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VIBRATION ANALYSIS OF COOLANT PUMP WITH TWO UNBALANCED DISKS BASED ON THE STATE-SPACE NEWMARK METHOD

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ABSTRACT

This paper discusses the vibration of the latest vertical coolant pump with two disks and nonlinear bearing oil film force, namely rotor-bearing-disk system. To analyze the vibration of the rotor, the finite element method is adopted with the Timoshenko beam, and the dynamical equations are solved by using state-space Newmark method. Results of this study reveals the different mass eccentricity and phase angle of two disks are the important variables to the rotor with two disks under bearing nonlinear oil film force and periodic force, and affect the critical speed obviously.

1 INTRODUCTION

The coolant pump, with a vertical rotor-bearing structure, has two disks made by tungsten alloy blocks, and it is more complex than the Jeffcot rotor. The coolant pump instabilities have become more and more important as the rotational speed and power. In this paper, the coolant pump is modeled as a continuum model of rotor-bearing system, and the mass unbalance response and bearing nonlinear oil film force is taken into account.

Because of the special structure, it is difficult to promise the coincidence between mass center and rotor axis, and the defect in coolant pump will affect the vibration behaviour. These instabilities can be erratic, and the vibrational amplitude will increase dramatically without indication, it also transmits rotational force to hydraulic bearings and to the pump shell. The mass unbalance response is believed to be one of the primary causes[1]. Sometimes the periodic force resulted from unbalanced mass damages the coolant pump and shortens its working life.

For the coolant pump with two mass unbalance disks, the response varies when the phase angle between two mass centers of disks is changed. The dynamics of a rotor-bearing model with two mass disks are investigated by Ding, he revealed that the pre-existing non-synchronous whirl/whip in a shaft can activate the onset of oil instability of its neighbour shaft [2]. Xie described two phenomenons in rotor-bearing system with two unbalanced disks: (1) the chaos with two attracting areas which cannot be distinguished from the stable period doubling motion on Poincare section; (2) for the two unbalanced disks, because of the phase angle between the eccentricities of disks, the response varies in a large extent [3].

State-space Newmark method is a direct integration scheme based on the average velocity concept. T.C.Fung presented State-space Newmark method, it is an unconditionally stable higher-order accurate time-step integration algorithms to solve the linear first-order differential equations, and simpler than the conventional Newmark method, and has a period error of second-order accuracy for small damping and fourth-order for large damping and an amplitude error of second-order, regardless of damping[4]. An Sung Lee and Byung Ok Kim applied the State-space Newmark method to analyse a rotor-bearing system considering a base transferred shock force, and estimated the quantitative error between analytical and experimental time response[5,6].

In this paper, a continuum rotor-bearing system with nonlinear oil film force is modeled by finite element method to analyse the influence of the phase angle and mass eccentricity to vibrational amplitude of rotor center under rotational speed(ω=0~400 rad/s). The results show that the different mass eccentricities affect the critical speed and the vibrational amplitude, and when the phase angle is 180^o in the model, the vibrational amplitude of rotor center is maximum in 0~360^o, but the different mass eccentricity and rotational speed cases are different .

2 MODELING

The vertical flexible rotor with two tungsten alloy disks(WHA) is supported by two radial water-lubricating bearings, as shown in the Fig.1.

The rotor of coolant pump is discretized by Timoshenko beams with two translational and two rotational degrees of freedom per node, The mass of the disk is treated as lumped mass, and is superimposed on the corresponding shaft nodes; the bearing nonlinear oil film force and the periodic force resulted from mass unbalance will be applied to corresponding shaft nodes. The phase angle φ between the eccentricities of two disks is modeled as the Fig.2. *m1* and *m2* denote the mass centers, *l1* and *l2* denote the eccentric distance.

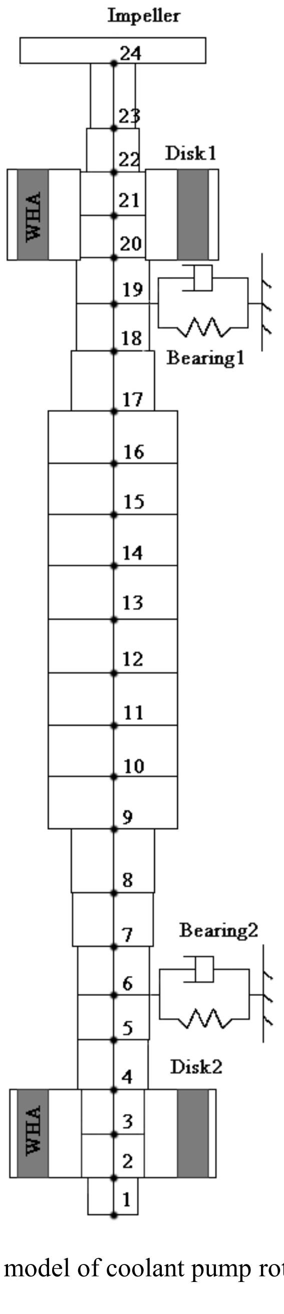


Fig.1. The model of coolant pump rotor

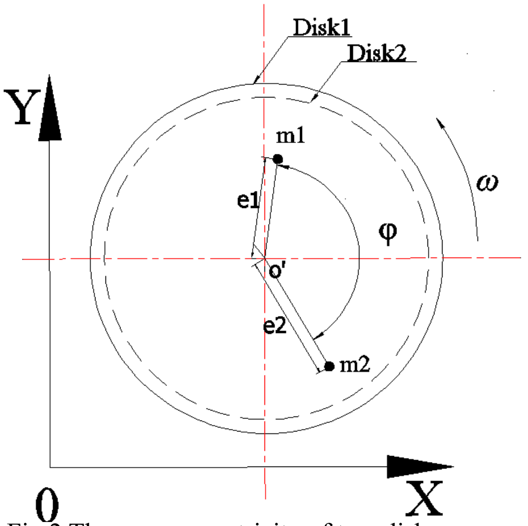


Fig.2 The mass eccentricity of two disks

3 MATHEMATICAL DESCRIPTION

In the absence of mass unbalanced, the system governing equation of motion for the rotor-bearing system is given by

$$M\ddot{u}(t)+B\dot{u}(t)+Ku(t)=F_e(t)+F_b(t) \qquad (1)$$

These can be rearranged as follow.

$$\begin{Bmatrix} \dot{u} \\ \ddot{u} \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}B \end{bmatrix} \begin{Bmatrix} u \\ \dot{u} \end{Bmatrix} + \begin{Bmatrix} 0 \\ M^{-1}(\{F_e\}+\{F_b\}) \end{Bmatrix} \qquad (2)$$

Where, *M*, *B* and *K* are the mass matrix , damping matrix and stiffness matrix when the eccentricity is zero, F_e is a periodic force per node resulted from mass unbalanced, and F_b denotes the bearing nonlinear oil film force per node.

$$\{F_e\} = \begin{Bmatrix} F_{ex} \\ F_{ey} \end{Bmatrix} = \omega^2 ml \begin{Bmatrix} \cos(\omega t+\varphi) \\ \sin(\omega t+\varphi) \end{Bmatrix} \qquad (3)$$

The water-lubricating bearing oil film force is obtained from short bearing theory, it can be expressed as (4)

$$\{F_b\} = \begin{Bmatrix} F_{bx} \\ F_{by} \end{Bmatrix} = -\zeta \frac{[(u_x-2\dot{u}_y)^2+(u_y-2\dot{u}_x)^2]^{0.5}}{1-u_x^{\,2}-u_y^{\,2}} \times \begin{pmatrix} 3u_x V(u_x,u_y,\alpha) \\ 3u_y V(u_x,u_y,\alpha) \\ -\sin\alpha G(u_x,u_y,\alpha) \\ +\cos\alpha G(u_x,u_y,\alpha) \\ -2\cos\alpha S(u_x,u_y,\alpha) \\ -2\sin\alpha S(u_x,u_y,\alpha) \end{pmatrix} \qquad (4)$$

Where ζ is Sommerfeld modifying parameter, u_x and u_y are the co-ordinates displacement of the rotor axial center, the superposed dot are the velocity variables after time derivated[7].

$$\zeta = \mu\omega RL(\frac{R}{c})^2(\frac{L}{2R})^2 \tag{5}$$

Where μ is viscosity of lubricating oil, R and L denotes radius and length of bearing, c is the clearance of bearing radius.

$$V(u_x,u_y,\alpha) = \frac{2+(u_y\cos\alpha - u_x\sin\alpha)}{1-u_x^{\ 2}-u_y^{\ 2}} \times G(u_x,u_y,\alpha) \tag{6}$$

$$S(u_x,u_y,\alpha) = \frac{u_x\cos\alpha + u_y\sin\alpha}{1-(u_x\cos\alpha + u_y\sin\alpha)^2} \tag{7}$$

$$G(u_x,u_{\mathrm{y}},\alpha) = \frac{2}{(1-u_x^{\ 2}-u_y^{\ 2})^{1/2}} \times \left(\frac{\pi}{2} + arctg\frac{u_y\cos\alpha - u_x\sin\alpha}{(1-u_x^{\ 2}-u_y^{\ 2})^{1/2}}\right) \tag{8}$$

$$\alpha = arctg\frac{u_y+2\dot{u}_x}{u_x+2\dot{u}_y} - \frac{\pi}{2}sign\left(\frac{u_y+2\dot{u}_x}{u_x-2\dot{u}_y}\right) - \frac{\pi}{2}sign(u_y+2\dot{u}_x) \tag{9}$$

4 STATE-SPACE NEWMARK METHOD

The State-space Newmark method assumes Δt between t_n and t_{n+1}, and defines a variable $\tau(0\le\tau\le\Delta t)$. The average velocity within t_n and t_{n+1} can be expressed as (10).

$$\{\dot{u}(\tau)\} = \frac{1}{2}\left[\{\dot{u}\}_{n+1} + \{\dot{u}\}_n\right] \tag{10}$$

When the initial condition is given, $\{\tau(0)\}=\{u\}_n$ integrating Eq.(10), the displacement at time τ can be obtained.

$$\{u(\tau)\} = \{u\}_n + \frac{\tau}{2}\left[\{\dot{u}\}_{n+1} + \{\dot{u}\}_n\right] \tag{11}$$

From Eq.10, the displacement of next time-step at $\tau=\Delta t$ is

$$\{u\}_{n+1} = \{u\}_n + \frac{\tau}{2}\left[\{\dot{u}\}_{n+1} + \{\dot{u}\}_n\right] \tag{12}$$

And then, the velocity of next time-step t_{n+1} is express as follow.

$$\{\dot{u}\}_{n+1} = \frac{2}{\Delta t}\left[\{u\}_{n+1} - \{u\}_n\right] - \{\dot{u}\}_n \tag{13}$$

Finally, considering Eq.(2) at t_{n+1}, and substituting Eq.(12) into it and simplify.

$$\begin{Bmatrix} u \\ \dot{u} \end{Bmatrix}_{n+1} = \left(I - \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}B \end{bmatrix}\right)^{-1} \times \left(\begin{Bmatrix} u \\ \dot{u} \end{Bmatrix}_n + \frac{\Delta t}{2}\begin{Bmatrix} \dot{u} \\ \ddot{u} \end{Bmatrix}_{n+1} + \frac{\Delta t}{2}\begin{Bmatrix} 0 \\ M^{-1}(\{F_e\}_{n+1} + \{F_b\}_{n+1}) \end{Bmatrix}\right) \tag{14}$$

Now, the state-space vector at next time-step t_{n+1} can be obtained after calculating Eq.(14). The novel Newmark method is much simpler and more straightforward than the conventional Newmark method, and it can be more readily carried out by coding[5,6].

5 NUMERICAL ANALYSIS AND DISCUSSION

For the mathematical model in section 3 the parameters of rotor and bearings are fixed, and the variables are the eccentricities of two disks and the phase angle between eccentricity of upper disk and lower disk.

In this section, the influences of different eccentricities and phase angle to vibrational amplitude under the rotational speed ascending process are discussed, and in this analysis ,the State-space Newmark method is adopted to calculate the governing equation.

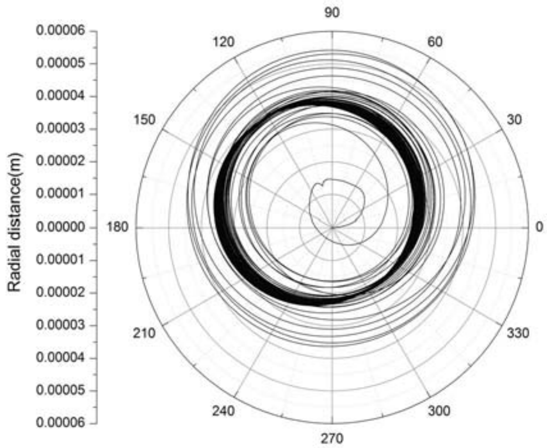
5.1 Eccentricity and vibrational amplitude

As shown in the Fig.3(a) and (b), the orbit of rotor center at upper and lower bearings is plotted in the cylindrical coordinate respectively. In this paper, the radial distance of rotor cent is regard as the vibrational amplitude.

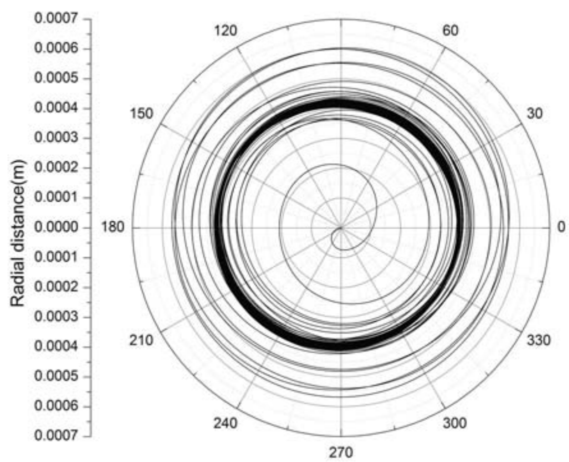
The vibrational amplitude is plotted in Fig.4 .When the rotational speed changes from 0 rad/s to 400 rad/s, the vibrational amplitude of rotor center in the location of bearings is different between two unbalanced disks.

There are two analysis cases of disk eccentricity. In the first case, the upper disk eccentricity(*m1*) changes from 0.2kg·m to 1kg·m by 0.2kg·m, and the lower disk eccentricity(m2) is 0kg·m. In the second case, the upper disk eccentricity(*m1*) is 0kg·m, and the lower disk eccentricity(*m2*) changes from 0.2kg·m to 1kg·m by 0.2kg·m. The results of the two cases are shown in the Fig.4(a) and Fig.4.(b).

In the Fig.4.(a), it shows the vibrational amplitude at the lower bearing under case 1 and case 2. Fig.4.(b) shows the vibrational amplitude at the upper bearing under case 1 and case 2. Comparing the results of case 1 and case 2, the critical speed of the first case is about 300rad/s, while the critical speed of second case is about 350rad/s. Besides, the peak of vibrational amplitude of first case is greater than that of second one. In another word, the lower unbalanced disk affects more distinctly than the upper unbalanced disk.

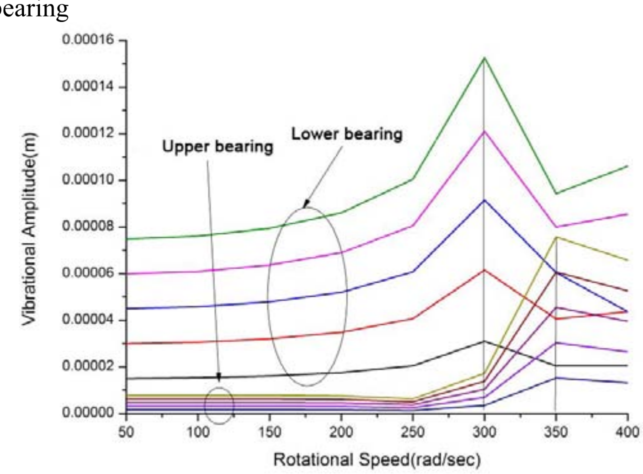


(a)

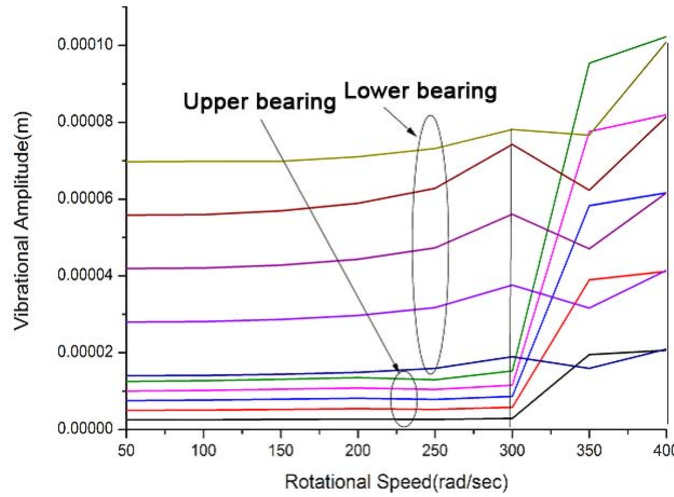


(b)

Fig.3 The orbit of rotor center at upper bearing and lower bearing



(a)



(b)

Fig. 4 The vibrational amplitude of rotor center in different mass eccentricity

5.2 Phase angle and vibrational amplitude

Phase angle is the angle between eccentricities of two disks. There are three different unbalanced mass of two disks and the phase angle varies from 0 to 360°.

There are three cases, the first case is that mass eccentricity of upper disk(m1) is greater than that of lower disk (m2); the second case is that m1 is equal to m2; and in the last case, m1 is lower than m2.

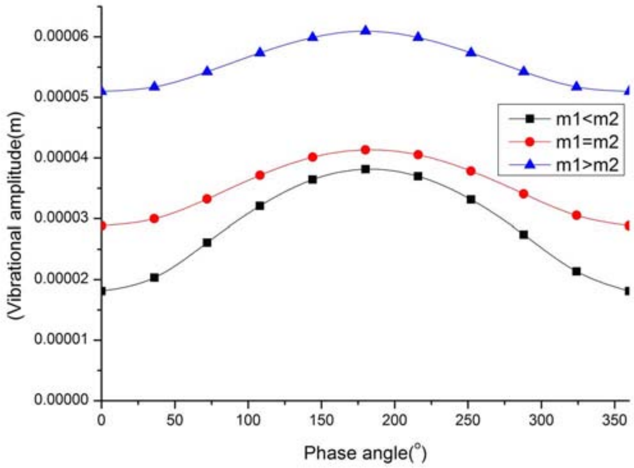
As shown in the Fig.5, Horizontal axis is the phase angle's range(0~360°), and the vertical axis is the vibrational amplitude. Fig.5.(a), (c), (e) and (g) are the vibrational amplitude of rotor center at upper bearing, (b), (d), (f) and (h) are the vibrational amplitude of rotor center at lower bearing. (a) and (b) are the results under the rotational speed 100rad/s, (c) and (d) are the results under the rotational speed 200rad/s, (e) and (f) are the results under the rotational speed 300rad/s, (g) and (h) are the results under the rotational speed 400rad/s.

From the results, there are two conclusions obtained.

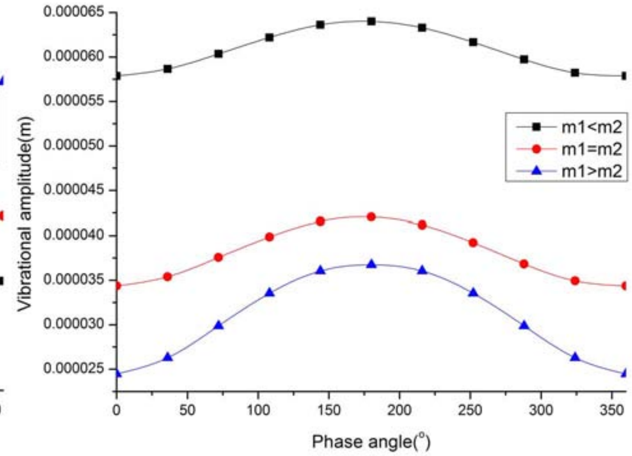
(1) Under different relationship between mass eccentricities of two disks, the degree of influence is not the same at the same location. From the Fig.5, when mass eccentricity of upper disk is greater than that of lower disk (m1>m2), the vibrational amplitude of rotor center at upper bearing is more obvious than the low disk, when mass eccentricity of upper disk is greater than that of lower disk (m1<m2), the vibrational amplitude of rotor center at lower bearing is more obvious than the upper disk; and when m1=m2, the vibration amplitude is not obvious.

(2) The relationship of phase angle and vibrational amplitude is like the sine curve or cosine curve under different rotational speed. When the rotational speed is not close to the critical rotational speed(300rad/s), the curve of phase angle and vibrational amplitude like 'peak', and the peak value of amplitude is at 180°, but when the rotational speed is close to the critical rotational speed(300rad/s), the curve of phase angle and vibrational amplitude like 'valley' at the upper bearing, and

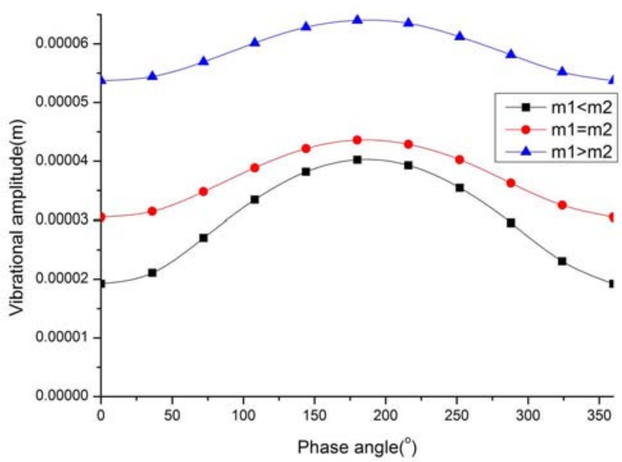
the valley value of amplitude is at 180º, but at the lower bearing, the curve is different from the others, and the influence of phase angle is not obvious.



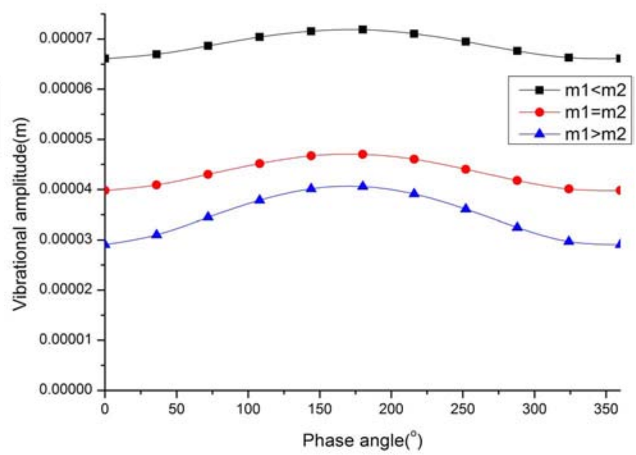
(a)



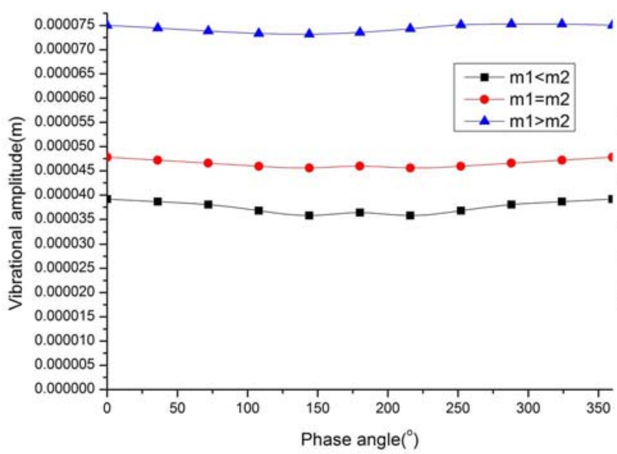
(b)



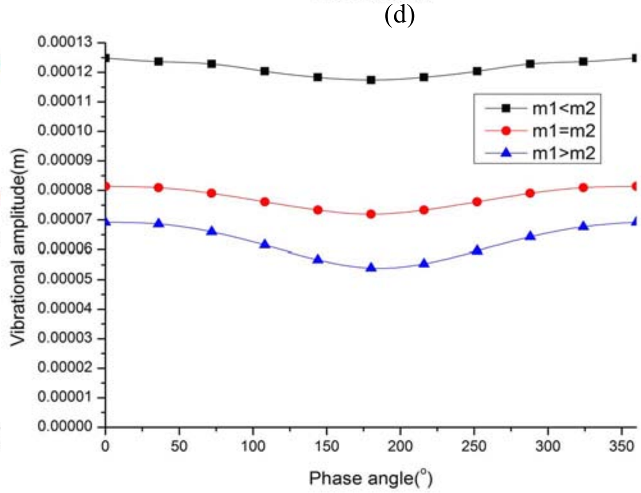
(c)



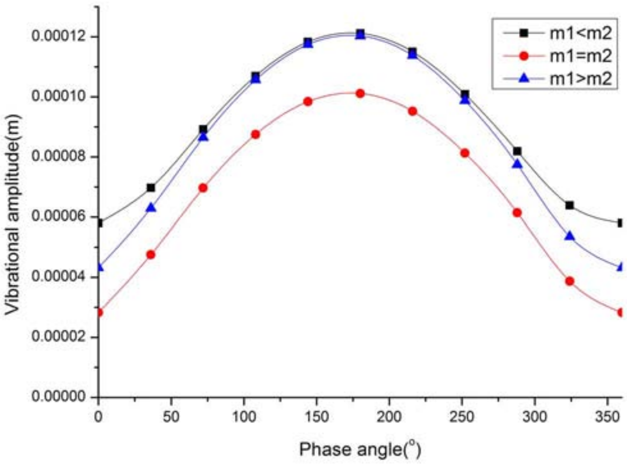
(d)



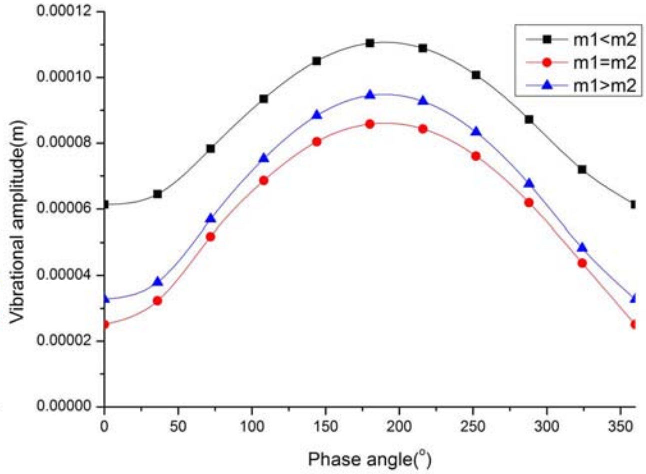
(e)



(f)



(g)



(h)

Fig.5 The vibrational amplitude of rotor center in different phase angle

6 CONCLUSIONS

This paper has studied the vibrational amplitude of rotor of coolant pump in different mass eccentricity and phase angle. Due to the bearing nonlinear oil film force and the periodic force resulted from unbalance response, the critical speed and vibrational amplitude are different linear rotordynamic, it is difficult to predict the obit of the vertical rotor.

From this study, the mass eccentricity and phase angle are variables. When the variables are changed, the vibrational behaviour of rotor will be not the same.

The critical speed is changed when the mass eccentricities of two disks are different. Under different relationship between mass eccentricities of two disks, the degree of influence is not the same at the same location. The relationship between phase angle and vibrational amplitude is like the sine curve when the rotational speed is not close to the critical speed, or else, the influence of phase angle cannot be predicted.

ACKNOWLEDGMENTS

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