Solution to Ex. 13.17

of *Turbulent Flows* by Stephen B. Pope, 2000

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Consider high-Reynolds-number homogeneous turbulence with the sharp spectral filter in the inertial subrange. Use the Kolmogorov spectrum to obtain the estimate for the mean residual kinetic energy.

$$\langle k_{\mathrm{r}} \rangle = \int_{\kappa_{\mathrm{c}}}^{\infty} E(\kappa)\,\mathrm{d}\kappa \approx \frac{3}{2} C \left(\frac{\varepsilon \Delta}{\pi} \right)^{\frac{2}{3}} \tag{1}$$

Solution

Using the Kolmogorov spectrum Eq. (6.239)

$$\begin{aligned}
\langle k_{\mathrm{r}} \rangle &= \int_{\kappa_{\mathrm{c}}}^{\infty} E(\kappa)\,\mathrm{d}\kappa \\
&\approx \int_{\kappa_{\mathrm{c}}}^{\infty} C \varepsilon^{\frac{2}{3}} \kappa^{-\frac{5}{3}}\,\mathrm{d}\kappa \\
&= -\frac{3}{2} C \varepsilon^{\frac{2}{3}} \kappa^{-\frac{2}{3}} \Big|_{\kappa_{\mathrm{c}} = \frac{\pi}{\Delta}}^{\infty} \\
&= \frac{3}{2} C \left(\frac{\varepsilon \Delta}{\pi} \right)^{\frac{2}{3}}
\end{aligned} \tag{2}$$