



Day 7: Pearson Correlation Coefficient I ★

22/27 challenges solved

Points: 22



10
Days of
Statistics

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Covariance

This is a measure of how two random variables change together, or the strength of their correlation.

Consider two random variables, \mathbf{X} and \mathbf{Y} , each with n values (i.e., $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ and $\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_n$). The covariance of \mathbf{X} and \mathbf{Y} can be found using either of the following equivalent formulas:

$$\text{cov}(\mathbf{X}, \mathbf{Y}) = \frac{1}{n} \sum_{i=1}^n (\mathbf{x}_i - \bar{\mathbf{x}}) \cdot (\mathbf{y}_i - \bar{\mathbf{y}})$$
$$\text{cov}(\mathbf{X}, \mathbf{Y}) = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n \frac{1}{2} (\mathbf{x}_i - \mathbf{x}_j) \cdot (\mathbf{y}_i - \mathbf{y}_j) = \frac{1}{n^2} \sum_i \sum_{j>i} (\mathbf{x}_i - \mathbf{x}_j) \cdot (\mathbf{y}_i - \mathbf{y}_j)$$

Here, $\bar{\mathbf{x}}$ is the mean of \mathbf{X} (or $\mu_{\mathbf{X}}$) and $\bar{\mathbf{y}}$ is the mean of \mathbf{Y} (or $\mu_{\mathbf{Y}}$).

Pearson Correlation Coefficient

The Pearson correlation coefficient, $\rho_{\mathbf{X}, \mathbf{Y}}$, is given by:

$$\rho_{\mathbf{X}, \mathbf{Y}} = \frac{\text{cov}(\mathbf{X}, \mathbf{Y})}{\sigma_{\mathbf{X}} \sigma_{\mathbf{Y}}} = \frac{\sum_i (\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{y}_i - \bar{\mathbf{y}})}{n \sigma_{\mathbf{X}} \sigma_{\mathbf{Y}}}$$

Here, $\sigma_{\mathbf{X}}$ is the standard deviation of \mathbf{X} and $\sigma_{\mathbf{Y}}$ is the standard deviation of \mathbf{Y} . You may also see $\rho_{\mathbf{X}, \mathbf{Y}}$ written as $r_{\mathbf{X}, \mathbf{Y}}$.

