Junyu Wang
jw6602

1.

Let's start. Below we have a table with the current **Old** and the proposed **New**.

Old	**New**
AC → E **AC → CB** **E → DE** D → B	AC → E **AC → B** **E → D** D → B

New was obtained by removing the "defective" parts of **Old** and therefore there is nothing to do for proving equivalence. **New** now becomes **Old**.

Old	**New**
AC → E **AC → B** E → D D → B	AC → E E → D D → B

We compute $\mathrm{AC}^{+} = \mathrm{AC} + \mathrm{E} \rightarrow \mathrm{D} = \mathrm{ACE} + \mathrm{D} \rightarrow \mathrm{B} = \mathrm{ACEDB}$, and as it contains B, we prove equivalence.

Therefore, our minimal cover ω is

AC → E
E → D
D → B

2.

Let's start. Below we have a table with the current **Old** and the proposed **New**.

Old	**New**
AB → C A → E DE → F **E → EF** F → B	AB → C A → E DE → F **E → F** F → B

New was obtained by removing the "defective" parts of **Old** and therefore there is nothing to do for proving equivalence. **New** now becomes **Old**.

Old	New
$AB \to C$ $A \to E$ $\mathbf{DE \to F}$ $E \to F$ $F \to B$	$AB \to C$ $A \to E$ $E \to F$ $F \to B$

We compute $E^+ = E + F \to B = EFB$, and as it contains F, we prove equivalence. **New** becomes **Old**.

Old	New
$\mathbf{AB \to C}$ $A \to E$ $E \to F$ $F \to B$	$\mathbf{A \to C}$ $A \to E$ $E \to F$ $F \to B$

We compute $A^+ = AC + E \to F = ACE + F \to B = ACEFB$, and as it contains C, we prove equivalence. **New** becomes **Old**.

Old	New
$\mathbf{A \to C}$ $\mathbf{A \to E}$ $E \to F$ $F \to B$	$\mathbf{A \to CE}$ $E \to F$ $F \to B$

This is an application of the union rule, which always produces an equivalent set.

Therefore, our minimal cover ω is

$A \to CE$
$E \to F$
$F \to B$

3.

(a)

1. Let's try with trivial removal and union rule. We can see that both operations are unavailable.
2. Let's try to simplify the LHS or RHS.

Attempt 1.

Old	**New**
AB → CD C → EF E → C F → G	**AB → C** C → EF E → C F → G

We compute $AB^+ = ABCEFG$, and as it does not contain D, we prove non-equivalence.

Attempt 2.

Old	**New**
AB → CD C → EF E → C F → G	**AB → D** C → EF E → C F → G

We compute $AB^+ = ABD$, and as it does not contain C, we prove non-equivalence.

Attempt 3.

Old	**New**
AB → CD C → EF E → C F → G	**A → CD** C → EF E → C F → G

We compute $A^+ = ACDEFG$, and as it does not contain B, we prove non-equivalence.

Attempt 4.

Old	**New**
AB → CD C → EF E → C F → G	**B → CD** C → EF E → C F → G

We compute $B^+ = BCDEFG$, and as it does not contain A, we prove non-equivalence.

Attempt 5.

Old	**New**
AB → CD	AB → CD

C → EF E → C F → G	**C → E** E → C F → G

We compute $C^+ = CE$, and as it does not contain F, we prove non-equivalence.

Attempt 6.

Old	New
AB → CD **C → EF** E → C F → G	AB → CD **C → F** E → C F → G

We compute $C^+ = CFG$, and as it does not contain E, we prove non-equivalence.

We prove that we cannot simplify the LHS or RHS. Therefore, the given set of FDs is already a minimal cover.

(b)

We classify the attributes based on where they appear in the FDs:

1. On both sides: C, E, F
2. On left side only: A, B
3. On right side only: D
4. Nowhere: H

ABH appear in every key. D does not appear in any key. CEF may appear in any key. We start with $ABCEFH$, which must contain a key. We cannot remove ABH but may try to remove CEF.

We attempt to remove C. We compute $ABEFH^+ = ABCDEFGH$. We can remove C and we continue with $ABEFH$.

We attempt to remove E. We compute $ABFH^+ = ABCDEFGH$. We can remove E and we continue with $ABFH$.

We attempt to remove F. We compute $ABH^+ = ABCDEFGH$. We can remove F and we continue with ABH.

As nothing else can be done, ABH is a global key. We can remove CE because it's a subset of CEF.

Therefore, our final decomposition is

ABCD
CEF
FG
ABH