

# CASCADE SYSTEM CALCULATION

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## 1 Introduction

Different types of collectors and different forms of electricity generation are suitable for different working temperature zones with different costs. A prototype of cascade collection and cascade utilization of solar energy with high efficiency and low cost is presented. Parabolic trough collectors are used to collect low temperature energy with low cost and dish collectors are used to collect high temperature energy with high efficiency. Rankine cycle is used to work in low temperature zone and Stirling cycle is used to work in high temperature zone. Furthermore, effective topological structures are considered to take full advantages of thermodynamic characters of different components of the system. The cold chamber of Stirling engine is cooled by condensed fluid of Rankine cycle to use the heat produced by Stirling engine.

## 2 Problem description

To realize the system, calculation of the system must be applied first. The scale of the system and the dimensions of the parameters must be evaluated.

Before detailed model of the system to be created, lots of simplifying assumptions are made:

- Steady state at nominal load
- Pressure losses negligible everywhere in the pressure circuit
- Simplified calculation of processes and equipment

For system 1, water is used as the working fluid of Rankine cycle. Feed water is used to cool the cold chamber of Stirling engine. Figure 1 shows the sketch of system 1. Air is heated in dish collectors, then used to heat the hot chamber of Stirling engine and air-water heat exchanger successively. Water is heated in the cold chamber of Stirling engine, preheater, evaporator, superheater and air-water heat exchanger successively, and then used to work in the turbine, cooled in the condenser. Trough collectors are used to provide heat for the preheater, evaporator and superheater. Pumps are used to change the pressure of



Table 1: System characteristics of System 1

Nominal electric power		
$P_{stirling} = 50\text{kW}$	$P_{generator} = 200\text{kW}$	
Solar data		
$DNI = 0.657\text{kW/m}^2$		
Dish		
$T_{dish,inlet} = 350^\circ\text{C}$	$T_{dish,outlet} = 800^\circ\text{C}$	$p_{dish} = 0.5\text{MPa}$
$\eta_{dish} = 0.85$		
Trough		
$\eta_{trough} = 0.6$		
Stirling engine		
$T_{1,afterstirling} = 400^\circ\text{C}$	$T_{environment} = 30^\circ\text{C}$	$\Delta T_{stirling,hot} = 30^\circ\text{C}$
$\Delta T_{stirling,cold} = 25^\circ\text{C}$	$\eta_{co} = 0.5$	$p_{2,stirling} = 0.1\text{MPa}$
Main steam and condensation		
$T_s = 340^\circ\text{C}$	$p_s = 2.797\text{MPa}$	$T_c = 50^\circ\text{C}$
Isentropic efficiency of steam turbine		
$\eta_{turbine} = 0.75$		

circulation, are built in some specific states and in some components. Known parameters of the states, efficiency of the system and the overall efficiency of separated systems can be achieved.

#### 4.1 Air circulation

For State 11,

$$\left. \begin{array}{l} p_{11} = p_{dish} \\ T_{11} = T_{dish,outlet} \end{array} \right\} \Rightarrow h_{11} \quad (1)$$

For state 12,

$$\left. \begin{array}{l} p_{12} = p_{11} \\ T_{12} = T_{1,afterstirling} \end{array} \right\} \Rightarrow h_{12} \quad (2)$$

For Stirling engine,

$$\left. \begin{array}{l} T_{hot} = T_{12} - \Delta T_{stirling,hot} \\ T_{cold} = T_{25} + \Delta T_{stirling,cold} \\ \eta_{stirling} = \eta_{co} \cdot (T_{hot} - T_{cold})/T_{hot} \\ Q_{stirling} = P_{stirling}/\eta_{stirling} \\ Q_{stirling} = m_1(h_{11} - h_{12}) \\ Q_{stirling,cold} = Q_{stirling} - P_{stirling} \\ Q_{stirling,cold} = m_2(h_{25} - h_{24}) \end{array} \right\} \Rightarrow h_{25} \quad (3)$$

For state 13,

$$\left. \begin{array}{l} p_{13} = p_{12} \\ T_{13} = T_{dish,inlet} \end{array} \right\} \Rightarrow h_{13} \quad (4)$$

For dish,

$$\begin{aligned} Q_{dish} &= m_1(h_{11} - h_{13}) \\ Q_{dish} &= DNI \cdot A_{dish} \cdot \eta_{dish} \end{aligned} \quad (5)$$

## 4.2 Water circulation

For state 21,

$$\left. \begin{array}{l} p_{21} = p_s \\ T_{21} = T_s \end{array} \right\} \Rightarrow h_{21} \quad (6)$$

For state 22,

$$\left. \begin{array}{l} T_{22} = T_c \\ x_{22} = 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} h_{22} \\ p_{22} \end{array} \right. \quad (7)$$

For state 23,

$$\left. \begin{array}{l} T_{23} = T_{22} \\ x_{23} = 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} h_{23} \\ s_{23} \\ p_{23} \end{array} \right. \quad (8)$$

For Rankine cycle,

$$\begin{aligned} P_{generator} &= \eta_{generator} P_{turbine} \\ P_{turbine} &= \eta_{turbine} E_{rankine} \\ E_{rankine} &= \eta_{rankine} Q_{2,total} \\ Q_{2,total} &= m_2(h_{21} - h_{23}) \\ \eta_{rankine} &= (h_{21} - h_{22}) / (h_{21} - h_{23}) \\ \eta_{2,total} &= \eta_{rankine} \eta_{turbine} \eta_{generator} \end{aligned} \quad (9)$$

For state 24,

$$\left. \begin{array}{l} s_{24} = s_{23} \\ p_{24} = p_{2,stirling} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} T_{24} \\ h_{24} \end{array} \right. \quad (10)$$

For state 25,

$$\left. \begin{array}{l} p_{25} = p_{24} \\ h_{25} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} T_{25} \\ s_{25} \end{array} \right. \quad (11)$$

For state 26,

$$\left. \begin{array}{l} p_{26} = p_s \\ s_{26} = s_{25} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} T_{26} \\ h_{26} \end{array} \right. \quad (12)$$

For state 27,

$$\left. \begin{array}{l} p_{27} = p_{26} \\ x_{27} = 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} T_{27} \\ h_{27} \end{array} \right. \quad (13)$$

For state 28,

$$\left. \begin{array}{l} p_{28} = p_{27} \\ x_{28} = 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} T_{28} \\ h_{28} \end{array} \right. \quad (14)$$

For air-water heat exchanger,

$$m_1(h_{12} - h_{13}) = m_2(h_{21} - h_{29}) \Rightarrow h_{29} \quad (15)$$

For state 29,

$$\left. \begin{array}{l} p_{29} = p_{28} \\ h_{29} \end{array} \right\} \Rightarrow T_{29} \quad (16)$$

For trough,

$$\begin{aligned} Q_{trough} &= m_2(h_{29} - h_{26}) \\ Q_{trough} &= DNI \cdot A_{trough} \cdot \eta_{trough} \end{aligned} \quad (17)$$

### 4.3 System efficiency calculation

$$\begin{aligned} E_{total} &= DNI \cdot (A_{dish} + A_{trough}) \\ \eta_{system} &= (P_{stirling} + P_{generator})/E_{total} \\ T_{cold,separate} &= T_{environment} + \Delta T_{stirling,cold} \\ \eta_{stirling,separate} &= \eta_{co} \cdot (T_{hot} - T_{cold,separate})/T_{hot} \\ \eta_{separate} &= (Q_{dish}\eta_{stirling,separate} + Q_{trough}\eta_{2,total})/E_{total} \end{aligned} \quad (18)$$

## 5 Results

Figure 2 shows main results of the system. From the results, we can see that the overall of the cascade system  $\eta_{system} = 0.09864$  is higher than that of separated system  $\eta_{separate} = 0.0946$ . The separated system uses the same collectors of the cascade system and recirculated cooling water with the same temperature of the environment is used to cool cold chamber of Stirling engine.

**Unit Settings: SI K MPa kJ mass deg**

$A_{dish} = 525.4$ [m <sup>2</sup> ]	$A_{trough} = 3332$ [m <sup>2</sup> ]	$C2K = 273.2$ [K]
$\Delta T_{stirling,cold} = 25$ [K]	$\Delta T_{stirling,hot} = 30$ [K]	$DNI = 0.657$ [kW/m <sup>2</sup> ]
$\eta_{2,total} = 0.1278$	$\eta_{co} = 0.5$	$\eta_{dish} = 0.85$
$\eta_{generator} = 0.975$	$\eta_{rankine} = 0.1748$	$\eta_{separate} = 0.0946$
$\eta_{stirling} = 0.1908$	$\eta_{stirling,separate} = 0.2449$	$\eta_{system} = 0.09864$
$\eta_{trough} = 0.6$	$\eta_{turbine} = 0.75$	$E_{rankine} = 273.5$ [kW]
$E_{total} = 2534$ [kW]	$F1\$ = 'air\_ha'$	$F2\$ = 'water'$
$m_1 = 0.5879$ [kg/s]	$m_2 = 0.542$ [kg/s]	$p_{2,stirling} = 0.1$ [MPa]
$p_{dish} = 0.5$ [MPa]	$P_{generator} = 200$ [kW]	$p_s = 2.797$ [MPa]
$P_{stirling} = 50$ [kW]	$P_{turbine} = 205.1$ [kW]	$Q_{2,total} = 1565$ [kW]
$Q_{dish} = 293.4$ [kW]	$Q_{stirling} = 262.1$ [kW]	$Q_{stirling,cold} = 212.1$ [kW]
$Q_{trough} = 1314$ [kW]	$T_{1,afterstirling} = 673.2$ [K]	$T_c = 323.2$ [K]
$T_{cold} = 397.8$ [K]	$T_{cold,separate} = 328.2$ [K]	$T_{dish,inlet} = 623.2$ [K]
$T_{dish,outlet} = 1073$ [K]	$T_{environment} = 303.2$ [K]	$T_{hot} = 643.2$ [K]
$T_s = 613.2$ [K]		

[Click on this line to see the array variables in the Arrays Table window](#)

No unit problems were detected.

Calculation time = .1 sec.

Figure 2: Main results

Sort	1 $h_i$ [kJ/kg]	2 $p_i$ [MPa]	3 $s_i$ [kJ/kg-K]	4 $T_i$ [K]	5 $x_i$
[11]	1131	0.5		1073.15	
[12]	685.1	0.5		673.15	
[13]	631.9	0.5		623.15	
[14]					
[15]					
[16]					
[17]					
[18]					
[19]					
[20]					
[21]	3096	2.797		613.15	
[22]	2591	0.01234		323.15	1
[23]	209.3	0.01234	0.7037	323.15	0
[24]	209.4	0.1	0.7037	323.15	
[25]	600.8	0.1	1.794	372.78	
[26]	614.7	2.797	1.794	418.72	
[27]	990.2	2.797		503.19	0
[28]	2803	2.797		503.19	1
[29]	3038	2.797		589.05	

Figure 3: Array table results

Figure 3 shows array table results of the system. The array shows the enthalpy, pressure, entropy, temperature and dryness of states.