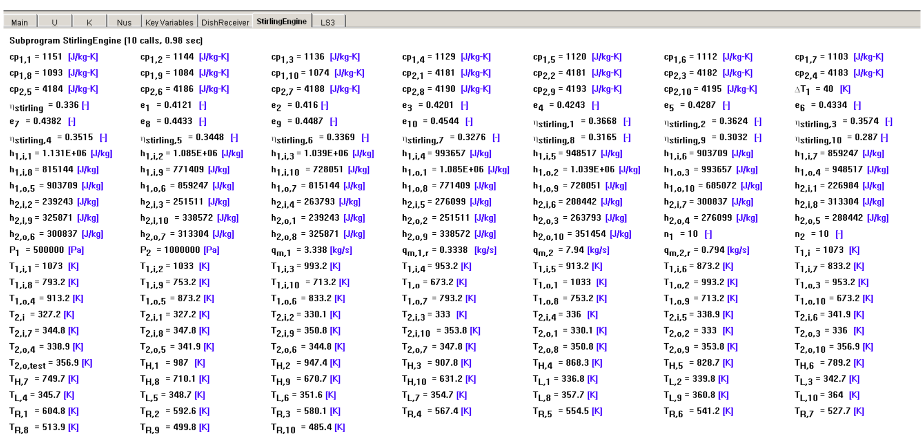

Figure 6: Results of the Stirling engine part

A Trough collector

LS-3 is used as the trough collector, the parameters are listed in Table 2.[4] Figure 7 shows the sketch of trough collector.

Table 2: Parameters of trough collector

$\rho = 0.94$	$\gamma = 0.93$	$\tau = 0.95$
$\alpha_{abs} = 0.96$	$w_{trough} = 5.76\,\mathrm{m}$	$F_e = 0.97$
$d_i = 0.066\,\mathrm{m}$	$d_o = 0.07\,\mathrm{m}$	$\phi = 70°$

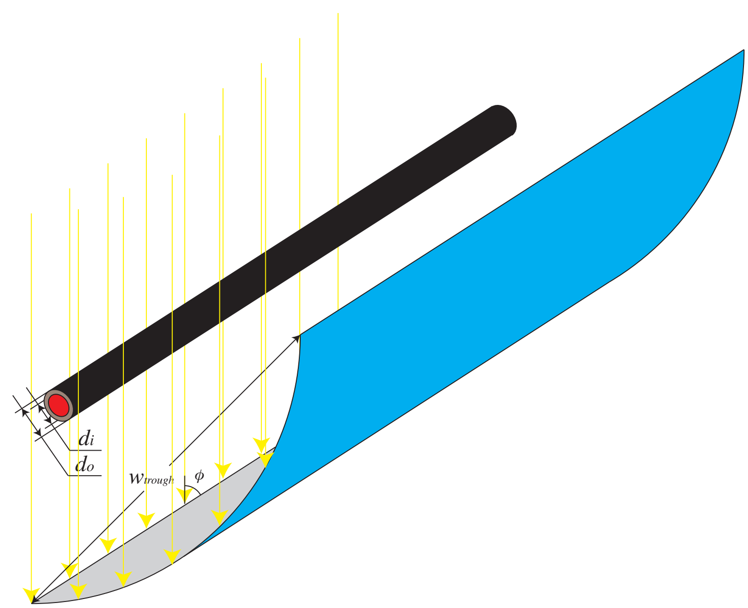

Figure 7: Sketch of trough collector

$$\left.\begin{array}{r} p_{trough}, T_i \Rightarrow h_i \\ p_{trough}, T_o \Rightarrow h_o \end{array}\right\} \Rightarrow q_{collector,fluid} = q_m(h_o - h_i) \tag{63}$$

$$P = \pi d_o \tag{64}$$

$$q^{''} = I_{DNI} \cdot w_{trough} \eta_{opt,0} K(\phi) F_e / P \tag{65}$$

where $K(\phi)$ is the incidence angle modifier, $\eta_{opt,0}$ is peak optical efficiency (optical efficiency with an incidence angle of 0)

$$\eta_{opt,0} = \rho\gamma\tau\alpha_{abs} \tag{66}$$

Assume overall heat transfer coefficient $U(T_{abs})$ is uniform for whole length, so that we can use the heat transfer correlation in Appendix F.

$$\frac{T_o - T_{amb} - \dfrac{q^{''}}{U(T_{abs})}}{T_i - T_{amb} - \dfrac{q^{''}}{U(T_{abs})}} = \exp(-\frac{U(T_{abs})PL}{q_m c_p}) \tag{67}$$

$q_{collector,fluid}$ can be obtained by calculating the heat transfer of pipe and fluid with the correlation [5]

$$Nu = 0.027 Re^{0.5} Pr^{1/3} (\mu/\mu_w)^{0.14} \tag{68}$$

which is written as a function $NulnPipe(Re, Pr, \mu, \mu_w)$. It returns a large value of Nu, so small difference exists between $T_{abs}(x)$ and $T(x)$. So $(T_i + T_o)/2$ can be used as the average value of T_{abs}, and $U(T_{abs})$ can be obtained. By using Equation (67), we can get the length L required. So

$$q_{sun,collector} = I_{DNI} \cdot w_{trough} L \tag{69}$$

$$\eta = \frac{q_{sun,collector}}{q_{collector,fluid}} \tag{70}$$

B Stirling engine array

The layout of Stirling engines is shown in Figure 8. n_1 is chosen to be 10 and can be optimized later.

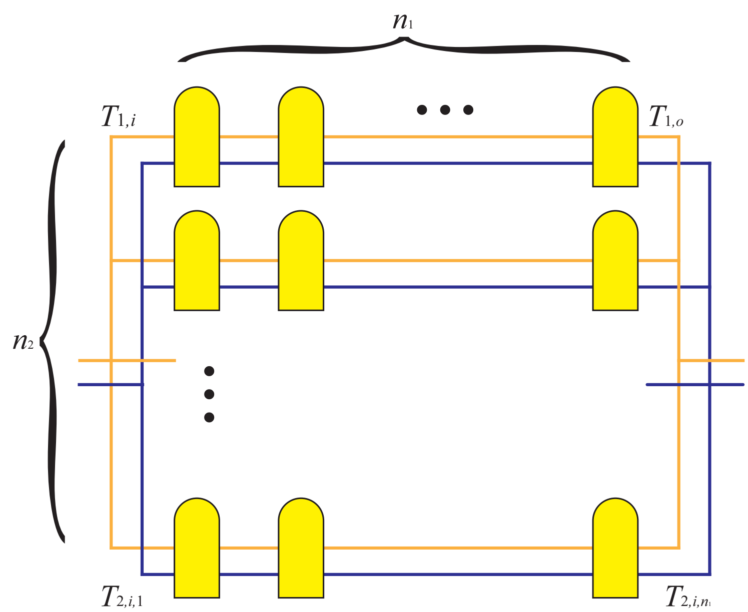

Figure 8: Layout of Stirling engines

$$T_{1,i,1} = T_{1,i} \tag{71}$$

$$q_{m,1,r} = q_{m,1}/n_2 \tag{72}$$

where $n_2 = n_{stirlingEngine}/n_1$.

If the flow type between the heating and cooling flows is parallel flow,

$$T_{2,i,1} = T_{2,i} \tag{73}$$

$$q_{m,2,r} = q_{m,2}/n_2 \tag{74}$$

if the flow type between the heating and cooling flows is counter flow,

$$T_{2,o,n_1} = T_{2,i} \tag{75}$$

$$q_{m,2,r} = -q_{m,2}/n_2 \tag{76}$$

For a Stirling engine in column x, x from 1 to n_1, $c_{p,1,x}$ and $c_{p,2,x}$ can be written as

$$c_{p,1,x} = g_1(T_{1,i,x}, T_{1,o,x}) \tag{77}$$

$$c_{p,2,x} = g_2(T_{2,i,x}, T_{2,o,x}) \tag{78}$$

According to Appendix D,

$$T_{H,x} = T_{1,i,x} - \frac{T_{1,i,x} - T_{1,o,x}}{1 - \exp(-\dfrac{U_{stirling,1}A_{stirling,1}}{q_{m,1,r}c_{p,1,x}})} \tag{79}$$

$$T_{L,x} = T_{2,i,x} - \frac{T_{2,i,x} - T_{2,o,x}}{1 - \exp(-\dfrac{U_{stirling,2}A_{stirling,2}}{q_{m,2,r}c_{p,2,x}})} \tag{80}$$

Because of Equation (77) and (78), Equation (79) and (80) can be written as

$$T_{H,x} = g_3(T_{1,i,x}, T_{1,o,x}) \tag{81}$$

$$T_{L,x} = g_4(T_{2,i,x}, T_{2,o,x}) \tag{82}$$

Choose $T_{R,x} = \dfrac{T_{H,x} - T_{L,x}}{\ln(T_{H,x}/T_{L,x})}$, and $e_x = \dfrac{T_{R,x} - T_{L,x}}{T_{H,x} - T_{L,x}}$, then

$$\eta_{stirling,x} = \frac{T_{H,x} - T_{L,x}}{T_{H,x} + \dfrac{1-e_x}{k-1} \cdot \dfrac{T_{H,x} - T_{L,x}}{\ln \gamma_{stirling}}} \tag{83}$$

which can be written as

$$\eta_{stirling,x} = g_5(T_{1,i,x}, T_{1,o,x}, T_{2,i,x}, T_{2,o,x}) \tag{84}$$

On the other way, for x from 1 to n_1,

$$h_{1,i,x} = h_1(T_{1,i,x}) \tag{85}$$

$$h_{1,o,x} = h_1(T_{1,o,x}) \tag{86}$$

$$h_{2,i,x} = h_2(T_{2,i,x}) \tag{87}$$

$$h_{2,o,x} = h_2(T_{2,o,x}) \tag{88}$$

For energy balance,

$$q_{m,1,r}(h_{1,i,x} - h_{1,o,x})(1 - \eta_{stirling,x}) = q_{m,2,r}(h_{2,o,x} - h_{2,i,x}) \tag{89}$$

which can be written as

$$h_3(\eta_{stirling,x}, T_{1,i,x}, T_{1,o,x}, T_{2,i,x}, T_{2,o,x}) = 0 \tag{90}$$

for $q_{m,1,r}$ and $q_{m,2,r}$ can be obtained once the flow type is known.

And in Stirling cycle, the heat absorbed by the working gas is

$$nR\left(T_{H,x}\ln\gamma_{stirling}+\frac{1-e_x}{k-1}\left(T_{H,x}-T_{L,x}\right)\right)=q_{m,1,r}(h_{1,i,x}-h_{1,o,x})/n_{se} \quad (91)$$

can be written as

$$f(T_{1,i,x},T_{1,o,x})=0 \quad (92)$$

And for x from 1 to n_1-1,

$$T_{1,i,x+1}=T_{1,o,x} \quad (93)$$

$$T_{2,i,x+1}=T_{2,o,x} \quad (94)$$

Solve the simultaneous equations for x from 1 to n_1: Equation (71), (73) (or (75)), (84), (90), (92), (93), (94) ($5n_1$ equations with $5n_1$ variables), we can get $T_{2,i,1}, T_{2,o,n_1}$ and then use $h_{2,i,1}, h_{2,o,n_1}$ to get $\eta_{stirling}$ by Equation (95)

$$\eta_{stirling}=1-\frac{q_{m,2}(h_{2,o,n_1}-h_{2,i,1})}{q_{m,1}(h_{1,i,1}-h_{1,o,n_1})} \quad (95)$$

and power of Stirling engine in column x

$$P_{stirlingEngine,x}=q_{m,1,r}(h_{1,i,x}-h_{1,o,x})\eta_{stirling,x} \quad (96)$$

and $P_{stirling}$ can be obtained by

$$P_{stirling}=\eta_{stirling}q_{m,1}(h_{1,i,1}-h_{1,o,n_1}) \quad (97)$$

C Dish receiver

A dish collector product of SES used in Fraser's paper, which is also used in this system, and its parameters are listed in Table 3.[6] The structure of the dish receiver is as shown in Figure 9.

Table 3: Parameters of the dish receiver

$d_{cav} = 0.46\,\mathrm{m}$	$\delta_{insu} = 0.075\,\mathrm{m}$	$l_{cav} = 0.23\,\mathrm{m}$	$d_{ap} = 0.184\,\mathrm{m}$
$\lambda_{insu} = 0.06\,\mathrm{W/(m{\cdot}K)}$	$\epsilon_{insu} = 0.6$	$\alpha_{cav} = 0.87$	$\delta_a = 0.005\,\mathrm{m}$
$d_{i,air} = 0.07\,\mathrm{m}$	$\theta_{dish} = 45°$	$\gamma = 0.97$	$\eta_{shading} = 0.95$
$\rho = 0.91$			

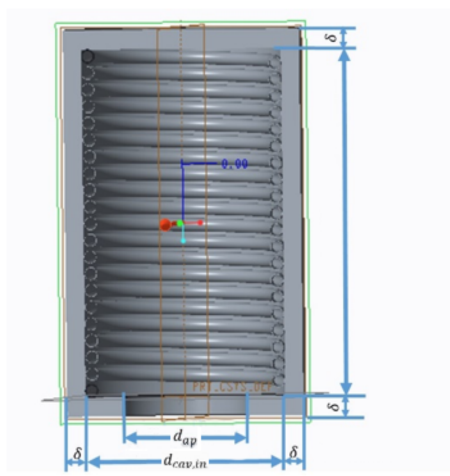

Figure 9: The structure of the dish receiver

Consider the situation that all the dish collectors are parallel, for energy balance of each dish collectors, the energy flow of the dish receiver is shown in Figure 10, q_{in} equals to the sum of energy absorbed by working hot air $q_{dish,air}$, reflected radiation $q_{rad,reflect}$, heat losses $q_{cond,tot} + q_{conv,tot} + q_{rad,emit}$.

$$q_{in} = q_{rad,reflect} + q_{dish,air} + (q_{cond,tot} + q_{conv,tot} + q_{rad,emit}) \tag{98}$$

$$q_{in} = I_{DNI} \cdot A_{dishCollector} \gamma \eta_{shading} \rho \tag{99}$$

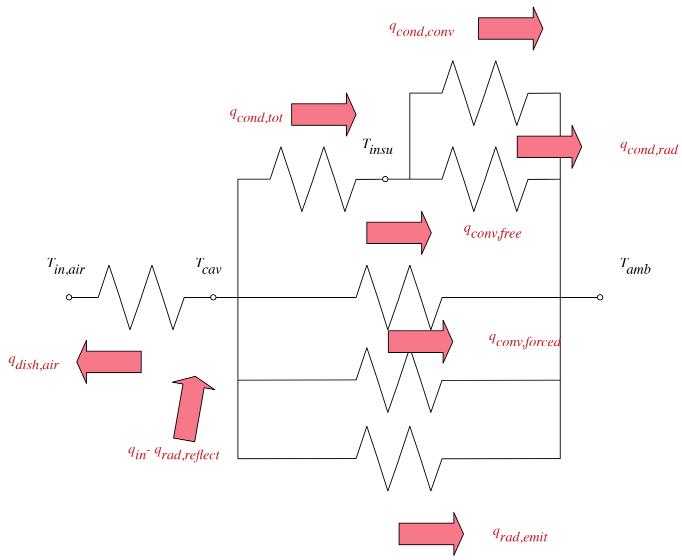

Figure 10: Heat network of the dish receiver

$$q_{rad,reflect} = (1 - \alpha_{eff})q_{in} \tag{100}$$

$$\alpha_{eff} = \frac{\alpha_{cav}}{\alpha_{cav} + (1 - \alpha_{cav})\dfrac{A_{ap}}{A_{cav}}} \tag{101}$$

where

$$A_{ap} = \pi d_{ap}^2/4 \tag{102}$$

$$A_{cav} = \frac{\pi \bar{d}_{cav}^2}{4} + \pi \bar{d}_{cav} l_{cav} + \frac{\pi(\bar{d}_{cav}^2 - d_{ap}^2)}{4} \Leftarrow \bar{d}_{cav} = d_{cav} - 2d_{i,air} - 4\delta_a \tag{103}$$

$$q_{dish,air} = q_{m,dish}(h_{dish,outlet} - h_{dish,inlet}) \tag{104}$$

which can be written as

$$q_{dish,air} = f_1(q_{m,dish}) \tag{105}$$

$$q_{dish,air} = h_{dish,air} A_{dish,air} \Delta T_{ln,Drec,air} \tag{106}$$

where

$$A_{dish,air} = \pi d_i L_c \Leftarrow L_c = N\sqrt{(\pi d_{cav})^2 + H_c^2} \Leftarrow \begin{cases} H_c^{'} = d_i + 2\delta_a \\ N = floor(l_{cav}/H_c^{'}) \\ H_c = l_{cav}/N \end{cases} \tag{107}$$

$$\Delta T_{ln,Drec,air} = \frac{(T_{cav} - T_{dish,inlet}) - (T_{cav} - T_{dish,outlet})}{\ln \dfrac{T_{cav} - T_{dish,inlet}}{T_{cav} - T_{dish,outlet}}} \tag{108}$$

To get $q_{dish,air}$, $h_{dish,air}$ is required

$$h_{dish,air} = Nu_{dish,air}\lambda_{dish,air}/d_i \tag{109}$$

$$Nu_{dish,air} = c_r Nu^{'}_{dish,air} \tag{110}$$

where

$$c_r = 1 + 1.77\frac{2d_i}{d_{cav} - d_i - 2\delta_a} \tag{111}$$

$$Nu^{'}_{dish,air} = NuInPipe(Re_{dish,air}, Pr_{dish,air}, \mu_{dish,air}, \mu_{cav}) \tag{112}$$

$\lambda_{dish,air}$, $Re_{dish,air}$, $Pr_{dish,air}$, $\mu_{dish,air}$ are evaluated at $T_{dish,air} = (T_{dish,inlet} + T_{dish,outlet})/2$, μ_{cav} is evaluated at T_{cav}.

Since $Re_{dish,air}$ can be written as

$$Re_{dish,air} = \frac{\rho_{dish,air} v_{dish,air} d_i}{\mu_{dish,air}} = \frac{4q_{m,dish}}{\pi\mu_{dish,air} d_i} \tag{113}$$

Equation (106) can be written as

$$q_{dish,air} = f_2(q_{m,air}, T_{cav}) \tag{114}$$

$q_{cond,tot}$ is the heat loss through the insulating layer, T_{insu} is outside temperature of the insulating layer.

$$q_{cond,tot} = \frac{T_{cav} - T_{insu}}{\dfrac{\ln(r_2 - r_1)}{2\pi\lambda_{insu} l_{cav}}} \tag{115}$$

can be written as

$$q_{cond,tot} = g(T_{cav}, T_{insu}) \tag{116}$$

where $r_1 = d_{cav}/2$, $r_2 = r_1 + \delta_{insu}$.

$$q_{cond,tot} = q_{cond,conv} + q_{cond,rad} \tag{117}$$

$$q_{cond,conv} = h_{insu} A_{insu}(T_{insu} - T_{amb}) \tag{118}$$

$$h_{insu} = k_{air,insu} Nu_{insu}/\left(2r_2\right) \tag{119}$$

In Equation (119), Nu_{insu} can be calculated by the formula of convection heat transfer over a cylinder, which is calculated by function $Nus(Re_{insu}, Pr_{insu}, Pr_{insu,w}, \theta_{dish})$.

$k_{air,insu}$, Re_{insu} and Pr_{insu} are evaluated at T_{amb}. $Pr_{insu,w}$ is evaluated at T_{insu}. So Equation (118) can be written as

$$q_{cond,vonv} = g_1(T_{insu}) \tag{120}$$

$$q_{cond,rad} = \epsilon_{insu} A_{insu} \sigma (T_{insu}^4 - T_{amb}^4) \tag{121}$$

can be written as

$$q_{cond,rad} = g_2(T_{insu}) \tag{122}$$

So Equation (117) can be written as

$$q_{cond,tot} = g_1(T_{insu}) + g_2(T_{insu}) \tag{123}$$

According to Fraser's paper[6], the convection loss $q_{conv,tot}$ can be written as the sum of free convection loss and forced convection loss, and the relationships are

$$q_{conv,tot} = (h_{nature} + h_{forced}) A_{cav} (T_{cav} - T_{amb}) \tag{124}$$

$$h_{nature} = k_{film} Nu_{nat,conv} / \overline{d_{cav}} \tag{125}$$

where k_{film} is the thermal conductivity of air evaluated by $T_{film} = (T_{cav} + T_{amb})/2$.

$$Nu_{nat,conv} = 0.088 Gr_{film}^{1/3} (T_{cav}/T_{amb})^{0.18} (\cos\theta_{dish})^{2.47} (\frac{d_{ap}}{\overline{\overline{d_{cav}}}})^S \tag{126}$$

$$S = -0.982(\frac{d_{ap}}{\overline{\overline{d_{cav}}}}) + 1.12 \tag{127}$$

Assume the wind is side-on, then

$$h_{forced} = 0.1967[W/m^2 \cdot K] \cdot (v_{wind}/1[m/s])^{1.849} \tag{128}$$

So Equation (124) can be written as

$$q_{conv,tot} = f_3(T_{cav}) \tag{129}$$

The emissivity is set equal to the effective absorptivity of the cavity (gray body),

$$\epsilon_{cav} = \alpha_{eff} \tag{130}$$

$$q_{rad,emit} = \epsilon_{cav} A_{ap} \sigma (T_{cav}^4 - T_{amb}^4) \tag{131}$$

can be written as

$$q_{rad,emit} = f_4(T_{cav}) \tag{132}$$

Solve the simultaneous equations: Equation (98), (99), (100), (105), (114), (116), (123), (129), (132), (9 equations with 9 variables) we can get $q_{dish,air}$.

so

$$\eta_{dish} = \frac{q_{dish,air}}{I_{DNI} \cdot A_{dishCollector}} \tag{133}$$

D Stirling engine model

A simple Stirling engine model is used for the system. (See Appendix G for the model efficiency calculation) The cycle efficiency is given by [7]

$$\eta = \frac{T_H - T_L}{T_H + \frac{1-e}{k-1} \cdot \frac{T_H - T_L}{\ln \gamma}} \tag{134}$$

where $e = \frac{T_R - T_L}{T_H - T_L}$,$T_R$ is the regenerator temperature, $k = \frac{c_p}{c_v}$ for the working gas, $\gamma = \frac{V_{max}}{V_{min}}$ is the compression ratio.

The heat transfer diagram is shown in Figure 11. Flow 1 is used for heating the hot chamber of Stirling engine, T_{1i} is the inlet temperature, T_{1o} is the outlet temperature. Flow 2 is used for cooling the cold chamber of Stirling engine, T_{2i} is the inlet temperature, T_{2o} is the outlet temperature. T_H is the highest temperature of expansion space, T_L is the lowest temperature of compression space.

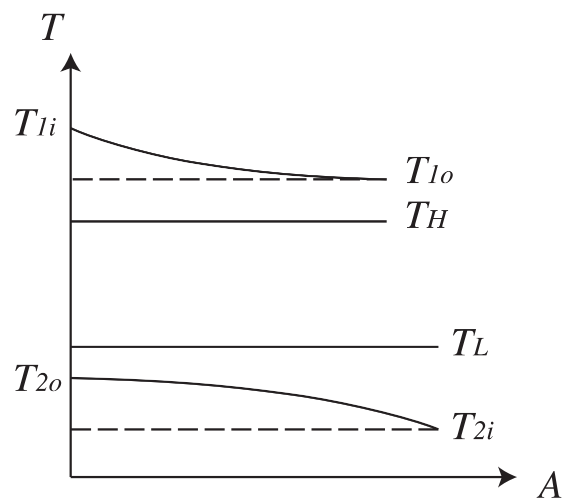

Figure 11: Heat transfer diagram of Stirling engine

According to Appendix E, we have

$$\frac{T_{1o} - T_H}{T_{1i} - T_H} = \exp(-\frac{U_1 A_1}{q_{m,1} c_{p1}}) \tag{135}$$

and

$$\frac{T_{2o} - T_L}{T_{2i} - T_L} = \exp(-\frac{U_2 A_2}{q_{m,2} c_{p2}}) \tag{136}$$

So known T_{1o}, T_{1i}, U_1, A_1, $q_{m,1}$, c_{p1}, T_{2o}, T_{2i}, U_2, A_2, $q_{m,2}$, c_{p2}, we can get T_H and T_L, and then use Equation (134) to get η.

E Ambient loss of a flow

Constant U, T_{amb}, q_m, c_p, for given T_i ,

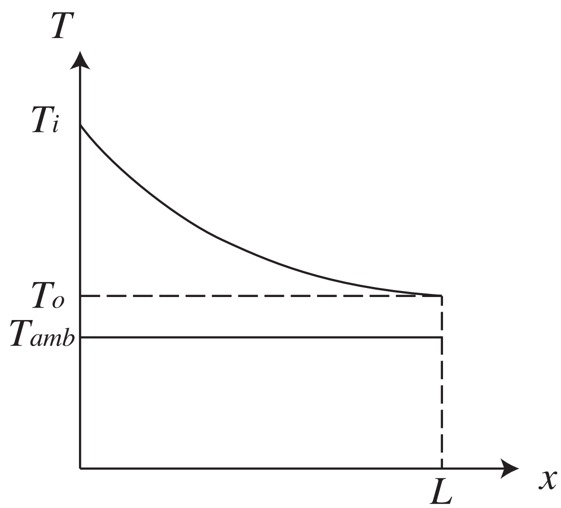

Figure 12: Diagram of ambient loss

if $A(x) = Px$, x from 0 to L, $T(x)$ from T_i to T_o,

$$q_m c_p dT(x) = (T_{amb} - T(x))UPdx \tag{137}$$

so,

$$\frac{dT(x)}{dx} = -\frac{UP}{q_m c_p}(T(x) - T_{amb}) \tag{138}$$

Set $F(x) = T(x) - T_{amb}$, then $F(0) = T_i - T_{amb}$, and

$$\frac{dF(x)}{F(x)} = -\frac{UP}{q_m c_p}dx \tag{139}$$

$$F(x) = F(0) \cdot \exp(-\frac{UP}{q_m c_p}x) \tag{140}$$

$$\frac{F(L)}{F(0)} = \exp(-\frac{UPL}{q_m c_p}) \tag{141}$$

That is

$$\frac{T_o - T_{amb}}{T_i - T_{amb}} = \exp(-\frac{UA}{q_m c_p}) \tag{142}$$

F Ambient loss of a flow under constant heat flux

Constant U, T_{amb}, q_m, c_p, $q^{''}$, for given T_i,

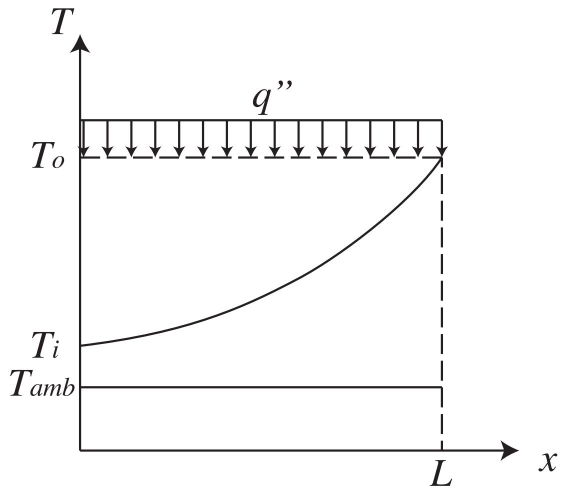

Figure 13: Diagram of ambient loss of a flow under constant heat flux

if $A(x) = Px$, x from 0 to L, $T(x)$ from T_i to T_o,

$$q_m c_p dT(x) = (T_{amb} - T(x))UPdx + q^{''}Pdx \tag{143}$$

so

$$\frac{dT(x)}{dx} = -\frac{UP}{q_m c_p}T(x) + \frac{q^{''}P + UPT_{amb}}{q_m c_p} \tag{144}$$

Set $G(x) = T(x) - T_{amb} - \frac{q^{''}}{U}$, then $G(0) = T_i - T_{amb} - \frac{q^{''}}{U}$ and

$$\frac{dG(x)}{G(x)} = -\frac{UP}{q_m c_p}dx \tag{145}$$

$$G(x) = G(0) \cdot \exp(-\frac{UP}{q_m c_p}x) \tag{146}$$

$$\frac{G(L)}{G(0)} = \exp(-\frac{UPL}{q_m c_p}) \tag{147}$$

That is

$$\frac{T_o - T_{amb} - \frac{q^{''}}{U}}{T_i - T_{amb} - \frac{q^{''}}{U}} = \exp(-\frac{UA}{q_m c_p}) \tag{148}$$

G Simple Stirling engine model efficiency calculation

Figure 14 shows thermodynamic diagrams of a Stirling cycle.

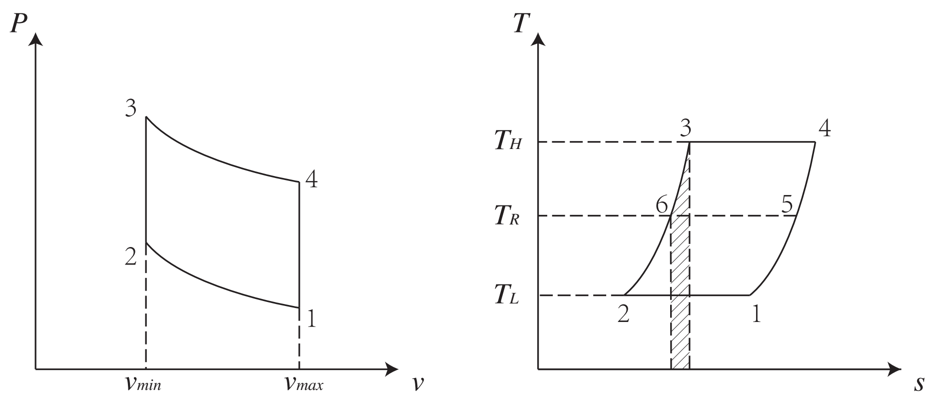

Figure 14: Thermodynamic diagrams of an Stirling Engine

The net work of a cycle

$$w_{net} = w_{12} + w_{34} \tag{149}$$

$$w_{12} = \int_{P_1}^{P_2} P dv = \int_{v_1}^{v_2} \frac{RT_L}{v} dv = RT_L \ln \frac{v_2}{v_1} = -RT_L \ln \gamma \tag{150}$$

where

$$\gamma = \frac{v_{max}}{v_{min}} = \frac{v_1}{v_2} = \frac{v_4}{v_3} \tag{151}$$

$$w_{34} = \int_{P_3}^{P_4} P dv = \int_{v_3}^{v_4} \frac{RT_H}{v} dv = RT_H \ln \frac{v_4}{v_3} = RT_H \ln \gamma \tag{152}$$

so

$$w_{net} = R(T_H - T_L) \ln \gamma \tag{153}$$

For an imperfect regeneration, the heat absorbed in a cycle

$$q_{abs} = q_{63} + q_{34} \tag{154}$$

$$q_{63} = c_v(T_H - T_R) \tag{155}$$

$$q_{34} = w_{34} = RT_H \ln \gamma \tag{156}$$

so

$$q_{abs} = RT_H \ln \gamma + c_v(T_H - T_R) \tag{157}$$

The cycle efficiency

$$\eta = \frac{w_{net}}{q_{abs}} = \frac{T_H - T_L}{T_H + \dfrac{c_v}{R} \cdot \dfrac{T_H - T_R}{\ln \gamma}} \tag{158}$$

for regeneration effectiveness $e = \dfrac{T_R - T_C}{T_H - T_C}$,

$$T_H - T_R = (1 - e)(T_H - T_L) \tag{159}$$

and

$$\frac{c_v}{R} = \frac{c_v}{c_p - c_v} = \frac{1}{\frac{c_p}{c_v} - 1} = \frac{1}{k - 1} \tag{160}$$

so Equation (158) can be written as

$$\eta = \frac{T_H - T_L}{T_H + \dfrac{1 - e}{k - 1} \cdot \dfrac{T_H - T_L}{\ln \gamma}} \tag{161}$$

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