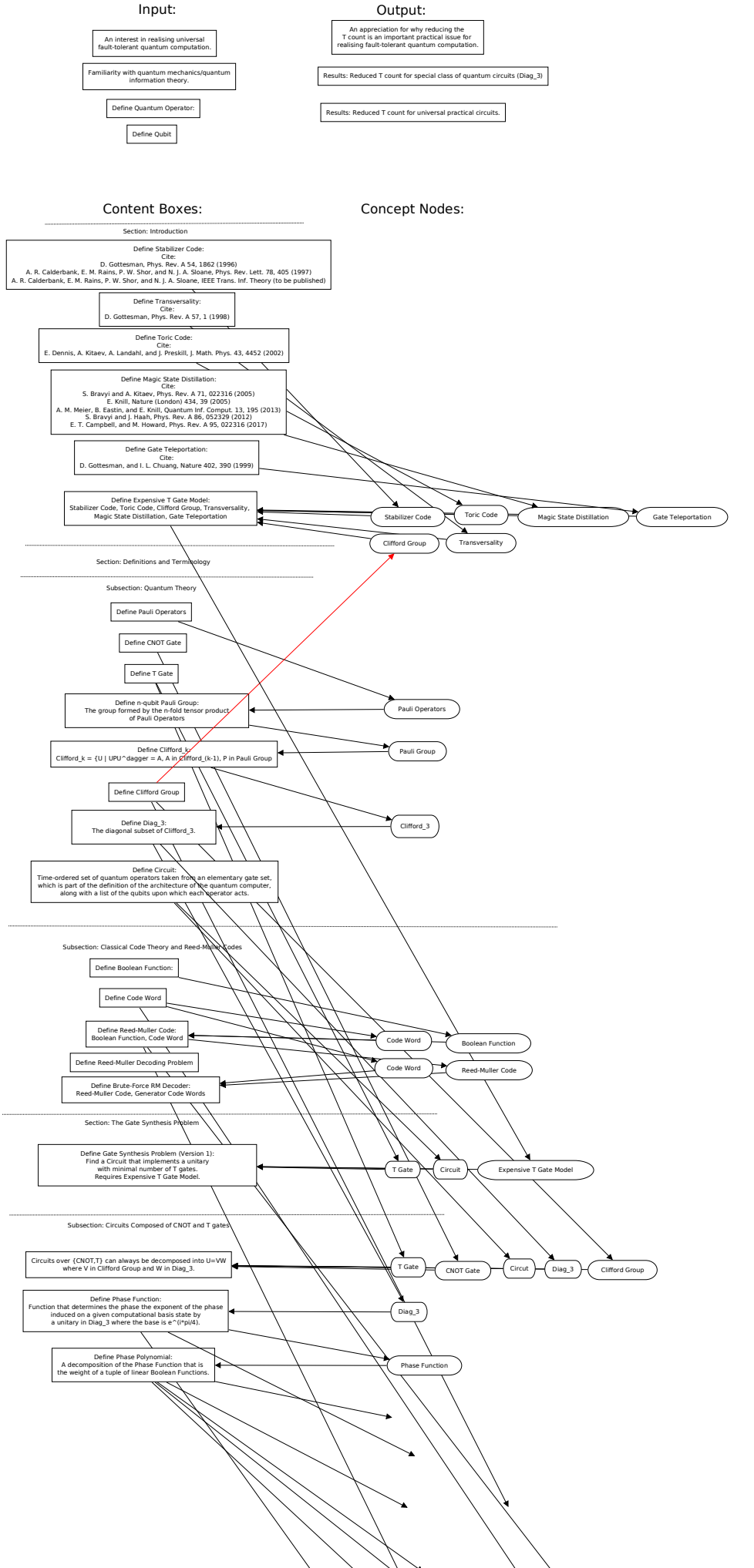
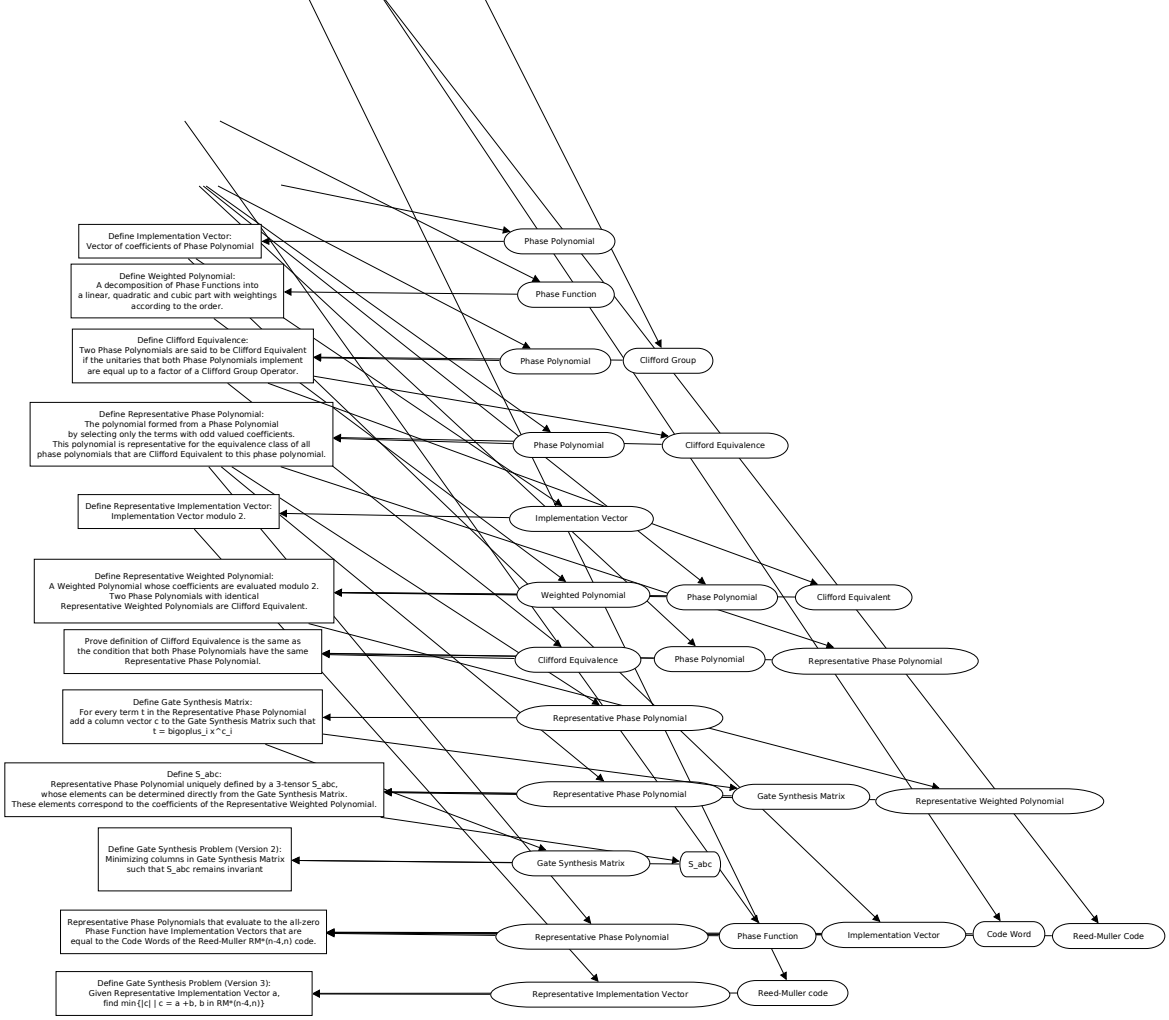

Input:
An interest in realising universal fault-tolerant quantum computation.
Familiarity with quantum mechanics/quantum information theory.
Define Quantum Operator:
Define Qubit
Output:
An appreciation for why reducing the T count is an important practical issue for realising fault-tolerant quantum computation.
Results: Reduced T count for special class of quantum circuits (Diag_3)
Results: Reduced T count for universal practical circuits.
Content Boxes:
Concept Nodes:
Section: Introduction
Define Stabilizer Code:
Cite:
D. Gottesman, Phys. Rev. A 54, 1862 (1996)
A. R. Calderbank, E. M. Rains, P. W. Shor, and N. J. A. Sloane, Phys. Rev. Lett. 78, 405 (1997)
A. R. Calderbank, E. M. Rains, P. W. Shor, and N. J. A. Sloane, IEEE Trans. Inf. Theory (to be published)
Define Transversality:
Cite:
D. Gottesman, Phys. Rev. A 57, 1 (1998)
Define Toric Code:
Cite:
E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, J. Math. Phys. 43, 4452 (2002)
Define Magic State Distillation:
Cite:
S. Bravyi and A. Kitaev, Phys. Rev. A 71, 022316 (2005)
E. Knill, Nature (London) 434, 39 (2005)
A. M. Meier, B. Eastin, and E. Knill, Quantum Inf. Comput. 13, 195 (2013)
S. Bravyi and J. Haah, Phys. Rev. A 86, 052329 (2012)
E. T. Campbell, and M. Howard, Phys. Rev. A 95, 022316 (2017)
Define Gate Teleportation:
Cite:
D. Gottesman, and I. L. Chuang, Nature 402, 390 (1999)
Define Expensive T Gate Model:
Stabilizer Code, Toric Code, Clifford Group, Transversality, Magic State Distillation, Gate Teleportation
Stabilizer Code
Toric Code
Magic State Distillation
Gate Teleportation
Clifford Group
Transversality
Section: Definitions and Terminology
Subsection: Quantum Theory
Define Pauli Operators
Define CNOT Gate
Define T Gate
Define n-qubit Pauli Group:
The group formed by the n-fold tensor product of Pauli Operators
Pauli Operators
Define Clifford_k:
Clifford_k = {U | UPU^dagger = A, A in Clifford_(k-1), P in Pauli Group
Pauli Group
Define Clifford Group
Define Diag_3:
The diagonal subset of Clifford_3.
Clifford_3
Define Circuit:
Time-ordered set of quantum operators taken from an elementary gate set, which is part of the definition of the architecture of the quantum computer, along with a list of the qubits upon which each operator acts.
Subsection: Classical Code Theory and Reed-Muller Codes
Define Boolean Function:
Define Code Word
Define Reed-Muller Code:
Boolean Function, Code Word
Code Word
Boolean Function
Define Reed-Muller Decoding Problem
Code Word
Reed-Muller Code
Define Brute-Force RM Decoder:
Reed-Muller Code, Generator Code Words
Section: The Gate Synthesis Problem
Define Gate Synthesis Problem (Version 1):
Find a Circuit that implements a unitary with minimal number of T gates.
Requires Expensive T Gate Model.
T Gate
Circuit
Expensive T Gate Model
Subsection: Circuits Composed of CNOT and T gates
Circuits over {CNOT,T} can always be decomposed into U=VW where V in Clifford Group and W in Diag_3.
T Gate
CNOT Gate
Circut
Diag_3
Clifford Group
Define Phase Function:
Function that determines the phase the exponent of the phase induced on a given computational basis state by a unitary in Diag_3 where the base is e^(i*pi/4).
Diag_3
Define Phase Polynomial:
A decomposition of the Phase Function that is the weight of a tuple of linear Boolean Functions.
Phase Function



Section: Solutions to the Gate Synthesis Problem

Subsection: The Lempel Algorithm

A = BB^T problem where A is a symmetric matrix.

Describe algorithm.

Show applicability to Gate Synthesis Problem

Deconstruct input Circuit into series of control-unitaries.

Minimize number of T gates within target block.

Add the control qubit back on to target phase polynomial.

Quote analytical upper limit on T count

Subsection: The Extended Lempel Algorithm

S_abc = sum(B_ka B_kb B_kc) problem
where S_abc is a symmetric tensor.

Describe algorithm

Algorithm acts directly on the Gate Synthesis Matrix
so no need for control-unitary decomposition.

Section: Experimental Results

Subsection: Random Circuits over CNOT and T

Subsection: Universal Practical Circuits

Generalised Toffoli-n
Standard method.

Hadamard Path Variables
Cite:
- A. Montanaro, e-print arXiv:1607.08473
- M. Bremner, R. Jozsa, and D. Shepherd, Proc. Roy. Soc. Ser. A
467 (2126):459-472 (2011). e-print arXiv:1005.1407

Section: Discussion and Conclusions