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Gradient Similarity in Lezgian Laryngeal Harmony: Representation and Computation

Huteng Dai

1. Introduction

Phonological similarity plays a key role in defining the classes of sound that interact in phonology (natural classes), which is often related to the phonetic information (Kaun, 1995; Burzio, 2002; Wayment, 2009); for this reason, any proposal on phonetics-phonology interface must define phonological similarity. In particular, the study of phonological similarity has been a critical part of Phonetically-based Phonology (Steriade, 2008; Hayes et al., 2004; Flemming, 2013) and Contrastive Hierarchy (Dresher, 2009; Mackenzie, 2011) under the name of "contrast", "distinctiveness", "perceptibility", and "perceptual distance/salience" depends on the contexts (Steriade, 2008, 2007; Hume & Johnson, 2001). Moreover, phonological similarity is the basis of the correspondence relation in any output-driven phonological maps (Tesar, 2014; Magri, 2018). The input-output correspondence (faithfulness) constraints in Optimality Theory (OT) (Prince & Smolensky, 2004) correlates the input-output similarity with the grammaticality of candidates. This evaluation extends to other correspondence relations, such as Base-Reduplicant correspondence (McCarthy & Prince, 1995), Output-output correspondence (Paradigm Uniformity) (Benua, 1997; Burzio, 2002), and Agreement-by-correspondence (ABC) (Rose & Walker, 2004; Hansson, 2010).

However, the **structural** properties of phonological representation necessary to compute similarity are often left undefined in previous works. Structure in this article indicates the **abstract** properties of any linguistic theory and description, and specifically multi-dimensional lattice structures for computing feature-based similarity. The current paper encodes similarity as a weighted featural similarity lattice. The gradient representational system proposed correctly predicts the behavior of stops in Lezgian, providing a reanalysis of the consonant agreement patterns in Ozburn & Kochetov (2018).

This paper is organized as follows: §2 introduces the basics of Agreement by Correspondence and previous similarity metrics; §3 shows the laryngeal harmony pattern and the special status of [CONSTRICTED GLOTTIS] in Lezgian; §4 and §5 show the representational and computational proposals in the current study.

2. Similarity and ABC

This section provides the background of Agreement-by-correspondence (ABC) (Rose & Walker, 2004; Hansson, 2010), and the traditional similarity metrics.

2.1. ABC: the basics

In ABC, the family of correspondence constraints CORR[Seg $\leftrightarrow$ Seg] ensures that two sufficiently similar segments must be in correspondence. IDENT-CC[FEATURE] (or CC-IDENT, IDENT-SegSeg) further enforces the agreement between correspondents on certain feature.

For example, Bolivian Aymara bans the cooccurence of homorganic aspirated stop and voiceless stop ($K^h \leftrightarrow K$; K'=Ejective, K=Voiceless, K^h=Aspirated, G=Voiced, G'=Implosive; T and K indicate different PLACE), but allows sufficiently dissimilar pairs, such as heterorganic $K^h \leftrightarrow T$.

* Huteng Dai, Rutgers University, huteng.dai@rutgers.edu. I would like to thank Adam McCollum, Bruce Tesar, Brian Pinsky, Adam Jardine, Robin Karlin, and audiences at LSA 2020, BLSW 2020, and WCCFL 2020 for their comments and insights. My special thanks are extended to Alan Yu for providing the valuable recordings of Lezgian.

(1) *ABC analysis in Bolivian Aymara* (Rose & Walker, 2004: modified)

/$\mathrm{k^h}$usku/	CORR[$\mathrm{K^h}$↔K]	IDENT-CC[SG]	IDENT-IO[SG]	CORR[$\mathrm{K^h}$↔T]
a. ☞ $\mathrm{k^h}_x\mathrm{us}_z\mathrm{k^h}_x\mathrm{u}$			*	
b. $\mathrm{k^h}_x\mathrm{us}_z\mathrm{k}_x\mathrm{u}$		*!		
c. $\mathrm{k^h}_x\mathrm{us}_z\mathrm{k}_y\mathrm{u}$	*!			

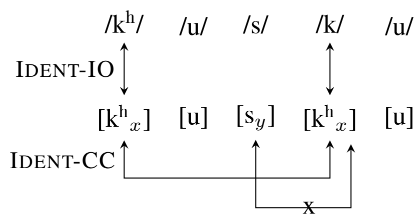


Figure 1: The evaluation of similarity in input-output and surface correspondence; X indicates dissimilar segments that are not in correspondence

As illustrated in (1) and Figure 1, the ABC analysis CORR[$\mathrm{K^h}$↔K] $\gg$ IDENT-IO[SG] correctly predicts the mapping /$\mathrm{k^h}$usku/→[$\mathrm{k^h}$usk$^\mathrm{h}$u].

In ABC, phonological similarity is encoded in the correspondence hierarchy (2) and grounded on the categorical feature-based similarity metrics (3).

(2) *Similarity-based correspondnece hierarchy* (Rose & Walker, 2004)
CORR[T↔T] $\gg$ CORR[T↔D] $\gg$ CORR[K↔T] $\gg$ CORR[K↔D] $\gg \ldots$

(3) *Categorical feature-based similarity metric* (Frisch et al., 2004)

$$\textit{similarity}(x,y) = \frac{\text{the number of shared features/natural classes}}{\text{the total number of shared and nonshared features/natural classes}}$$

Correspondence hierarchy compresses the computed similarity to a one-dimensional segmental similarity scale (Rose & Walker, 2004), e. g. t-d-k, which is visualized in Figure 2. On the one-dimensional similarity scale, each slot represents a segment. CORR[Seg ↔ Seg] scans through this scale and evaluates the distance between two segments. The similarity scale encodes **adjacency**: adjacent pairs are crucially more similar than nonadjacent pairs (Rose & Walker, 2004).

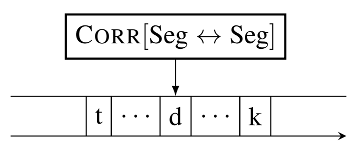


Figure 2: One-dimensional similarity scale encoded by correspondence hierarchy

However, the similarity scale leads to incorrect predictions. For instance, the scale in Figure 2 successfully characterizes that t↔d is more similar than t↔k; however, it also incorrectly predicts that k↔d is more similar than k↔t.

2.2. The problem of categorical feature-based similarity metrics

Categorical feature-based similarity metrics maps the phonological similarity on to a $[0, 1]$ probability scale. This probabilistic nature is one of the crucial distinctions between phonological and phonetic similarity/distance (Mielke, 2012), and is inherited in the current proposal.

From the perspective of Bayesian inference (Tenenbaum & Griffiths, 2001), defined similarity is the belief that two segments x and y are (non-)identical, and it's updated by the observed shared features. When x and y have no shared features, the belief is 0; when x and y are identical, the belief is 1. For example, t↔d is more similar than k↔d, because t↔d has only one nonshared feature [VOICE], while k↔d has three [VOICE], [DORSAL], and [CORONAL].

The categorical feature system entails that the distance from [+] to [-] is 1 step for any feature. Therefore, the categorical feature-based similarity metric predicts that, any segmental pairs with the same amount of shared features have the same similarity. This prediction is further reflected in ABC. When the same similarity, the CORR[Seg ↔ Seg] of segmental pairs will share the ranking status. Therefore, if T↔T' is sufficiently similar to be in agreement, then T↔T^{h}, T↔D must be in agreement as well. However, this prediction is falsified by the typology of laryngeal harmony, as illustrated in Table 1. In fact, the pairs with different [CONSTRICTED GLOTTIS] always trigger harmony, such as *T↔T' and *D↔D', in contrast to the pairs with different [VOICE] and [SPREAD GLOTTIS]. Hansson (2004) provides similar counter-evidence from voice agreement in Kera and Ngizim.

Inventory	minimally dissimilar pairs	Languages
T', T, T^{h}, D	***T↔T'**, ✓T↔D, ✓T^{h}↔T ...	Lezgian, Ndebele
T', T, D	***T↔T'**, *T↔D	Amharic, Chaha, Chontal
T', T, T^{h}	***T↔T'**, *T↔T^{h}	Peruvian & Bolivian Aymara
T', T, D'	***T↔T'**, ✓T'↔D'	Tzotzil, Tzutujil, Yucatec
T', T, D, D'	***T↔T'**, ***D↔D'**, ✓T↔D ...	Hausa
T, D, D'	***D↔D'**, ✓T↔D	Bumo Izon, Kalabari Ijo

Table 1: The typology of laryngeal harmony; $*$ shows the pairs which trigger harmony

The incorrect prediction results in consequences on correspondence hierarchy: there are no rankings among CORR[T↔T^{h}], CORR[T↔T'], and CORR[T↔D]. The same prediction holds when other feature classes are taken into consideration, such as PLACE: there are no rankings among CORR[T↔K^{h}], CORR[T↔K'], and CORR[T↔G]. In other words, if voiceless-ejective pairs are sufficiently similar to trigger the agreement by correspondence, then voiceless-aspirated, voiceless-voiced are also in agreement by correspondence.

Tableaux (4, 5) show the consequence of categorical similarity metrics in ABC. The dominant CORR constraints ensures that T↔T^{h}, T↔T', and T↔D are in correspondence, as indicated by the indices x. IDENT-CC constraints penalize any difference between correspondents. Therefore, consider a set of cooccurrences which have the same amount of non-shared features {K. . .T', K. . .T^{h}, K. . .D}, **if one of the cooccurences does not surface, none of them will surface**.

(4)

/k. . .t'/	CORR[T↔T^{h}]	CORR[T↔T']	CORR[T↔D]	ID-CC[SG]	ID-CC[CG]	ID-CC[VOICE]	ID-IO[CG]
a. ☞ $k_x \ldots t_x$							*
b. $k_x \ldots d_x$						*!	
c. $k_x \ldots t^h{}_x$					*!		
d. $k_x \ldots t'_x$				*!			
e. $k_x \ldots t'_y$		*!					

(5)

/k. . .t^{h}/	CORR[T↔T^{h}]	CORR[T↔T']	CORR[T↔D]	ID-CC[SG]	ID-CC[CG]	ID-CC[VOICE]	ID-IO[SG]
a. ☞ $k_x \ldots t_x$							*
b. $k_x \ldots d_x$						*!	
c. $k_x \ldots t^h{}_x$					*!		
d. $k_x \ldots t'_x$				*!			
e. $k_x \ldots t^h{}_y$		*!					

This implication does not fit natural languages, e. g. {k. . .d, *k. . .t^{h}, *k. . .t'} in Chaha (Rose & King, 2007). A common practice is to appeal to the mechanism outside of ABC, such as IO-faithfulness, to rule out the exceptions, as illustrated in Tableau 6.

(6) *Exception handling with IO-faithfulness*

/k. . .t'/	ID-IO[SG]	ID-IO[CG]	ID-CC[SG]	ID-CC[CG]	ID-CC[VOICE]	ID-IO[VOICE]
a. $\mathrm{k}_x \ldots \mathrm{t}_x$		*!				
b. $\mathrm{k}_x \ldots \mathrm{d}_x$		*!				*
c. $\mathrm{k}_x \ldots \mathrm{t^h}_x$	*!		*			
d. ☞ $\mathrm{k}_x \ldots \mathrm{t'}_x$				*		

However, it's hard to explain why [VOICE], [CG], and [SG] in the same feature class behave differently on IO-faithfulness when their relative phonological similarities are the same. Moreover, the insight of **similarity**-driven agreement lost in entertaining non-ABC mechanisms.

3. Lezgian laryngeal harmony

This section shows the special status of [CONSTRICTED GLOTTIS] in Lezgian laryngeal harmony (Ozburn & Kochetov, 2018).

In Lezgian, there are four classes of stops: T, T', $\mathrm{T^h}$, and D. Co-occurring stops generally agree in LARYNGEAL. *T↔T' is a categorical constraint in the corpus ($N = 0$).

Yet surprisingly $\mathrm{T^h}$↔T and T↔D are also overrepresented in the corpus.

(7) *Underrepresented laryngeal co-occurrences in Lezgian* ($O/E < 1$)
T↔T', T'↔T, T'↔D, D↔T', T'↔$\mathrm{T^h}$, $\mathrm{T^h}$↔T', D↔$\mathrm{T^h}$, $\mathrm{T^h}$↔D, **$\mathbf{T^h}$↔T**, **D↔T**

(8) *Overrepresented laryngeal co-occurrences in Lezgian* ($O/E \geq 1$)

T'↔T'	[q'ats'un]	'get dirty'	T↔T	[qaqa]	'ready'
$\mathrm{T^h}$↔$\mathrm{T^h}$	[$\mathrm{ʧ^h}$i$\mathrm{p^h}$]	'fool'	D↔D	[midad]	'grieve'
$\mathrm{T^h}$↔T	[$\mathrm{k^h}$utsun]	'to flush'	T↔D	[etsigun]	'put'

As a type of phonological evidence, the underrepresented patterns doesn't always undergo agreement patterns, while overrepresented patterns are impossible to undergo agreement patterns. As illustrated in the detailed morphophonological analysis from Ozburn & Kochetov (2018), *T'↔ T is a categorical constraint in Lezgian (0 occurrence in the corpus). The generalization is that, comparing to *T'↔T, $\mathrm{T^h}$↔T and T↔D are sufficiently **dissimilar** to escape the impetus to agree. The challenge is to define similarity as to allow **overrepresented but disharmonic** exceptions $\mathrm{T^h}$↔T and T↔D, while penalizing the underrepresented cooccurrences, such as T↔T'. This scalar similarity motivates the gradient representations. The directionality of agreement is omitted in this article for the convenience of discussion.

4. Representation: weighted featural lattice

The idea of weighted/gradient/real numbered (sub-)featural representation has been proposed since 2000 (Bachra, 2000; Burzio, 2002; Frisch et al., 2004; Rose & Walker, 2004; Wilson & Obdeyn, 2009; Lionnet, 2017). However, weighted features were not well-motivated by empirical and experimental data at that time (Frisch et al., 2004: P204). And even if the experimental/empirical data is available, people couldn't find a lawful way to constrain those weights on features with **representational structures**. Without the structural restrictiveness, it's difficult to incorporate gradient featural representations into phonological generalization and typology.

4.1. Lattice structure and phonetic distance

The categorical feature-based similarity metrics implicate a relative similarity lattice (Tesar, 2014), as illustrated in Figure 3.

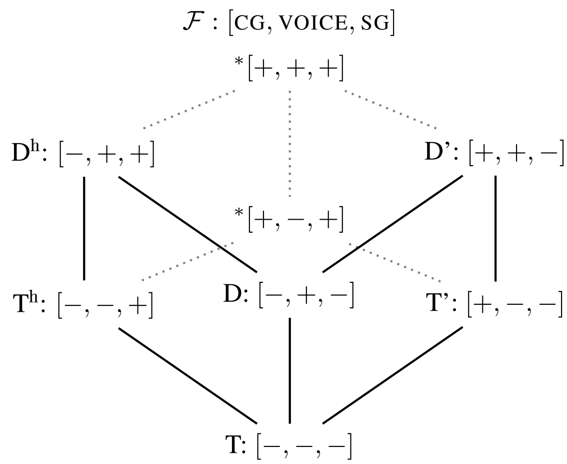


Figure 3: Categorical feature-based similarity lattice for the LARYNGEAL class

The lattice represents the universal structure of a binary feature system, and shows the 8 logically possible combinations of the three LARYNGEAL features. Unattested stop segments in human languages are marked by asterisks. Solid and dashed lines indicate the attested and unattested paths. The lattice precisely characterizes the categorical similarity metric in which the distances from [+] to [-] are exactly the same for any features. For example, the distances from [+VOICE] to [-VOICE], [+SONORANT] to [-SONORANT] are both 1 step. The lattice structure ensures that, for instance, T^{h} is always more similar to T than D, because the distance between T^{h}↔D is always longer than between T^{h}↔T. The current study proposes weighted featural lattice representation. The distance between two segments x and y is defined as the summed weights of their unshared features with respect to the feature set ($\mathcal{F}$) and a weighted featural lattice (**w**):

$$\textit{distance}_{\mathbf{w}}(x, y) = \sum_{f \in \mathcal{F}} \left(w_f \cdot \delta_f \left(x, y \right) \right), \tag{1}$$

$$\delta_f(x, y) = \begin{cases} 0, & \text{if } x \text{ and } y \text{ share the feature } f \\ 1, & \text{else} \end{cases} \tag{2}$$

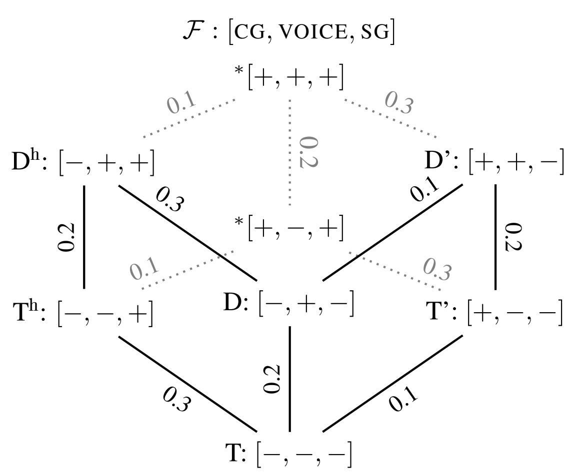


Figure 4: Weighted featural lattice **w** of LARYNGEAL class

The term $\delta_f(x, y)$ is 0 when x and y share the feature f, and 1 elsewhere, which conceptually is the **categorical** distance provided by the lattice structure. The featural weight w_f encodes the **gradient** phonetic distance between [+] and [-] feature values. I assume the featural weights are constant over the lattice and independent from other features, otherwise the system predicts the arbitrary patterns such as the saliency of [DORSAL] depends on value of [SG].

The specific weights in use are $w_{[\text{CG}]} = 0.1$, $w_{[\text{VOICE}]} = 0.2$, $w_{[\text{SG}]} = 0.3$, and the rest featural weights are assumed as 0 at present. This representation captures the intuition that the contrast between T↔T' is less salient than other pairs. The lattice also encodes that, for instance, the distance between T'↔T is always longer than T'↔T^{h}.

Conceptually, the *phonetic distance* here represents the phonetic information encoded in the feature system. The correlation between weights and phonetic similarity is expected (Johnson & Babel, 2010; Johnson, 2012). The next section provides a restrictive method to convert this **phonetic** representation into the **phonological** similarity.

4.2. Phonological similarity as probability

The computed *phonetic distance* on weighted featural lattice is normalized by the maximal distance between two segments (the sum of all featural weights $\sum_{f\in\mathcal{F}} w_f$). The term $\textit{distance}_{\mathbf{w}}(x, y)/\sum_{f\in\mathcal{F}} w_f$ is essentially the **dissimilarity** between x and y. The defined similarity converts the *phonetic distance* to a well-formed probability.

$$\textit{similarity}(x, y) = 1 - \frac{\textit{distance}_{\mathbf{w}}(x, y)}{\sum_{f\in\mathcal{F}} w_f} \tag{3}$$

The motivation of this transformation is similar to the categorical similarity metric illustrated in §2 (Frisch et al., 2004). Similarity is the belief that x and y are (non-)identical, which is updated by the observed phonetic distance between two segments. When *distance* $= 0$, two segments are identical; when *distance* is equal to the maximal distance between two segments, two segments are maximally non-identical. As illustrated in 2, the computed similarity naturally produces five thresholds, which are used in phonological computation: 0.8, 0.6, 0.4. 0.2, 0.

C1↓C2→	T'	T	D'	D	T^{h}	D^{h}
T'	1	0.8	0.6	0.4	0.4	0
T	0.8	1	0.4	0.6	0.6	0.2
D'	0.6	0.4	1	0.8	0	0.4
D	0.4	0.6	0.8	1	0.2	0.6
T^{h}	0.4	0.6	0	0.2	1	0.6
D^{h}	0	0.2	0.4	0.6	0.6	1

Table 2: The computed similarity between LARYNGEAL pairs

5. Computation: Agreement by similarity

The current study proposes a family of IDENT-CC[FEATURE/CLASS, *similarity* $\geq k$] constraints, which merges CORR and IDENT-CC constraints, and directly incorporates the computed gradient distance/similarity into the evaluation of surface correspondence. 'CC' here indicates the length of window that the constraint operates on. Instead of some specific feature, this constraint provides the capacity to characterize the agreement pattern over a whole class, such as LARYNGEAL and PLACE. Moreover, the elimination of CORR constraint avoids the pathological **agreement by proxy** patterns (Hansson, 2014; McMullin, 2016), in which two dissimilar segments are in correspondence because of the transitivity of CORR.

(9) IDENT-CC(LARYNGEAL, *similarity* $\geq k$)
if *similarity* $\geq$ k in a 2-long **subsequence**, assign one violation to any difference in LARYNGEAL.

The term **subsequence** from formal language theory describes precedence relation, and has been successfully adopted to the study of non-local phonotactics (Heinz, 2010). For a string $abcd$, the 2-long subsequences include $\{a \ldots b,\ a \ldots c,\ a \ldots d,\ b \ldots c,\ b \ldots d,\ c \ldots d\}$. Ident-CC(Laryngeal, *similarity* $\geq k$) enforces the agreement on Laryngeal class of the subsequences in which *similarity* is lower than certain threshold k. This constraint is referred to as Agreemeent-by-similarity (ABS) contraint henceforth.

A faithfulness constraint Ident-IO[Laryngeal] on Laryngeal class is proposed to evaluate the similarity between input and output. If a language has any long-distance laryngeal agreement pattern, this constraint must be dominated by some ABS constraint.

(10) Ident-IO[Laryngeal]
Assign one violation to any input-output difference in Laryngeal.

(11) Ranking in Lezgian laryngeal harmony
Id-CC(Laryn., *s.* ≥ 0.8) $\gg$Id-IO[Laryn.] $\gg$Id-CC(Laryn., *s.* ≥ 0.6)

Consider a Lezgian example /qats'un/→[q'ats'un] 'get dirty' (Ozburn & Kochetov, 2018). The 2-long subsequences in *[qats'un] include {q. . .a, q. . .ts', q. . .u, q. . .n, a. . .ts', a. . .u, a. . .n, ts'. . .u, ts'. . .n, u. . .n}. The similarity between q and ts' is below the threshold $s. \geq 0.8$ which triggers Laryngeal agreement, while the similarity between ts' and n is not. Henceforth, only the similarity between stops in following tableaux are exhibited to save the space.

(12)

/qats'un/	Id-CC(Laryn., *s.* ≥ 0.8)	Id-IO[Laryn.]	Id-CC(Laryn., *s.* ≥ 0.6)
a. qats'un [*s.* $= 0.8$]	*!		*
b. ☞ q'ats'un [*s.* $= 1$]		*	

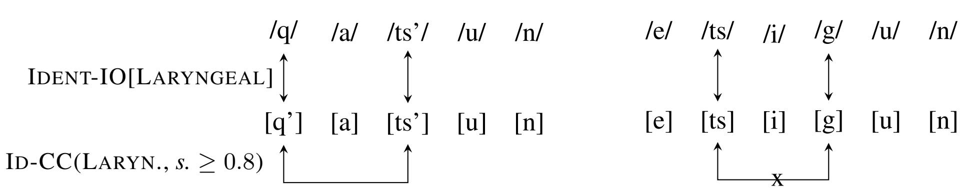
The ranking also correctly predicts the overrepresented but non-identical patterns T↔D and T↔T^h. As illustrated in Tableaux (13,14), the similarities in ts↔g and k^h↔ts are lower than the threshold ~ 0.8 to trigger Laryngeal agreement. This ABS analysis is further exhibited in Figure 5.

(13)

/etsigun/	Id-CC(Laryn., *s.* ≥ 0.8)	Id-IO[Laryn.]	Id-CC(Laryn., *s.* ≥ 0.6)
a. edzigun [*s.* $= 1$]		*!	
b. ☞ etsigun [*s.* $= 0.6$]			*

(14)

/k^{h}utsu/	Id-CC(Laryn., *s.* ≥ 0.8)	Id-IO[Laryn.]	Id-CC(Laryn., *s.* ≥ 0.6)
a. k^{h}utsu [*s.* ≥ 1]		*!	
b. ☞ k^{h}utsu [*s.* ≥ 0.43]			*



(a) **(b)**

Figure 5: The analysis of Agreement by similarity in Lezgian laryngeal harmony

A hypothetical example [tit'e] is considered to test if the current proposal incorrectly generalizes that some similar vowels also undergo agreement. The following analysis shows that even if there is enough evidence of *similarity*(i,e) $\geq$ *similarity*(t,t') = 0.8, the unattested vowel harmony pattern can be eliminated by high-ranked faithfulness constraint Id-IO[V-Place.]. It's very common that the different feature classes Laryngeal and V-place behave differently on IO-faithfulness.

(15)

/tit'e/	Id-CC(Laryn., *s.* ≥ 0.8)	Id-IO[Laryn.]	Id-IO[V-Place]	Id-CC(V-Place, *s.* ≥ 0.8)
a. tit'e	*!			
b. t'it'i		*	*!	
c. ☞ t'it'e		*		*

As illustrated in tableau 15, if the similarity of two vowels i and e is also above the similarity threshold *similarity* ≥ 0.8, the Ident-CC(Laryngeal, *similarity* ≥ 0.8) will enforce the Laryngeal but not V-Place agreement. In this case, Ident-CC(V-Place, *similarity* ≥ 0.8) must be ranked below the faithfulness Ident-IO[V-Place] to eliminate unattested vowel harmony patterns.

$$
\begin{tikzcd}
\text{Id-CC(Laryn., } s. \geq 1) \arrow[d] & \text{Id-IO(V-Place)} \arrow[d] \\
\text{Id-CC(Laryn., } s. \geq 0.8) \arrow[d] & \text{Id-CC(V-Place, } s. \geq 1) \arrow[d] \\
\text{Id-IO[Laryngeal]} \arrow[d] & \text{Id-CC(V-Place, } s. \geq 0.8) \arrow[d] \\
\text{Id-CC(Laryn., } s. \geq 0.6) & \text{Id-CC(V-Place, } s. \geq 0.6)
\end{tikzcd}
$$

Figure 6: The Hasse diagram of constraint ranking in Lezgian laryngeal harmony

6. Discussion

This section discusses the typological prediction and theoretical implication of the current proposal.

6.1. Typological prediction

The gradient properties of the current proposal provides the capacity to handle patterned exceptions, while the restrictiveness of lattice structure enables the typological predictions.

The lattice structure entails that the salience of one different feature must be lower than two or more. This stringency relation further predicts that *similarity*(T', D) and *similarity*(T', T^h) is always larger than *similarity*(T', T). Thus, if T'↔D, T'↔T^h are sufficiently similar to trigger agreement, T'↔T must also in agreement. This aligns with the typology of laryngeal harmony exhibited in Table 3. Moreover, by varying the thresholds of similarity for triggering agreement, we can predict the formal typology of laryngeal harmony.

6.2. Theoretical implication

The current study critically examines the similarity metrics in Agreement-by-correspondence, and shows the importance of a correct representational structure for phonological similarity. This paper further proposes a weighted featural lattice as the representational structure which is sufficiently robust to characterize the patterned exceptions, and restrictive to predict the formal typology in laryngeal harmony.

Unlike other proposals of gradient phonological representation, gradient information, the phonetic distance, is not directly represented in phonology, but in phonetic module. In the weighted featural lattice, the lattice structure represents the phonological, symbolic, universal structure of feature system, while the weights on the edges represent the phonetic, gradient, language-specific details, which is only available in the computation of similarity. Moreover, the computation of similarity is **only** triggered in the evaluation of IO- and CC-correspondence. This proposal implicates that the phonetic knowledge is only accessible by phonology in the phonological computation.

An alternative of representation is to posit new features to capture the gradient similarity, such as a language-specific feature [tense] which groups the ejective and voiceless stop in Legzian (Ozburn & Kochetov, 2018). The current proposal is skeptical about this proposal since it collapse the universal and language-specific information into phonology module. Moreover, the lattice structure necessary

Inventory	Thresholds	Cooccurrences in Agreement	Languages
T', T	0.8	T'↔T	Gitksan
T', T, D	0.8	T'↔T	Amharic, Chol, Chontal
	0.6	T'↔T, T'↔D	Chaha
T', T, T^h	0.8	T'↔T	
	0.43	T'↔T, T'↔T^h	Peruvian Aymara, Bolivian Aymara
T', T, D'	0.8	T'↔T	Tzotzil, Tzutujil, Yucatec
	0.6	T'↔T, T'↔D'	
T', T, D, D'	0.8	T'↔T, D'↔D	Hausa
	0.6	T'↔T, D'↔D, T↔D, T'↔D'	
	0.57	T'↔T, D'↔D, T↔D, T'↔D', T↔D', T'↔D	
T', T, T^h, D	0.8	T'↔T	Ndebele, Lezgian
	0.6	T'↔T, T↔D	
	0.57	T'↔T, T↔D, T↔T^h,T'↔D,	
	0.43	T'↔T, T↔D, T↔T^h, T'↔D, T'↔T^h	
	0.29	T'↔T, T↔D, T↔T^h, T'↔D, T'↔T^h, D↔T^h	
T, D, D'	0.8	D'↔D	Bumo Izon, Kalabari Ijo
	0.6	D'↔D, T↔D	
	0.57	D'↔D, T↔D, T↔D'	

Table 3: The typology of laryngeal harmony ($w[\textsc{cg}] > w[\textsc{voice}] > w[\textsc{sg}]$); (Hansson, 2010: modified)

to define feature-based similarity disfavors language-specific features with low functional load, which creates many unattested featural combinations in the lattice. The evaluation metrics for feature economy has been proposed since Clements (2003).

7. Conclusion

The current study accounts for laryngeal co-occurrence patterns in Lezgian by introducing a gradient featural similarity lattice, over which the intra-featural similarity is evaluated. Moreover, the proposed lattice offers a universal structure to link language-specific phonetics to phonological features. The proposal successfully predicts the typology of laryngeal harmony. Future work is needed to verify the induced similarity in perceptual and production experiments.

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