Gradient similarity in Lezgian laryngeal harmony: representation & computation

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Take-home message

Given a well-defined **representational structure** and **similarity metric**, gradient representation is not orthogonal to universal feature system and discrete symbolic computation.

Find the slides and code on `http://hutengdai.com`

What's gradience?

- Given x (a segment/feature/string/...)
  - Categoricity: map x to $\{\mathrm{True}, \mathrm{False}\}$;
  - Gradience: map x to a continuous value, e. g. $[0, 1]$, natural number.
- This formal property underlies diverse research programs: fine-grained similarity, well-formedness, variability ...
- Gradience is **abstraction**!

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Why similarity matters?

Similarity defines the natural classes that interact in **phonology**, which is directly connected to **phonetic** information.

#	Related research programs	Selected works
1.	Output-driven Phonology	Tesar (2014)
2.	Base-Reduplicant Correspondence	McCarthy & Prince (1995)
3.	Paradigm Uniformity	Benua (1997)
4.	**Agreement by Correspondence**	Rose & Walker (2004)
5.	Dispersion Theory	Flemming (2013)
6.	P-map	Steriade (2001)
7.	Similarity avoidance principle	Frisch et al. (2004)
8.	Contrastive Hierarchy	Dresher (2009)
9.	Learning bias	Wilson (2006)
10.	Exemplar phonology	Bybee (2003)

Why similarity matters?

The linguistic inquiry of similarity is under the name of “contrast”, “perceptibility”, “perceptual distance/salience/distinctness” ...

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Lesson from ABC

$$
\begin{array}{lcccccc}
 & /k^h/ & /u/ & /s/ & /k/ & /u/ \\
\textsc{Ident-IO}\ \checkmark & \updownarrow & & & \updownarrow & \\
 & [k^h] & [u] & [s] & [k] & [u]
\end{array}
$$

Bolivian Aymara (Rose & Walker, 2004)

IDENT-IO[SG]

Lesson from ABC

/k^h/ /u/ /s/ /k/ /u/

IDENT-IO $\checkmark$

[$k^h{}_x$] [u] [s] [k_x] [u]

IDENT-CC: $k \leftrightarrow k^h$

Bolivian Aymara (Rose & Walker, 2004)

CORR($T^h \leftrightarrow T$) > IDENT-IO[SG]

Lesson from ABC

/kʰ/ /u/ /s/ /k/ /u/

IDENT-IO: k↔k^h

[$k^h{}_x$] [u] [s] [$k^h{}_x$] [u]

IDENT-CC: ✓

Bolivian Aymara (Rose & Walker, 2004)

IDENT-CC[SG], CORR(Tʰ↔T) > IDENT-IO[SG]

Lesson from ABC

$/k^h/$ $/u/$ $/s/$ $/k/$ $/u/$

IDENT-IO: $k \leftrightarrow k^h$

$[k^h{}_x]$ $[u]$ $[s_y]$ $[k^h{}_x]$ $[u]$

IDENT-CC: ✓

X

Bolivian Aymara (Rose & Walker, 2004)

IDENT-CC[SG], CORR($T^h \leftrightarrow T$) > IDENT-IO[SG]

Similarity and ABC

Similarity is encoded in the **correspondence (Corr) hierarchy**:

$$\textsc{Corr}[\mathrm{T}\leftrightarrow\mathrm{T}] \gg \textsc{Corr}[\mathrm{T}\leftrightarrow\mathrm{D}] \gg \textsc{Corr}[\mathrm{K}\leftrightarrow\mathrm{T}] \gg \textsc{Corr}[\mathrm{K}\leftrightarrow\mathrm{D}] \gg \ldots$$

T'=Ejective, T=Voiceless, T^h=Aspirated, D=Voiced, D'=Implosive, T vs. K: the difference on Place.

(Rose & Walker, 2004)

The probabilistic nature of similarity

- CORR hierarchy is grounded on **categorical** featural similarity metrics:

$$\textit{similarity}(x, y) = \frac{\text{the number of shared features between } x \text{ and } y}{\text{the total number of shared and nonshared features}}$$

as in natural classes-based metrics (Frisch et al., 2004)

- A Bayesian perspective:
  - Similarity is the **belief** that two segments x and y are **(non-)identical**;
  - This belief is updated by the observed shared features.

(Tenenbaum & Griffiths, 2001; Jaynes, 2003)

Structural assumption

- The distance from [+] to [-] is 1 step for any feature.
  - Any pairs of phonemes with the same amount of shared features have exactly the same similarity;
  - If T↔T’ is sufficiently similar to be in agreement, then T↔T^h, T↔D, and T↔K must be in agreement as well.

Lezgian laryngeal harmony

- *T'↔T is a categorical constraint in Lezgian ($N = 0$), and always triggers laryngeal harmony, while Tʰ↔T and T↔D are sufficiently **dissimilar** to escape the impetus to agree.
  - *Underrepresented co-occurrences* ($O/E < 1$)

    T↔T', T'↔T, T'↔D, T'↔Tʰ, D↔T', D↔Tʰ, Tʰ↔D, Tʰ↔T', ...
  - *Overrepresented co-occurrences* ($O/E \geq 1$)

T'↔T'	[q'at͡s'un]	'get dirty'	T↔T	[qaqa]	'ready'
Tʰ↔Tʰ	[tʃʰipʰ]	'fool'	D↔D	[midad]	'grieve'

(Ozburn & Kochetov, 2018)

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Tʰ↔T	[kʰut͡sun]	'to flush'	T↔D	[et͡sigun]	'put'

(Ozburn & Kochetov, 2018)

Challenge to categorical similarity metrics

The calculated similarity neither aligns with the co-occurrence constraints, nor fits the distribution of speech errors.

(Rose & King, 2007)

Inventory	minimally dissimilar pairs	Languages
T', T, T^h, D	*$\mathbf{T{\leftrightarrow}T'}$, ✓T↔D, ✓$T^h$↔T …	Lezgian, Ndebele
T', T, D	*$\mathbf{T{\leftrightarrow}T'}$, ✓T↔D	Amharic, Chaha, Chontal
T', T, T^h	*$\mathbf{T{\leftrightarrow}T'}$, *T↔$T^h$	Peruvian & Bolivian Aymara
T', T, D'	*$\mathbf{T{\leftrightarrow}T'}$, ✓T'↔D'	Tzotzil, Tzutujil, Yucatec
T', T, D, D'	*$\mathbf{T{\leftrightarrow}T'}$, *$\mathbf{D'{\leftrightarrow}D}$, ✓T↔D …	Hausa
T, D, D'	*$\mathbf{D'{\leftrightarrow}D}$, ✓T↔D	Bumo Izon, Kalabari Ijo

Analysis: the special status of [CG]

- Cross-linguistically, we found that different features play different roles in similarity.
- Only the difference on [CG] **always** triggers harmony
- Hypothesis: the distance from [+CG] to [-CG] is systematically shorter than in other LARYNGEAL features.

(Gallagher & Coon, 2009; Kochetov & Ozburn, 2014)

Acoustic cues

Cross-linguistically, the difference of VOT and preceding vowel duration on [CG] is less distinctive than [VOICE] and [SG].

(Beguš, 2017; Gallagher, 2010a)

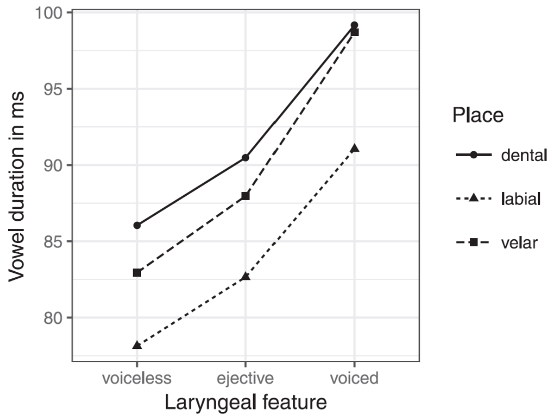


FIG. 2. Estimates of the effects of Laryngeal Features and Place of articulation on preceding vowel duration in ms (from a linear mixed effects model).

Georgian (Beguš, 2017)

Gradient representation: Pros and Cons

- Weighted/gradient/valued (sub-)featural representation can easily handle language-specific phonetic variations;

  (Ladefoged, 1969, 1972, 1973; Keating, 1985)

- However, even if the empirical/laboratory evidence is available, gradient representations are at the cost of the **generality**.

  e. g. universal feature system; typology; similarity metrics; modular representation ...

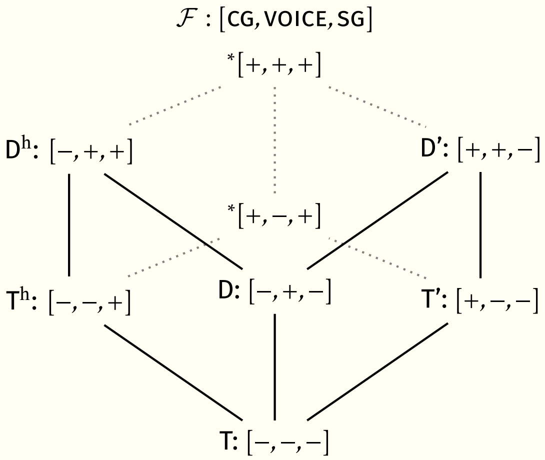
  see criticism in Mackenzie (2009)

Representation

Proposal: weighted featural lattice

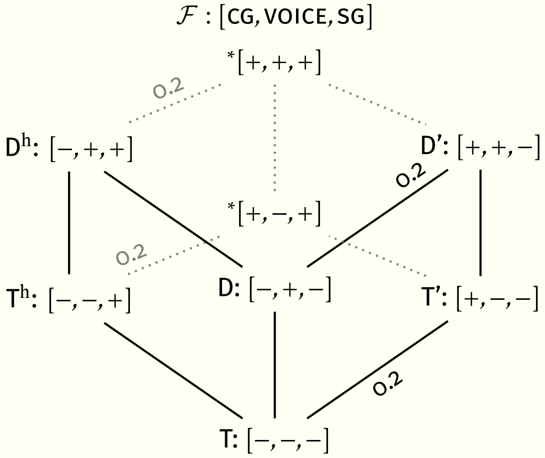
- Lattice: the universal feature system:
  - properties: partial order and bounds;
  - asterisks: the unattested groups in human languages.

(Tesar, 2014)



Proposal: weighted featural lattice

- Weight: the phonetic distance between [+] and [-]
- Restriction: $0 < w_f < 1$
  "How likely two **features** are non-identical?"
- $w_{[\text{CG}]} < w_{[\text{VOICE}]}, w_{[\text{SG}]}$



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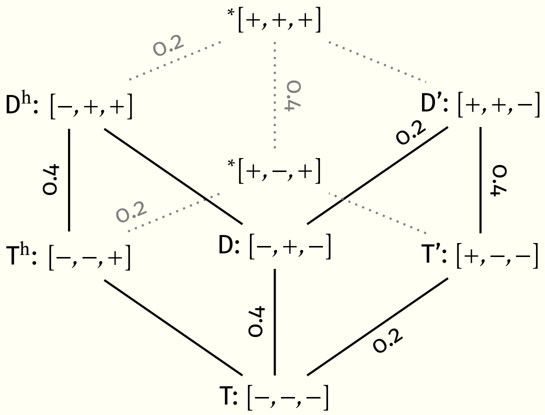
$\mathcal{F}$: [CG, VOICE, SG]



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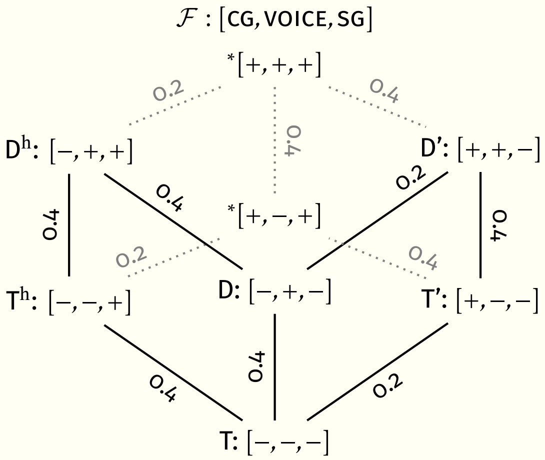
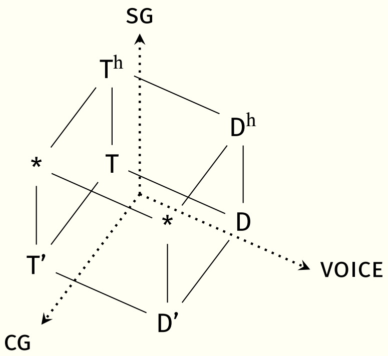
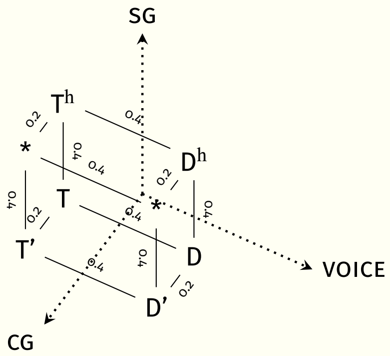


Representational space



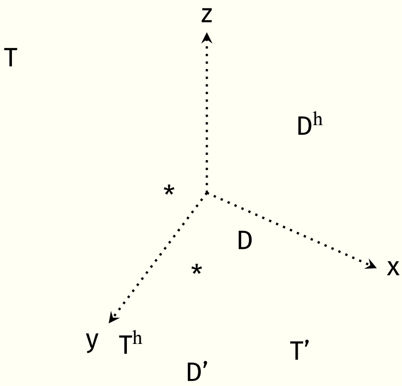
phonological structure $\Longleftrightarrow$ discrete lattice

Representational space



phonetic substance $\Longleftrightarrow$ weighted lattice: scaled by [0.2, 0.4, 0.4]

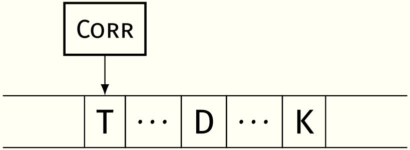
Unconstrained representational space



e. g. *Hilbert space* in Smolensky et al. (2014)

Alternative: segmental similarity scale

- One-dimensional totally-ordered similarity scale;
- The relative similarity is encoded by **adjacency**.



(Rose & Walker, 2004, P.505)

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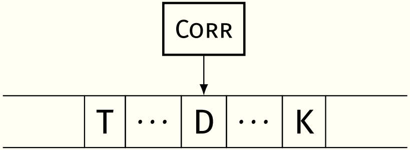

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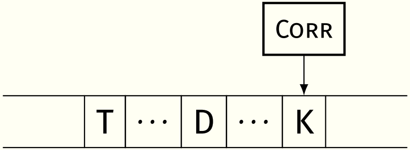

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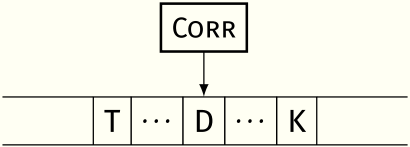
Alternative: segmental similarity scale

$\textsc{Corr}[\mathrm{T}\leftrightarrow\mathrm{D}] \gg \textsc{Corr}[\mathrm{K}\leftrightarrow\mathrm{T}] \gg \textsc{Corr}[\mathrm{K}\leftrightarrow\mathrm{D}]$

$\checkmark$ *similarity*(T,D) > *similarity*(T,K)



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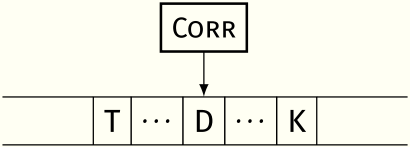
✓*similarity*(T,D) > *similarity*(T,K)

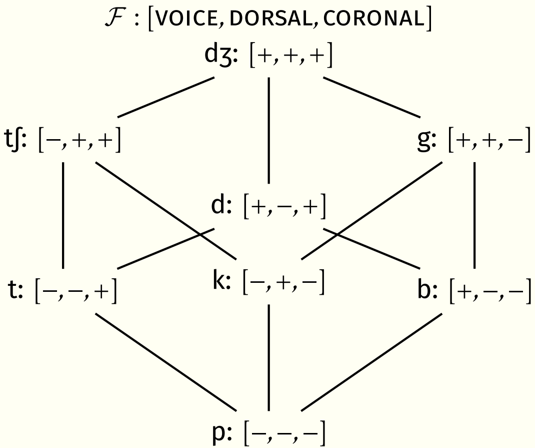
**similarity*(K,D) > *similarity*(K,T)



Alternative: segmental similarity scale

A lattice represents a higher dimensional space which captures the insights lost in an one-dimensional similarity scale.

Similar problem exists in many other scales.



Computation

Phonetic distance

- The phonetic distance between two segments x and y is computed over a weighted featural lattice $\mathbf{w}$:

$$\mathit{distance}_{\mathbf{w}}(x,y) = \sum_{f \in \mathcal{F}} w_f \cdot \delta_f(x,y), \qquad \text{(summed weights of nonshared features)}$$

$$\delta_f(x,y) = \begin{cases} 0, & \text{if } x \text{ and } y \text{ share the feature } f \\ 1, & \text{else} \end{cases}$$

(Wilson & Obdeyn, 2009)

Similarity as Bayesian probability

- **Phonological similarity** is the belief that x and y are (non-)identical, which is updated by the observed **phonetic distance** between two segments.

$$\begin{aligned} \mathit{similarity}_{\mathbf{w}}(x,y) &= 1 - \mathit{dissimilarity}_{\mathbf{w}}(x,y) \\ &= 1 - \frac{\mathit{distance}_{\mathbf{w}}(x,y)}{\sum_{f\in\mathcal{F}} w_f} \end{aligned}$$

  - This function converts the *phonetic distance* to a probability in $[0,1]$.
  - The sum of all featural weights $\sum_{f\in\mathcal{F}} w_f$ is the **maximal distance** between two segments (0 shared features). This knowledge is encoded in the lattice.

Similarity of LARYNGEAL pairs

C1↓C2→	T’	T	D’	D	T^h	D^h
T’	1	0.8	0.6	0.4	0.4	0
T	0.8	1	0.4	0.6	0.6	0.2
D’	0.6	0.4	1	0.8	0	0.4
D	0.4	0.6	0.8	1	0.2	0.6
T^h	0.4	0.6	0	0.2	1	0.6
D^h	0	0.2	0.4	0.6	0.6	1

Whatever the alphabet is, the set of thresholds is always **finite**.

Agreement by similarity

- IDENT-CC(LARYNGEAL, *SIMILARITY* $\geq k$):
  if *similarity* $\geq k$ in a 2-long **subsequence**, penalize any difference in LARYNGEAL.
- IDENT-IO[LARYNGEAL]:
  penalize any input-output difference in LARYNGEAL.
- The critical threshold is determined by the lowest similarity that triggers harmony.

ID-CC(LARYN., *S.* ≥ 1)

$\downarrow$

ID-CC(LARYN., *S.* ≥ 0.8)

$\downarrow$

ID-IO[LARYNGEAL]

$\downarrow$

ID-CC(LARYN., *S.* ≥ 0.6)

$\downarrow$

ID-CC(LARYN., *S.* ≥ 0.4)

Formal language-theoretic computation

- The 2-long **subsequences** in *[qats'un] include {q…a, **q**…**ts'**, q…u, q…n, a…ts', a…u, a…n, ts'…u, ts'…n, u…n} (Heinz, 2010)

/q/ /a/ /ts'/ /u/ /n/

IDENT-IO[LARYNGEAL]

[q'] [a] [ts'] [u] [n]

ID-CC(LARYN., S. ≥ 0.8)

/e/ /ts/ /i/ /g/ /u/ /n/

[e] [ts] [i] [g] [u] [n]

x

(Q & A: "Agreement by similarity vs. by projection")

Constraint-based analysis

/qats'un/	ID-CC(LARYN., $s. \geq 0.8$)	ID-IO[LARYN.]	ID-CC(LARYN., $s. \geq 0.6$)
a. qats'un [s. = 0.8]	*!		*
b. ☞ q'ats'un [s. = 1]		*	

/etsigun/	ID-CC(LARYN., $s. \geq 0.8$)	ID-IO[LARYN.]	ID-CC(LARYN., $s. \geq 0.6$)
a. edzigun [s. = 1]		*!	
b. ☞ etsigun [s. = 0.6]			*

Classical OT (Prince & Smolensky, 2004)

Typology

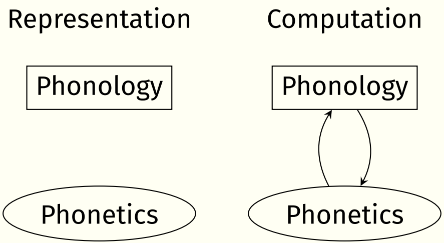
The typology of laryngeal harmony is predicted by varying critical thresholds.

Inventory	Thresholds	Pairs	Languages
T', T	0.8	*T↔T'	Gitksan, Chol
T', T, D'	0.8	*T↔T', ✓T'↔D', ✓T↔D'	Tzotzil, Tzutujil, Yucatec
T', T, D, D'	0.8	*T↔T', *D'↔D, ✓T↔D, ...	Hausa
T', T, T^h, D	0.8	*T↔T', ✓T↔D, ✓T^h↔T ...	Ndebele, Lezgian
T, D, D'	0.8	*D'↔D, ✓T↔D, ✓T↔D'	Bumo Izon, Kalabari Ijo
T', T, D	0.8	*T↔T', ✓T↔D, ✓T'↔D	Amharic
	0.4	*T↔T', *T↔D, *T'↔D	Chaha
T', T, T^h	0.4	*T↔T', *T↔T^h, *T'↔T^h	Peruvian & Bolivian Aymara

Theoretical consequences

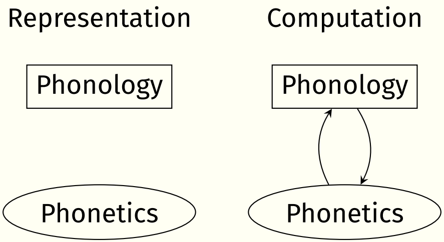
Modular representation

- Lattice:
  - the phonological, abstract, symbolic, universal structure of feature system;
  - it tells you how many (non-)shared features between two segments.



Modular representation

- Weight:
  - the phonetic, fine-grained, gradient, language-specific **substance**;
  - not in UR, and **only** available to SR in the **computation** of input-output and surface correspondence.
  - The only addition is one vector!



Structure + Gradience

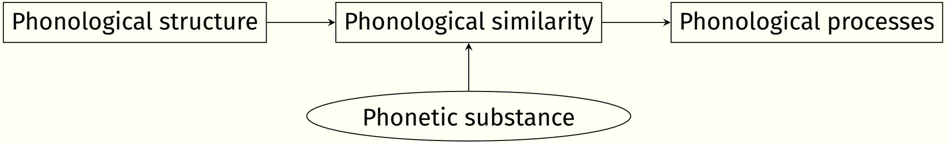
- Contrastive hierarchy: **structure** matters – phonetic similarity doesn't participate in phonological processes at all;

  (Mackenzie, 2009, 2011)

- Perceptual grounding approach: supplement the universal feature system with **language-specific** auditory features, such as [long VOT], to account for perceptual similarity.

  (Gallagher, 2010a,b, 2012)

The interplay of phonology and phonetics



The similarity is computed w.r.t phonological structure and phonetic substance, and this information is further used in phonological computation.

Future directions

- How language-specific/universal is the weighting relation $w_{[\text{CG}]} < w_{[\text{VOICE}]}, w_{[\text{SG}]}$?
- Laboratory evidence:
  - Confusion matrix (Miller & Nicely, 1955; Johnson & Babel, 2010)
  - Neural featural encoding (Mesgarani et al., 2014, Q & A)
- Extension in general cognitive science.

  Tversky's set-theoretic model vs. Shepard's continuous metric space model

  (Tversky, 1977; Shepard, 1980; Tenenbaum & Griffiths, 2001)
- ……

Take-home message

- Given a well-defined **representational structure** and **similarity metric**, gradient representation is not orthogonal to universal feature system and discrete symbolic computation;

Find the slides and code on `http://hutengdai.com`, and feel free to contact me for questions and collaborations!

Acknowledgement

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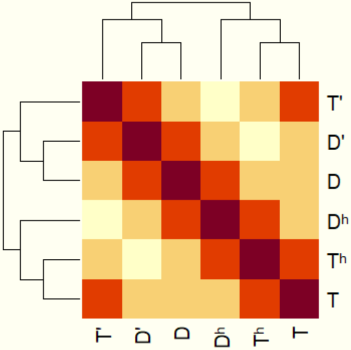
Q&A

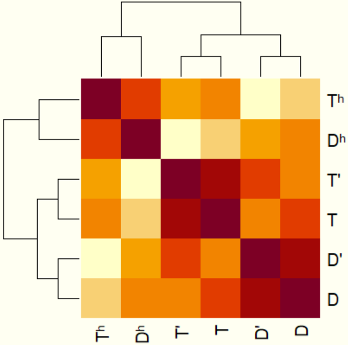
Abstractness of gradient representation

- Phonetic measurement ≠ weight on featural lattice:
  - Phonetic invariance doesn't exist; (Pierrehumbert, 2016; Zellou & Tamminga, 2014)
  - Real-numbered representation is still an **abstraction**!

Categorical vs. weighted similarity metrics

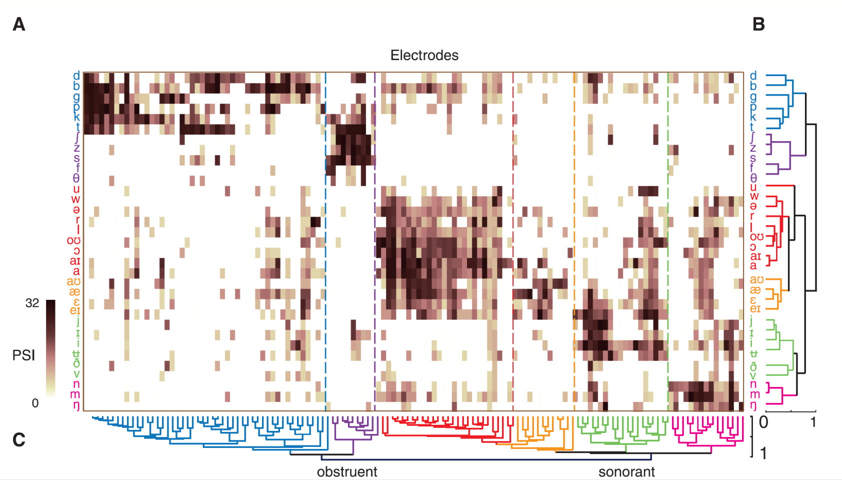


Categorical similarity metrics ☹



Gradient similarity metrics ☺

Neural featural encoding



Mesgarani et al. (2014)

Reference

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