# Day 2: Basic Probability ★



Terms you'll find helpful in completing today's challenge are outlined below.

## Event, Sample Space, and Probability

In probability theory, an experiment is any procedure that can be infinitely repeated and has a well-defined set of possible outcomes, known as the sample space, $S$. We define an event to be a set of outcomes of an experiment (also known as a subset of $S$) to which a probability (numerical value) is assigned.

The probability of the occurrence of an event, $A$, is:

$$P(A) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of outcomes}}$$

Here are the first two fundamental rules of probability:

1. Any probability, $P(A)$, is a number between $0$ and $1$ (i.e., $0 \le P(A) \le 1$).
2. The probability of the sample space, $S$, is $1$ (i.e., $P(S) = 1$).

So how do we bridge the gap between the value of $P(A)$ and the sample space? Quite simply, since we know that $P(A)$ is the probability that event $A$ will occur, then we define $P(A')$ (also written as $P(A^c)$) to be the probability that event $A$ will not occur (the complement of $P(A)$). If our sample space is composed of the probabilities of $A$'s occurrence and non-occurrence, we can then say $P(A) + P(A') = 1$, or the sum of all possible outcomes of $A$ in the sample space is equal to $1$. This is the third fundamental rule of probability: $P(A^c) = 1 - P(A)$.

**Question 1**

Find the probability of getting an odd number when rolling a $6$-sided fair die.

Given the above question, we can extract the following:

- Experiment: rolling a $6$-sided die.
- Sample space ($S$): $S = \{1, 2, 3, 4, 5, 6\}$.
- Event ($A$): that the number rolled is odd (i.e., $A = \{1, 3, 5\}$).

If we refer back to the basic formula for the probability of the occurrence of an event, we can say:

$$P(A) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of outcomes}} = \frac{|A|}{|S|} = \frac{3}{6} = \frac{1}{2}$$

## Compound Events, Mutually Exclusive Events, and Collectively Exhaustive Events

Let's consider $2$ events: $A$ and $B$. A compound event is a combination of $2$ or more simple events. If $A$ and $B$ are simple events, then $A \cup B$ denotes the occurrence of either $A$ or $B$. Similarly, $A \cap B$ denotes the occurrence of $A$ and $B$ together.

$A$ and $B$ are said to be mutually exclusive or disjoint if they have no events in common (i.e., $A \cap B = \varnothing$ and $P(A \cap B) = 0$). The probability of any of $2$ or more events occurring is the union ($\cup$) of events. Because disjoint probabilities have no common events, the probability of the union of disjoint events is the sum of the events' individual probabilities. $A$ and $B$ are said to be collectively exhaustive if their union covers all events in the sample space (i.e., $A \cup B = S$ and $P(A \cup B) = 1$). This brings us to our next fundamental rule of probability: if $2$ events, $A$ and $B$, are disjoint, then the probability of either event is the sum

probabilities of the $2$ events (i.e., $P(A \textit{ or } B) = P(A) + P(B)$).

If the outcome of the first event ($A$) has no impact on the second event ($B$), then they are considered to be independent (e.g., tossing a fair coin). This brings us to the next fundamental rule of probability: the multiplication rule. It states that if two events, $A$ and $B$, are independent, then the probability of both events is the product of the probabilities for each event (i.e., $P(A \; and \; B) = P(A) \times P(B)$). The chance of all events occurring in a sequence of events is called the intersection ($\bigcap$) of those events.

**Question 2**

Find the probability of getting $1$ head and $1$ tail when $2$ fair coins are tossed.

Given the above question, we can extract the following:

- Experiment: tossing $2$ coins.
- Sample space ($S$): The possible outcomes for the toss of $1$ coin are $\{H, T\}$, where $H = heads$ and $T = tails$. As our experiment tosses $2$ coins, we have to consider all possible toss outcomes by finding the Cartesian Product of the possible outcomes for each coin: $S = \{\{H, T\} \times \{H, T\} = \{(H, H), (H, T), (T, H), (T, T)\}$.
- Event ($A \cap B$): that the outcome of $1$ toss will be $H$, and the outcome of the other toss will be $T$ (i.e., $A = \{(H, T), (T, H)\}$.

Connecting this information back to our basic formula for $P(A)$, we can say:

$$P(A) = \frac{Number \; of \; favorable \; outcomes}{Total \; number \; of \; outcomes} = \frac{|A|}{|S|} = \frac{2}{4} = \frac{1}{2}$$

**Question 3**

Let $A$ and $B$ be two events such that $P(A) = \frac{2}{5}$ and $P(B) = \frac{4}{5}$. If the probability of the occurrence of either $A$ or $B$ is $\frac{3}{5}$, find the probability of the occurrence of both $A$ and $B$ together (i.e., $A \cap B$).

We can use our fundamental rules of probability to solve this problem:

$|A \cup B| = |A| + |B| - |A \cap B|$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\Rightarrow P(A \cap B) = \frac{2}{5} + \frac{4}{5} - \frac{3}{5} = \frac{3}{5}$