

Review:

Regular expressions define string patterns

Concatenation: **ab**  $\Rightarrow$  **a must be followed by b**

Alternation: **a|b**  $\Rightarrow$  **a or b**

Closure: **a\***  $\Rightarrow$  **a; zero or more times**

Given some finite alphabet of symbols, a language is all the possible sequences of symbols, which can be infinite. Regular expressions are a way to define a language.

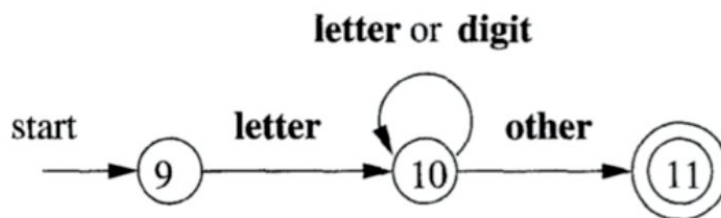
Regular languages can be represented one for one with finite automata.

Finite automata: A finite set of symbols, and FA can process them one at a time and changes its state depending on what symbols it sees, also how causes it to transition to a new state.

Computational model of turnstile:

Two states: locked and unlocked; changing its state depending on what input/symbol you give.

Example of a State Diagram:



**Circles** represent the **states**, they can be labeled, such as 9, etc. **Transitions** between states are represented by **arrows** between states. Seeing a **letter** transitions us to the next state in our language. Accepting state is represented by a **double circle**.

Epsilon represents empty string in our language.

Deterministic FA: one state at a time

Non-deterministic FA: multiple states at once, the same symbol, multiple transitions, epsilon transitions

Example of FA with Transition Table:

STATE	<i>a</i>	<i>b</i>	$\epsilon$
0	{0, 1}	{0}	$\emptyset$
1	$\emptyset$	{2}	$\emptyset$
2	$\emptyset$	{3}	$\emptyset$
3	$\emptyset$	$\emptyset$	$\emptyset$

$\Rightarrow$  this ex. Non-deterministic FA.

A non-deterministic FA can be converted to DFA via **subset construction**.

If we have some number  $n$  of states, how many subsets of those states are there?  $2^n$

We can turn all those subsets into a single states in a DFA (subset construction).

Regex Operations as NFAs:

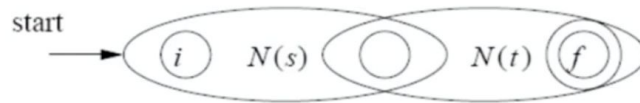
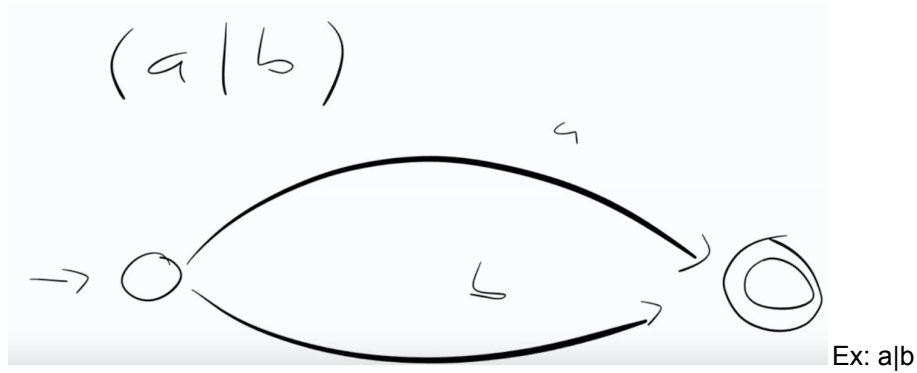


Figure 3.41: NFA for the concatenation of two regular expressions

Dragon book-concatenation.

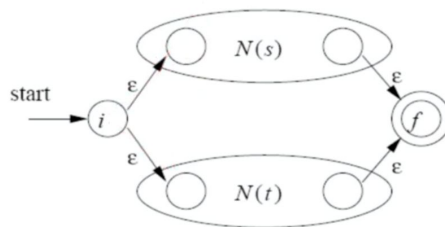


Figure 3.40: NFA for the union of two regular expressions

Dragon book-alternation.

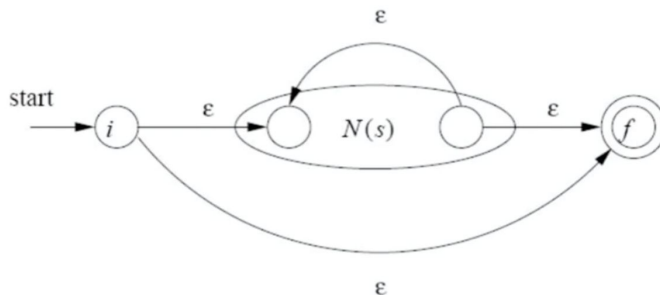
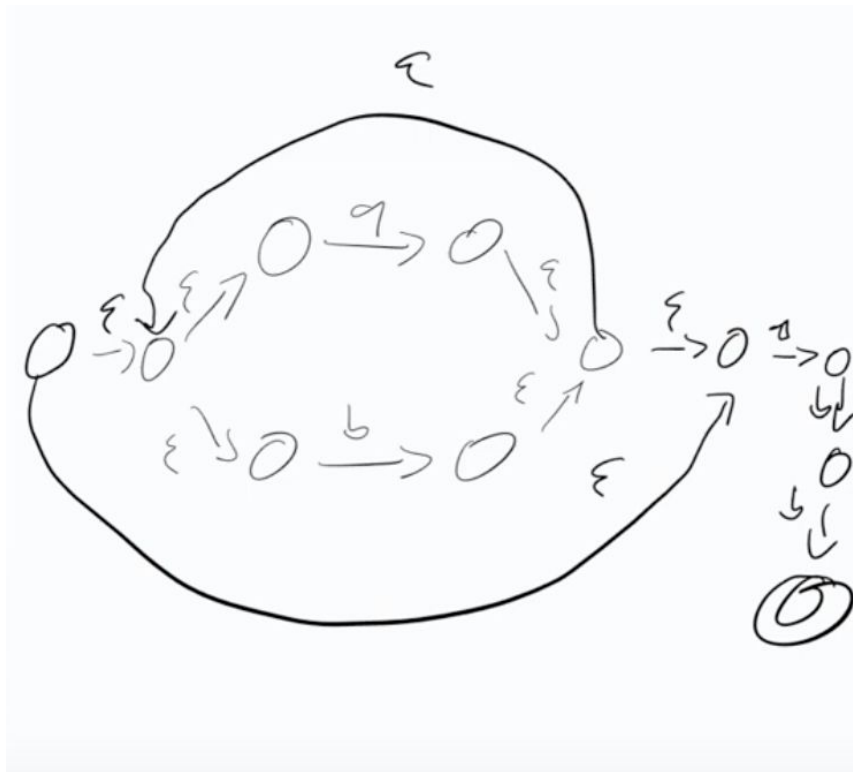


Figure 3.42: NFA for the closure of a regular expression

Dragon book-kleene closure.

Ex.  $(a|b)^*abb$



Dragon book.

How to construct a DFA from an NFA systematically:

Each DFA state created by subset of NFA states, can be in multiple states

Simulate being in multiple states using a single state

Multiple states are a subset of the NFA states

Create the DFA by calling each subset a single DFA state

Subset Construction Algorithm:

Start at the starting state of NFA

Group all states reachable by epsilon (epsilon closure); call this group as initial state of DFA

For each symbol  $s$  in the alphabet;

    Get all states that  $s$  transitions to

    Find epsilon-closure of those states

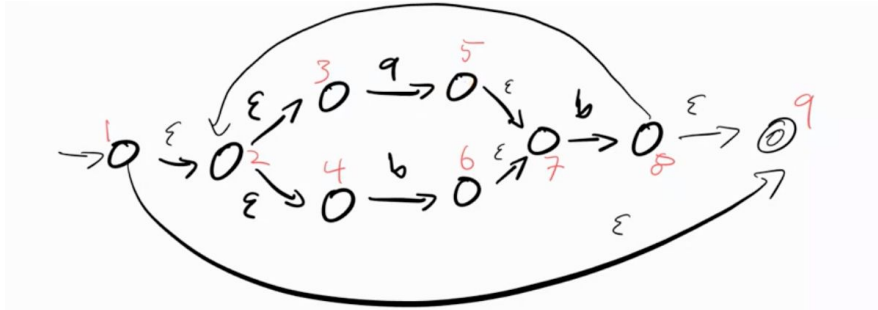
    Call this group as a single state of DFA

Repeat for all combinations of NFA states and symbols

Stop when you have covered them all.

Ex of converting from NFA to DFA.

$((a|b)b)^*$



DFA	NFA Subsets	a	b
A $\odot$	$\{1, 9, 2, 3, 4\}$	$\{5, 7\}$	$\{6, 7\}$
B	$\{5, 7\}$	<del><math>\emptyset</math></del>	$\{8, 9, 2, 3, 4\}$
C	$\{6, 7\}$	<del><math>\emptyset</math></del>	$\{8, 9, 2, 3, 4\}$
D $\odot$	$\{8, 9, 2, 3, 4\}$	$\{5, 7\}$	$\{6, 7\}$

