5 more challenges to get your gold badge!

Points: 15/20

10 Days of Statistics

Day 5: Poisson Distribution I ★

Problem | Submissions | Leaderboard | Editorial | Tutorial

Terms you'll find helpful in completing today's challenge are outlined below.

Poisson Random Variables

We've already learned that we can break many problems down into terms of n, x, and p and use the following formula for binomial random variables:

$$p(x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$$

But what do we do when $p(x)$ cannot be calculated using that formula? Enter the Poisson random variable.

Poisson Experiment

A Poisson experiment is a statistical experiment that has the following properties:

- The outcome of each trial is either success or failure.
- The average number of successes (λ) that occurs in a specified region is known.
- The probability that a success will occur is proportional to the size of the region.
- The probability that a success will occur in an extremely small region is virtually zero.

Poisson Distribution

A Poisson random variable is the number of successes that result from a Poisson experiment. The probability distribution of a Poisson random variable is called a Poisson distribution:

$$P(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Here,

- $e = 2.71828$
- λ is the average number of successes that occur in a specified region.
- k is the actual number of successes that occur in a specified region.
- $P(k, \lambda)$ is the Poisson probability, which is the probability of getting exactly k successes when the average number of successes is λ.

Example

Acme Realty company sells an average of 2 homes per day. What is the probability that exactly 3 homes will be sold tomorrow?

Here, $\lambda = 2$ and $k = 3$, so $P(k = 3, \lambda = 2) = \frac{\lambda^k e^{-\lambda}}{k!} = 0.180$

Example

Suppose the average number of lions seen by tourists on a one-day safari is 5. What is the probability that tourists will see fewer than 4 lions on the next one-day safari?

$P(k \le 3, \lambda = 5) = \sum^{3} \frac{\lambda^r e^{-\lambda}}{\ldots} = 0.2650$

$P(k \leq 3, \lambda = 5) = \sum_{r=0} \frac{}{r!} = 0.2650$

Special Case

Consider some Poisson random variable, X. Let $E[X]$ be the expectation of X. Find the value of $E[X^2]$.

Let $\textbf{Var}(X)$ be the variance of X. Recall that if a random variable has a Poisson distribution, then:

- $E[X] = \lambda$
- $\textbf{Var}(X) = \lambda$

Now, we'll use the following property of expectation and variance for any random variable, X:

$$\textbf{Var}(X) = E[X^2] - (E[X])^2$$

$$\Rightarrow E[X^2] = \textbf{Var}(X) + (E[X])^2$$

So, for any random variable X having a Poisson distribution, the above result can be rewritten as:

$$\Rightarrow E[X^2] = \lambda + \lambda^2$$