Electric Networks: Computer Aided Analysis and Simulation
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Problem Sheet 3   November 2020

Consider the circuit in figure 1. The resistance is $R = 1\,\Omega$, the capacitance is $C = 10^{-3}\,F$, the initial condition of the capacitive voltage at $t = 0\,\text{s}$ is $v_C(0) = 10\,\text{V}$.
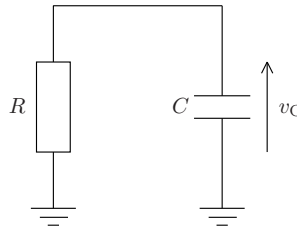

Figure 1

a) **1 points** Derive an analytic expression for $v_C(t)$. Draw the waveform in the interval from $t = 0\,\text{ms}$ to $t = 10\,\text{ms}$ in a diagram. The use of a computational tools such as MATLAB is recommended.

b) **4 points** Using the backward-Euler method, derive a difference equation to calculate $v_C(k)$. Select an appropriate time step size. Draw the graph of $v_C(t)$ over time t in the interval from $t = 0\,\text{ms}$ to $t = 10\,\text{ms}$ in the diagram of a). Discuss the selection of the time step size.

c) **4 points** Perform the same tasks as described under b), but use the trapezoidal and forward-Euler methods.

d) **8 points** Draw all curves of a), b) and c) in the same diagram and zoom in, i. e. consider for example the region $0 \le v_C \le 10\,\text{V}$ and $0.5\,\text{ms} \le t \le 1\,\text{ms}$. Consider the accuracy of the results. Discuss how these experimental results relate to the analytical accuracy considerations of backward-Euler, forward-Euler and trapezoidal methods studied in problem sheet 1 and chapter 2 of the lecture notes.

e) **2 points** Determine the value of the time step size above which the forward-Euler method becomes unstable. Tau > |1 / lambda|

f) **2 points** Determine the value of the time step size above which numerical oscillations are observed when using the trapezoidal method. Tau > |2 / lambda|

g) **4 points** Select the following time step size: $\tau = 2RC$. Draw the curves obtained for the backward-Euler, trapezoidal and forward-Euler methods over time t in the interval from $t = 0\,\text{ms}$ to $t = 20\,\text{ms}$. Discuss the results.

h) **4 points** Select the following time step size: $\tau = 4RC$. Draw the curves obtained for the backward-Euler, trapezoidal and forward-Euler methods over time t in the interval from $t = 0\,\text{ms}$ to $t = 20\,\text{ms}$. Discuss the results.