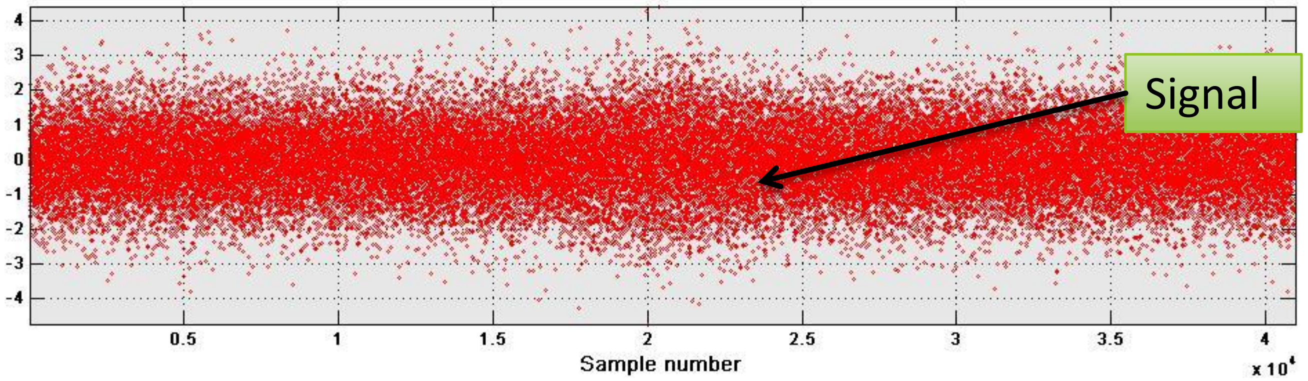
Beyond MLE and GLRT

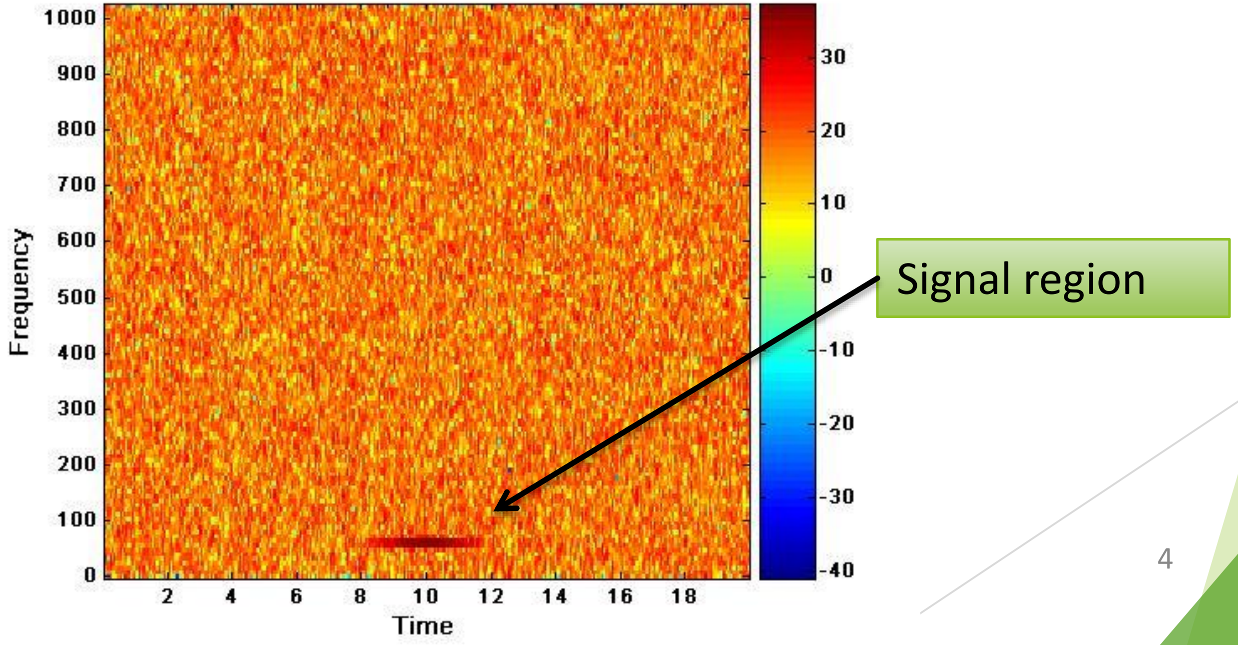
Signal models

- MLE and GLRT both require reliable signal models
- Many anticipated GW sources have unpredictable signals (e.g., core-collapse supernova) or signals that are extremely computationally expensive to use MLE/GLRT on (e.g., Extreme mass ratio inspirals)
- It is possible to extend MLE and GLRT to the case of where signal models are not known: Requires regularization techniques
- Various types of time-frequency analysis methods may also be used
- MLE and GLRT also require reliable noise models
- Data analysis techniques (e.g., vetoes) are needed to bring the performance of MLE and GLRT closer to the ideal one

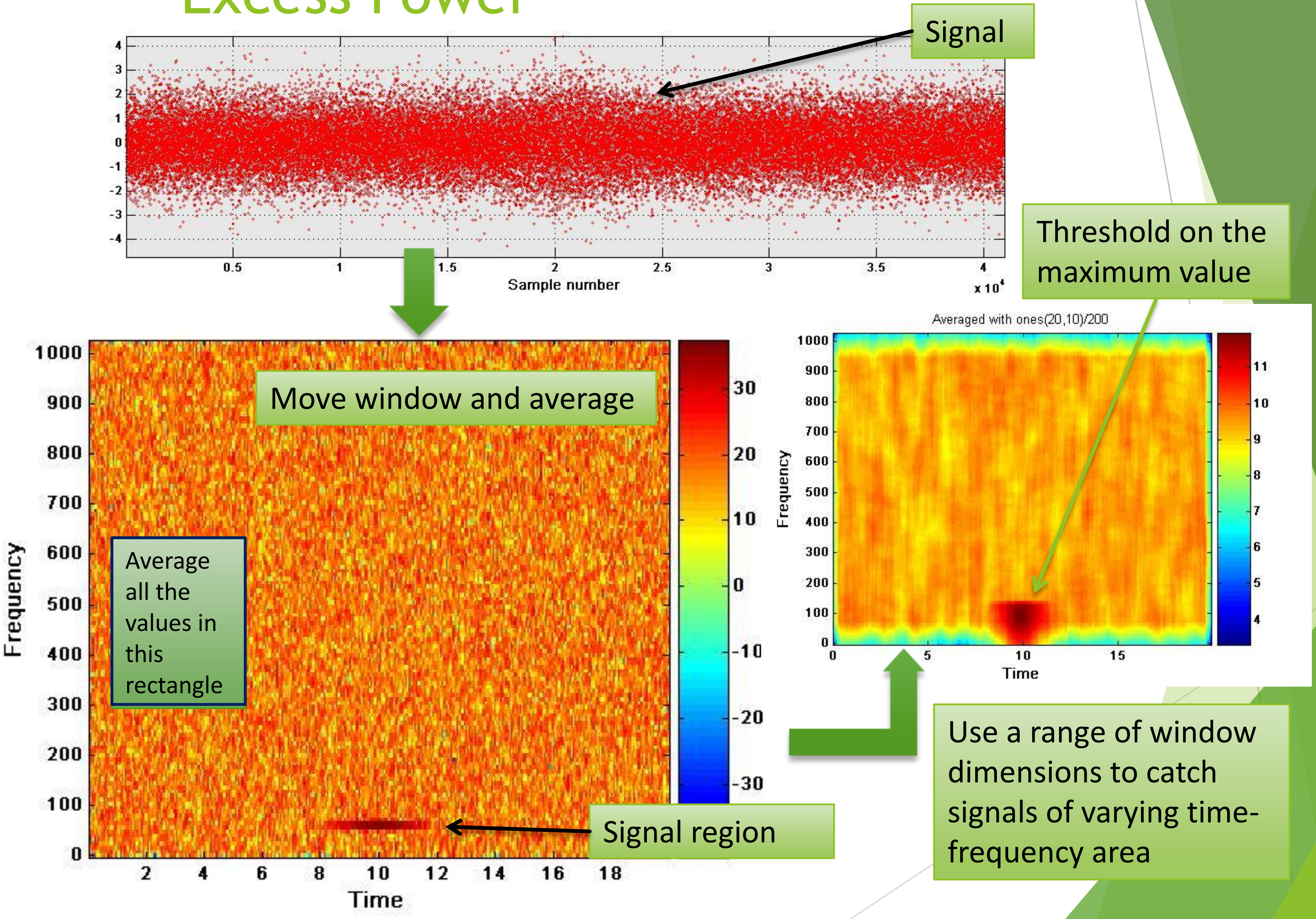
Time-frequency analysis

Time frequency analysis of noisy data



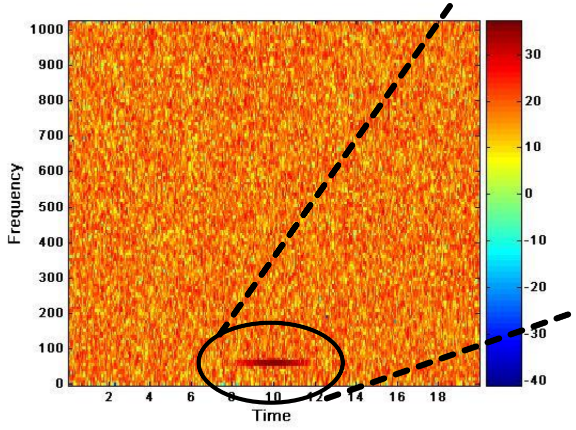


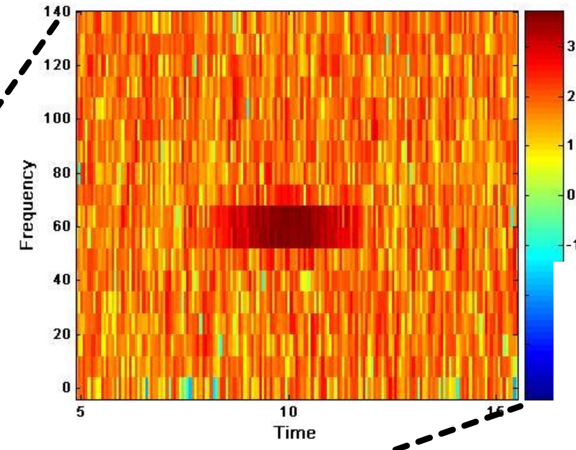
Excess Power



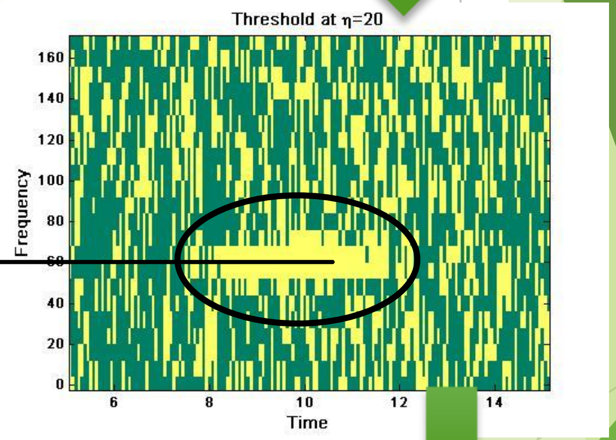
TF clustering

Assume that a signal occupies a simply connected region of the time-frequency plane





•Apply a threshold η: set pixels whose value is $< \eta$ to zero



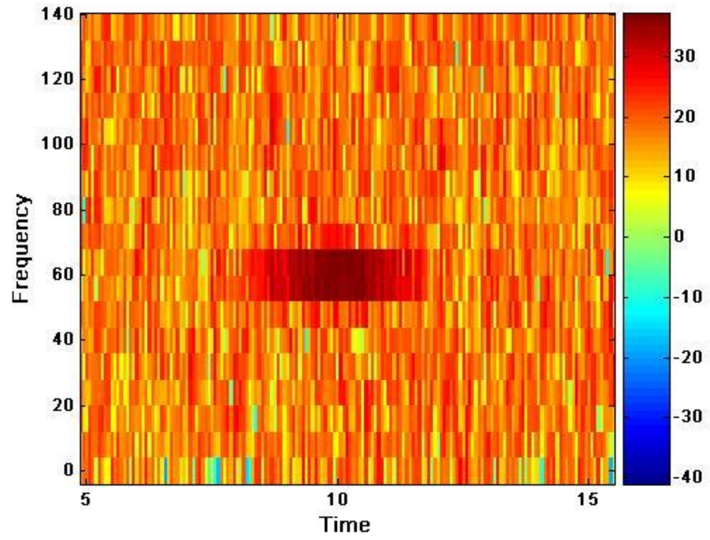
Cluster
Find groups of pixels that are "connected" to each other
•A binary metric: distance 0 or 1

Retain only clusters above a certain threshold size

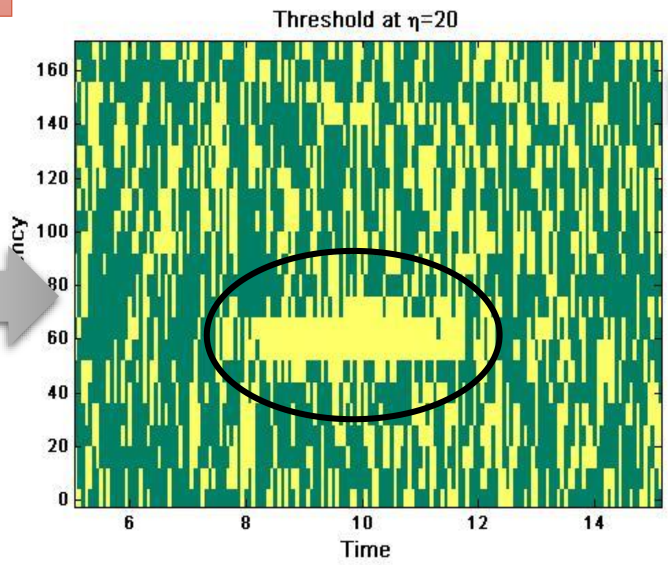
Further threshold on average value of pixels in the cluster

TF clustering

Assume that a signal occupies a simply connected region of the time-frequency plane



Apply a threshold: set pixels below threshold to 0 and above to 1



Cluster
Find groups of pixels that are "connected" to each other

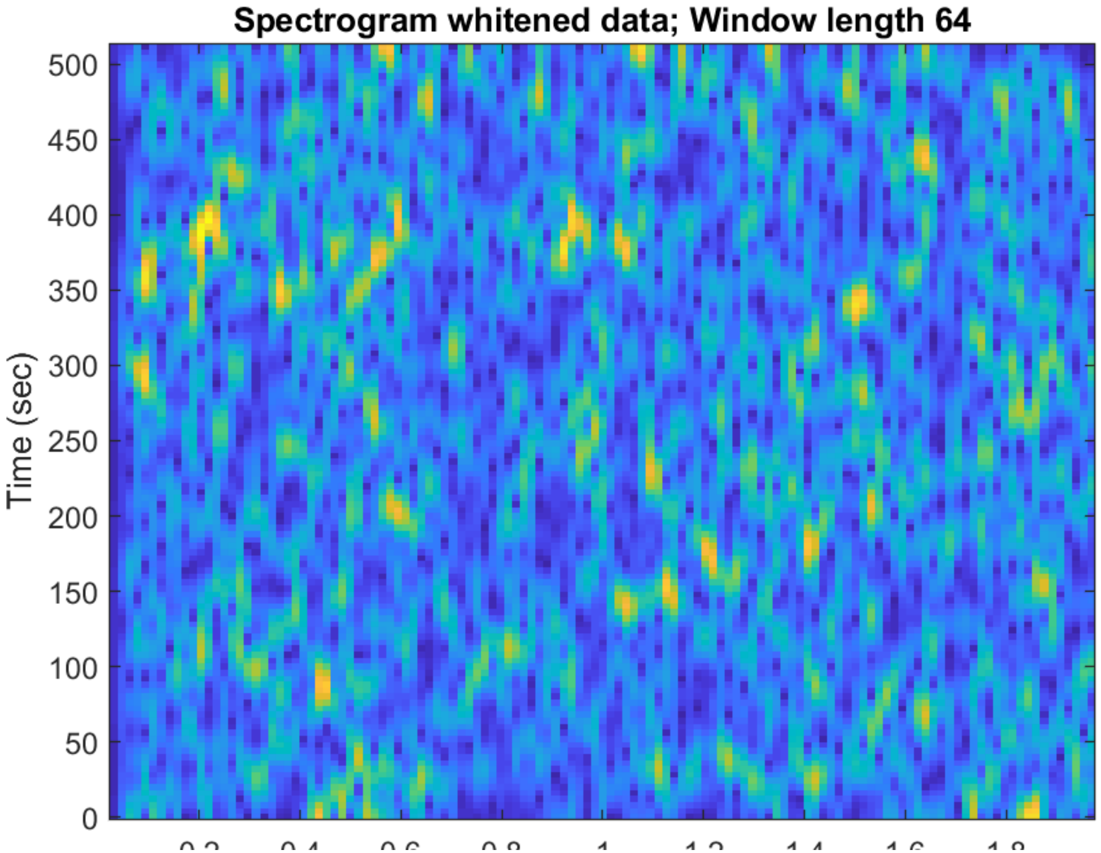
Retain only clusters above a certain threshold size

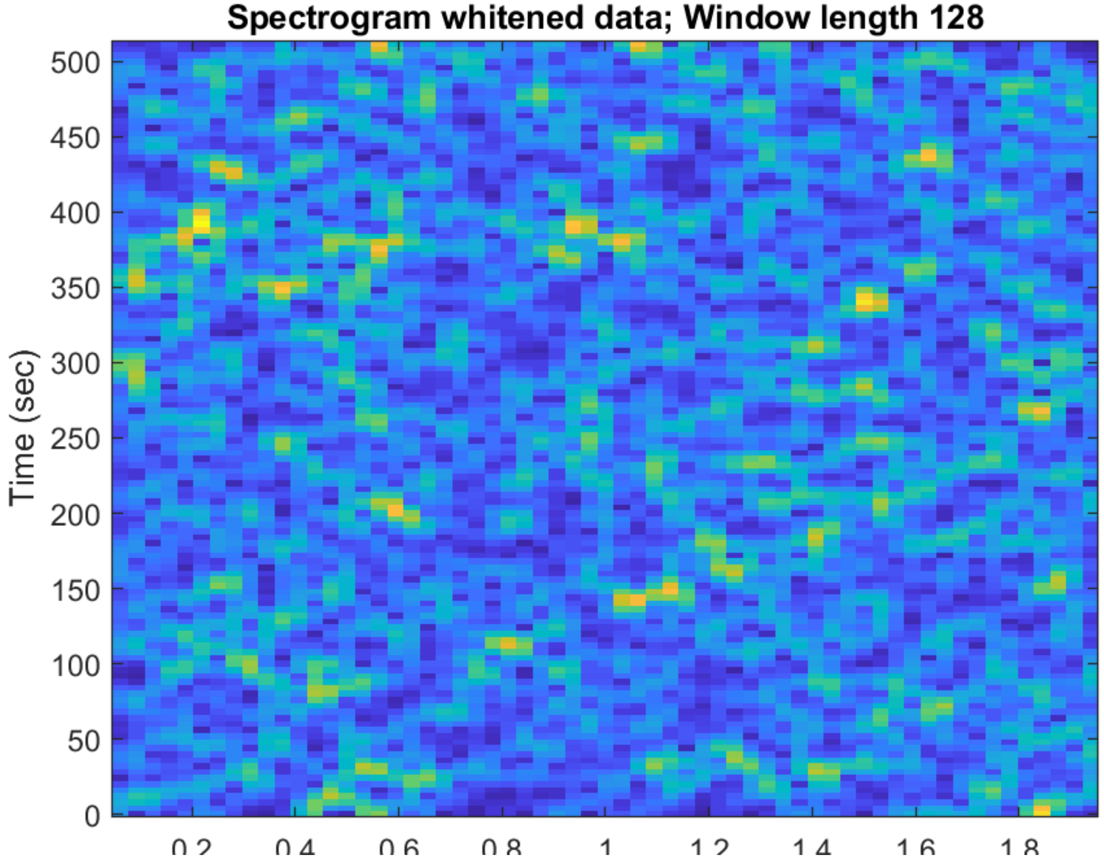
Further threshold on average value of pixels in the cluster

TF methods in GW data analysis

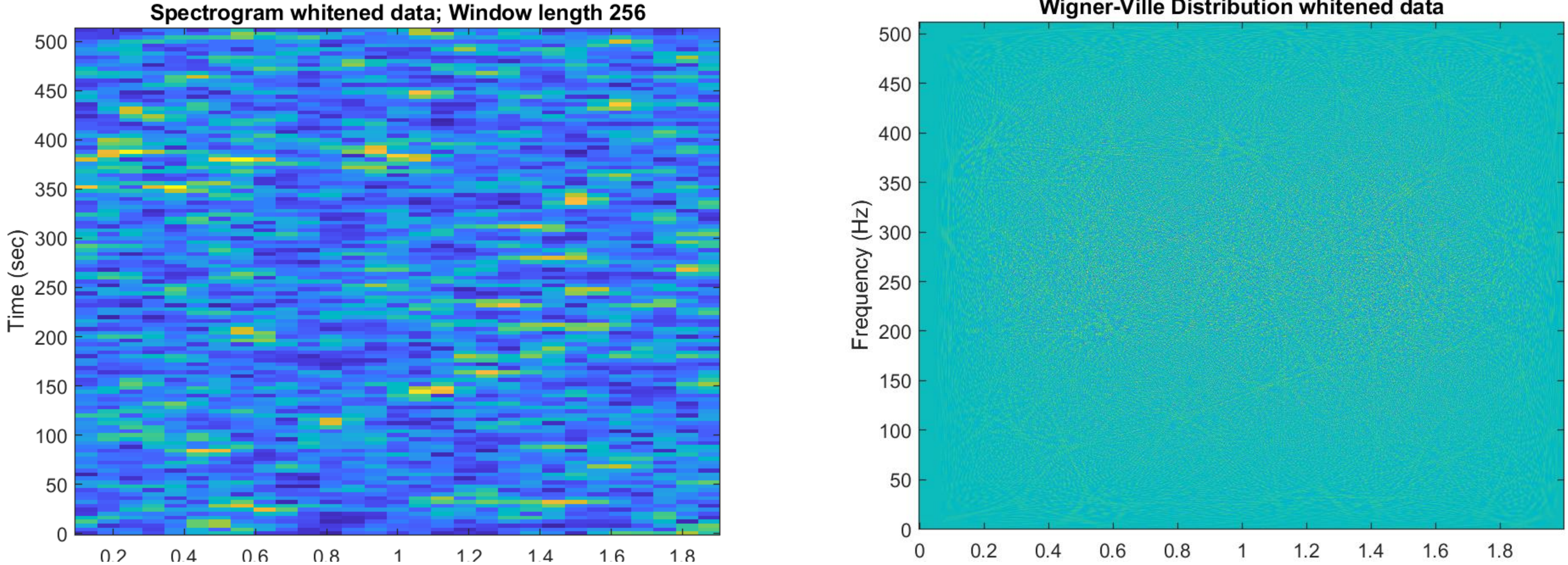
- Time-frequency methods plays an important role in searches for unmodeled gravitational wave signals
  - GW150914 discovered with Coherent WaveBurst (CWb): TF analysis with Wavelets (*Klimenko, Yakushin, Rakhmanov, Mitselmakher, CQG, 2004)* combined with regularized network analysis (*Klimenko, Mohanty, Rakhmanov, Mitselmakher, PRD, 2005)*
  - *Klimenko et al, PRD, 2016*
- Q-transform (used in Omicron)
  - *Brown, JASA, 1991; Chatterji, Blackburn, Martin, Katsavounidis, CQG, 2004*

TF analysis: Mock data

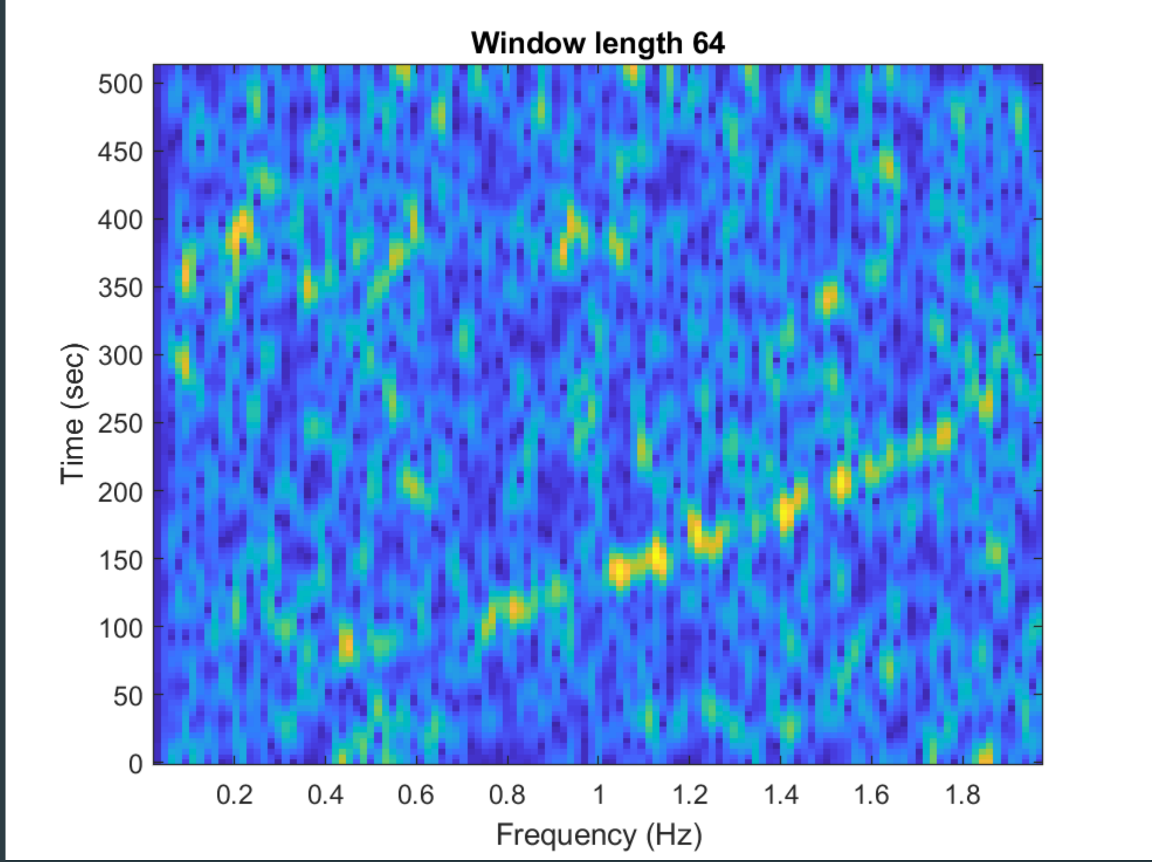


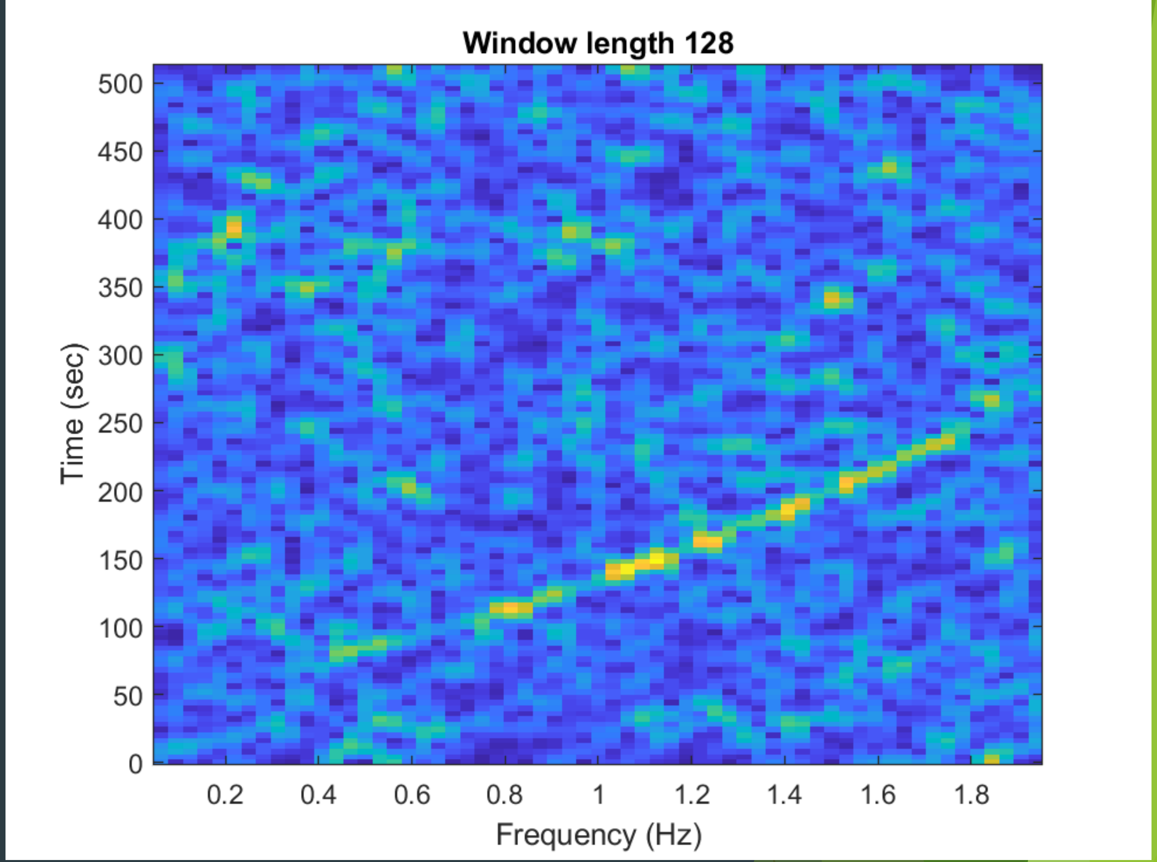


Quadratic chirp in (whitened) initial LIGO noise (SNR=8.3)

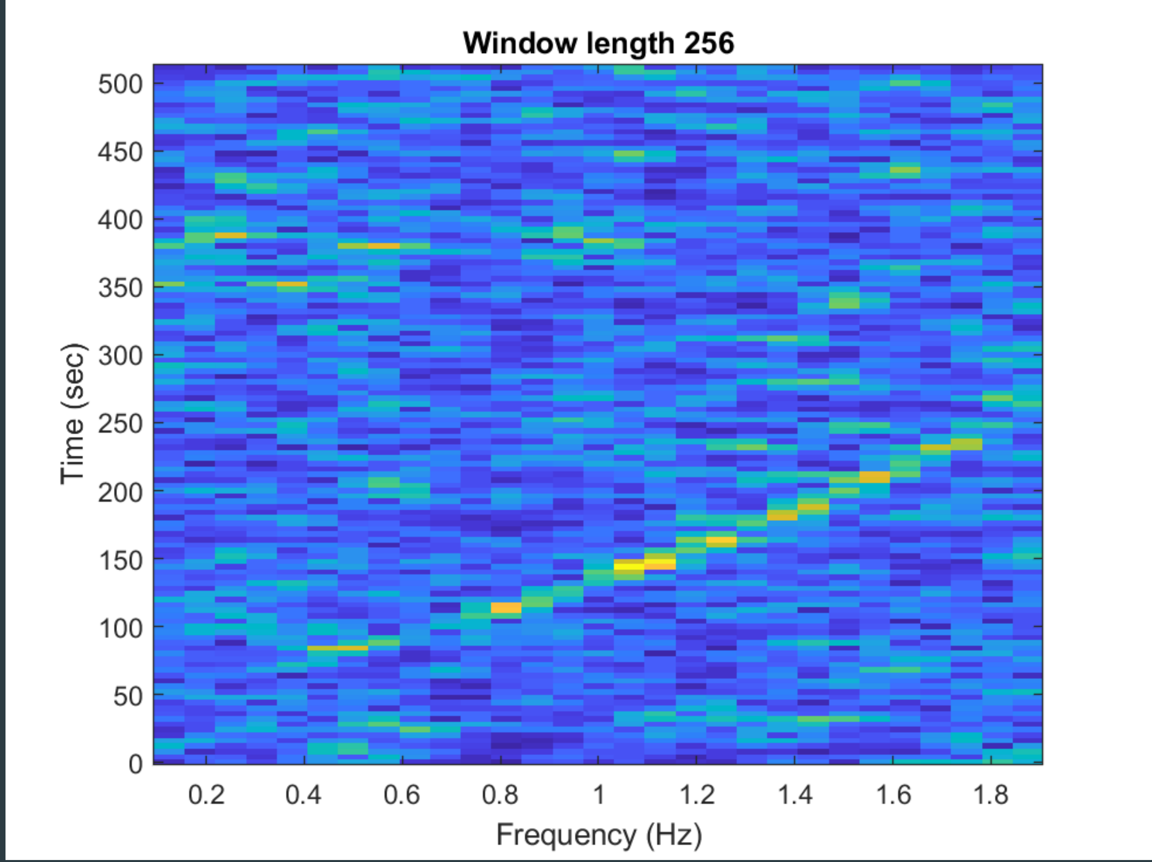


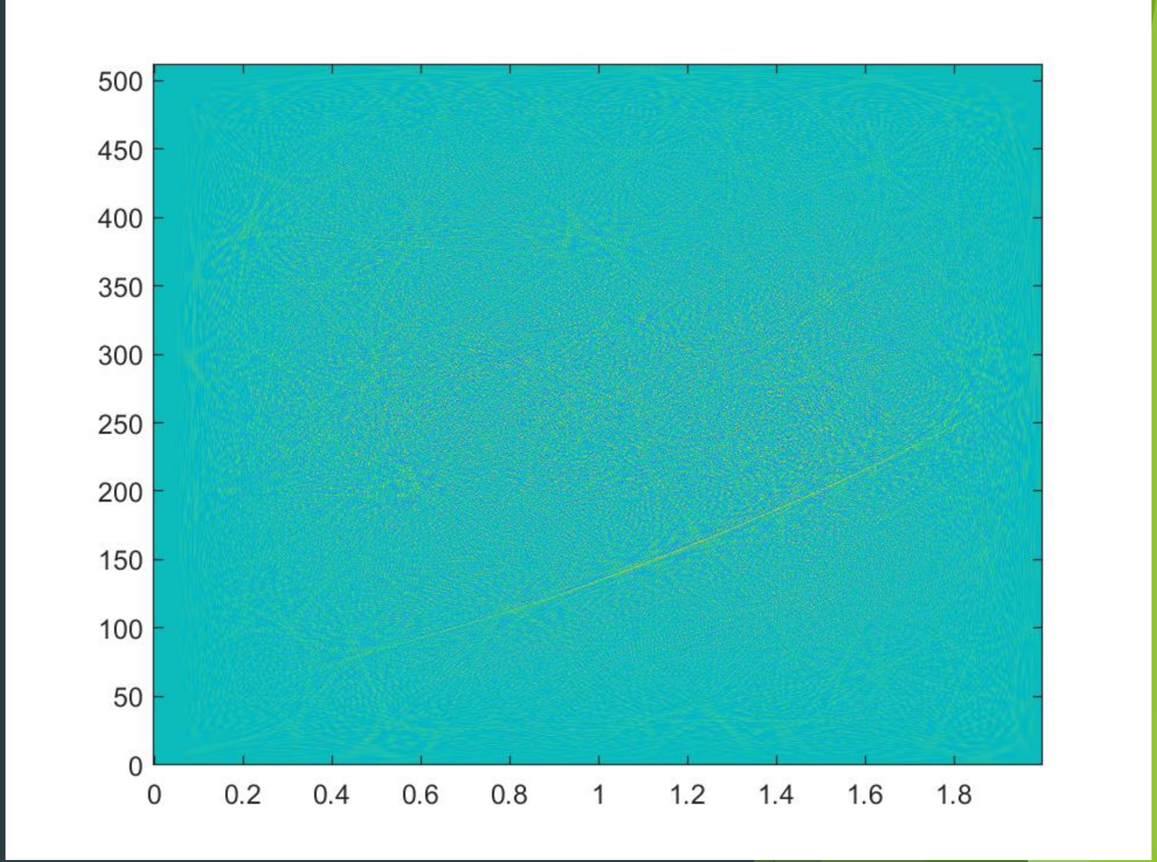
Quadratic chirp in (whitened) initial LIGO noise (SNR=8.3)



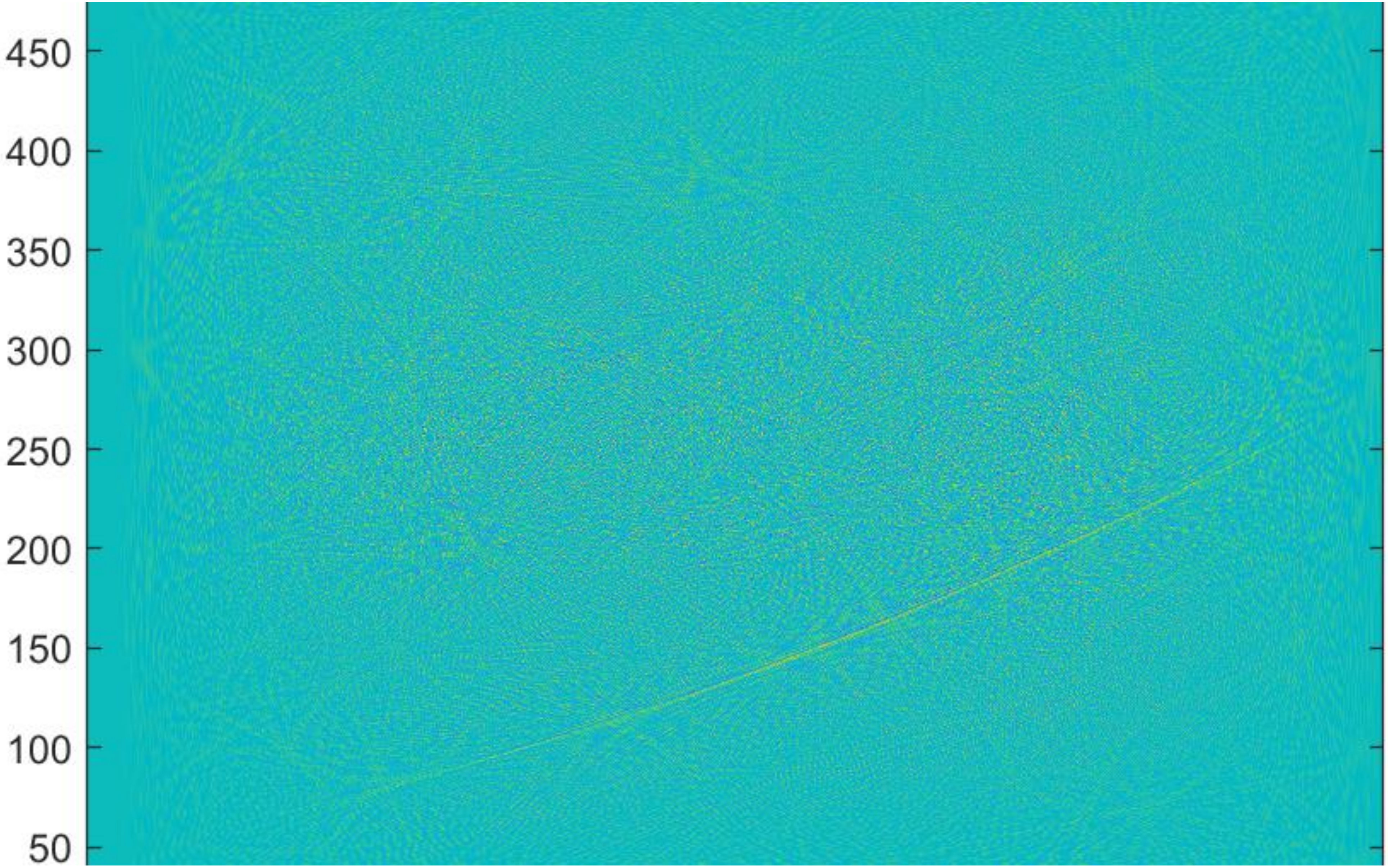


Quadratic chirp in (whitened) initial LIGO noise (SNR=15)



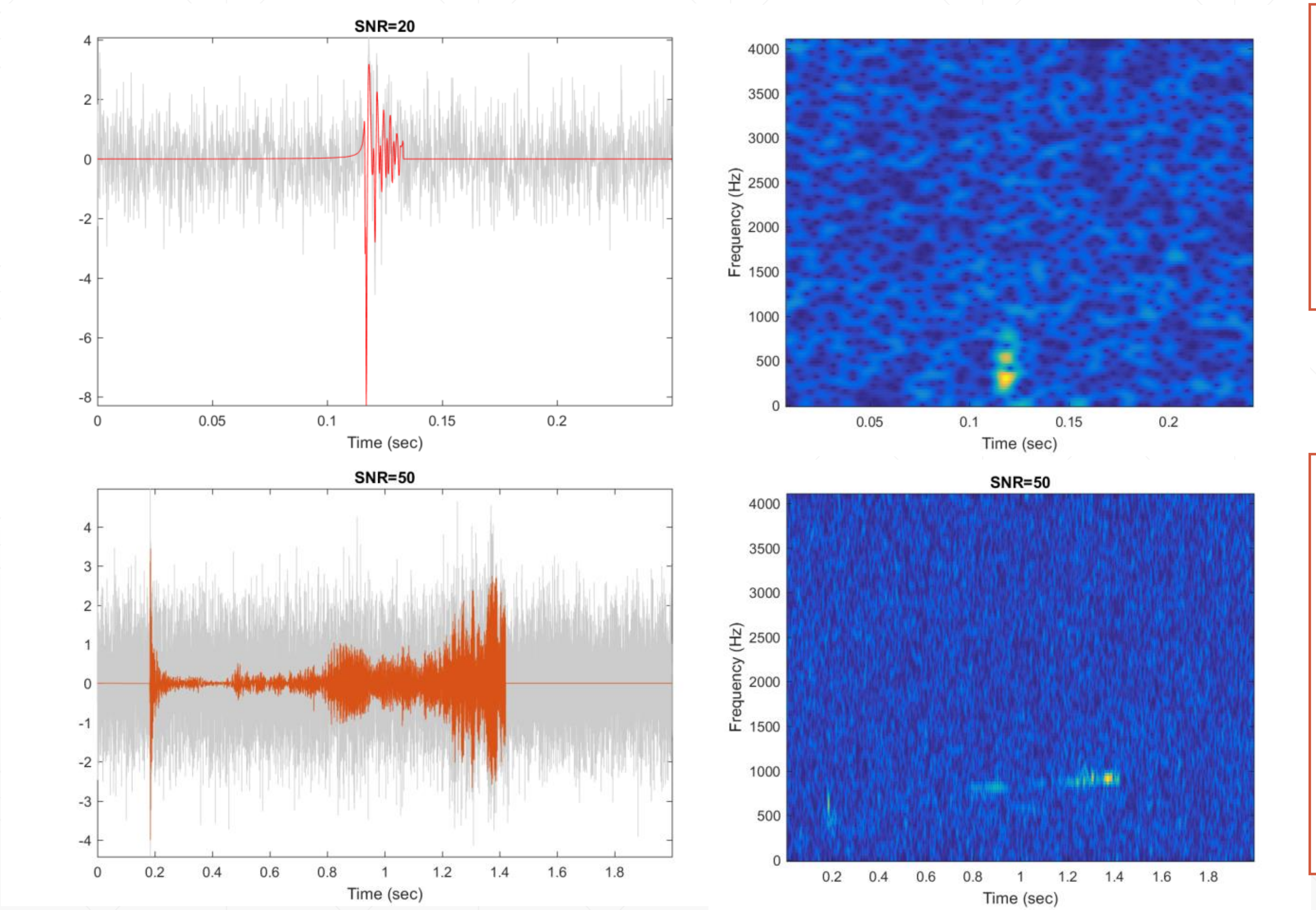


Quadratic chirp in (whitened) initial LIGO noise (SNR=15)


450
400
350
300
250
200
150
100
50

Issues with TF analysis

TF clustering and CCSN signals



Core bounce

- Single Cluster
- Signal power well-localized ⇒ cluster easily detectable

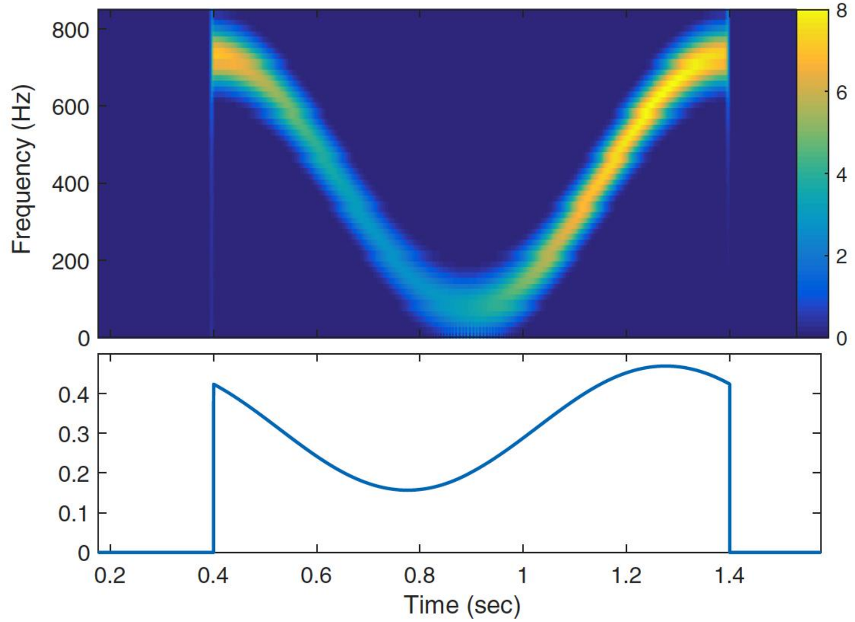
Post-Shock

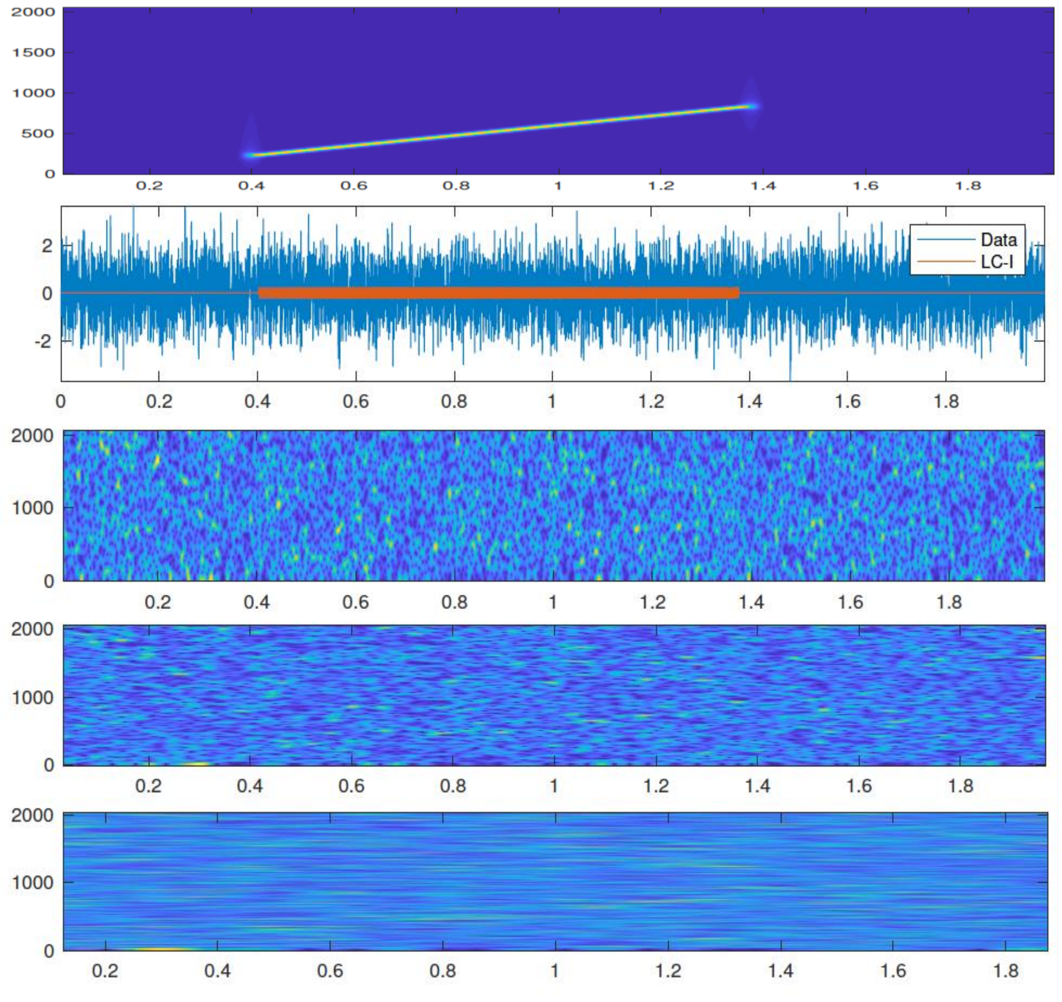
- Fragmented Cluster
- Individual clusters can be too weak or too small
- Aggregation of multiple clusters ⇒ additional ad hoc algorithm parameters

Note: Multi-resolution TF analysis is a must (not shown here)

UNMODELED CHIRPS

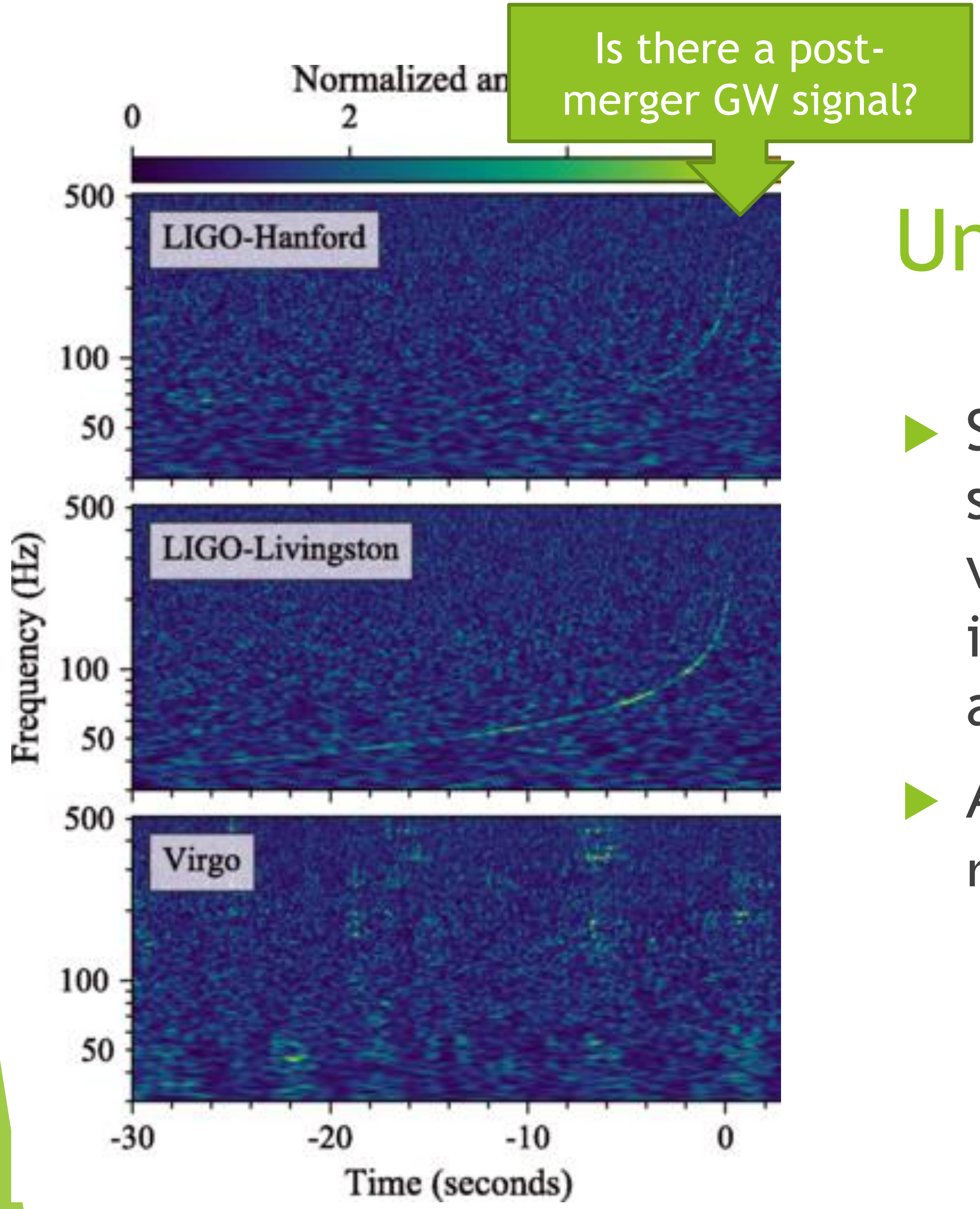
- Chirp signal: $f(x) = a(x)\sin(\Phi(x))$,
  - Where the instantaneous frequency, $\frac{d\Phi}{dx}$, changes adiabatically on timescales of the instantaneous period
  - $\Rightarrow$ Track in the TF plane
- Unmodeled chirp signal: $a(x)$ and $\Phi(x)$ have unknown functional forms





TIME-FREQUENCY ANALYSIS

- At signals strengths expected for GW signals, noise can completely mask chirp signals in a time-frequency transform



Unmodeled GW chirps

- Since all LIGO methods for unmodeled signals (bursts) are based on some variation of Time-frequency clustering, it is possible that unmodeled chirp signals are being missed
- Alternative to time-frequency methods are required for unmodeled chirps

Example: Mohanty, Phys Rev D, 2017

Regularization

COHERENT NETWORK ANALYSIS

- Coherent network analysis for burst signals: MLE without signal model

$$\begin{pmatrix} x_1(t) \\ \vdots \\ x_N(t) \end{pmatrix} = \begin{pmatrix} F_{+,1}(\theta,\phi)B(\tau_1(\theta,\phi)) & F_{\times,1}(\theta,\phi)B(\tau_1(\theta,\phi)) \\ \vdots & \vdots \\ F_{+,N}(\theta,\phi)B(\tau_N(\theta,\phi)) & F_{\times,N}(\theta,\phi)B(\tau_N(\theta,\phi)) \end{pmatrix} \begin{pmatrix} h_+(t) \\ h_\times(t) \end{pmatrix} + \begin{pmatrix} n_1(t) \\ \vdots \\ n_N(t) \end{pmatrix}$$

GW detector data

(θ, φ) : source position
Strain signals for each detector with time delays
(B(τ)[ ])

Noise

$$\mathbf{x}(t) = \mathbf{A}(\theta,\phi)\mathbf{h}(t) + \mathbf{n}(t)$$

$$\mathbf{h}_{ML}(t) = \mathbf{M}\mathbf{A}^{\mathrm{T}}(\theta_{ML},\phi_{ML})\mathbf{x}(t)$$

$$\mathbf{M} = \left(\mathbf{A}^{\mathrm{T}}(\theta,\phi)\mathbf{A}(\theta,\phi)\right)^{-1}$$

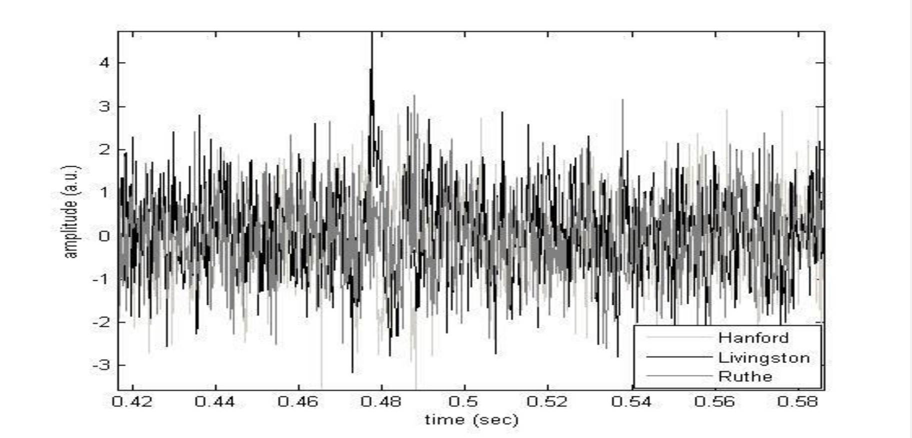
But the problem is ill-posed!
A(θ,φ) can become rank-deficient

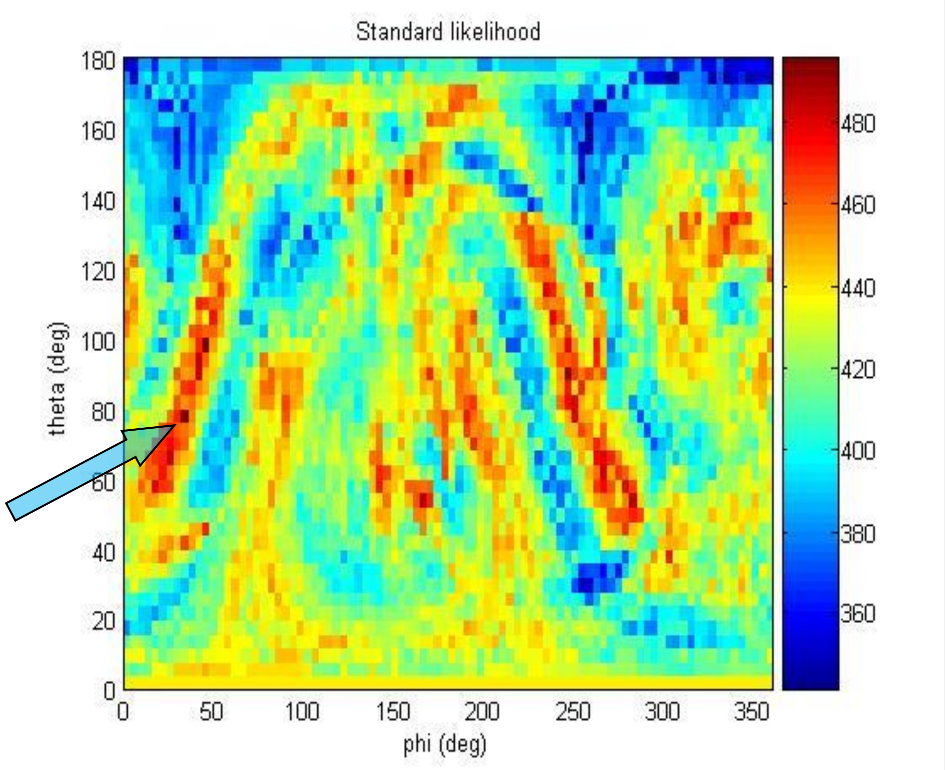
REGULARIZATION: CHANGING THE FITNESS FUNCTION

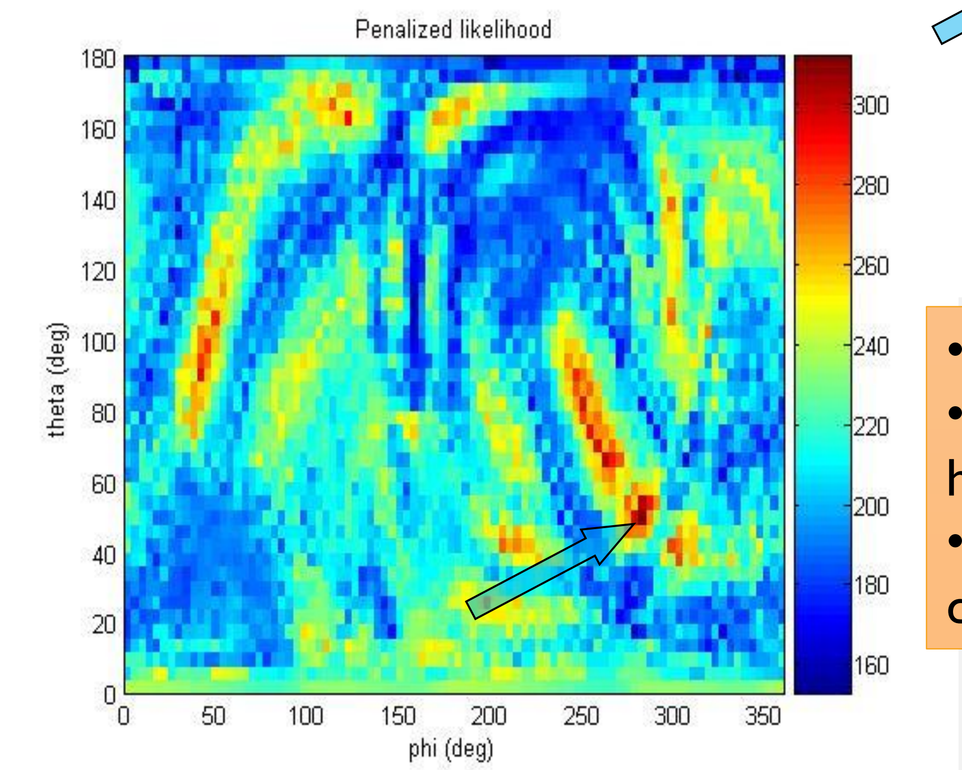
$$\max_{\bar{h}} \|\bar{y} - A(\theta, \phi)h\|^2 \rightarrow \max_{\bar{h}} \left[\left\|\bar{y} - A(\theta, \phi)\bar{h}^T\right\|^2 + \underbrace{\lambda\, P(\bar{h})}_{Regulator} \right]$$

- The regulator pushes the solution towards a more desirable one that is less sensitive to noise and ill-posedness
- Singular value decomposition
  - Klimenko, Mohanty, Rakhmanov, Mitselmakher, PRD, 2005→ Basis of the current main analysis pipeline – Coherent Wave Burst (CWb) -- for burst signals in LIGO
- SNR-variability
  - Mohanty, Rakhmanov, Klimenko, Mitselmakher, CQG, 2006

REGULARIZED MLE DEMO: LIGO AND GEO600







- Likelihood of source at (θ, φ)
- Without regulator, the source location has a large error
- Regulator introduces a small bias but drastically reduces the variance

Sparsity regularization (LASSO)

- General form of regularization

$$\max_{\bar{s}} \left[\|\bar{y} - \bar{s}\,\|^2 + \underbrace{\lambda\, P(\bar{s})}_{Regulator} \right]$$

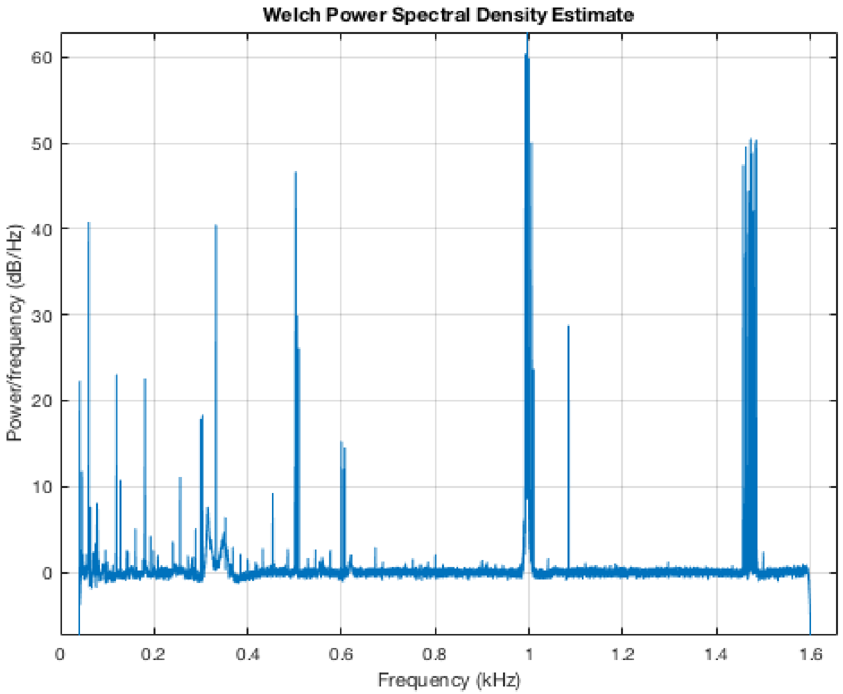
- Linear model: $\bar{B}_k$ are given vectors

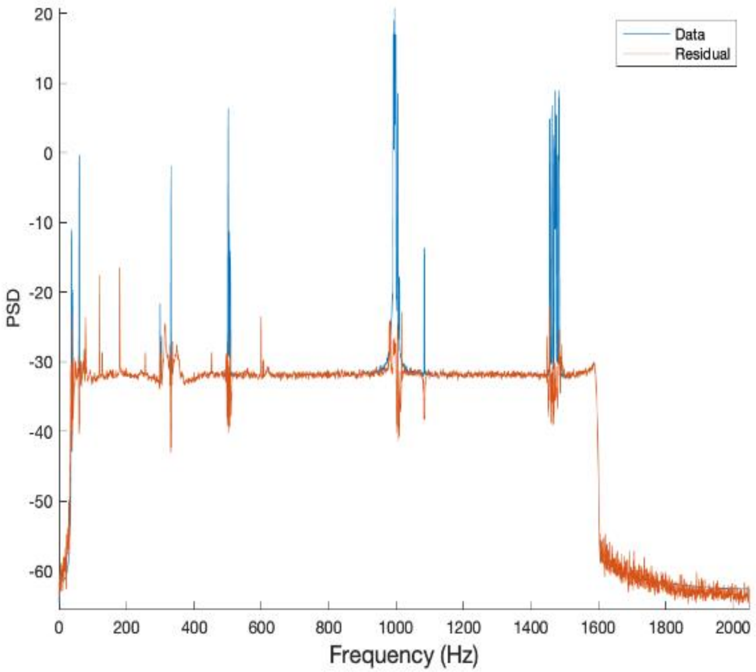
$$\bar{s} = \sum_{k=0}^{M-1} \alpha_k \ \bar{B}_k\,;$$

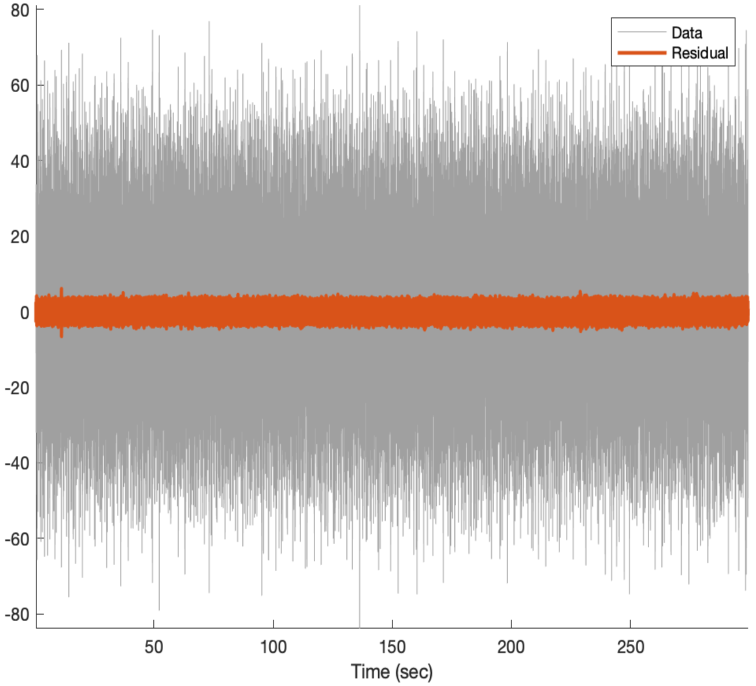
- Sparsity regularization:

$$\max_{\bar{\alpha}} \left[\left\| \bar{y} - \sum_{k=0}^{M-1} \alpha_k \ \bar{B}_k \right\|^2 + \underbrace{\lambda \sum_{k=0}^{M-1} |\alpha_k|}_{Regulator} \right]$$

- Forces the solution towards the case where only a few α_k are large and rest are close to zero







LASSO for Line Removal

Mohanty, 2019 (In progress; Unpublished)

Handling real GW detector noise

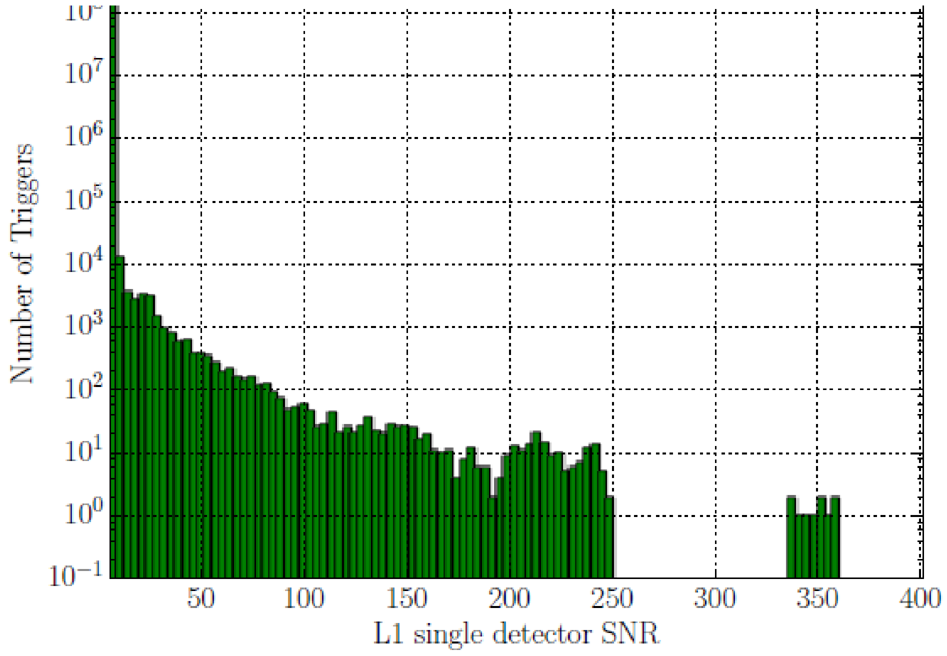
Vetoes

- GLRT/MLE assume a noise model
- The noise in real data never follows any noise model exactly: Instrumental artifacts, changing environment etc all contribute to deviations
- $\Rightarrow$A large number of events at the output of any GW search are not GW signals
- Vetoes are required to increase the rejection of spurious signals
  - Using detector characterization$\rightarrow$ Data quality vetoes
  - Using auxiliary channels
  - Using consistency tests
    - Example: χ^2-veto in binary inspiral search
- Veto safety: We do not want too many GW signals to be removed accidentally
  - Hardware and software signal injections are needed to test safety

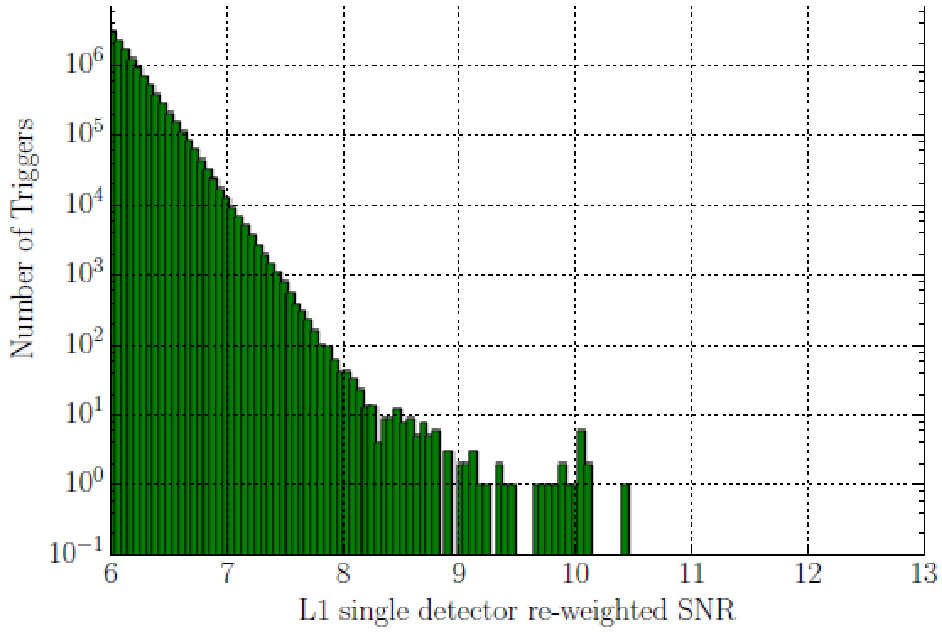
Effect of vetoes

arXiv:1710.02185v3 [gr-qc]

Histograms of single detector PyCBC triggers from the Livingston (L1) detector. These triggers were generated using data from September 12 to October 20, 2015. These histograms contain triggers from the entire template bank, but exclude any triggers found in coincidence between the two detectors. (1a) A histogram of single detector triggers in SNR. The tail of this distribution extends beyond SNR = 100. (1b) A histogram of single detector triggers in re-weighted SNR. The chi-squared test down-weights the long tail of SNR triggers in the re-weighted SNR distribution. The triggers found using only the Hanford detector have a similar distribution.

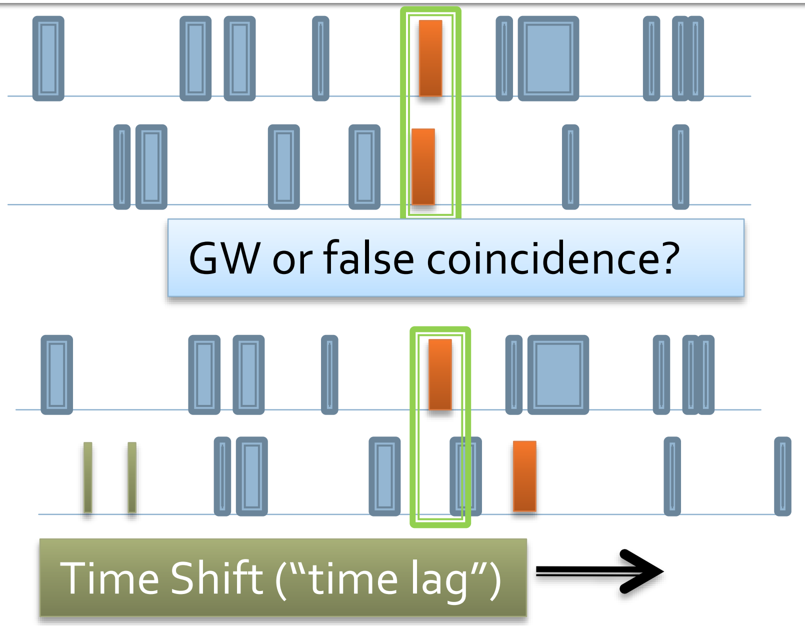


(a)



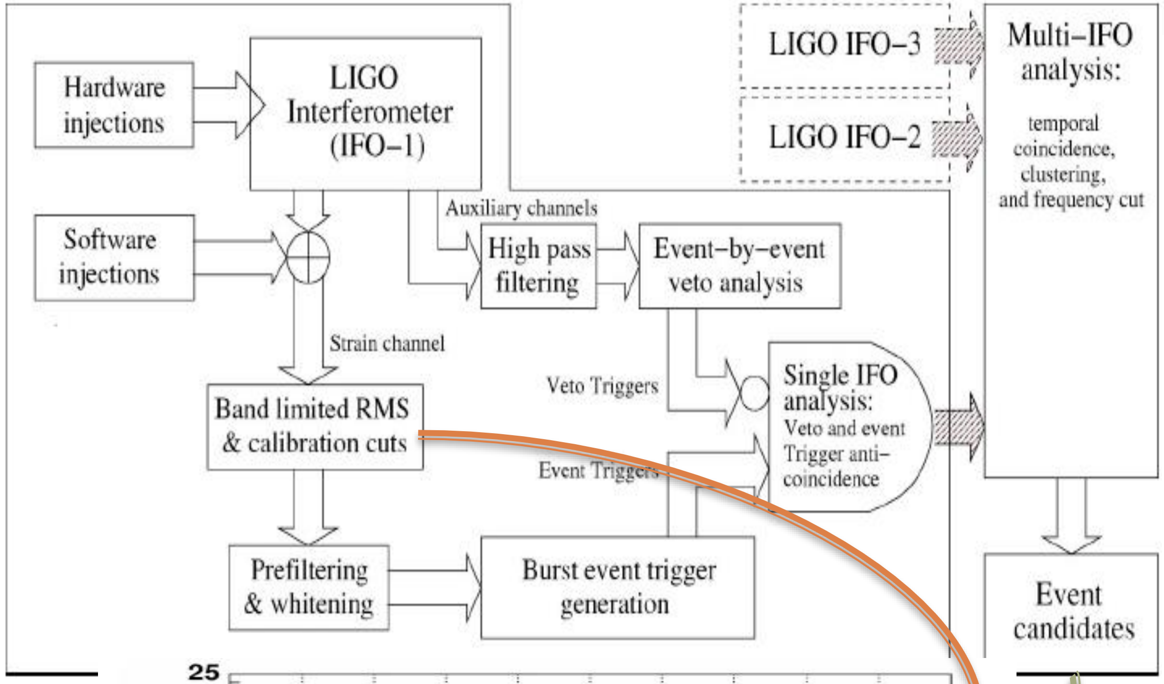
Background rate (Significance) estimation

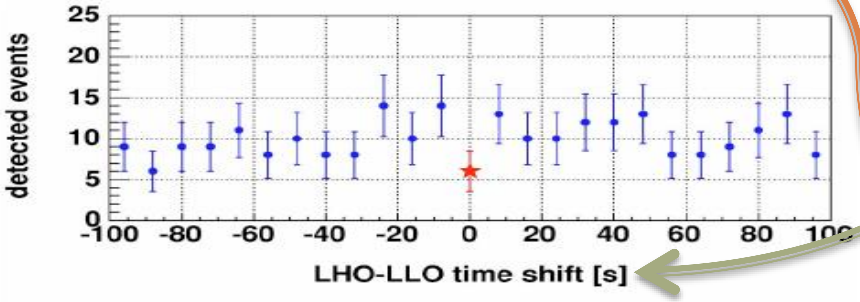
- The false alarm rate for GW detections must be very small
- How can we measure small false alarm rates?
  - We do not have 50 years of data to measure a rate of 1/50 years!
  - Real noise does not follow a Gaussian behavior, so we cannot calculate the rate theoretically
- Solution: artificial time-lags
  - Create new data sets where one detector's data is shifted in time relative to another by a large amount
  - Re-analyze the new data sets using the same search method: All events must be coming from noise, not GWs
  - Estimate false alarm rate (Assumption: shifted dataset are statistically independent)

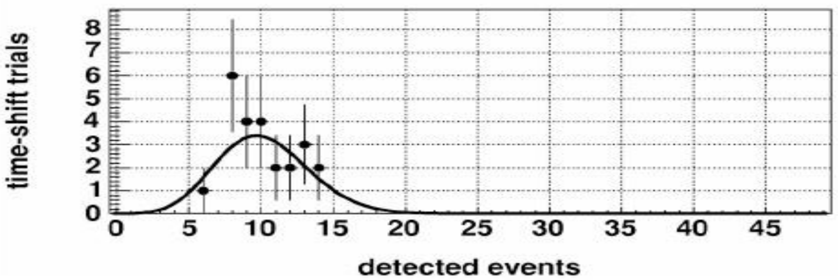


•GW signals will not coincide in time-shifted data, so all observed coincidences are random
•Measure the number of coincidences for a set of time-lags
•Fit a poisson distribution and get rate

A real pipeline







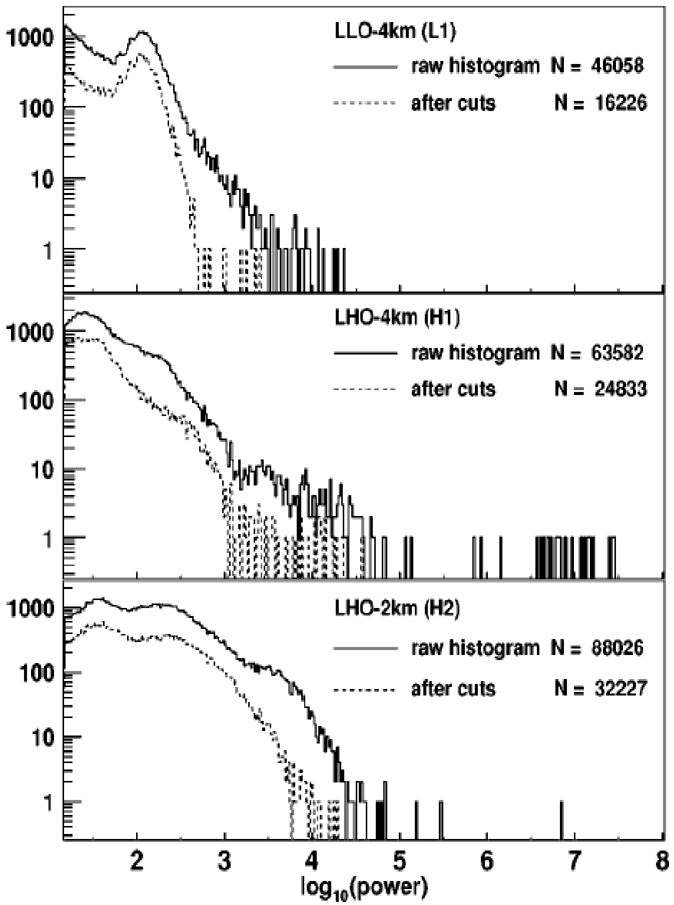


FIG. 4. Histogram of TFCLUSTERS event triggers from the three LIGO detectors, before and after the BL rms and calibration cuts.

Topics not covered

GW data analysis methods not covered

- Bayesian analysis: Mathematically identical to regularization but different choice of regulators (= prior probability)
- Model selection
  - Bayesian model selection: Odds ratio
  - Frequentist model selection: Information criterion
- Machine learning applications in GW data analysis: Classification of glitches
- Cross-channel regression: Removing noise in the GW strain time series using measurements from auxiliary sensors