44. $$X = \begin{pmatrix} 1 & x_{11} & \cdots & x_{1p} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{np} \end{pmatrix} \in \mathbb{R}^{N \times (p+1)}$$

$$\beta = \begin{pmatrix} \beta_0 \\ \vdots \\ \beta_p \end{pmatrix} \in \mathbb{R}^{p+1}$$ とおくと.

線形回帰

$$L := \frac{1}{2} \sum_{i=1}^{N} \left(y_i - (\beta_0 + \beta^T x_i) \right)^2$$

$$= \frac{1}{2} (y - X\beta)^T (y - X\beta)$$

$$\nabla L = \frac{1}{2} (-X^T)(y - X\beta) + \frac{1}{2} (y - X\beta)^T (-X)$$

$$= -X^T (y - X\beta)$$

ロジスティック回帰

$$L := \sum_{i=1}^{n} \log\left(1 + \exp\left(-y_i (\beta_0 + \beta_1 x_{i1} + \cdots + \beta_p x_{ip})\right)\right)$$

$$\frac{\partial L}{\partial \beta_j} = \sum_{i=1}^{n} \frac{-y_i x_{ij} \exp(-y_i(\beta_0 + \beta^T x_i))}{1 + \exp(-y_i(\beta_0 + \beta^T x_i))}$$

$$= -\sum_{i=1}^{n} \frac{x_{ij} y_i}{1 + \exp(y_i(\beta_0 + \beta^T x_i))}$$

$$= -X^T \left(\frac{y_i}{1 + \exp(y_i(\beta^T x)_i)} \right)_i$$