



$$\begin{aligned}
 \text{Loss: } & \min L_{VAE} - MI(x, c) \\
 & = L_{VAE} + H(X|C) - H(X) \\
 & = L_{VAE} - E_{c \sim p(c)} E_{x \sim p(x|c)} [\log p(x|c)] + \text{const} \\
 & = E_q [\log p(x|z, c)] - \beta D_{KL}[q(z, c) || p(z, c)] - E_{c \sim p(c)} E_{x \sim p(x|c)} [\log p(x|c)] + \text{const}
 \end{aligned}$$

To enable back prop,  $q(c|x)$  is approximated by continuous Gumbel-softmax distribution

Combine VAE and Infomax, maximize the mutual information between data and cluster assignment