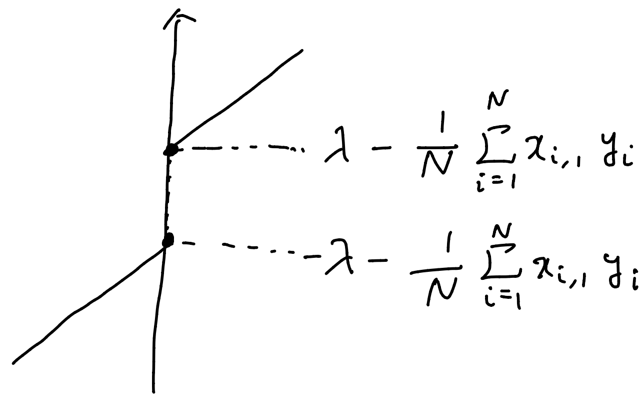
15. $\frac{1}{N}\sum_{i=1}^{N} x_{i,1}^2 = 1$ のとき

$$\frac{1}{2N}\sum_{i=1}^{N}(y_i-\beta_1 x_{i,1})^2+\lambda|\beta_1|$$

$\beta_1 \neq 0$ の点で微分して 0 とおくと.

$$-\frac{1}{N}\sum_{i=1}^{N} x_{i,1}(y_i-\hat{\beta}_1 x_{i,1})+\lambda\,\mathrm{sign}(\hat{\beta}_1)=0.$$

$$-\frac{1}{N}\sum_{i=1}^{N} x_{i,1}y_i+\hat{\beta}_1+\lambda\,\mathrm{sign}(\hat{\beta}_1)=0$$



よって、

$$\begin{cases} 0>\lambda-\frac{1}{N}\sum_{i=1}^{N}x_{i,1}y_i \text{ のとき、} & \hat{\beta}_1=\frac{1}{N}\sum_{i=1}^{N}x_{i,1}y_i-\lambda \\ \lambda-\frac{1}{N}\sum_{i=1}^{N}x_{i,1}y_i>0>-\lambda-\frac{1}{N}\sum_{i=1}^{N}x_{i,1}y_i \text{ のとき} & \hat{\beta}_1=0 \\ \lambda-\frac{1}{N}\sum_{i=1}^{N}x_{i,1}y_i>0 \text{ のとき.} & \hat{\beta}_1=\frac{1}{N}\sum_{i=1}^{N}x_{i,1}y_i+\lambda. \end{cases}$$