Minimal Cover (III)

- Algorithm for computing a minimal cover

  F_c *CalculateMinimalCover*(F)
  // Input: A set F of FDs
  // Output: A minimal cover F_c

  // Step 1: Initialize F_c
  $F_c := F$

  // Step 2: Perform a left reduction of the FDs in F_c, i.e., identify and remove all attributes on the left-hand sides of FDs in F_c that are extraneous
  for each $A \rightarrow B \in F_c$ **do**
  for each $a \in A$ **do**
  if $A - \{a\} \neq \emptyset$ **and** $B \subseteq CalculateAttributeClosure(F_c, A - \{a\})$ **then**
  $F_c := F_c - \{A \rightarrow B\} \cup \{(A - \{a\}) \rightarrow B\}$

Minimal Cover (IV)

- Algorithm for computing a minimal cover (*continued*)

// Step 3: Perform a right reduction of the remaining FDs in F_c, i.e., identify and remove all attributes on the right-hand sides of FDs in F_c that are extraneous

for each $A \rightarrow B \in F_c$ **do**

for each $b \in B$ **do**

if $b \in \mathit{CalculateAttributeClosure}(F_c - \{A \rightarrow B\} \cup \{A \rightarrow (B - \{b\})\}, A)$ **then**

$F_c := F_c - \{A \rightarrow B\} \cup \{A \rightarrow (B - \{b\})\}$

// Step 4: Remove all FDs of the form $A \rightarrow \emptyset$ from F_c, which have perhaps been produced in the previous step, since they are meaningless

for each $A \rightarrow B \in F_c$ **do**

if $B = \emptyset$ **then** $F_c := F_c - \{A \rightarrow \emptyset\}$

Minimal Cover (V)

- Algorithm for computing a minimal cover (*continued*)

  // Step 5a: If the goal is to obtain a minimal cover in *standard form*,
  decompose the right-hand sides of all FDs in F_c such that each
  FD in F_c has a single attribute on its right-hand side

  for each $A \to B \in F_c$ **do**

  if $B = \{b_1, \ldots, b_n\}$ **and** $n > 1$ **then**

  $F_c := F_c - \{A \to B\} \cup \{A \to \{b_1\}, \ldots, A \to \{b_n\}\}$

  return F_c

Minimal Cover (VI)

- Algorithm for computing a minimal cover (*continued*)

// Step 5b: If the goal is to obtain a minimal cover in *nonstandard form*,
apply the union rule to all FDs with equal left-hand sides

$H := F_c$

$F_c := \varnothing$

for each $A \rightarrow B \in H$ **do**

$G := \varnothing$ // FDs that have been processed and that have to be deleted from
// H at the end of each loop

$X := \varnothing$ // Union of all right-hand sides of FDs with A on their left-hand side

for each $C \rightarrow D \in H$ **do**

if $A = C$ **then**

$G := G \cup \{C \rightarrow D\}$

$X := X \cup D$

$H := H - G$

$F_c := F_c \cup \{A \rightarrow X\}$

return F_c

Minimal Cover (VII)

- Example 1
  - Compute a minimum cover for the set $F = \{B \rightarrow A, D \rightarrow A, AB \rightarrow D\}$ of FDs on $R(A, B, D)$
  - Step 1
    - $F_c := \{B \rightarrow A, D \rightarrow A, AB \rightarrow D\}$
  - Step 2
    - Only $AB \rightarrow D$ has more than one attribute on its left-hand side
    - To check whether B can be removed, we compute whether $D \subseteq \mathit{CalculateAttributeClosure}(F_c, A)$ holds
    - This is not the case since $A^+ = A$ and $D \not\subseteq A$ holds
    - To check whether A can be removed, we compute whether $D \subseteq \mathit{CalculateAttributeClosure}(F_c, B)$ holds
    - This is the case since $B^+ = ABD$ and $D \subseteq ABD$
    - Hence, A can be removed, and we obtain $F_c := \{B \rightarrow A, D \rightarrow A, B \rightarrow D\}$

Minimal Cover (VIII)

- Example 1 (*continued*)
  - Step 3
    - To check whether A can be removed from $B \rightarrow A$, we check whether $A \subseteq CalculateAttributeClosure(\{B \rightarrow \emptyset, D \rightarrow A, B \rightarrow D\}, B)$ holds
    - This is the case since $B^+ = ABD$ and $A \subseteq ABD$
    - Hence, A can be removed, and we obtain $F_c := \{B \rightarrow \emptyset, D \rightarrow A, B \rightarrow D\}$
    - To check whether A can be removed from $D \rightarrow A$, we check whether $A \subseteq CalculateAttributeClosure(\{B \rightarrow \emptyset, D \rightarrow \emptyset, B \rightarrow D\}, D)$ holds
    - This is not the case since $D^+ = D$ and $A \not\subset D$ holds
    - To check whether D can be removed from $B \rightarrow D$, we check whether $D \subseteq CalculateAttributeClosure(\{B \rightarrow \emptyset, D \rightarrow A, B \rightarrow \emptyset\}, B)$ holds
    - This is not the case since $B^+ = B$ and $D \not\subset B$ holds
    - After this step we have: $F_c := \{B \rightarrow \emptyset, D \rightarrow A, B \rightarrow D\}$
  - Step 4: We obtain $F_c := \{D \rightarrow A, B \rightarrow D\}$
  - Step 5a/5b: $F_c := \{D \rightarrow A, B \rightarrow D\}$ is in both forms

Minimal Cover (IX)

- Example 2
  - Compute a minimum cover for the set $F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C\}$ of FDs on $R(A, B, C)$
  - Step 1
    - $F_c := \{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C\}$
  - Step 2
    - Only $AB \rightarrow C$ has more than one attribute on its left-hand side
    - To check whether A can be removed, we compute whether $C \subseteq CalculateAttributeClosure(F_c, B)$ holds
    - This is the case since $B^+ = BC$ and $C \subseteq BC$
    - Hence, A can be removed, and we obtain $F_c := \{A \rightarrow BC, B \rightarrow C, A \rightarrow B\}$
    - This also means that the number of FDs in F_c has been reduced by 1

Minimal Cover (X)

- Example 2 (*continued*)
  - We have so far: $F_c := \{A \rightarrow BC, B \rightarrow C, A \rightarrow B\}$
  - Step 3
    - To check whether C can be removed from $A \rightarrow BC$, we check whether $C \subseteq \mathit{CalculateAttributeClosure}(\{A \rightarrow B, B \rightarrow C\}, A)$ holds
    - This is the case since $A^+ = ABC$ and $C \subseteq ABC$
    - Hence, C can be removed, and we obtain $F_c := \{A \rightarrow B, B \rightarrow C\}$
    - To check whether B can be removed from $A \rightarrow B$, we check whether $B \subseteq \mathit{CalculateAttributeClosure}(\{A \rightarrow \emptyset, B \rightarrow C\}, A)$ holds
    - This is not the case since $A^+ = A$ and $B \not\subseteq A$ holds
    - To check whether C can be removed from $B \rightarrow C$, we check whether $C \subseteq \mathit{CalculateAttributeClosure}(\{A \rightarrow B, B \rightarrow \emptyset\}, B)$ holds
    - This is not the case since $B^+ = B$ and $C \not\subseteq B$ holds
    - After this step we have: $F_c := \{A \rightarrow B, B \rightarrow C\}$
  - Step 4: Nothing to do since there is no FD with an $\emptyset$ on its right-hand side
  - Step 5a/5b: $F_c := \{A \rightarrow B, B \rightarrow C\}$ is in both forms

Minimal Cover (XI)

- Example 3
  - This example shows that more than one minimal cover can exist for the *same* set F of FDs
    - The minimal covers computed for the same F of FDs depend on the order in which the FDs are processed
    - Different orders can lead to different minimal covers
    - However, the algorithm computes exactly one of them; they are all equivalent
  - Compute a minimum cover for the set $F = \{A \rightarrow BC, C \rightarrow AB, B \rightarrow AC\}$ of FDs on $R(A, B, C, D)$
  - Step 1
    - $F_c := \{A \rightarrow BC, C \rightarrow AB, B \rightarrow AC\}$
  - Step 2
    - There is no FD that has more than one attribute on its left-hand side
    - Therefore, nothing has to be done

Minimal Cover (XII)

- Example 3 (*continued*)
  - We have so far: F_c := $\{A \rightarrow BC, C \rightarrow AB, B \rightarrow AC\}$
  - Step 3
    - In $A \rightarrow BC$ both B and C are extraneous under F_c
      - C can be removed since $C \subseteq CalculateAttributeClosure(\{A \rightarrow B, C \rightarrow AB, B \rightarrow AC\}, A)$ holds: $A^+ = ABC$ and $C \subseteq ABC$
      - B can be removed since $B \subseteq CalculateAttributeClosure(\{A \rightarrow C, C \rightarrow AB, B \rightarrow AC\}, A)$ holds: $A^+ = ABC$ and $B \subseteq ABC$
    - We are not allowed to remove B and C at the same time since the algorithm picks one of the two and deletes it
    - Case 1: C is removed; we get $F_c^1 = \{A \rightarrow B, C \rightarrow AB, B \rightarrow AC\}$
      - B is now not extraneous in $A \rightarrow B$ since $A^+ = A$ under $\{A \rightarrow \emptyset, C \rightarrow AB, B \rightarrow AC\}$ holds and $B \not\subset A$ holds
      - Continuing the algorithm, we find that A and B are extraneous in the right-hand side of $C \rightarrow AB$ under F_c^1
      - B can be removed since $B \subseteq CalculateAttributeClosure(\{A \rightarrow B, C \rightarrow A, B \rightarrow AC\}, C)$ holds: $C^+ = ABC$ and $B \subseteq ABC$

Minimal Cover (XIII)

- Example 3 (*continued*)
  - Step 3 (*continued*)
    - A can be removed since $B \subseteq CalculateAttributeClosure(\{A \to B, C \to B, B \to AC\}, C)$ holds: $C^+ = ABC$ and $A \subseteq ABC$
    - Case 1.1: B is removed; we get $F_c^2 = \{A \to B, C \to A, B \to AC\}$
      - A is now not extraneous in $C \to A$ since $C^+ = C$ under $\{A \to B, C \to \varnothing, B \to AC\}$ holds and $A \not\subset C$ holds
      - C is not extraneous in $B \to AC$ since $B^+ = AB$ under $\{A \to B, C \to A, B \to A\}$ holds and $C \not\subset AB$ holds
      - A is extraneous in $B \to AC$ since $B^+ = ABC$ under $\{A \to B, C \to A, B \to C\}$ holds and $A \subseteq ABC$ holds
      - We get $F_c^3 = \{A \to B, C \to A, B \to C\}$
      - C is not extraneous in $B \to C$ since $B^+ = B$ under $\{A \to B, C \to A, B \to \varnothing\}$ holds and $C \not\subset B$ holds
      - The algorithm terminates, and we obtain the first minimal cover $F_{c1} = \{A \to B, C \to A, B \to C\}$

Minimal Cover (XIV)

- Example 3 (*continued*)
  - Step 3 (*continued*)
      - Case 1.2: A is removed; we get $F_c^4 = \{A \rightarrow B, C \rightarrow B, B \rightarrow AC\}$
        - B is now not extraneous in $C \rightarrow B$ since $C^+ = C$ under $\{A \rightarrow B, C \rightarrow \varnothing, B \rightarrow AC\}$ holds and $B \not\subset C$ holds
        - C is not extraneous in $B \rightarrow AC$ since $B^+ = AB$ under $\{A \rightarrow B, C \rightarrow B, B \rightarrow A\}$ holds and $C \not\subset AB$ holds
        - A is not extraneous in $B \rightarrow AC$ since $B^+ = BC$ under $\{A \rightarrow B, C \rightarrow B, B \rightarrow C\}$ holds and $A \not\subset BC$ holds
        - The algorithm terminates, and we obtain the second minimal cover $F_{c2} = \{A \rightarrow B, C \rightarrow B, B \rightarrow AC\}$
    - Case 2: B is removed; we get $F_c^5 = \{A \rightarrow C, C \rightarrow AB, B \rightarrow AC\}$
      - Similarly to case 1, we obtain two further minimal covers:
        - $F_{c3} = \{A \rightarrow C, C \rightarrow B, B \rightarrow A\}$
        - $F_{c4} = \{A \rightarrow C, C \rightarrow AB, B \rightarrow C\}$

Minimal Cover (XV)

- Example 3 (*continued*)
  - Step 3 (continued)
    - For $F = \{A \rightarrow BC, C \rightarrow AB, B \rightarrow AC\}$ we have detected the following four minimal covers:
      - $F_{c1} = \{A \rightarrow B, C \rightarrow A, B \rightarrow C\}$
      - $F_{c2} = \{A \rightarrow B, C \rightarrow B, B \rightarrow AC\}$
      - $F_{c3} = \{A \rightarrow C, C \rightarrow B, B \rightarrow A\}$
      - $F_{c4} = \{A \rightarrow C, C \rightarrow AB, B \rightarrow C\}$
    - Note that more minimal covers can be found for F
  - Step 4
    - There is no FD with an $\emptyset$ on its right-hand side
  - Step 5a
    - We have to modify F_{c2} and F_{c4} and obtain

      $F_{c2}' = \{A \rightarrow B, C \rightarrow B, B \rightarrow A, B \rightarrow C\}$

      $F_{c4}' = \{A \rightarrow C, C \rightarrow A, C \rightarrow B, B \rightarrow C\}$
  - Step 5b: F_{c1}, F_{c2}, F_{c3}, and F_{c4} are already in nonstandard form