CASCADE SYSTEM CALCULATION

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1 Introduction

Different types of collectors and different technologies for electricity generation are suitable for different working temperature zones with different costs. A prototype of cascade collection and cascade utilization of solar energy with high efficiency and low cost is presented. Parabolic trough collectors are used to collect low temperature energy with low cost and dish collectors are used to collect high temperature energy with high efficiency. Rankine cycle is used to work in low temperature zone and Stirling cycle is used to work in high temperature zone. Furthermore, effective topological structures are considered to take full advantages of thermodynamic characters of different components of the system. The cold chamber of Stirling engine is cooled by condensed fluid of Rankine cycle to use the heat released by Stirling engine.

2 Problem description

To characterize the system, calculation of the system must be applied first. The scale of the system and the dimensions of the parameters must be evaluated.

Before detailed model of the system can be created, lots of simplifying assumptions are made:

- Steady state at nominal load
- Pressure losses negligible everywhere in the pressure circuit
- Simplified calculation of processes and equipment

For system 1, water is used as the working fluid of Rankine cycle. Feed water is used to cool the cold chamber of Stirling engine. Figure 1 shows the sketch of system 1. Air is heated in the dishes, then used to heat the hot chamber of Stirling engine and air-water heat exchanger successively. Water is heated in the cold chamber of Stirling engine, preheater, evaporator, superheater and air-water heat exchanger successively, and then used to work in the turbine, cooled in the condenser. Troughs are used to provide heat for the preheater, evaporator and superheater. Pumps are used to change the pressure of fluids.

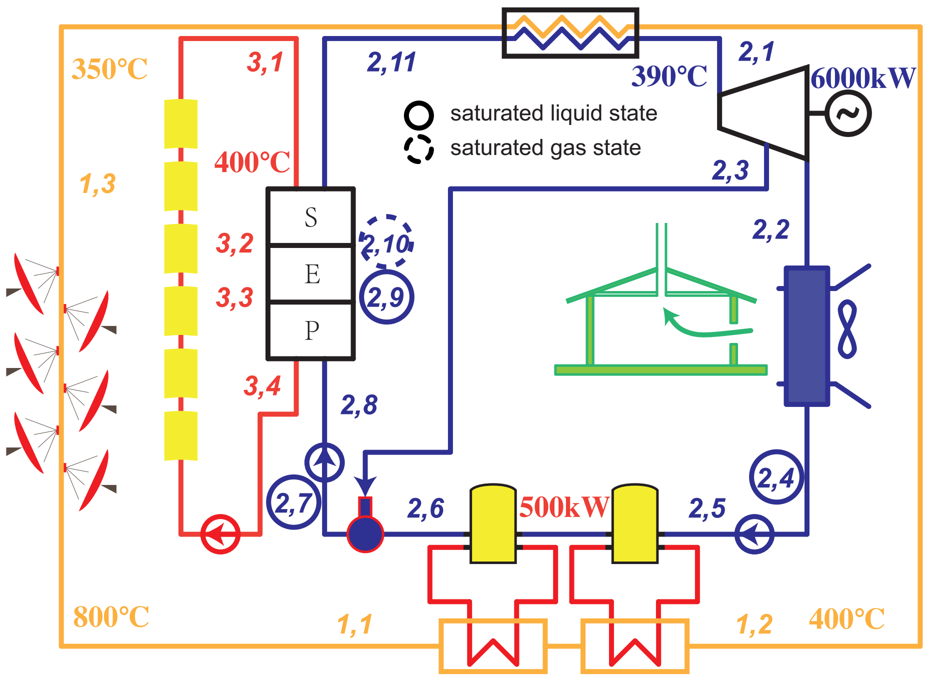

Figure 1: Sketch of system 1

State numbers are marked on the sketch according to its logical position. Numbers with solid circle means saturated liquid states ($x = 0$), and with dotted circle means saturated gas states ($x = 1$).

3 Data

Table 1 shows the basic design data of system 1, such as the nominal electric power of Stirling engine and generator, temperatures of inlet and outlet of dishes, design parameters of turbine. According to these data, some important parameters of the system, such as mass flow rate, temperature of a state, system efficiency, can be obtained by calculation. To calculate the system, the model must be built first. EES is used for the model construction and calculation.

In Table 1, DNI value reference to SAM website[1], p_{dish} is according to the design value of air pressure in cavity absorber.

[1] Website of DOE(US)

Table 1: System characteristics of System 1

Nominal electric power		
$P_{stirling} = 500\,\text{kWe}$	$P_{generator} = 6000\,\text{kWe}$	
Solar data		
$DNI = 700\ \text{W/m}^2$		
Dish		
$T_{dish,inlet} = 350℃$	$T_{dish,outlet} = 800℃$	$p_{dish} = 0.5\,\text{MPa}$
Dish type: SES		
Trough		
$\Delta T_{oil,water,min} = 20℃$	$T_{trough,outlet} = 350℃$	$p_{trough} = 2\,\text{MPa}$
Trough type: LS-3		
Stirling engine		
$T_{1,afterstirling} = 400℃$	$T_{environment} = 20℃$	$\Delta T_{stirling,hot} = 90℃$
$\Delta T_{stirling,cold} = 25℃$	$p_{2,stirling} = 1\,\text{MPa}$	Engine type: from an example
Steam turbine		
$T_s = 340℃$	$p_s = 2.35\,\text{MPa}$	$p_c = 0.015\,\text{MPa}$
$P_{generator} = 6\,\text{MW}$	$T_{s,d} = 390℃$	Turbine type: a product

4 Model of the system

The system is built in several blocks. These blocks are made of circulations and efficiency calculations. Two circulations, air circulation and water circulation, are built in some specific states and in some components. Known parameters of the states, we can get the efficiency of the system and the overall efficiency of separated systems.

4.1 Air circulation

Calculate η_{dish} first,
Call DishReceiver$(DNI, A_{dishCollector}, T_{dish,inlet}, T_{dish,outlet}, T_{amb} : \eta_{dish})$
For state $[1,1]$,

$$\left.\begin{array}{l} p_{1,1} = p_{dish} \\ T_{1,1} = T_{dish,outlet} \end{array}\right\} \Rightarrow h_{1,1} \tag{1}$$

For state $[1,2]$,

$$\left.\begin{array}{l} p_{1,2} = p_{1,1} \\ T_{1,2} = T_{1,afterstirling} \end{array}\right\} \Rightarrow h_{1,2} \tag{2}$$

For Stirling engine,

$$T_{hot} = T_{1,2} - \Delta T_{stirling,hot} \tag{3}$$

$$T_{cold} = T_{2,6} - \Delta T_{stirling,cold} \tag{4}$$

$\eta_{stirling}$ is calculated by using

$Call\ StirlingEngine(\frac{T[1,1]+T[1,2]}{2}, \frac{T[2,5]+T[2,6]}{2}, \Delta T_{stirling,hot}, \Delta T_{stirling,cold} : \eta_{stirling})$.

$$Q_{stirling} = P_{stirling}/\eta_{stirling} \tag{5}$$

$$Q_{stirling} = m_1\left(h_{1,1} - h_{1,2}\right) \tag{6}$$

$$Q_{stirling,cold} = Q_{stirling} - P_{stirling} \tag{7}$$

For state $[1,3]$,

$$\left.\begin{array}{l} p_{1,3} = p_{1,2} \\ T_{1,3} = T_{dish,inlet} \end{array}\right\} \Rightarrow h_{1,3} \tag{8}$$

For dish,

$$Q_{dish} = m_1\left(h_{1,1} - h_{1,3}\right) \tag{9}$$

$$Q_{dish} = DNI \cdot A_{dish} \cdot \eta_{dish} \tag{10}$$

4.2 Water circulation

Calculate $\eta_{i,turbine}$ first,

For state $d[2,1]$ on design:

$$\left.\begin{array}{l} p_{d,2,1} = p_s \\ T_{d,2,1} = T_{s,d} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{d,2,1} \\ s_{d,2,1} \end{array}\right. \tag{11}$$

For state $d[2,2]$ on design:

$$\left.\begin{array}{l} P_{turbine,d} = m_{2,d}(h_{d,2,1} - h_{d,2,2}) \Rightarrow h_{d,2,2} \\ \left.\begin{array}{l} p_{i,d,2,2} = p_c \\ s_{i,d,2,2} = s_{d,2,1} \end{array}\right\} \Rightarrow h_{i,d,2,2} \end{array}\right\} \Rightarrow \eta_{i,turbine} \tag{12}$$

For state $[2,1]$,

$$\left.\begin{array}{l} p_{2,1} = p_s \\ T_{2,1} = T_s \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{2,1} \\ s_{2,1} \end{array}\right. \tag{13}$$

For state $[2,2]$ and state $[2,3]$,

$$P_{turbine} = (1-y)\, m_2\left(h_{2,1} - h_{2,2}\right) + y m_2\left(h_{2,1} - h_{2,3}\right) \tag{14}$$

$$\left.\begin{array}{l} p_{2,2} = p_c \\ h_{2,2} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,2} \\ x_{2,2} \end{array}\right. \tag{15}$$

$$\left.\begin{array}{l} p_{2,3} = p_b \\ h_{2,3} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,3} \\ x_{2,3} \end{array}\right. \tag{16}$$

$$\left.\begin{array}{l} s_{i,2,2} = s_{2,1} \\ p_{i,2,2} = p_c \end{array}\right\} \Rightarrow h_{i,2,2} \tag{17}$$

$$\left.\begin{array}{l} s_{i,2,3} = s_{2,1} \\ p_{i,2,3} = p_b \end{array}\right\} \Rightarrow h_{i,2,3} \tag{18}$$

$$\eta_{i,turbine} = \frac{h_{2,1} - h_{2,2}}{h_{2,1} - h_{i,2,2}} \tag{19}$$

$$\eta_{i,turbine} = \frac{h_{2,1} - h_{2,3}}{h_{2,1} - h_{i,2,3}} \tag{20}$$

For state $[2, 4]$,

$$\left.\begin{array}{l} p_{2,4} = p_{2,2} \\ x_{2,4} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{2,4} \\ s_{2,4} \end{array}\right. \tag{21}$$

For Rankine cycle,

$$\eta_{rankine} = \frac{(1-y)\left(h_{2,1} - h_{2,2}\right) + y\left(h_{2,1} - h_{2,3}\right)}{(1-y)\left(h_{2,6} - h_{2,4}\right) + h_{2,1} - h_{2,7}} \tag{22}$$

For state $[2, 5]$,

$$\left.\begin{array}{l} p_{2,5} = p_{2,stirling} \\ s_{2,5} = s_{2,4} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,5} \\ h_{2,5} \end{array}\right. \tag{23}$$

For state $[2, 6]$,

$$Q_{stirling,cold} = (1-y)\, m_2 \left(h_{2,6} - h_{2,5}\right) \tag{24}$$

$$\left.\begin{array}{l} p_{2,6} = p_{2,5} \\ h_{2,6} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,6} \\ s_{2,6} \end{array}\right. \tag{25}$$

For state $[2, 7]$,

$$\left.\begin{array}{l} p_{2,7} = p_{deaerator} \\ x_{2,7} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{2,7} \\ s_{2,7} \end{array}\right. \tag{26}$$

$$y h_{2,3} + (1-y)\, h_{2,6} = h_{2,7} \tag{27}$$

For state $[2, 8]$,

$$\left.\begin{array}{l} p_{2,8} = p_s \\ s_{2,8} = s_{2,7} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,8} \\ h_{2,8} \end{array}\right. \tag{28}$$

For state $[2,9]$,

$$\left.\begin{array}{l} p_{2,9} = p_{2,8} \\ x_{2,9} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,9} \\ h_{2,9} \end{array}\right. \tag{29}$$

For state $[2,10]$,

$$\left.\begin{array}{l} p_{2,10} = p_{2,9} \\ x_{2,10} = 1 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{2,10} \\ h_{2,10} \end{array}\right. \tag{30}$$

For state $[2,11]$,

$$\left.\begin{array}{l} p_{2,11} = p_{2,10} \\ m_1\left(h_{1,2} - h_{1,3}\right) = m_2\left(h_{2,1} - h_{2,11}\right) \end{array}\right\} \Rightarrow T_{2,11} \tag{31}$$

$$Q_{trough} = m_2\left(h_{2,11} - h_{2,8}\right) \tag{32}$$

$$Q_{trough} = DNI \cdot A_{trough} \cdot \eta_{trough} \tag{33}$$

$n_{troughCollector} = Ceil(A_{trough}/A_{troughCollector})$

4.3 Oil circulation

Calculate trough collector thermal efficiency first,
$Call\, TroughCollector(DNI, T_{trough,outlet}, T_{amb}, q_{m.3}, L : T_{trough,outlet}, P_{Q,sun,collector}, P_{Q,collector,fluid})$

$$\eta_{trough} = P_{Q,collector,fluid}/P_{Q,sun,collector} \tag{34}$$

For state $[3,1]$,

$$\left.\begin{array}{l} p_{3,1} = p_{trough} \\ T_{3,1} = T_{trough,outlet} \end{array}\right\} \Rightarrow h_{3,1} \tag{35}$$

For state $[3,2]$,

$$\left.\begin{array}{l} p_{3,2} = p_{3,1} \\ m_3\left(h_{3,1} - h_{3,2}\right) = m_2\left(h_{2,11} - h_{2,10}\right) \\ h_{3,2} = h\left(F\$_3, T_{3,2}, p_{3,2}\right) \end{array}\right\} \Rightarrow T_{3,2} \tag{36}$$

For state $[3,3]$,

$$\left.\begin{array}{l} p_{3,3} = p_{3,2} \\ m_3\left(h_{3,2} - h_{3,3}\right) = m_2\left(h_{2,10} - h_{2,9}\right) \\ h_{3,3} = h\left(F\$_3, T_{3,3}, p_{3,3}\right) \end{array}\right\} \Rightarrow T_{3,3} \tag{37}$$

$$T_{3,3} = T_{2,9} + \Delta T_{oil,water,min} \tag{38}$$

For state $[3,4]$,

$$\left.\begin{array}{l} p_{3,4} = p_{3,3} \\ m_3\left(h_{3,3} - h_{3,4}\right) = m_2\left(h_{2,9} - h_{2,8}\right) \\ h_{3,4} = h\left(F\$_3, T_{3,4}, p_{3,4}\right) \end{array}\right\} \Rightarrow T_{3,4} \tag{39}$$

4.4 System efficiency

$$E_{total} = DNI \cdot (A_{dish} + A_{trough}) \tag{40}$$

$$\eta_{system} = (P_{stirling} + P_{generator}) / E_{total} \tag{41}$$

4.5 Separate dish system

$$T_{hot,s} = 720℃^{2}$$

$$T_{cold,s} = T_{environment} + \Delta T_{stirling,cold} \tag{42}$$

$Call\ StirlingEngine(T_{hot,s}, T_{cold,s}, \Delta T_{stirling,hot}, \Delta T_{stirling,cold} : \eta_{stirling,s})$

$$Q_{dish,s} = DNI \cdot A_{dish} \cdot \eta_{dish,s} \tag{43}$$

$\eta_{dish,s}$ uses the value 0.8.[1]

$$P_{stirling,s} = Q_{dish,s}\eta_{stirling,s} \tag{44}$$

4.6 Separate trough system

$p_{b,s} = 0.1\,\text{MPa}$[3],$p_{condenser,s} = 0.1\,\text{MPa}$[4] , $p_{deaerator,s} = 0.12\,\text{MPa}$[5].
For state $s[2,1]$,

$$\left.\begin{array}{l} p_{s,2,1} = p_s \\ T_{s,2,1} = T_s \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{s,2,1} \\ s_{s,2,1} \end{array}\right. \tag{45}$$

For state $s[2,2]$ and state $s[2,3]$,

$$P_{turbine,s} = (1-y)\, m_{2,s}\, (h_{s,2,1} - h_{s,2,2}) + y_s m_{2,s}\, (h_{s,2,1} - h_{s,2,3}) \tag{46}$$

$$\left.\begin{array}{l} p_{s,2,2} = p_c \\ h_{s,2,2} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{s,2,2} \\ x_{s,2,2} \end{array}\right. \tag{47}$$

$$\left.\begin{array}{l} p_{s,2,3} = p_{b,s} \\ h_{s,2,3} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{s,2,3} \\ x_{s,2,3} \end{array}\right. \tag{48}$$

$$\left.\begin{array}{l} s_{i,s,2,2} = s_{s,2,1} \\ p_{i,s,2,2} = p_c \end{array}\right\} \Rightarrow h_{i,s,2,2} \tag{49}$$

[2]Value 993K is used in SAM as the default heater heat set temperature.
[3]?Don't need high pressure.
[4]?The pressure after condensate pump.
[5]Atmospheric deaerator.

$$\left.\begin{array}{l} s_{i,s,2,3} = s_{s,2,1} \\ p_{i,s,2,3} = p_{b,s} \end{array}\right\} \Rightarrow h_{i,s,2,3} \tag{50}$$

$$\eta_{i,turbine} = \frac{h_{s,2,1} - h_{s,2,2}}{h_{s,2,1} - h_{s,i,2,2}} \tag{51}$$

$$\eta_{i,turbine} = \frac{h_{s,2,1} - h_{s,2,3}}{h_{s,2,1} - h_{s,i,2,3}} \tag{52}$$

For state $s[2,4]$,

$$\left.\begin{array}{l} p_{s,2,4} = p_{s,2,2} \\ x_{s,2,1} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{s,2,4} \\ s_{s,2,4} \end{array}\right. \tag{53}$$

For Rankine cycle,

$$\eta_{rankine,s} = \frac{(1-y_s)\left(h_{s,2,1} - h_{s,2,2}\right) + y\left(h_{s,2,1} - h_{s,2,3}\right)}{(1-y_s)\left(h_{s,2,5} - h_{s,2,4}\right) + h_{s,2,1} - h_{s,2,6}} \tag{54}$$

For state $s[2,5]$,

$$\left.\begin{array}{l} p_{s,2,5} = p_{condenser,s} \\ s_{s,2,5} = s_{s,2,4} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{s,2,5} \\ h_{s,2,5} \end{array}\right. \tag{55}$$

For state $s[2,6]$,

$$\left.\begin{array}{l} p_{s,2,6} = p_{deaerator,s} \\ x_{s,2,6} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} h_{s,2,6} \\ s_{s,2,6} \end{array}\right. \tag{56}$$

$$y_s h_{s,2,3} + (1-y_s)\, h_{s,2,5} = h_{2,6} \tag{57}$$

For state $s[2,7]$,

$$\left.\begin{array}{l} p_{s,2,7} = p_s \\ s_{s,2,7} = s_{s,2,6} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{s,2,7} \\ h_{s,2,7} \end{array}\right. \tag{58}$$

For state $s[2,8]$,

$$\left.\begin{array}{l} p_{s,2,8} = p_{s,2,7} \\ s_{s,2,8} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{s,2,8} \\ h_{s,2,8} \end{array}\right. \tag{59}$$

For state $s[2,9]$,

$$\left.\begin{array}{l} p_{s,2,9} = p_{s,2,8} \\ x_{s,2,9} = 1 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_{s,2,9} \\ h_{s,2,9} \end{array}\right. \tag{60}$$

$$Q_{trough} = m_{2,s}\left(h_{s,2,1} - h_{s,2,7}\right) \tag{61}$$

$$P_{generator,s} = P_{turbine,s}\eta_{generator} \tag{62}$$

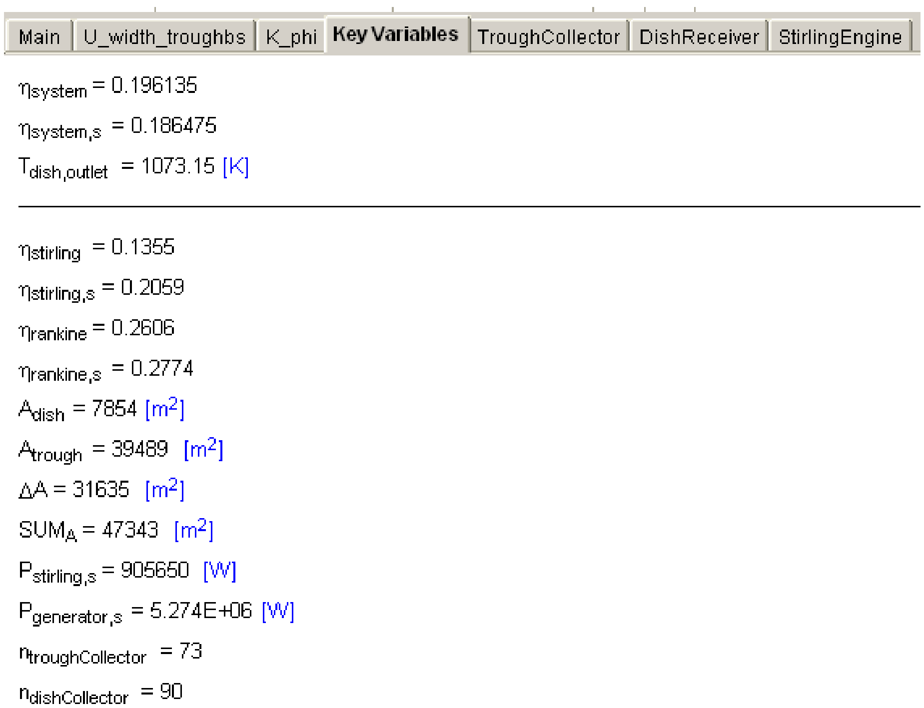

Figure 2: Some improtant results

4.7 Separate system efficiency

$$\eta_{system,s} = (P_{stirling,s} + P_{generator,s})/E_{total} \tag{63}$$

5 Results

Figure 2 shows some important results of the system. From the results, we can see that the overall of the cascade system $\eta_{system} = 0.196135$ is higher than that of separated system $\eta_{system,s} = 0.186475$. The separated system uses the same collectors of the cascade system and recirculated cooling water with the same temperature of the environment is used to cool cold chamber of Stirling engine.

Figure 3 shows results of trough collector part, Figure 4 shows results of dish receiver part, Figure 5 shows results of Stirling engine part. The detail explaination of these parts can be found in Appendix.

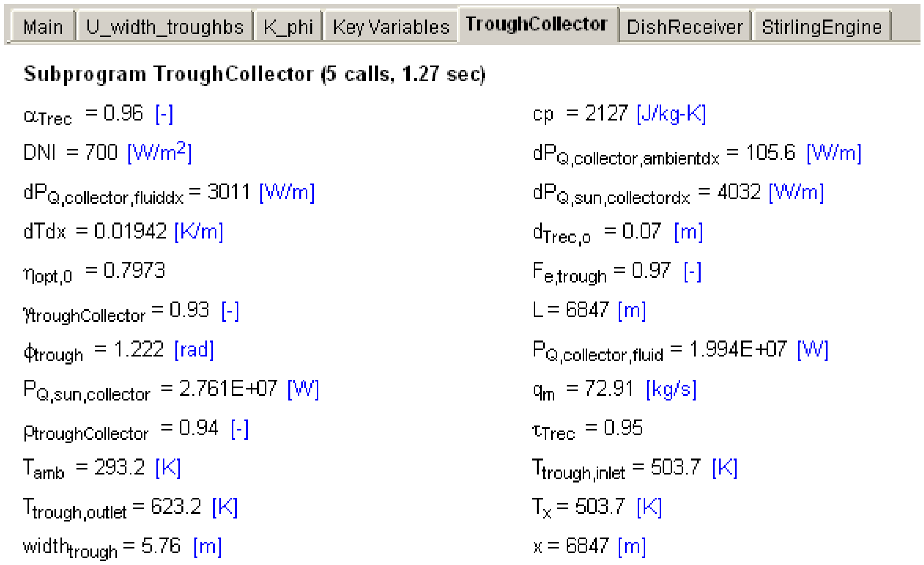

Figure 3: Results of the trough collector part

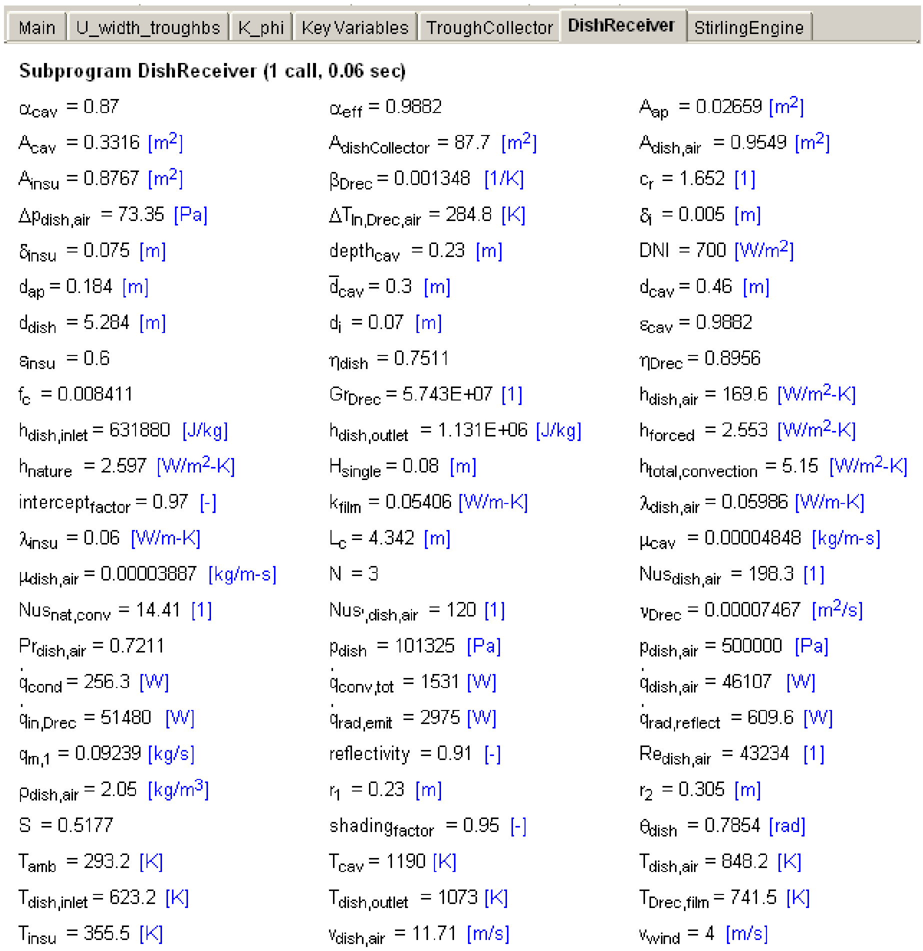

Figure 4: Results of the dish receiver part

Figure 5: Results of the Stirling engine part

Appendix A Trough collector part

Appendix B Dish receiver part

Appendix C Stirling engine part

References

[1] Shuang-Ying Wu, Lan Xiao, Yiding Cao, and You-Rong Li. A parabolic dish/AMTEC solar thermal power system and its performance evaluation. *Applied Energy*, 87(2):452–462, 2010.