Database Management Systems

(COP 5725)

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Homework 4

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Pledge (Must be signed according to UF Honor Code)

On my honor, I have neither given nor received unauthorized aid in doing this assignment.

Xiao Hu.

Signature

For scoring use only:

	Maximum	Received
Exercise 1	35	
Exercise 2	20	
Exercise 3	35	
Exercise 4	10	
Total	100	

Exercise 1 [35 points]

1. [5 points] Consider the relation schema R = (A, B, C, D, E, F, G, H) with the set of functional dependencies K = {A→B, CH→A, B→E, BD→C, EG→H, DE→F}. Show for each of the following FDs whether they can be inferred from K.
   - BFG→AE
   - CGH→BF
   - CEG→AB
   - ADE→CH
   - ABF→CE
   - ADF→CE

for K : $(BFG)^+ = BEFGH.$ $\quad AE \nsubseteq (BFG)^+$

$(CGH)^+ = ABCEGH.$ $\quad BF \nsubseteq (CGH)^+$

$(CEG)^+ = ABCEGH.$ $\quad AB \subseteq (CEG)^+$

$(ADE)^+ = ABCDEF$ $\quad CH \nsubseteq (ADE)^+$

$(ABF)^+ = ABEF.$ $\quad CE \nsubseteq (ABF)^+$

$(ADF)^+ = ABCDEF$ $\quad CE \subseteq (ADF)^+$

thus CEG→AB & ADF→CE can be inferred from K

2. [5 points] Consider a relation schema R(X, Y, Z) with the functional dependencies XY→Z and Z→X. Can we conclude that Y→XZ holds? If yes, please argue why. If no, please argue why not by giving a counterexample.

$K: \{XY \to Z, Z \to X\}$

$Y \to XZ.$

for K: $Y^+ = Y$. $XZ \nsubseteq Y$

so $Y \to XZ$ can't hold

X	Y	Z
a_1	b_1	c_1
a_2	b_1	c_2.

$(a_1, b_1) \to c_1$

$(a_2, b_1) \to c_2$.

$c_1 \to a_1$

$c_2 \to a_2$

but

$b_1 \nrightarrow c_1$

3. [5 points] Consider the relation schema R(A, B, C, D, E, F) with the functional dependencies K = {CDE→B, ACD→F, BEF→C, B→D}. Which of the following attribute sets is a superkey? Show each step.
   - ABCE
   - ABDF
   - ACDE
   - BEF
   - ABEF
   - BCDF

left: ABCDEF

right: BCDF

compute the candidate first:

① $\emptyset$

② AE

③ $\emptyset$

④ AE

⑤ $(AE)^+ = AE$ do not include all attributs of schema R.

⑥ BCDF.

⑦ the attributes A and E belong to all candidate keys.

construct the power set of the set.

BCDF

< B, C, D, F, BC, BD, BF, CD, CF, DF, BCD, BCF, BDF, CDF, BCDF >.

<1> consider the sets with three attributes by adding the sets with one element to AE.

$ABE^+ = ABDE \subset R$

$ACE^+ = ACE \subset R$

$ADE^+ = ADE \subset R$

$AEF^+ = AEF \subset R$.

<2> add two

$ABCE^+ = ABCDEF = R$, $ACDE^+ = ABCDEF = R$

$ABDE^+ = ABDE \subseteq R$, $ACEF^+ = ACEF \subseteq R$

$ABEF^+ = ABCDEF = R$, $ADEF^+ = ADEF \subseteq R$

<3> add three.

$ABCEF^+ = R$, $ABDEF^+ = R$.

$ABCDE^+ = R$. $ACDEF^+ = R$.

We see we only get superkeys that properly contain candidate keys

<4> add four

ABCDEF default superkey.

so the superkeys are:

{ABCE, ACDE, ABEF, ABCEF, ABDEF, ABCDE,

ACDEF, ABCDEF}

so the following attribute sets:

$\boxed{\text{ABCE, ACDE, ABEF are superkeys.}}$

4. [5 points] Consider the relation schema R(A, B, C, D, E, F, G, H, I, J) with functional dependencies K = {B→E, E→FH, BCD→G, CD→A, A→J, I→BCDE, H→I}. Determine if B→J holds and list every candidate key. Show each step.

- for K, B^+ = ABCDEFGHIJ.

  $J \subseteq B^+$ so B→J holds.

- candidate key

  ① neither left nor right. ϕ

  left: ABCDEHI

  right: ABCDEFGHIJ.

  ②. only left: ϕ

  ③ only right: FGJ

  ~~④ ϕ~~

  ⑤ $\phi^+ = \phi$ continue.

⑥ ABCDEHI

⑦ <A, B, C, D, E, H, I,

AB, AC, AD, AE, AH, AI, BC, BD, BE, BH, BI

CD, CE, CH, CI, DE, DH, DI, EH, EI, HI.

ABC, ABD, ABE, ABH, ABI, ACD, ACE, ACH, ACI.

ADE, ADH, ADI, AEH, AEI, AHI., BCD, B····

··

⑧ 1) $A^+ = AJ \subseteq R$

$B^+ = AB\,CDEFGHIJ = R$

$C^+ = C$ $\quad$ $E^+ = ABCDEFGHIJ = R$

$D^+ = D$ $\quad$ $H^+ = ABCDEFGHIJ = R$

$I^+ = ABCDEFGHIJ = R$

2) <A, C, D, AC, AD, CD, ACD>.

$AC^+ = ACJ \neq R$ $\quad$ $AD^+ = ADJ \neq R$

$CD^+ = ACDJ \neq R$ $\quad$ $ACD^+ = ACDJ \neq R$

in summary the candidate keys are.

B, E, H, I

5. [15 points] We have a set of functional dependencies $F = \{A \to B, B \to C\}$ for a relation schema R(A, B, C, D). Write down all the functional dependencies of the closure F^+ of F and count them.

power set of ABCD $\{\phi, A, B, C, D, AB, AC, AD, BC, BD, CD, ACD, ABC, ABD, BCD, ABCD\}$

$A^+ = ABC$

$\Rightarrow A \to A, A \to B, A \to C, A \to AB, A \to AC, A \to BC, A \to ABC$ (7)

$B^+ = BC$

$\Rightarrow B \to B, B \to C, B \to BC.$ (3).

$C^+ = C$

$\Rightarrow C \to C$ (1)

$D^+ = D$

$\Rightarrow D \to D$ (1)

$AB^+ = ABC$

$\Rightarrow AB \to A, AB \to B, AB \to C, AB \to AB, AB \to AC, AB \to ABC, AB \to BC$ (7)

$AC^+ = ABC$

$\Rightarrow AC \to A, AC \to B, AC \to C, AC \to AB, AC \to AC, AC \to ABC, AC \to BC$ (7)

$AD^+ = ABCD$

$\Rightarrow AD \to A, AD \to B, AD \to C, AD \to D, AD \to AB, AD \to AC, AD \to AD, AD \to BC$
$AD \to BD, AD \to CD, AD \to ABC, AD \to ABD, AD \to BCD, AD \to ABCD$

(15) $AD \to ACD$

$BC^+ = BC$

⇒ BC→B, BC→C, BC→BC (3)

$BD^+ = BCD$

⇒ BD→B, BD→C, BD→D, BD→BC, BD→BD, BD→BCD, BD→CD

(7).

$CD^+ = CD$

⇒ CD→C, CD→D, CD→CD (3)

$ABC^+ = ABC$

⇒ ABC→A, ABC→B, ABC→C, ABC→AB, ABC→AC, ABC→BC, ABC→ABC

(7)

$ABD^+ = ABCD$

⇒ ABD→A, ABD→B, ABD→C, ABD→D, ABD→AB, ABD→AC, ABD→AD

ABD→BC, ABD→BD, ABD→CD, ABD→ABC, ABD→ABD, ABD→BCD

ABD→ACD

ABD→ABCD (15).

$BCD^+ = BCD$

⇒ BCD→B, BCD→C, BCD→D, BCD→BC, BCD→BD, BCD→BCD

BCD→CD

(7).

$ABCD^+ = ABCD$

⇒ ABCD→A, ABCD→B, ABCD→C, ABCD→D, ABCD→AB, ABCD→AC

ABCD→AD, ABCD→BC, ABCD→BD, ABCD→CD, ABCD→ABC, ABCD→ABD

ABCD→BCD, ABCD→ABCD (15).

$ACD^+ = ABCD$

ACD→A, ACD→B, ACD→C, ACD→D, ACD→AB

ACD→AC, ACD→AD, ACD→BC, ACD→BD, ACD→CD

ACD→ABC, ACD→ABD, ACD→ACD, ACD→BCD, ACD→ABCD

(15)

total is 113 FDs.

Exercise 2 [20 points]

1. [5 points] Consider the relation schema R(A, B, C, D, E) with functional dependencies F = {A→B, AB→C, D→ACE} and G = {A→BC, D→AE}. Are the two sets F and G equivalent? Show each step.

check if G cover F:

$A^+ = ABC$. $B \subseteq A^+$ $D^+ = ABCDE$. $ACE \subseteq D^+$

$AB^+ = ABC$ $C \subseteq AB^+$

check if F cover G:

$A^+ = ABC$ $BC \subseteq A^+$

$D^+ = ABCDE$ $AE \subseteq D^+$

So F and G are equivalent

2. [5 points] Consider the relation schema R(A, B, C, G, H, I) and the set of functional dependencies F = {A→B, A→C, CG→H, CG→I, B→H}. Infer at least five new FDs by using five different Armstrong's axioms. Show each step.

augmentation rule: $AC \to BC$

transitivity rule: $A \to H$.

union rule: $A \to BC$.

decomposition rule: $C \to I$, $C \to G$.

pseudotransitivity rule: $AG \to H$

3. [5 points] Consider the relation schema R(A, B, C, D, E) with the set of functional dependencies F = {A→BC, CD→E, B→D, E→A}. List all candidate keys of R by using Armstrong's Axioms. Show each step.

$A^+ = ABCDE$. $\quad C^+ = C \quad E^+ = ABCDE$

$B^+ = BD \quad D^+ = D$

$BC^+ = ABCDE$

$BD^+ = BD$

$CD^+ = ABCDE$.

all candidate keys are

A, E, BC, CD,

1) augmentation rule: $A \to BC \Rightarrow A \to ABC$.

decomposition rule: $A \to BC \Rightarrow A \to B, A \to C$

transitivity rule: $A \to B, B \to D \Rightarrow A \to D$

augmentation rule: $A \to ABC, A \to D \Rightarrow A \to ABCD$

decomposition: $A \to ABCD \Rightarrow A \to AB, A \to CD$

transitivity: $A \to CD, CD \to E \Rightarrow A \to E$

Union: $A \to ABCD, A \to E \Rightarrow A \to ABCDE$.

2) pseudotransitivity: $CD \to E, B \to D \Rightarrow BC \to E$.

union: $BC \to E, B \to D \Rightarrow BC \to DE$.

decomposition: $BC \to DE \Rightarrow BC \to D, BC \to E$

transitivity $BC \to E, E \to A \Rightarrow BC \to A \Rightarrow BC \to ABCDE$

$A \to ABCDE$

3) transivity: $E \to A$, $A \to ABCDE \Rightarrow E \to ABCDE$

4) transivity: $CD \to E$, $E \to ABCDE \Rightarrow CD \to ABCDE$

transivity. $CDE \to A$, $A \to ABCDE \Rightarrow CDE \to ABCDE$

in summary all the candidate keys are {A, E, BC, CD.}

4. [5 points] Consider the relation schema R(A, B, C, D, E, F) with the set of functional dependencies F = {A→B, A→C, CD→E, CD→F, B→E}. Apply Armstrong's Axioms to find one candidate key for R. Show each step.

Pseudotransivity: $A \to C$, $CD \to E \Rightarrow AD \to E$.

Pseudotransivity: $A \to C$, $CD \to F \Rightarrow AD \to F$.

union: $AD \to E$, $AD \to F \Rightarrow AD \to EF$

union: $AD \to EF$, $A \to B$, $A \to C$, $A \to A$, $D \to D$. $\Rightarrow AD \to ABCDEF$

also. A^+ and $D^+ \neq R$

so AD is a candidate key for R

Exercise 3 [35 points]

1. [15 points] Find a minimal cover for the relation R(A, B, C, D, E, G) with the set K = {AB→C, C→A, BC→D, ACD→B, D→E, D→G, BE→C, CG→B, CG→D, CE→A, CE→G} of functional dependencies. Show each step.

① $K = \{AB \to C, C \to A, BC \to D, ACD \to B, D \to E, D \to G, BE \to C,$
$CG \to B, CG \to D, CE \to A, CE \to G\}.$

② . $AB \to C$

B: $A^+ = A \quad C \notin A^+$

A: $B^+ = B \quad C \notin B^+.$ A, B can not remove

$BC \to D$

B: $C^+ = AC. \quad D \notin C^+$

C: $B^+ = B \quad D \notin B^+$ B, C can not remove.

$ACD \to B$

A: $CD^+ = ABCDEG. \quad B \in CD^+$

thus $CD \to B$ ⇒ $CD \to B$

C: $D^+ = DEG. \quad B \notin D^+$

D: $C^+ = AC \quad B \notin C^+.$

$BE \to C$:

B: $E^+ = E \quad C \notin E^+$

E: $B^+ = B \quad C \notin B^+$

$CG \to B$:

C: $G^+ = G \quad B \notin G^+$

G: $C^+ = AC \quad B \notin C^+$

$CG \to D$:

C: $G^+ = G$ $\quad D \nsubseteq G^+$

G: $C^+ = AC$ $\quad D \nsubseteq C^+$

$CE \to A$:

C: $E^+ = E$ $\quad A \nsubseteq E^+$

E: $C^+ = AC$ $\quad A \subseteq C^+$ $\quad \Rightarrow \quad C \to A$

$CE \to G$.

C: $E^+ = E$ $\quad G \nsubseteq E^+$

E: $C^+ = AC$ $\quad G \nsubseteq C^+$

$$K = \{AB \to C,\ C \to A,\ BC \to D,\ \cancel{CD \to B},\ D \to E,\ D \to G,\ BE \to C,$$
$$CG \to B,\ \cancel{CG \to D},\ \cancel{C \to A},\ CE \to G\}.$$

③

$AB \to C$: $\quad AB^+ = AB$ $\quad C \nsubseteq AB^+$.

$C \to A$: $\quad C^+ = C$ $\quad A \nsubseteq C$.

$BC \to D$: $\quad BC^+ = ABC$ $\quad D \nsubseteq BC^+$

~~$CD \to B$: $\quad CD^+ = ABCDEG$. $\quad B \subseteq CD^+$.~~

$D \to E$: $\quad D^+ = DG$. $\quad E \nsubseteq D^+$

$D \to G$: $\quad D^+ = DE$ $\quad G \nsubseteq D^+$

$BE \to C$: $\quad BE^+ = BE$. $\quad C \nsubseteq BE^+$

$CG \to B$: $\quad CG^+ = ACDEG$. $\quad B \nsubseteq CG^+$

$CG \to D$: $\quad CG^+ = ABCDEG$. $\quad D \subseteq CG^+$

$C \to A$: $\quad C^+ = AC$ $\quad A \subseteq C^+$

$CE \to G$: $\quad CE^+ = ACE$ $\quad G \nsubseteq CE^+$

④ $K = \{AB \to C,\ C \to A,\ BC \to D,\ D \to E,\ D \to G,\ BE \to C,$
$CG \to B,\ CE \to G\}$

2. [10 points] Find a minimal cover for the relation R(A, B, C, D, E, F, G, H) with the set K = {A→B, ABCD→E, EF→GH, ACDF→EG} of functional dependencies. Show each step.

① $K = \{A \to B,\ ABCD \to E,\ EF \to GH,\ ACDF \to EG\}$

② $ABCD \to E$:

$A: BCD^+ = BCD \quad E \nsubseteq BCD^+$

$B: ACD^+ = ABCDE \quad E \subseteq ACD^+$

$ACD \to E$: $\Rightarrow ACD \to E$

$A: CD^+ = CD \quad E \nsubseteq CD^+$

$C: AD^+ = ABD \quad E \nsubseteq AD^+$

$D: AC^+ = ABC \quad E \nsubseteq AD^+$

$EF \to GH$.

$E: F^+ = F. \quad GH \nsubseteq F^+$

$F: E^+ = E \quad GH \nsubseteq E^+$

$ACDF \to EG$

$A: CDF^+ = CDF \quad EG \nsubseteq CDF^+$

$C: ADF^+ = ABDF. \quad EG \nsubseteq ADF^+$

$D: ACF^+ = ABCF. \quad EG \nsubseteq ACF^+$

$F: ACD^+ = ABCDE \quad EG \nsubseteq ACD^+$

$K = \{A \to B,\ ACD \to E,\ EF \to GH,\ ACDF \to EG\}$

③ $EF \to GH$

remove G: $\{A \to B,\ ACD \to E,\ EF \to H,\ ACDF \to EG\}$

$(EF)^+ = EFH \quad GH \nsubseteq (EF)^+$

remove H: $\{A \to B,\ ACD \to E,\ EF \to G,\ ACDF \to EG\}$

$(EF)^+ = EFG \quad GH \nsubseteq EFG$

$ACDF \to EG.$

remove E: $\{A \to B,\ ACD \to E,\ EF \to GH,\ ACDF \to G\}$

$(ACDF)^+ = ABCDEFG \quad EG \subseteq (ACDF)^+$

remove G: $\{A \to B,\ ACD \to E,\ EF \to GH,\ ACDF \to E\}$

$(ACDF)^+ = ABCDEFGH \quad EG \subseteq (ACDF)^+.$

(3.1)

$K = \{A \to B,\ ACD \to E,\ EF \to GH,\ ACDF \to G\}$

for $ACDF \to G$:

remove G: $ACDF^+ = ABCDEFGH$

$G \subseteq (ACDF)^+.$

$K = \{A \to B,\ ACD \to E,\ EF \to GH,\ ACDF \to \phi\}$

(3.2)

$K = \{A \to B,\ ACD \to E,\ EF \to GH,\ ACDF \to E.\}$

for $ACDF \to E$:

remove E: $ACDF^+ = ABCDEFGH$

$E \subseteq (ACDF)^+$.

$K = \{A \to B,\ ACD \to E,\ EF \to GH,\ ACDF \to \phi\}$

$A \to B$: $A^+ = A$ $\quad B \nsubseteq A^+$

$ACD \to E$: $ACD^+ = ABCD$ $\quad E \nsubseteq ACD^+$

$EF \to GH$: $EF^+ = EF$ $\quad GH \nsubseteq EF^+$

standard form

(4). $F_C = \{A \to B,\ ACD \to E,\ EF \to G,\ EF \to H\}$

3. [10 points] Find a minimal cover for the relation R(A, B, C, D, E, F) with the set K = {A→D, AC→DE, B→F, D→CE} of functional dependencies. Show each step.

(1) $K = \{A \to D,\ AC \to DE,\ B \to F,\ D \to CE\}$.

(2) $AC \to DE$.

A: $C \to DE$ $\quad C^+ = C$ $\quad DE \nsubseteq C^+$.

C: $A \to DE$ $\quad A^+ = ACDE$ $\quad DE \subseteq A^+$. $\Rightarrow A \to DE$.

$K = \{A \to D,\ A \to DE,\ B \to F,\ D \to CE\}$

(3) $A \to DE$

D: $\{A \to D,\ A \to E,\ B \to F,\ D \to CE\}$

$A^+ = ACDE$. $\quad DE \subseteq A^+$.

for $A \to E$: $\{A \to D,\ A \to \phi,\ B \to F,\ D \to CE\}$.

$A^+ = ACDE$ $\quad E \subseteq A^+$

so: $\{A \to D,\ A \to \phi,\ B \to F,\ D \to CE\}$

E: $\{A \to D,\ A \to D,\ B \to F,\ D \to CE\}$

$A^+ = ACDE \quad DE \subseteq A^+.$

for $A \to D$ $\{A \to D, A \to \phi, B \to F, D \to CE\}.$

$A^+ = ACDE \quad D \subseteq A^+.$

So: $\{A \to D, A \to \phi, B \to F, D \to CE\}$

$D \to CE$

C: $\{A \to D, A \to \phi, B \to F, D \to E\}.$

$D^+ = DE \quad CE \nsubseteq D^+$

E: $\{A \to D, A \to \phi, B \to F, D \to C\}.$

$D^+ = CD \quad CE \nsubseteq D^+$

4 So. $\{A \to D, B \to F, D \to C, D \to E\}.$

standard form

Exercise 4 [10 points]

1. [5 points] Consider the relation schema R(A, B, C, D, E, F) with the set of functional dependencies K = {CF→D, AE→F, D→A, AB→C}. List all candidate keys of R in a systematic manner (do not use Armstrong's Axioms) and explain how you determine them. Show each step.

1 neither : ϕ

left: A B C D E F   right: A C D F

2 left: BE

3 right: ϕ

4 BE

5 $BE^+ = BE \neq R$ so continue.

6 A CDF

① $\langle A, C, D, F, AC, AD, AF, CD, CF, DF, ACD, ACF,$
$CDF, ACDF \rangle$

add one:

$ABE^+ = ABCDEF = R$

$CBE^+ = CBE \subseteq R$

$DBE^+ = ABCDEF = R$

$BEF^+ = BEF \subseteq R$

CBE, BEF

add one:

$ABCE^+ = ABCDEF = R.$

$BCDE^+ = ABCDEF = R.$

$BCEF^+ = ABCDEF = R$

$ABEF^+ = ABCDEF = R$

$BCEF^+ = ABCDEF = R$

$BDEF^+ = ABCDEF = R.$

In summary, the candidate keys are:

ABE, DBE, $BCEF$

2. [5 points] Consider the relation schema R(A, B, C, D, E, H) with the set of functional dependencies K = {A→B, BC→D, E→C, D→A}. Determine all candidate keys of R in a systematic manner (do not use the Armstrong's Axioms) and explain how you determine them. Show each step.

① neither : H

② left = E   left: ABCDE   right : ABCD.

③ right = $\varnothing$

④ EH

⑤ EH^+ = CEH. ≠ R   continue

⑥ ABCD

⑦ <A, B, C, D, AB, AC, AD, BC, BD, CD, ABC, ABD, ACD, BCD, ABCD>

add one :

AEH^+ = ABCDEH = R

BEH^+ = ABCDEH = R.

CEH^+ = CEH ⊆ R

DEH^+ = ABCDEH. = R

CEH : add one :

$ACEH^+$ = R

$BCEH^+$ = R

$CDEH^+$ =. R.   only supper key

In summary, the candidate keys are:

AEH, BEH, DEH.