

Configuration optimization of Stirling engine array in a cascade solar system

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Abstract

Cascade solar thermal systems provide a new direction for solar power generation. This paper focuses on the configuration optimization of a cascade solar system in which a Stirling engine array (SEA) is applied. The SEA has multiple configurations. To find out the influence of the configuration on the performance of SEA, five basic connection types were proposed. A Stirling engine model considering various losses and irreversibilities were developed. The model was evaluated by considering the prototype GPU-3 Stirling engine as a case study. SEA models were developed based on the Stirling engine model. Global efficiency and power of different connection types of SEAs with the same hot and cold flows were evaluated. Effects of different factors on the performance of SEA were considered. Result shows that flow order has little influence on the SEA performance, and serial flows connection type is the best for an SEA to obtain best performance and adaptability for given heating and cooling fluids.

Keywords: Stirling cycle, Stirling engine model, Stirling engine array, connection type

1. Introduction

With the emphasis on energy and environmental impact, solar energy, which has the advantages of widely distribution, huge amount, inexhaustible and no pollution, has received much attention by many countries and been regarded as the best potential candidate of the fossil energy. Concentrating solar thermal power generation is another form of power generation technology except solar photovoltaic power generation. Compared to solar photovoltaic, solar thermal power is gaining more attention for its advantages as higher energy density, smooth power generation, good grid compatibility, easy to integrate with existing fossil power plant. There are three types of concentrating solar power (CSP) technologies being commercially applied: parabolic trough, parabolic dish and power tower. Among the three solar thermal power technologies, parabolic trough is the most mature and commercially deployed technology. However, it has a low concentration ratio, the receiver's temperature is relatively low, the solar-to-electric efficiency is relatively low. Parabolic dish can

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obtain high temperature thermal energy, its solar-to-electric efficiency is higher than parabolic trough. Besides, one advantage of parabolic dish is that it requires much less water for power generation. However, solar parabolic dish is not a large-scale application, it's mainly applied for distributed power generation for its compact structure and easy installation. Solar power tower has a very high concentration ratio when more heliostats are used, the receiver's temperature can be very high and it can be applied for large-scale application. However, it has some disadvantages such as high investment and high system complexity.

It is very important to find out a way to utilize the advantages of existing solar thermal power technologies and overcome their disadvantages. In other words, to find out a new technology with higher efficiency and lower cost is urgent.

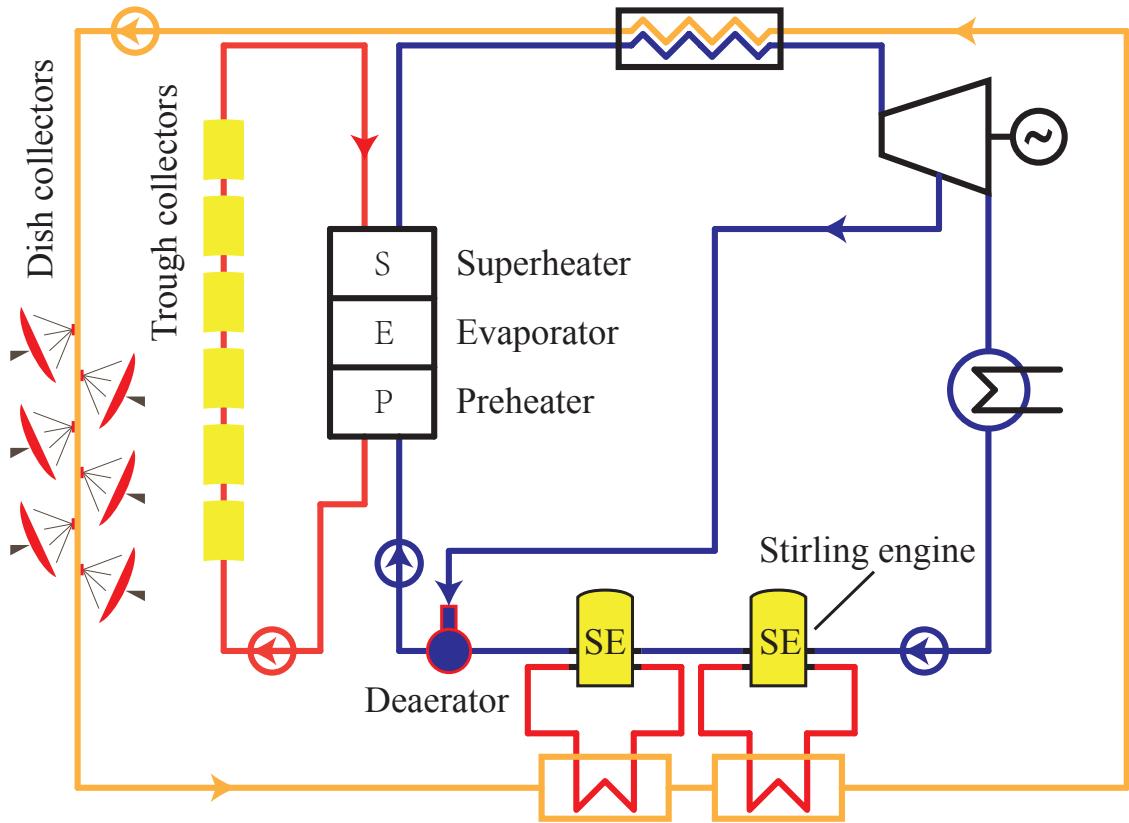


Figure 1: A cascade system using both trough system and dish system

Zhang et al. [1] proposed a novel solar thermal cascade system using both parabolic trough system and parabolic dish system as described in Figure 1. This cascade system has several features. First, it uses multiple types of collectors. Trough collectors are applied for lower temperature heat collection and dish collectors are applied for higher temperature heat collection. This helps to reduce the cost and improve

the efficiency. Second, it uses multiple types of thermodynamic cycles. Rankine cycle is applied for lower temperature heat utilization. Stirling cycle is applied for higher temperature heat utilization. Third, an air-water heat exchanger is applied to increase the temperature of the main steam, which helps to improve the efficiency of Rankine cycle. On the other hand, it can overcome the disadvantage of low limit temperature of heat transfer oil in conventional solar trough systems, which helps to achieve higher main steam parameters than traditional solar trough systems. Fourth, condensate of the Rankine cycle is used to cool the Stirling engine array. Rejected heat of the Stirling cycle can be reused by Rankine cycle, which helps to improve the overall system efficiency. Besides, the Stirling engine array may provide a large-scale capacity of Stirling cycle power generation.

In a conventional dish-Stirling system, the Stirling engine capacity is mainly constrained by two factors: dish collector size and Stirling engine size. The dish size is limited for the cost and difficulty of production of large mirrors. The Stirling engine size is limited for its low power-to-weight ratio. The tracking feature of dish collector limits the weight of engine on the focal point. Besides, the size of engine is limited for it overlaps part of the collector.

In the cascade system, Stirling engines can be put on the ground as a Stirling engine array (SEA). Heat collected by multiple dish collectors is supplied to the SEA by heating fluid. Due to the heat transfer fluid, a heat storage system becomes feasible. On the other hand, since the engines are static installations on the ground where space and weight are not at a premium any more, they can reach higher capacities for larger scale applications.

Great attention has focused on the application using parabolic dish to collect heat. Some researchers investigated the impact of various parameters on the optical and thermal performance of the solar dish receivers using Monte Carlo Ray Tracing Method (MCRTM) and/or numerical modeling methods. Some researchers considered the applications with different ways to use the collected heat. Loni et al. [2] considered a system using parabolic dish for an organic Rankine cycle. In the proposed system, thermal oil is used as the working fluid to transport the collected heat for the organic Rankine cycle. Wang et al. [3] proposed an inverse design method for a cavity receiver used in solar dish Brayton system. Craig et al. [4] evaluated a parabolic dish tubular receiver used in a dish Brayton cycle. An approach for incorporating a complex geometry like a tubular receiver generated using CFD software into SolTrace was developed. Aichmayer et al. [5] presented a hybrid solar micro gas-turbine system integrating volumetric solar dish receiver. Concerning the solar dish receiver integration, both pressurized and atmospheric configurations have been considered. Lovegrove et al. [6] presented an idea of using parabolic dish array to provide heat for ammonia based thermochemical energy storage. In this regard, using parabolic dishes to provide heat for SEA is applicable.

A large number of studies have been done on Stirling engine analysis. To describe a Stirling engine's behavior precisely is a difficult task due to the various losses and irreversibilities in the engine. Researchers have done a lot of work to build a precise Stirling engine model. Different models of Stirling engines were

developed using empirical, analytical and numerical methods .

Among these methods, numerical methods obtain the most accurate models. Urieli and Berchowitz [7] proposed an adiabatic model of Stirling engine considering some irreversible effects such as non-ideal heat transfer processes and pressure drop effect using numerical methods. The model is known as Simple model. Many researchers developed more accurate models based on the Simple model by using alternative methods or including more loss mechanisms. Ni et al. [8] proposed an improved Simple analytical model which considers the influence mechanism of rotary speed, pressure and working gas in the view of heat/power losses for Stirling engine performance. Jia et al. [9] developed a numerical model of free-piston engine generator. The dynamic equations have been linearized to simplify the model to a one-degree forced vibration system with various damping. The solving time of the proposed fast response model can be significantly reduced comparing to previous numerical models. Strauss and Dobson [10] proposed an alternative method to calculate the regeneration heat loss and pumping losses, which is more suitable for preliminary engine design and optimization, known as Simple II model. Abbas [11] considered the effects of non-ideal regeneration, shuttle loss and heat conduction losses based on Simple model. Araoz et al. [12] developed a rigorous Stirling engine model with adiabatic working spaces, isothermal heat exchangers. It considers dead volumes, and imperfect regeneration, mechanical pumping losses due to friction, limited heat transfer and thermal losses on the heat exchangers. The model is suitable for different engine configurations (α , β and γ engines). Babaelahi and Syyaadi [13] proposed a new numerical thermal model based on polytropic expansion/compression processes. Differential equations in the expansion/compression processes were modified to polytropic processes in the new model. New model shows a better performance prediction compared with previous models.

With the development of finite-time thermodynamics, many researchers studied the finite-time thermodynamic performance of the Stirling engine. This analysis can also be used in the case of irreversible machines further considering the internal irreversibilities of a Stirling engine such as friction, pressure drop and entropy generation [14]. Wu et al. [15] developed a numerical model considering finite-time effect to find out the relationship between the net power output and thermal efficiency of the engine. Li et al. [16] developed a mathematical model of a high temperature differential dish-Stirling system with finite-time thermodynamics. Finite-rate heat transfer, regenerative heat losses, conductive thermal bridging losses and finite regeneration processes of the Stirling engine were considered in the model. Hosseinzade [17] presented a new closed-form thermal model for the thermal simulation of Stirling engines based on the combination of polytropic analysis of expansion/compression processes and the concept of finite speed thermodynamics. Instead of finite-time method, Ahmadi et al. [18] proposed a finite-speed thermodynamic analysis based on the first law of thermodynamics for a closed system with finite speed and the direct method. The effects of heat source temperature, regenerating effectiveness, volumetric ratio, piston stroke as well as rotational speed are included in the analysis. Chen et al. [19] developed a heat-engine cycle model using finite-time thermodynamics. The model, considered the losses due to heat-resistance, heat leaks and internal irreversibility, is

applicable for generalized irreversible universal steady-flow heat-engine cycles.

On the other side of the researches, multi-objective optimization algorithms were used considering multi-variables to obtain a better performance was paid for attention by numbers of researchers recently. Ahmadi et al. [20] performed the thermodynamic analysis of solar dish Stirling engine based on the finite-time thermodynamics. Then the NSGA-II algorithm was applied. Three objectives, thermal efficiency, entransy loss rate and power output, were set as the objectives and three well known decision making methods have been employed in the algorithm. Li et al. [21] developed a multi-objective optimization model of a solar energy powered gamma type Stirling engine using Finite Physical Dimensions Thermodynamics (FPDT) method by multi-objective criteria. Genetic algorithm was used to get the Pareto frontier, and optimum points were obtained using the decision different making methods. Results show that total thermal conductance, hot temperature, stroke and diameter ratios can be improved. Patel and Savsani [22] developed a strategy for multi-objective optimization for Stirling engines using TS-TLBO (tutorial training and self learning inspired teaching-learning-based optimization) algorithm. An application example with eleven decision variables and three objectives are considered. Luo et al. [23] proposed a multiple optimization method that combines multiple optimization algorithms including differential evolution, genetic algorithm and adaptive simulated annealing. The optimization considers five decision variables, including engine frequency, mean effective pressure, temperature of heating source, number of wires in regenerator matrix, and the wire diameter of regenerator for maximum efficiency and output power.

In the cascade system, the performance of the Stirling engines is related with their configuration. However, the literature review indicates that the analysis of arrangements of Stirling engines, classification and performance of the SEA, has not been reported till now. In this regard, this paper investigated the connection types of SEA and its influence on SEA performance. According to Organ's theory [24], one equivalent analytical Stirling engine model always exists for different types (α type, β type and γ type) of engines. To find out the influence of connection type of SEA and to avoid falling into the problem of developing specific Stirling engine model, a Stirling engine model based on some assumptions and simplifications was developed. This model was evaluated using experimental data of the General Motor GPU-3 Stirling engine prototype. Imperfect regeneration and some irreversibilities were considered. Heat transfer analysis of Stirling engine with heating and cooling fluids was also included. SEA models of different connection types were built depending on the engine model. Impacts of different parameters on the performance of these models were analyzed.

2. Connection types of SEA

For a single Stirling engine, the heat transfer processes between fluids and engine are independent and irrelevant with the direction of the flows, which means the efficiency and power are not affected by the

direction of fluids. However, for an SEA, the connection type will affect the temperature profiles through the array and the specific work production, both of which will determine the efficiency and power of the SEA. It is practically significant to investigate the influence of connection type of an SEA on its performance. Using parallel flow, on the one hand, will reduce the flow rate of the fluid, which will reduce the power of each engine; however, on the other hand, will take the advantage of higher inlet heating fluid temperature (or lower inlet cooling fluid temperature), which may increase the power of each engine. friend who is Using serial flow, on the one hand, will increase the flow rate of the fluid, which will increase the power of each engine; however, on the other hand, the inlet heating fluid temperature reduces with the flow direction (or the inlet cooling fluid temperature increases with the flow direction), which leads to lower engine power along the flow direction. Using the same order will lead to largest fluid temperature difference (temperature difference of the heating and cooling fluids) at the first engines and smallest fluid temperature difference at the last engines. Using the reverse order will lead to more averaged fluid temperature differences of the engines. For a heat exchanger, the reverse order (counterflow), which leads to a smaller fluid temperature difference, has a better heat transfer effect for its lower exergy loss. However, for a Stirling engine, the smaller fluid temperature difference leads to lower performance due to the lower temperature difference of the working gas in the hot space and cold space. To find out the influence of connection types on the performance of SEA, it is essential to classify the connection types.

Five basic connection types of SEA were summarized according to the direction-irrelevant feature of Stirling engine, as shown in Figure 2. Type 1 is parallel flow, Type 2 is serial flows in the same order, Type 3 is serial flows in the reverse order, Type 4 is heating fluid in serial flow and cooling fluid in parallel flow and Type 5 is heating fluid in parallel flow and cooling fluid in serial flow. All other connection types are the combination of these five basic connection types. For instance, an SEA in Figure 3 is the combination of Type 2 and Type 4.

3. Thermodynamic analysis of Stirling engine

3.1. Stirling engine model

3.1.1. Theoretical Stirling cycle

In a Stirling cycle, there are two isothermal processes that exchange heat with heating and cooling fluids, two isochoric processes that exchange heat with regenerator. Figure 4 shows the schematic of a Stirling cycle, process 1-2 and process 3-4 are the two isothermal processes, process 2-3 and process 4-1 are the two isochoric processes. The heat absorbed by regenerator in process 4-1 is reused in process 2-3, but only able to heat the working gas from 2 to 3' due to the imperfect regeneration. e is defined as the regenerator effectiveness

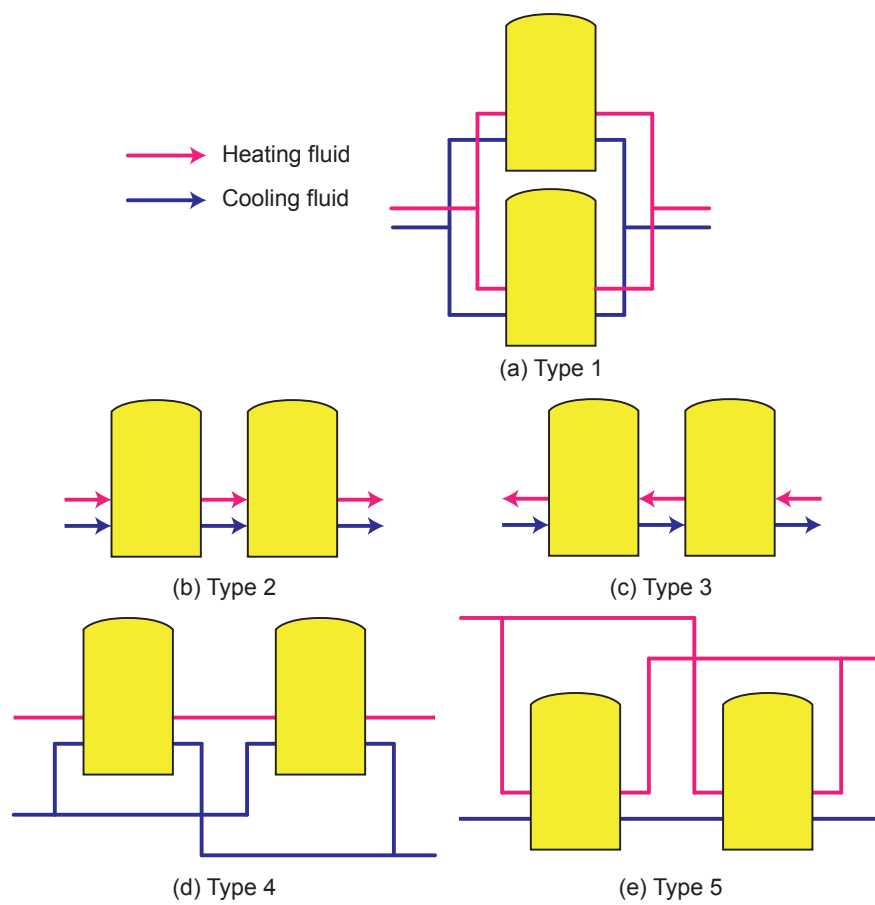


Figure 2: Five basic connection types of SEA

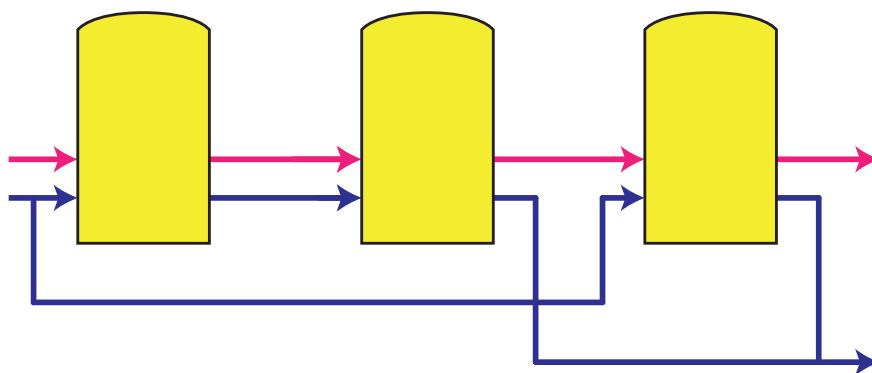


Figure 3: An instance of connection type of an SEA

for the imperfect regeneration [25], $e = \frac{T_R - T_L}{T_H - T_L}$, where T_H is the temperature in the hot space, T_L is the temperature in the cold space, T_R is the effective working fluid temperature in the regenerator.

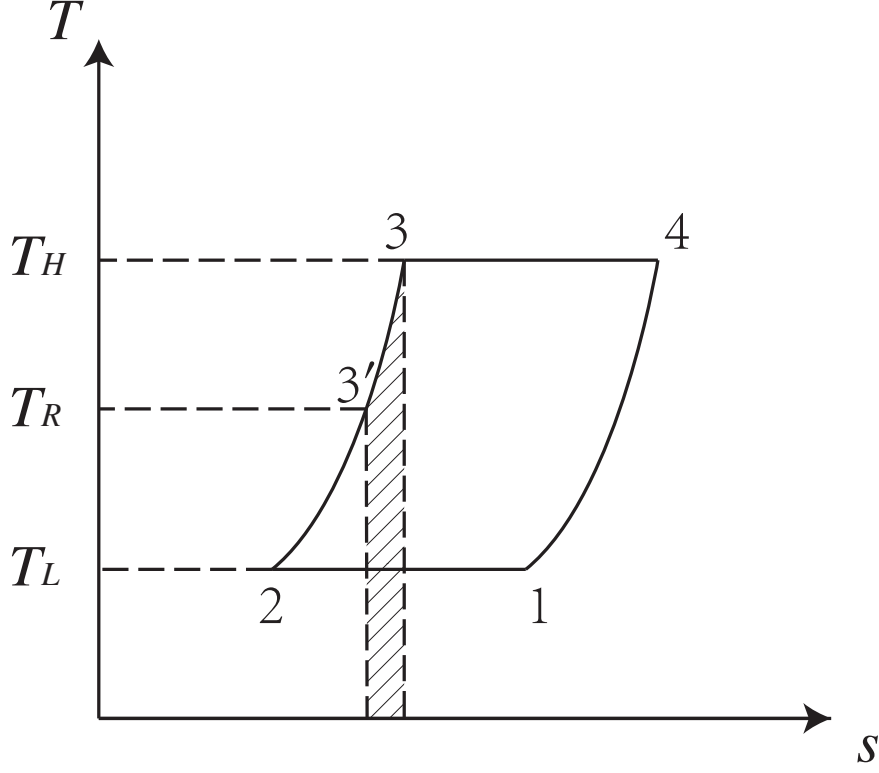


Figure 4: T - s diagram of a Stirling cycle

In order to obtain a simplified analytical model, several simplifications were made:

- The working gas in Stirling engines obeys the idea gas law.
- No heat loss to the environment for Stirling engines.
- Overall heat transfer coefficients of the fluids are constant.
- A symmetrical regenerator behavior is assumed [25] so that a simple effectiveness can be obtain by

$$T_R = \frac{T_H - T_L}{\ln(T_H/T_L)}.$$

To consider internal irreversibilities in Stirling cycle made by dead volumes, as described in [26], total dead volume V_D is divided into heater dead volume V_{DH} , regenerator dead volume V_{DR} and cooler dead

volume V_{DC} . There exists a factor K to describe the dead volumes under different temperatures. K is relevant with temperatures in the process and regenerator effectiveness.

$$K = \frac{V_{DH}}{T_H} + \frac{V_{DR}}{T_R} + \frac{V_{DC}}{T_L} \quad (1)$$

For the isothermal compression process 1-2, the output work

$$W_{12} = \int_{V_E+V_C}^{V_E} p_{12} dV = -mRT_L \ln \frac{V_E + V_C + KT_L}{V_E + KT_L} \quad (2)$$

For the isothermal expansion process 3-4, the output work

$$W_{34} = \int_{V_E}^{V_E+V_C} p_{34} dV = mRT_H \ln \frac{V_E + V_C + KT_H}{V_E + KT_H} \quad (3)$$

Define $\gamma_H = \frac{V_E+V_C+KT_H}{V_E+KT_H}$, and $\gamma_L = \frac{V_E+V_C+KT_L}{V_E+KT_L}$, so in a cycle, the theoretical output work

$$W_{th} = W_{12} + W_{34} = mR(T_H \ln \gamma_H - T_L \ln \gamma_L) \quad (4)$$

For the isochoric heating process 3'-3, the absorbed heat

$$Q_{3'3} = nc_v(T_H - T_R) = \frac{1-e}{k-1} mR(T_H - T_L) \quad (5)$$

For the isothermal expansion process 3-4, the absorbed heat

$$Q_{34} = W_{34} = mRT_H \ln \gamma_H \quad (6)$$

In a cycle, the theoretical absorbed heat

$$Q_{th} = Q_{3'3} + Q_{34} = \frac{1-e}{k-1} mR(T_H - T_L) + mRT_H \ln \gamma_H \quad (7)$$

3.1.2. Irreversibilities and losses

a. Non-ideal heat transfer effect

Because of non-ideal heater and cooler, the working fluid temperature (T_H , T_L) in these two heat exchangers is less/higher than the wall temperature (T_{hw} , T_{cw}), respectively. And T_H and T_L can be corrected by the wall temperatures as follows:

$$T_H = T_{hw} - \frac{Q s_{se}}{h_h A_{hw}} \quad (8)$$

$$T_L = T_{cw} + \frac{(Q - W) s_{se}}{h_c A_{cw}} \quad (9)$$

The heat transfer coefficient can be obtained using the following correlation [13]:

$$h_{h,c} = \frac{\mu c_p f_{Re}}{2 D_{h,c} Pr_{h,c}} \quad (10)$$

where f_{Re} is a Reynolds friction factor defined as:

$$f_{Re} = 0.0791 Re_{h,c}^{-0.75} \quad (11)$$

$Re_{h,c}$, $Pr_{h,c}$ and $D_{h,c}$ are Reynolds number, Prandtl number and hydraulic diameter of the heater/cooler exchanger.

b. Effect of pressure drop

Pressure drops in the heat exchangers cause power losses of the Stirling engine. The pressure drops can be obtained by [7]:

$$\Delta p = - \frac{2 f_{Re} \mu u V}{d^2 A} \quad (12)$$

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where u is the working gas speed, V is volume, A is flow cross-section area.

The net power loss of the Stirling engine due to pressure drop of the heat exchangers can be evaluated by:

$$W_{pd} = \oint \sum_{i=E,C} (\Delta p_i \frac{dV_i}{d\theta}) d\theta \quad (13)$$

c. Effect of finite speed of piston and mechanical friction

Due to the finite speed of piston, the pressure on the piston surface is different from the pressure of expansion and compression spaces. It has been demonstrated that the pressure on the piston surface in the expansion process is less than the mean pressure in the expansion space. Similarly, the pressure on

the piston surface in the compression process is greater than the mean pressure in the compression space. This means the output work is less than the theoretical value. Besides, The output work also reduces due to mechanical friction. The output work loss due to finite speed of piston and mechanical friction can be obtained as follows [13]:

$$W_{fs} = \oint p \left(\pm \frac{au_p}{c} \pm \frac{\Delta p_f}{p} \right) dV \quad (14)$$

where the sign (+) is used in the compression space, and the sign (-) is used in the expansion space. p is the mean pressure in the compression/expansion space, u_p is velocity of the piston, c is the average speed of molecules and Δp_f is the pressure loss due to mechanical friction. Δp_f , a and c can be obtained by [27]:

$$\Delta p_f = 0.97 + 0.009s_{se} \quad (15)$$

$$a = \sqrt{3k} \quad (16)$$

$$c = \sqrt{3RT} \quad (17)$$

d. Energy losses due to internal conduction

The temperature differs from the heater and cooler, heat losses from heater to cooler exists due to internal conduction through the walls of regenerator. [10] The internal conduction loss in a cycle can be obtained by follows:

$$Q_{id} = \frac{k_r A_r}{L_r s_{se}} (T_{hw} - T_{cw}) \quad (18)$$

where, k_r , A_r and L_r denote the regenerator matrix conductivity, regenerator length, and regenerator conductive area respectively.

e. Energy losses due to shuttle conduction

The displacer shuttles between the expansion and compression space. It absorbs heat during the hot end of its stroke and releases it during the cold end of its stroke. This heat loss can be estimated as [28]:

$$Q_{sc} = 0.4 \frac{Z^2 k_p D_p}{J L_d s_{se}} (T_H - T_L) \quad (19)$$

where, Z , k_p , D_p , J and L_d denote the displacer stroke, piston thermal conductivity, displacer diameter, gap between the displacer and the cylinder, and length of the displacer respectively.

So, in a Stirling engine, the total absorbed heat in a cycle

$$Q = Q_{th} + Q_{id} + Q_{sc} \quad (20)$$

the output work

$$W = W_{th} - W_{pd} - W_{fs} \quad (21)$$

Power of the Stirling engine

$$P = W_{se} \quad (22)$$

Efficiency of the Stirling engine

$$\eta = W/Q \quad (23)$$

3.2. Model validation

Evaluation of the developed thermal model was performed by considering the GPU-3 Stirling engine as a case study. Design specifications of the GPU-3 Stirling engine are indicated in Table 1. The thermal efficiency and power of the proposed Stirling engine model was compared with previous thermal models and experimental data as shown in Table 2 and Table 3.

It can be found that the proposed model has much better agreement with the experimental results than previous thermal models at various rotation speeds and mean effective pressures. It is required to mention that in all thermal models both power W and input heat Q were determined by the thermal process of heat transfer between the wall and working gas. In the proposed model, W and Q are obtained by Equation 8 and 9. Therefore, all the three parameters W , Q and η are determined by the thermal model and input parameters to the model. These input parameters include heater, cooler, mean effective pressure, type of working gas and geometrical specification of the engine.

Table 2 and 3 indicate that when mean effective pressure of the engine increases from 2.76 MPa to 6.90 MPa, best performance (efficiency and power) prediction of the proposed model exists. When rotation speed increases from 16.67 Hz to 58.33 Hz, error in prediction of performance of the proposed model increases. The proposed model may have the best performance prediction at a low rotation speed, with mean effective pressure between 4.14 MPa and 5.52 MPa.

However, there is still some discrepancy between the simulation results of proposed model and the experimental data. In the future researches, more accurate models of Stirling engine may be developed by considering other irreversibilities such as heat loss to the environment, gas spring hysteresis, and etc. It is worth pointing that there are more accurate Stirling engine models. For example, polytropic simulation

Table 1: Design specifications of the GPU-3 Stirling engine [13, 29]

Parameter	Value
Engine type	β
Working gas	Helium
Mass of the working gas	1.136 g
<i>Heater</i>	
Number of tubes	40
Tube external diameter	4.83×10^{-3} m
Tube internal diameter	3.02×10^{-3} m
Tube length (cylinder side)	0.1164 m
Tube length (regenerator side)	0.1289 m
<i>Cooler</i>	
Number of tubes	312
Tube external diameter	1.59×10^{-3} m
Tube internal diameter	1.09×10^{-3} m
Average tube length	4.61×10^{-2} m
<i>Regenerator</i>	
Number of regenerator	8
Regenerator internal diameter	2.26×10^{-2} m
Regenerator length	2.26×10^{-2} m
Diameter of regenerator tube	4×10^{-5} m
Material	Stainless steel
<i>Volume</i>	
Swept Vol. (expansion/compression)	120.82/114.13 cm ³
Clearance Vol. (expansion/compression)	30.52/28.68 cm ³
Dead Vol. (heater/cooler/regenerator)	70.28/13.18/50.55 cm ³

Table 2: Thermal efficiency of the proposed model, previous thermal models and experimental data(at $T_{hw} = 922$ K and $T_{cw} = 288$ K)

Rotation speed (Hz)	Mean effective pressure (MPa)	Thermal efficiency predicted by the simple analysis (variable Pr [7])			Thermal efficiency predicted by the adiabatic analysis (simple II [10])			Thermal efficiency predicted by the proposed Stirling Engine model			Experimental efficiency [13]
		Value (%)	Error (%)	Average error (%)	Value (%)	Error (%)	Average error (%)	Value (%)	Error (%)	Average error (%)	Actual value (%)
16.67	2.76	38.72	18.22	17.90	32.48	11.98	12.85	28.16	7.66	12.10	20.50
25.00		36.16	15.46		31.21	10.51		27.75	7.05		20.70
33.33		33.79	15.79		29.45	11.45		27.43	9.43		18.00
41.67		31.48	16.28		27.45	12.25		27.17	11.97		15.20
50.00		29.12	17.32		25.21	13.41		26.94	15.14		11.80
58.33		29.74	24.34		22.89	17.49		26.74	21.34		5.40
25.00	4.14	35.65	10.85	11.46	32.29	7.49	8.28	27.29	2.49	6.65	24.80
33.33		33.52	9.62		30.40	6.50		26.94	3.04		23.90
41.67		31.48	10.18		28.39	7.09		26.65	5.35		21.30
50.00		29.45	11.25		26.33	8.13		26.39	8.19		18.20
58.33		27.40	15.40		24.21	12.21		26.17	14.17		12.00
41.67	5.52	31.20	8.70	10.82	28.59	6.09	8.11	26.24	3.74	7.48	22.50
50.00		29.33	10.53		26.62	7.82		25.97	7.17		18.80
58.33		27.44	13.24		24.62	10.42		25.73	11.53		14.20
50.00	6.90	29.07	10.37	11.73	26.61	7.91	9.19	25.62	6.92	9.05	18.70
58.33		27.29	13.09		24.67	10.47		25.37	11.17		14.20

Table 3: Output power of the proposed model, previous thermal models and experimental data(at $T_{hw} = 922\text{ K}$ and $T_{cw} = 288\text{ K}$)

Rotation speed (Hz)	Mean effective pressure (MPa)	Output power predicted by the simple analysis (variable Pr [7])			Output power predicted by the adiabatic analysis (simple II [10])			Output power predicted by the proposed Stirling Engine model			Experimental output power (kW) [13]
		Value (kW)	Error (%)	Average error (%)	Value (kW)	Error (%)	Average error (%)	Value (kW)	Error (%)	Average error (%)	Actual value (kW)
16.67	2.76	1.796	119.02	272.03	1.772	116.10	254.71	0.861	4.98	104.84	0.82
25.00		2.555	128.13		2.500	123.21		1.253	11.88		1.12
33.33		3.215	165.70		3.117	157.60		1.632	34.88		1.21
41.67		3.769	211.49		3.615	198.76		2.001	65.37		1.21
50.00		4.195	303.37		3.973	282.08		2.362	127.12		1.04
58.33		4.505	704.46		4.203	650.54		2.715	384.82		0.56
25.00	4.14	3.844	114.75	259.70	3.761	110.11	158.41	1.818	1.56	39.83	1.79
33.33		4.856	120.73		4.708	114.00		2.362	7.36		2.20
41.67		5.734	136.94		5.501	127.31		2.890	19.42		2.42
50.00		6.462	174.98		6.126	160.68		3.405	44.89		2.35
58.33		7.030	306.36		6.573	279.94		3.908	125.90		1.73
41.67	5.52	7.645	133.08	180.02	7.334	123.60	164.91	3.742	14.09	43.68	3.28
50.00		8.655	163.87		8.206	150.18		4.401	34.18		3.28
58.33		9.470	243.12		8.858	220.94		5.045	82.79		2.76
50.00	6.90	10.788	174.50	287.04	10.223	160.13	263.63	5.362	36.44	97.75	3.93
58.33		11.840	399.58		11.071	367.13		6.140	159.07		2.37

models of Stirling engine show higher accuracy than our proposed model [13, 17]. However, the model needs more costly calculations and the polytropic indexes are engine-specific.

3.3. Heat transfer between the engine and the fluids

For a Stirling engine thermal process, the wall temperatures of the heater and cooler are considered to be uniform and constant. The heat transferred between the wall and the fluids is

$$(T_w - T)UdA = q_m c_p dT \quad (24)$$

with $T(0) = T_i$, $T(A) = T_o$,

$$\frac{T_o - T_w}{T_i - T_w} = \exp\left(-\frac{UA}{q_m c_p}\right) \quad (25)$$

For a Stirling engine, T_{hw} or T_{cw} can be used to substitute T_w to get the relationships between $T_{i,h}$, $T_{o,h}$ and T_{hw} , or $T_{i,c}$, $T_{o,c}$ and T_{cw} respectively.

$$\frac{T_{o,h} - T_{hw}}{T_{i,h} - T_{hw}} = \exp\left(-\frac{U_h A_h}{q_{m,h} c_{p,h}}\right) \quad (26)$$

$$\frac{T_{o,c} - T_{cw}}{T_{i,c} - T_{cw}} = \exp\left(-\frac{U_c A_c}{q_{m,c} c_{p,c}}\right) \quad (27)$$

Heat transferred from heating fluid to Stirling engine in a cycle

$$q_{m,h} c_{p,h} (T_{i,h} - T_{o,h}) / s_{se} = Q \quad (28)$$

Heat transferred from Stirling engine to cooling fluid in a cycle

$$q_{m,c} c_{p,c} (T_{o,c} - T_{i,c}) / s_{se} = Q - W \quad (29)$$

4. Modeling of the SEAs

As mentioned in section 2, there are five basic connection types for an SEA. All other connection types are the combination of these five basic connection types. This paper investigates the five basic connection types.

To determine the performance of an SEA, models of all the Stirling engines need to be built depending on their thermodynamic characteristic. Stirling engines are chosen to have the same parameters including the same speed s_{se} . This is a reasonable assumption when using SEA for power generation, where the output power frequency should be constant. The speed of Stirling engine can be calibrated by speed controller

system [30]. To eliminate interference of other factors, heating and cooling fluids are chosen to have same parameters for different connection types of SEAs. To clearly find out the performance differences of different SEAs, large temperature differences of the heating/cooling fluids after heat exchange with the engines are preferred. Air was chosen as the cooling fluid instead of commonly used water to avoid small temperature rise and evaporation in the cooling process. Design parameters of Stirling engines are the same as shown in Table 1. Other parameters of Stirling engines and heating/cooling fluids in SEAs are shown in Table 4. Rotation speed of the engines and mean effective pressure were chosen to be 25 Hz and 5 MPa respectively to get the best Stirling engine model for performance prediction, as pointed in section 3.2.

Table 4: Parameters of SEA models

Parameter	Value	Parameter	Value
Heating fluid	Air	$q_{m,h}$	0.4 kg/s
Cooling fluid	Air	$T_{i,h}$	1000 K
n_{se}	6	$p_{i,h}$	5×10^5 Pa
s_{se}	25 Hz	$q_{m,c}$	0.4 kg/s
p_{se}	5 MPa	$T_{i,c}$	300 K
$U_h A_h$	180 W/K	$p_{i,c}$	5×10^5 Pa
$U_c A_c$	180 W/K		

In an SEA, there are 2 flows as shown in Figure 2. In a serial flow, each engine's mass flow rate is q_m , and from the flow's direction, for $2 \leq x \leq n_{se}$, $T_{i,x} = T_{o,x-1}$. In a parallel flow, each engine's mass flow is q_m/n_{se} , for $2 \leq x \leq n_{se}$, $T_{i,x} = T_{i,h}$.

MATLAB was used as the programming tool to build the model of SEAs, and CoolProp was used to provide fluid properties for MATLAB program. Five basic SEA models composed of the aforementioned Stirling engines and fluids were built. To compare SEA connection types under various conditions, several parameters are investigated to find out their effects on SEA performance.

5. Result Analysis

SEA models with specified parameters in Table 4 were built and calculated. Results of the performances of the SEAs are shown in Table 5, it can be found that under specified parameters Type 3 has the highest efficiency and output power, while Type 1 has the lowest efficiency and output power.

Table 5: Results of SEA models under specified parameters

Parameter	Value	Parameter	Value
η_1	0.2215	P_1	8022 W
η_2	0.2273	P_2	8483 W
η_3	0.2277	P_3	8512 W
η_4	0.2227	P_4	8116 W
η_5	0.2263	P_5	8399 W

5.1. Effects of $T_{i,h}$

According to Carnot cycle efficiency formula, the temperature of heating fluid determines the efficiency of Stirling engine array. For a Stirling engine, lower temperature heating fluid leads to a lower efficiency. The efficiency and output power may drop to 0 due to its insufficient heating fluid temperature to drive the engine.

Curves of performance of SEAs and $T_{i,h}$ are shown in Figure 5. As it is shown, with the increase of $T_{i,h}$, both η and P increase for all SEAs. For some types of SEA, when $T_{i,h}$ is lower than a critical temperature, some of the engines in the SEA will not work and there will be turning points on the $\eta - T_{i,h}$, $P - T_{i,h}$ curves. For example, in SEA of Type 1, when $T_{i,h}$ is lower 820 K, all the engines stop working, turning points at 820 K can be found on the $\eta - T_{i,h}$, $P - T_{i,h}$ curves in Figure 5.

From curves in Figure 5, it can be concluded that Type 2 and Type 3 have the best performance, and Type 2 has the best adaptability for lower $T_{i,h}$. All engines in Type 2 work at 730 K.

5.2. Effects of $q_m c_p$

According to Equation 28, 29, $q_m c_p$ (both $q_{m,h} c_{p,h}$ and $q_{m,c} c_{p,c}$) will affect the heat transfer process, which is one of the vital factor for the performance of SEA.

Curves of performance of SEAs and $q_{m,h} c_{p,h}$ are shown in Figure 6. For a large $q_{m,h} c_{p,h}$ (> 800 W/K), Type 2, Type 3 and Type 5 have similar performance, which can be interpreted as the cooling fluid has the same properties for the two types of SEAs, and for a large $q_{m,h} c_{p,h}$, the heating fluid has similar effect after diverged. Similar performance of Type 1 and Type 4 can be also interpreted for the same reason.

Curves of performance of SEAs and $q_{m,c} c_{p,c}$ are shown in Figure 7. For a connection type of SEA, the performance improves with the increase of $q_{m,c} c_{p,c}$. For a large $q_{m,c} c_{p,c}$ (> 800 W/K), Type 2 and Type 3 have similar performance, which means the flow order doesn't affect the performance of SEA with a large $q_{m,c} c_{p,c}$. There exists an intersection point (at 830 W/K) of curves of Type 4 and Type 5. For a larger $q_{m,c} c_{p,c}$, Type 4 has a better performance, and vice versa. This can be interpreted that larger $q_{m,c} c_{p,c}$

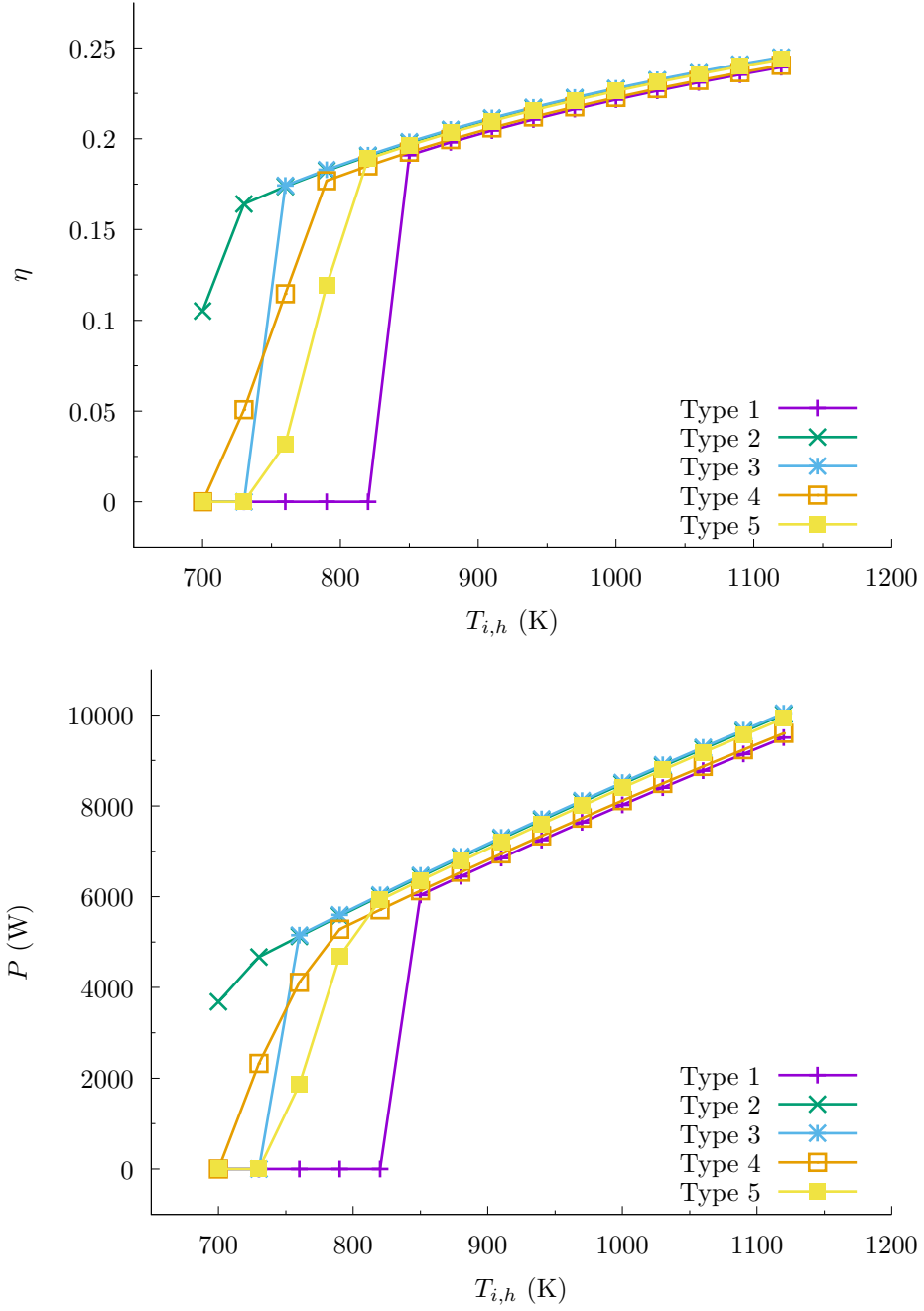


Figure 5: Influence of $T_{i,h}$ on efficiency and power of SEA

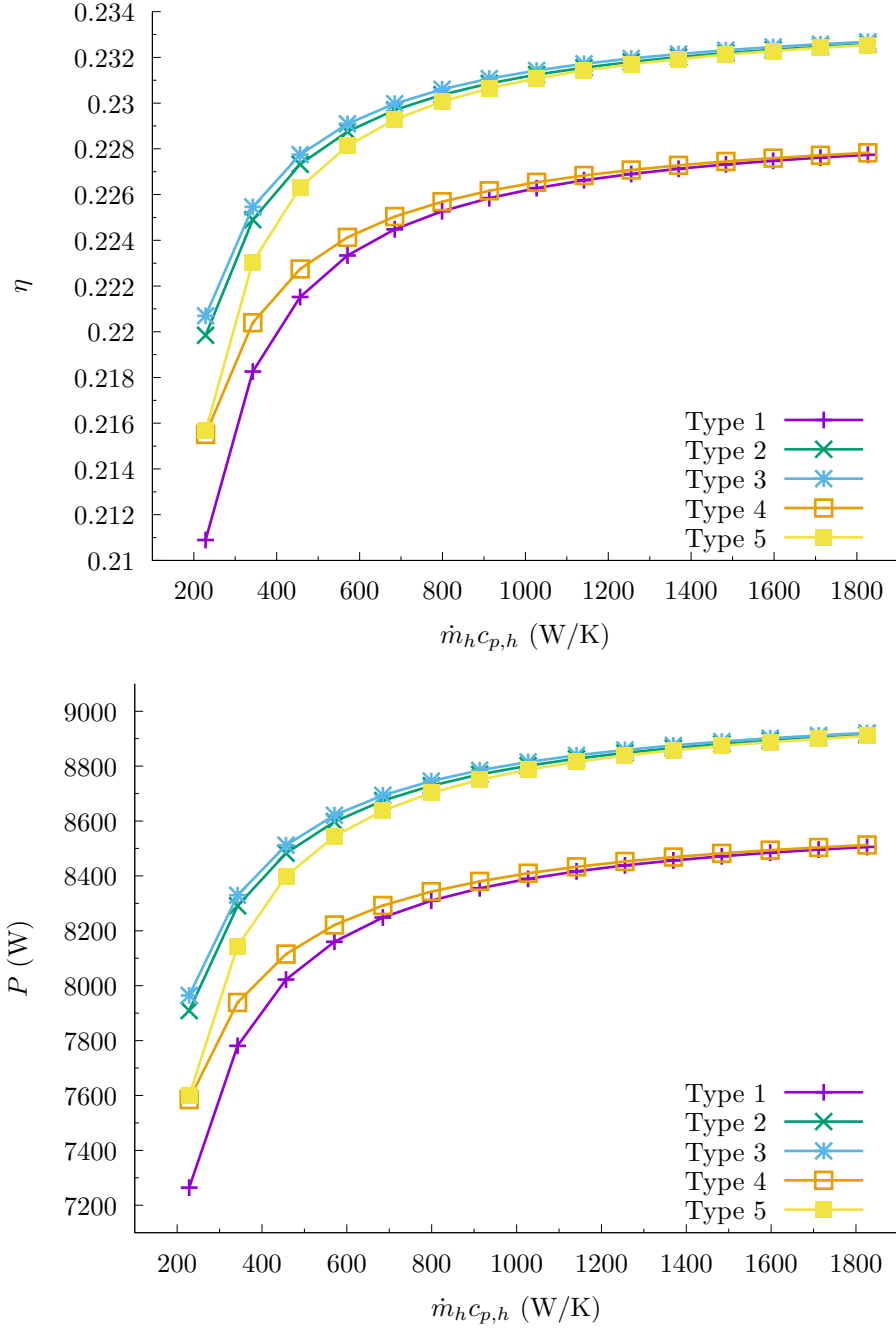


Figure 6: Influence of $q_{m,h}c_{p,h}$ on efficiency and power of SEA

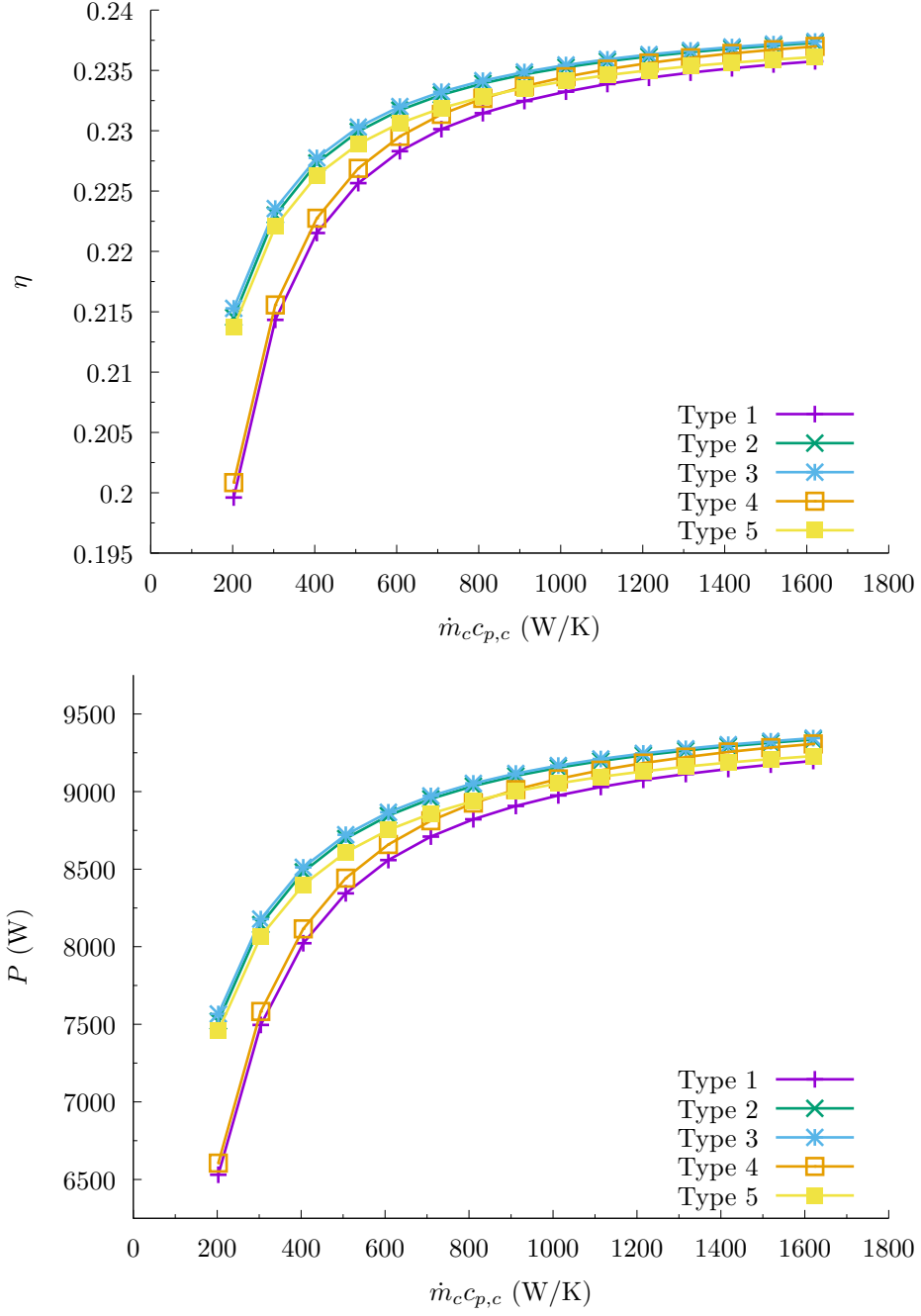


Figure 7: Influence of $\dot{m}_c c_{p,c}$ on efficiency and power of SEA

weaken the drawback of larger temperature rise of parallel flow, while for the heating fluid, temperature drop of serial flow is smaller than parallel flow.

5.3. Effects of n_{se}

By varying the number of engines in SEA, the performance levels changed accordingly. n_{se} may affect both the flow rates and temperatures of fluids of each engine. Figure 8 shows curves of performance of SEAs with different n_{se} . As it is shown, with an increase of n_{se} leads to a reduction of η for all SEAs due to smaller heating and cooling average temperature difference for more engines. For some types of SEA, when n_{se} is larger than a critical value, some of the engines in the SEA will not work and the curves will dive. E.g. for SEA of Type 1, when n_{se} is larger than 9, all the engines stop working, turning points at 9 can be found on the η - n_{se} , P - n_{se} curves in Figure 8.

For a certain connection type, increase n_{se} will reduce the efficiency of SEA. For some connection types, increase n_{se} will reduce the output power P due to inoperative engines and smaller output power engines. It is important to choose the number of engines for some connection types of SEA.

For Type 1, when $n_{se} \geq 10$, all engines stop working for given heating and cooling fluids due to small $\dot{m}c_p$. For Type 2 and Type 3, every engine in the SEAs works. η reduces with increasing n_{se} due to smaller temperature difference of the fluids, and P increases due to more operating engines. For Type 4, by checking results, it can be found that when $n_{se} = 13$, the last engine doesn't work; when $n_{se} = 14$, only the first 10 engines will work; when $n_{se} = 15$, the working engine number drops to 9. For Type 5, by checking results, it can be found that when $n_{se} = 12$, the last 2 engines stop working; when $n_{se} = 13$, only the first 8 engines will work; when $n_{se} = 14$, the working engine number drops to 6; when $n_{se} = 15$, the working engine number drops to 4. The aforementioned strategy is applied to achieve maximum total output power. For Type 4, when $n_{se} \geq 13$, the number of the operating engines is changed to be 12 to achieve maximum output power. For Type 5, when $n_{se} \geq 12$, the number of the operating engines is changed to be 11 to achieve maximum output power. Horizontal lines in Figure 8 shows the application results of the strategy.

6. Conclusion

This paper focused on the configuration optimization of Stirling engine array that can be used in a cascade system. Connection type of the engines may change the flow rates and temperatures of the fluids, as a result the performance of the SEA will be different depending on the connection schemes. In order to compare performance of SEAs with different arrangements, five basic connection types of SEA were summed up according to flow type and flow order.

Analytical Stirling engine model was created to develop the SEA models for the investigation of influence of connection types. Imperfect regeneration and cycle irreversibility of Stirling engine cycle and heat

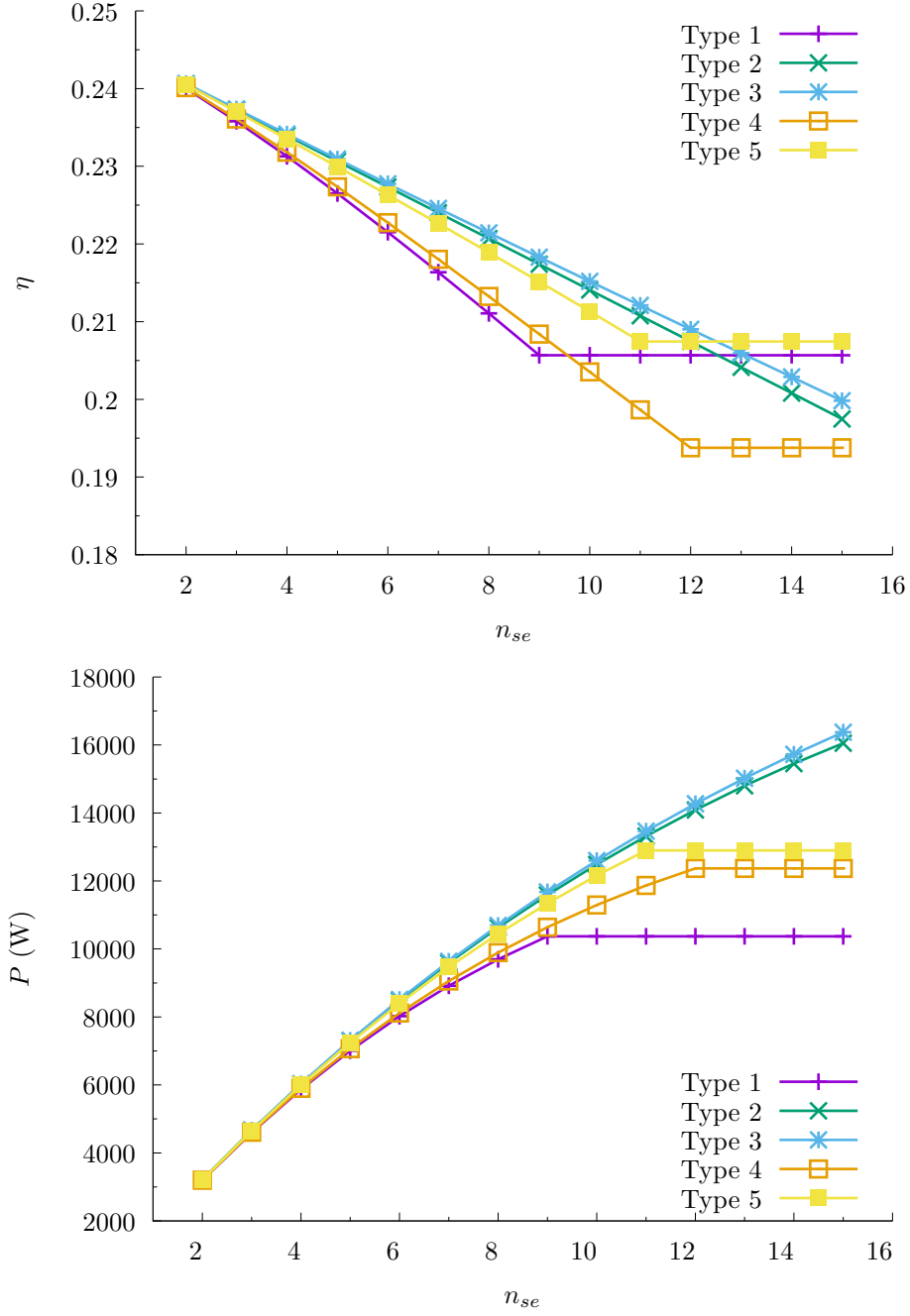


Figure 8: Influence of n_{se} on efficiency and power of SEA

exchange process between fluids and engine were considered in the model. Algorithm to numerically solve different connection types of SEA was developed. The model was evaluated by considering the prototype GPU-3 Stirling engine as a case study. Result shows that the proposed model predicted the performance with higher accuracy than Simple model [7] and Simple II model [10].

Models of SEAs were developed to calculate the performance under different parameters to find out the impacts of $T_{i,h}$, $q_{m,h}c_{p,h}$, $q_{m,c}c_{p,c}$ and n_{se} on different connection types. It was found that, as expected, decrease $T_{i,h}$ and $q_{m,c}c_{p,c}$ will weaken the performance of SEA of all connection types. However, for some connection types, there exists a critical temperature below which some engines stop working. This needs to be considered for SEA connection type selection, especially when $T_{i,h}$ is low. For given heating and cooling fluids, Type 2 has the best performance and adaptability. Type 2 and Type 3 have similar performance under different parameters ($T_{i,h}$, $T_{i,c}$ and $q_{m,c}c_{p,c}$), which means the flow order has little influence on the performance of an SEA. SEA of serial flows (Type 3) has the best performance and adaptability under different parameters. Given heating and cooling fluids, using serial flow is the best choice for the connection type of an SEA.

It is important to note that, in the future researches, the experiments of influence of connection type on SEA's performance can be carried out to verify the conclusions in this paper.

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Nomenclature

A	heat transfer area (m^2)
c_p	specific heat at constant pressure ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)
c_v	specific heat at constant volume ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)
e	regenerator effectiveness
J	annular gap cylinder displacer (m)
K	dead volume factor
k	specific heat ratio (c_p/c_v), thermal conductivity ($\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$)
m	mass of working fluid in Stirling engine (kg)
n_{se}	number of Stirling engine in SEA
P	power of Stirling engine (W)
p	pressure (Pa)
Q	absorbed heat (J)
q_m	mass flow rate ($\text{kg}\cdot\text{s}^{-1}$)
R	gas constant ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)
s_{se}	speed of Stirling engine (Hz)
T_H	working fluid temperature in the hot space (K)
T_L	working fluid temperature in the cold space (K)
T_R	effective working fluid temperature in regenerator (K)
T_w	wall temperature (K)
U	overall heat transfer coefficient ($\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$)
V_C	compression volume (m^3)
V_D	total dead volume (m^3)
V_E	expansion volume (m^3)

V_{DC}	cold space dead volume (m^3)
V_{DH}	hot space dead volume (m^3)
V_{DR}	regenerator dead volume (m^3)
W	output work (J)
Z	displacer stroke (m)

Abbreviations

SEA	Stirling engine array
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Greek Symbols

γ_H	space ratio in process 12
⁴⁰⁰ γ_L	space ratio in process 34
μ	dynamic viscosity ($\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$)

Subscripts

c	cooling fluid
cw	cooler wall
h	heating fluid
hw	heater wall
i	inlet
o	outlet
p	piston
r	regenerator
th	theoretical

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