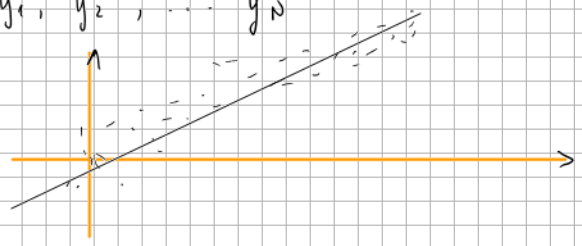
Training Samples

Inputs : $x_1, x_2, \dots x_N$

Outputs : $y_1, y_2, \dots y_N$

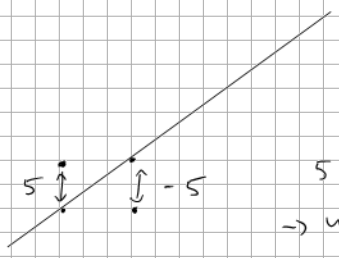


A line:

$$y = ax + b$$

Objective / Error / Cost function

$$E = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$



$5 + (-5) = 0$

→ want positive → 2

We want to minimize E by using derivative

$$E = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

$$\Rightarrow \frac{\partial E}{\partial a} = 0$$

$$\Rightarrow \frac{\partial E}{\partial a} = \sum_{i=1}^{N} 2(y_i - \hat{y}_i) \frac{\partial \hat{y}_i}{\partial a}$$

$$= \sum_{i=1}^{N} 2(y_i - \hat{y}_i) x_i = 0$$

$$\Leftrightarrow \sum y_i x_i = \sum \hat{y}_i x_i$$

$$\Leftrightarrow \sum y_i x_i = \sum (a x_i + b) x_i$$

$$= a \sum x_i^2 + b \sum x_i$$

$\hat{y}_i = a x_i + b$

$\frac{\partial E}{\partial b} = 0$

$\Leftrightarrow \sum 2(y_i - \hat{y}_i) = 0$

$\Leftrightarrow \sum y_i = \sum (a x_i + b)$

$= a \sum x_i + bN$

$\Leftrightarrow \frac{\sum y_i}{N} = \frac{a \sum x_i}{N} + b$

$\Leftrightarrow \bar{y} = a \cdot \bar{x}_i + b$

$\Rightarrow$

$$a = \frac{\sum y_i x_i - \bar{y} \sum x_i}{\sum x_i^2 - \bar{x} \sum x_i}$$

$$b = \frac{\bar{y} \sum x_i^2 - \bar{x} \sum y_i x_i}{\sum x_i^2 - \bar{x} \sum x_i}$$