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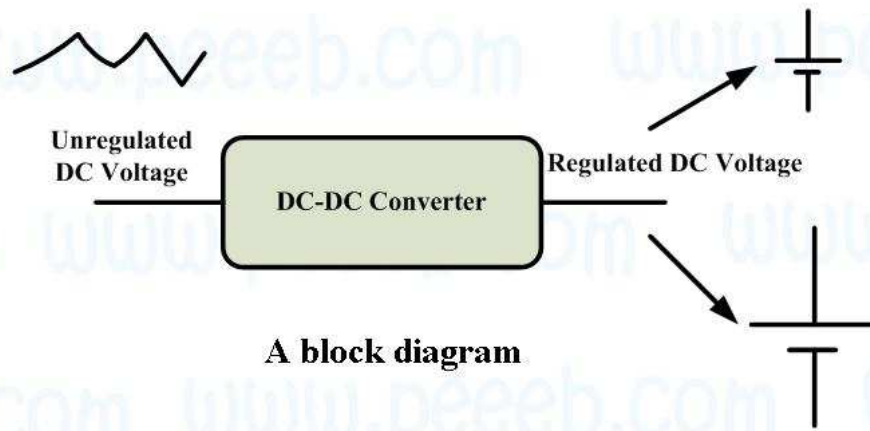


Lecture 5: Non-isolated DC-DC Converters

Presenter: Dr. Firuz Zare

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DC-DC Converters



A block diagram

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DC-DC Converters: Some applications

- **Power Supply with Constant or Adjustable Magnitude**
- **DC Motor Drives**
- **Improving Power Factor**

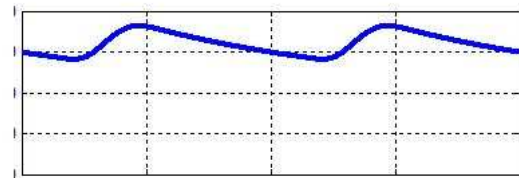
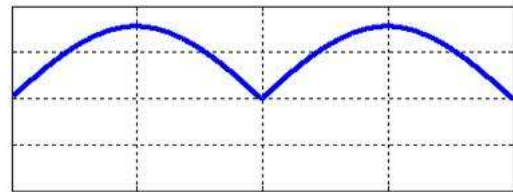
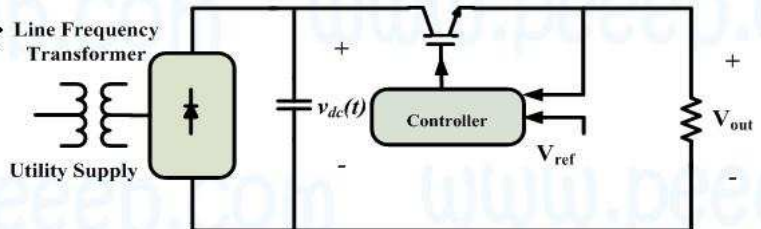
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Low Power Linear Power Supply (Voltage Regulator)

Low efficiency

Heavy and bulky transformer



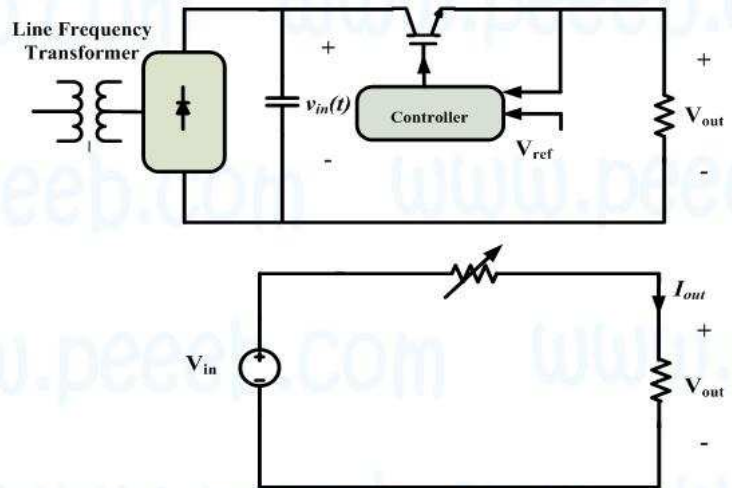
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Low Power Linear Power Supply (Voltage Regulator)

Low efficiency

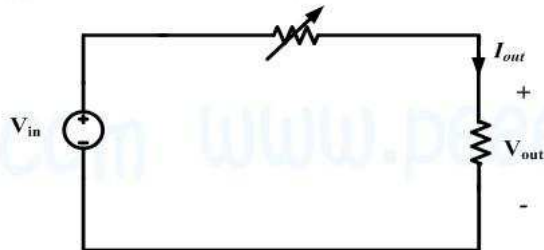
Heavy and bulky transformer



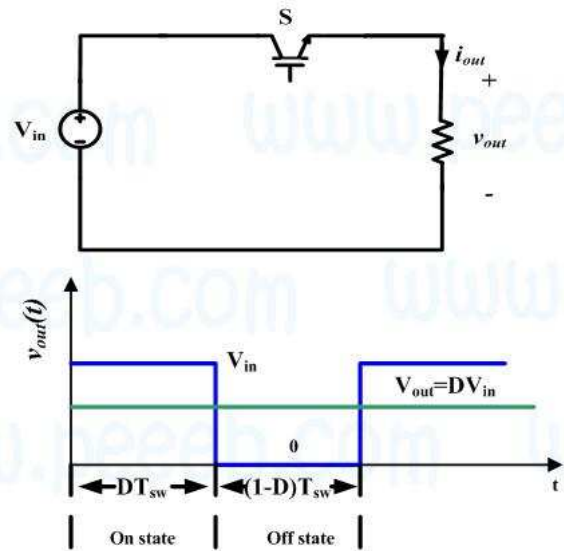
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Power Supply: Linear \Leftrightarrow Switched Mode



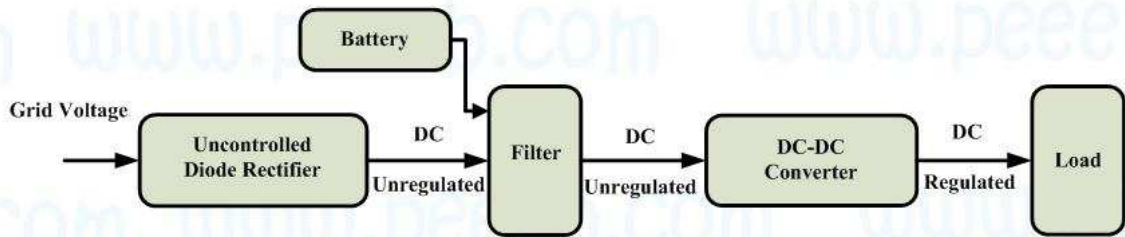
High efficiency
No heavy and bulky transformer
Suitable for high power applications



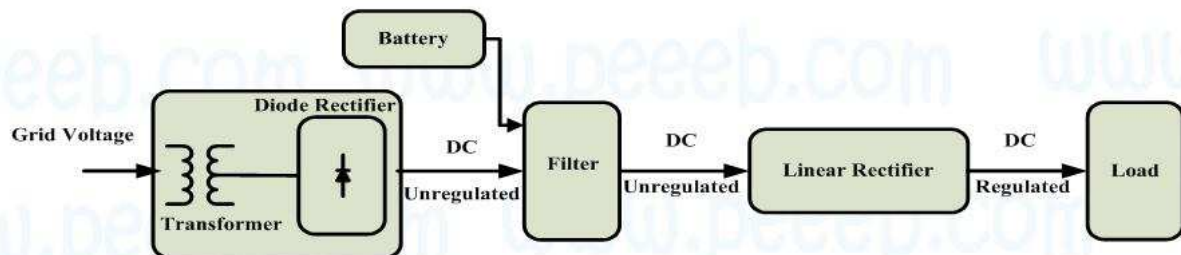
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Power Supply: Linear \Leftrightarrow Switched Mode



A Switched Mode Power Supply with AC-DC and DC-DC Converters

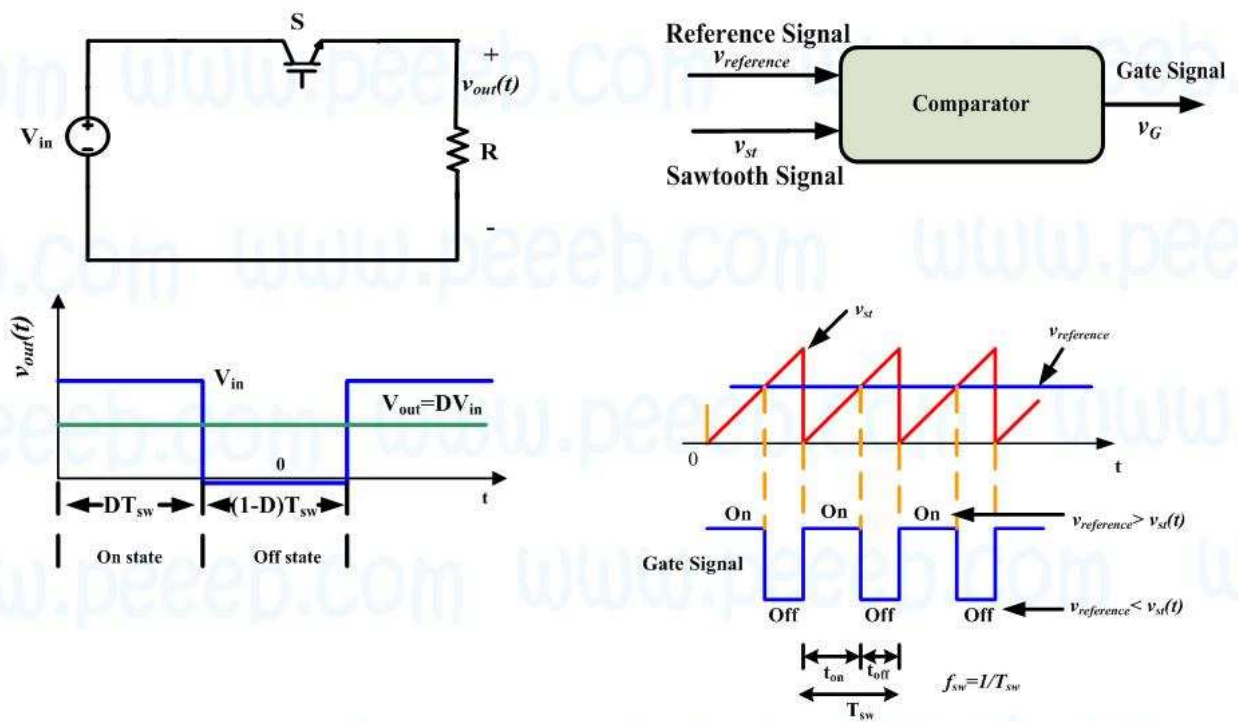


A Linear Mode Power Supply with AC-DC Converter and Transformer

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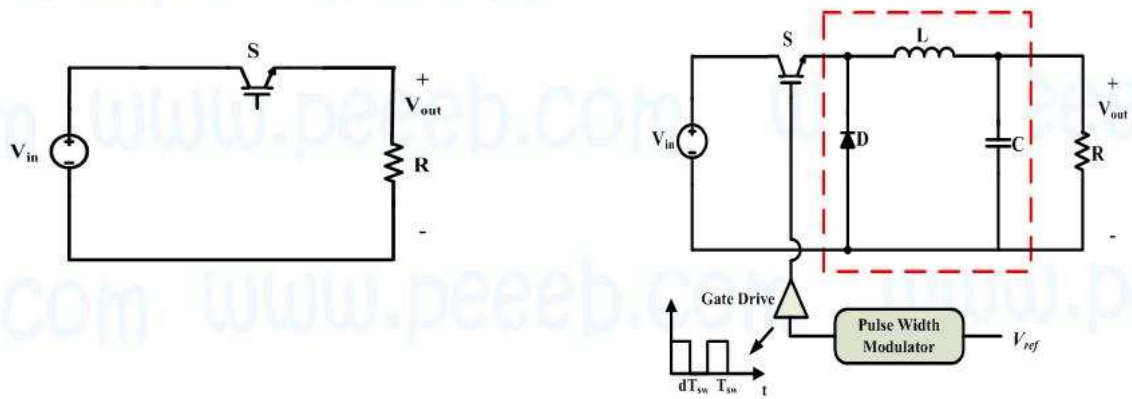
Pulse Width Modulator and Control



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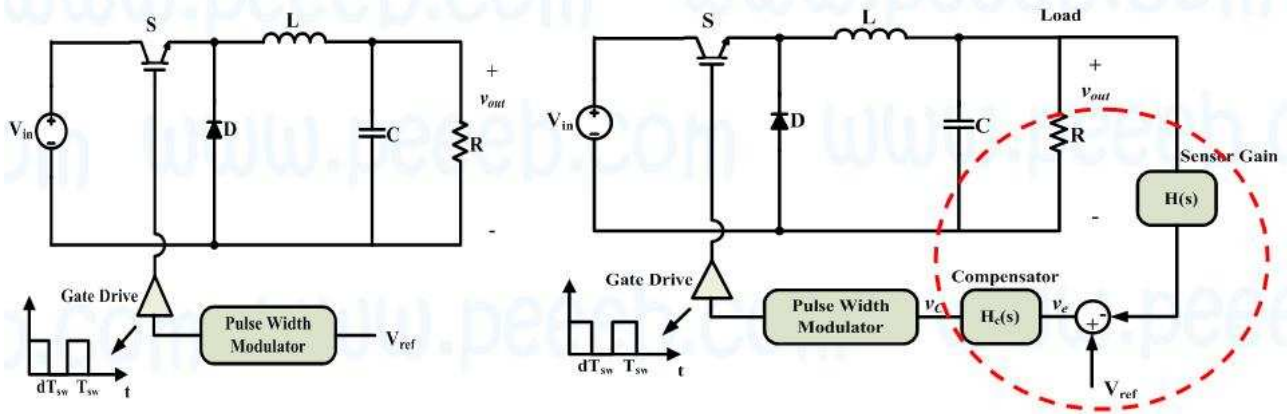
High Frequency Filter



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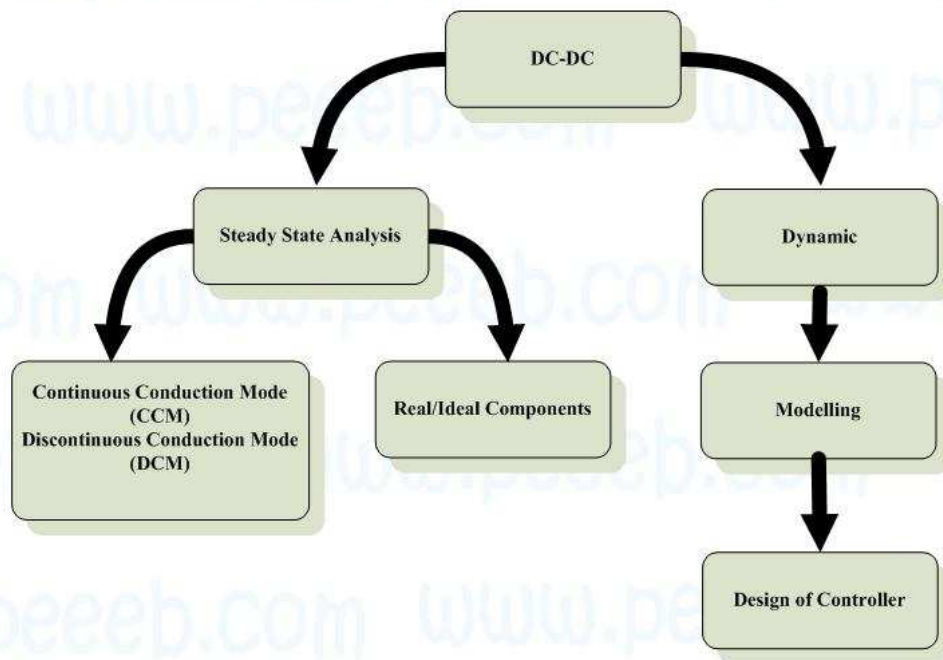
Control System: Open Loop or Close Loop



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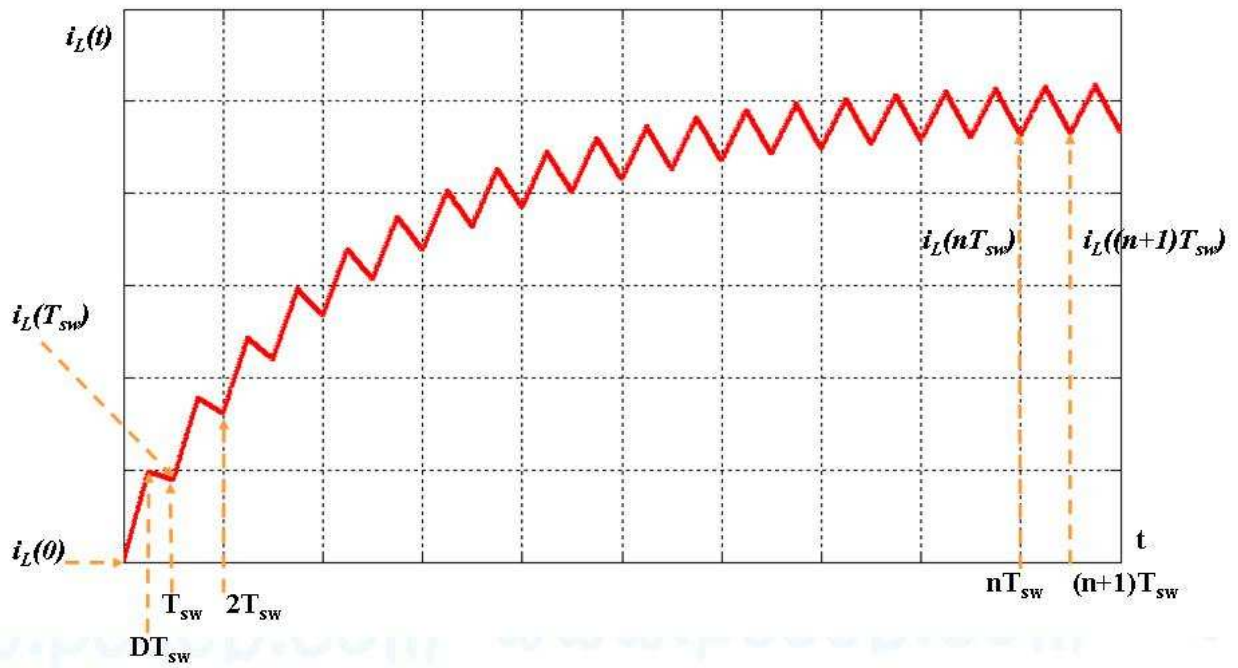
Steady State or Transient Analysis



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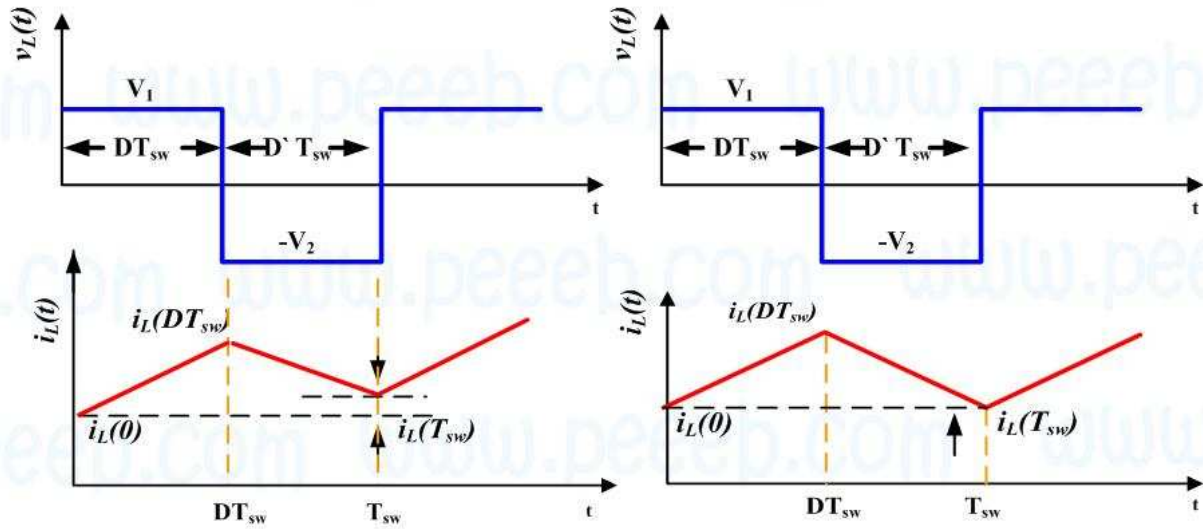
Steady State or Transient Analysis



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Key Rules in DC-DC Converters to Extract Steady State and Dynamic Equations



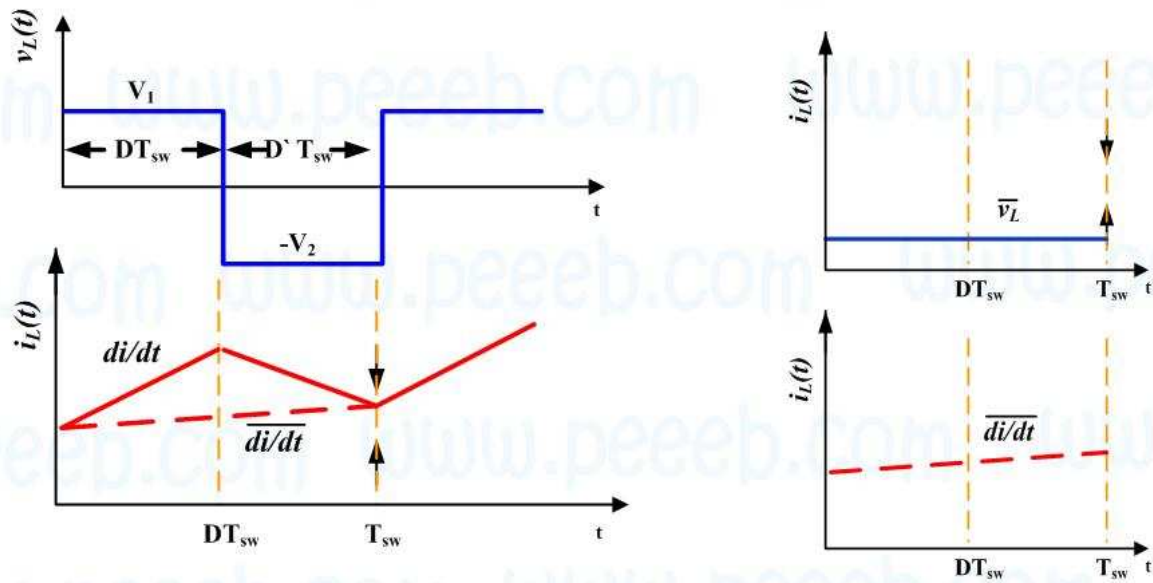
$$\overline{v_L(t)} = \frac{1}{T_{sw}} \int_0^{T_{sw}} v_L(t) dt \neq 0$$

$$\overline{v_L(t)} = \frac{1}{T_{sw}} \int_0^{T_{sw}} v_L(t) dt = 0$$

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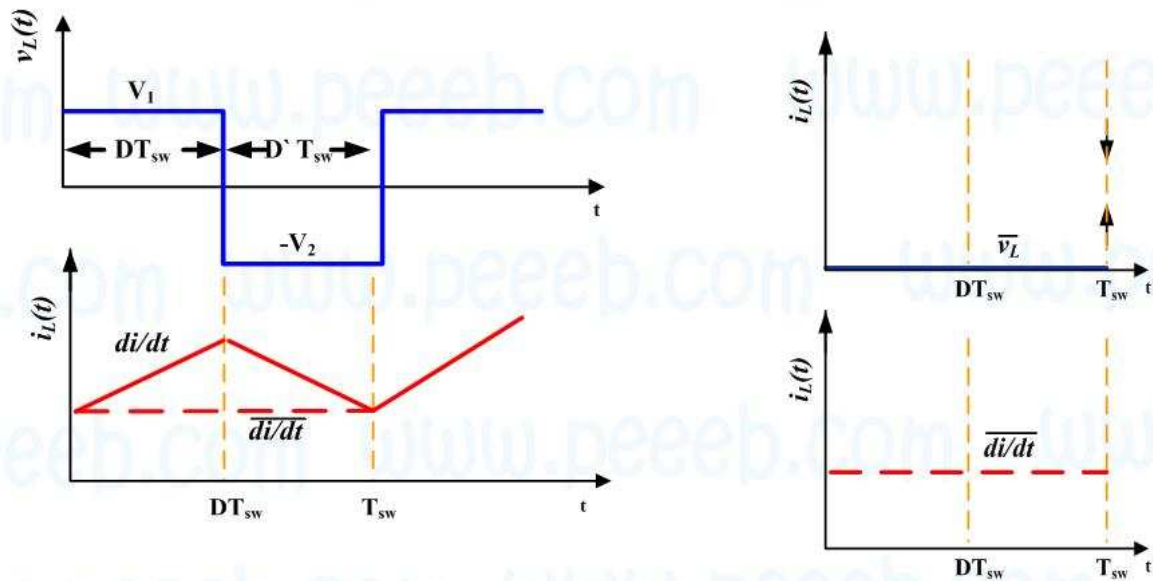
Key Rules in DC-DC Converters to Extract Steady State and Dynamic Equations



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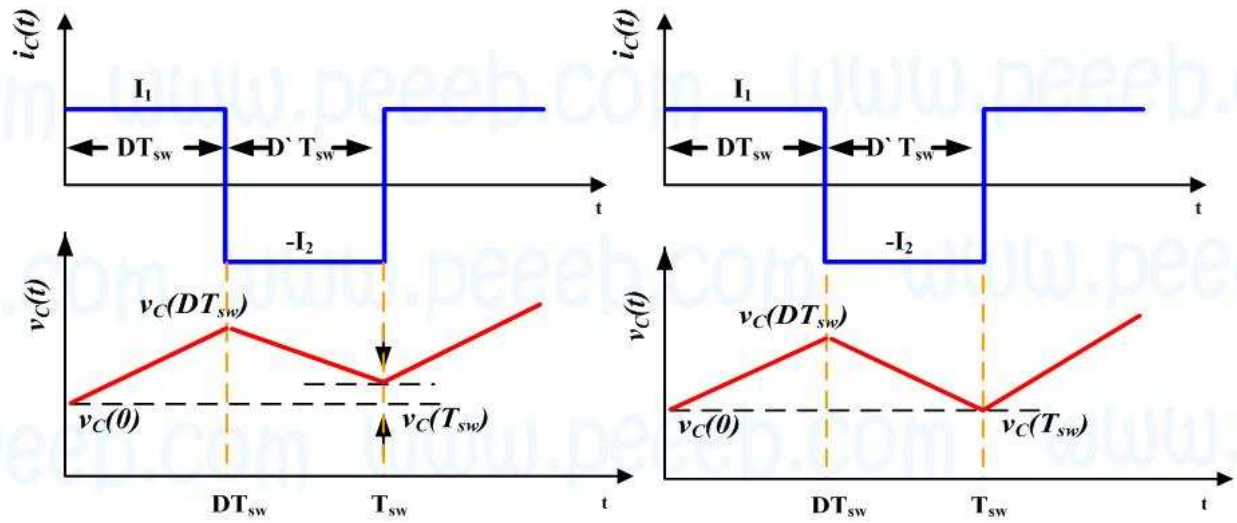
Key Rules in DC-DC Converters to Extract Steady State and Dynamic Equations



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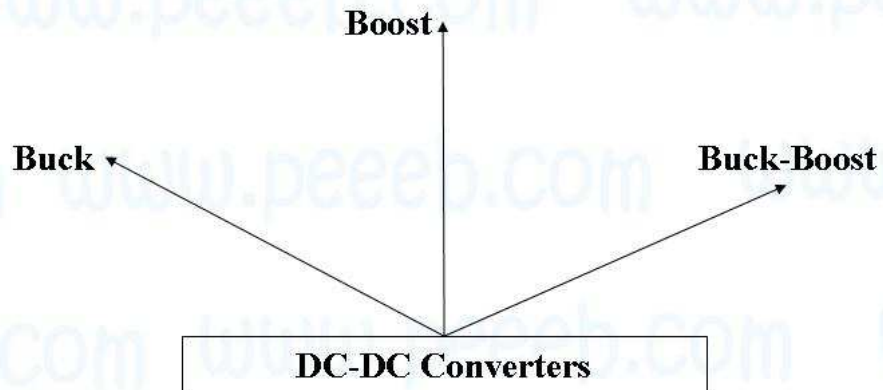
Key Rules in DC-DC Converters to Extract Steady State and Dynamic Equations



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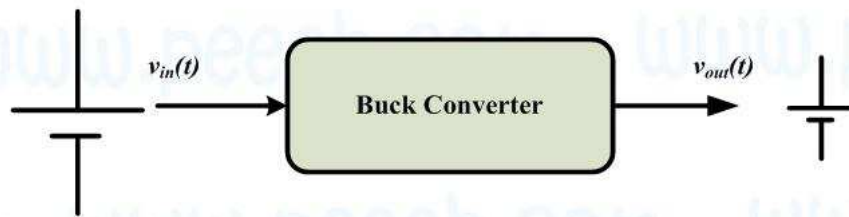
Three Different DC-DC Converters



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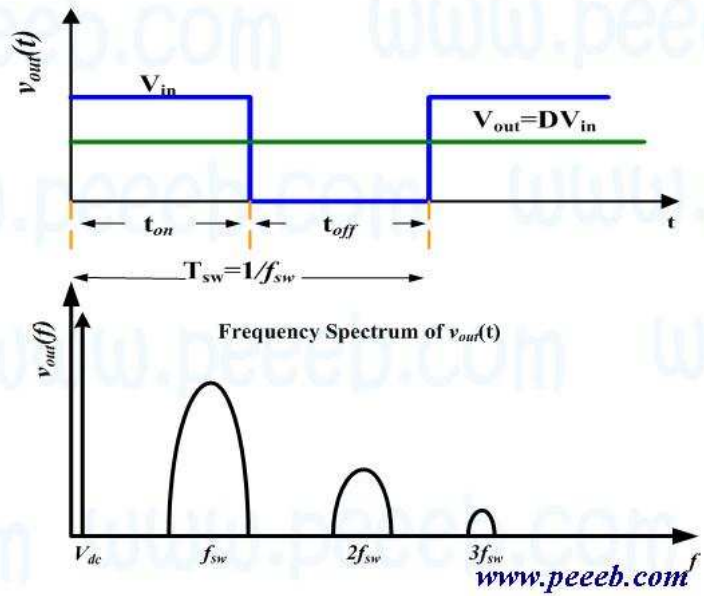
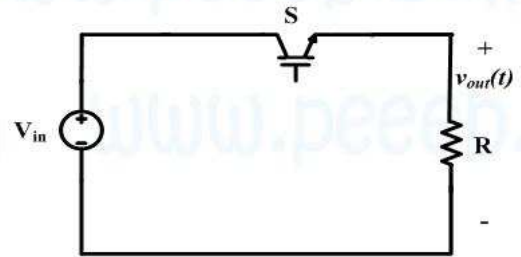
Buck Converter



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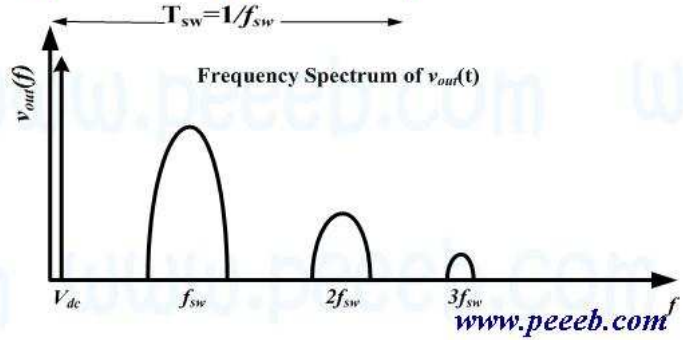
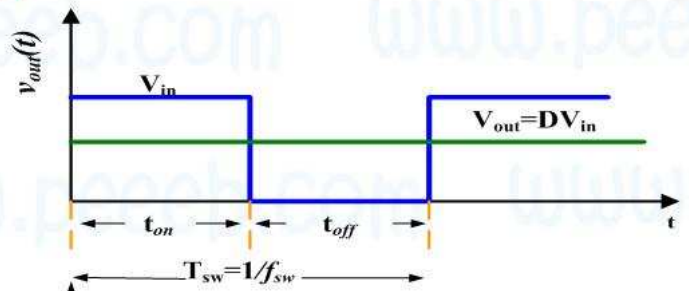
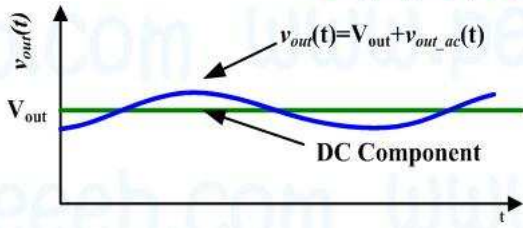
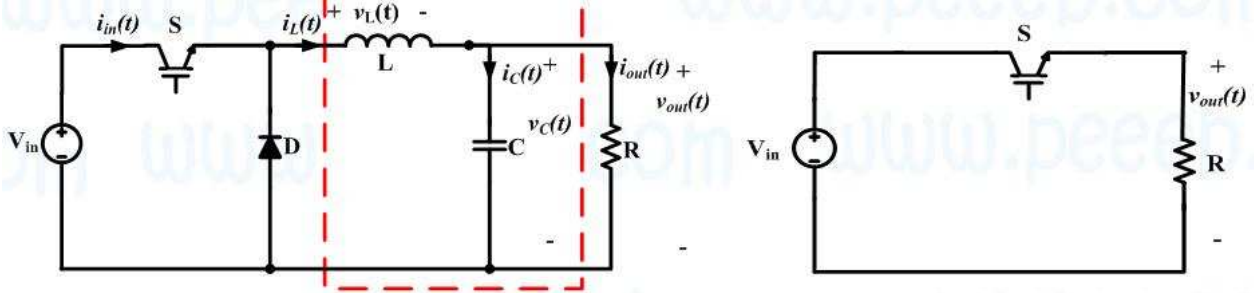
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Buck Converter



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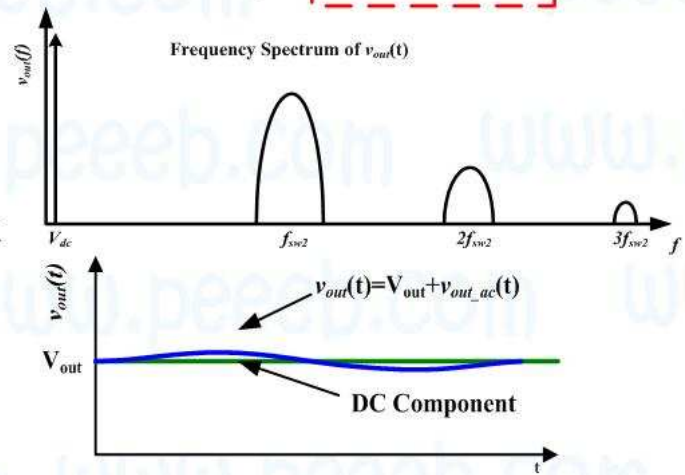
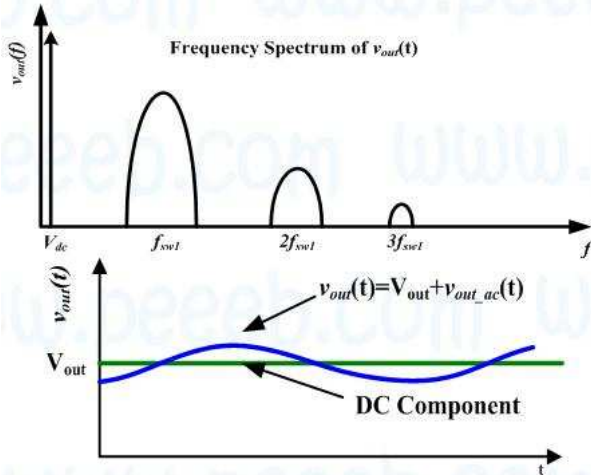
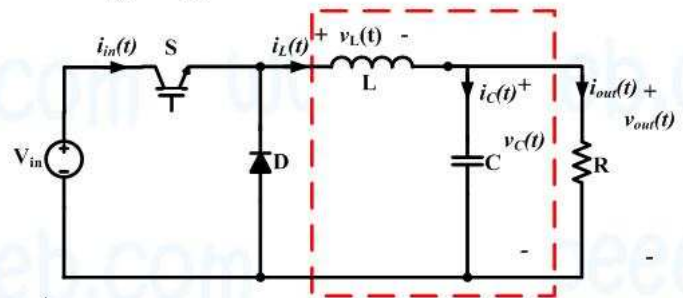
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Buck Converter

Effects of f_{sw} and filter size on output voltage ripple!

1: different filters

2: different f_{sw}



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Buck Converter

It is clear that by increasing the switching frequency, the output voltage ripple or filter size is decreased.

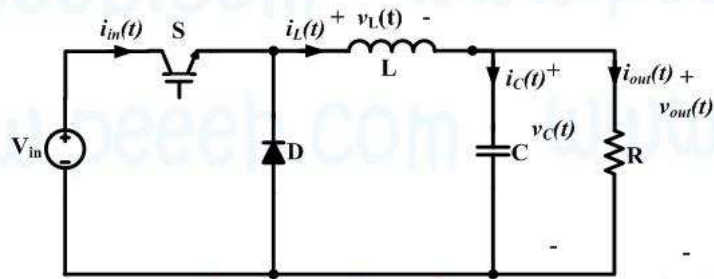
A larger LC filter can reduce the output voltage ripple magnitude but the size, cost and weight of system are increased.

Thus different applications have different requirements which almost define the system performance and cost!

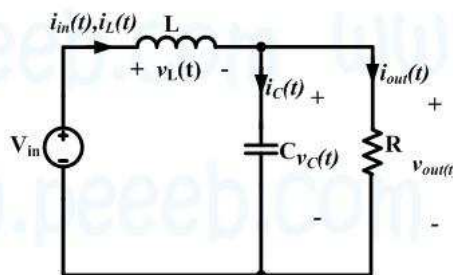
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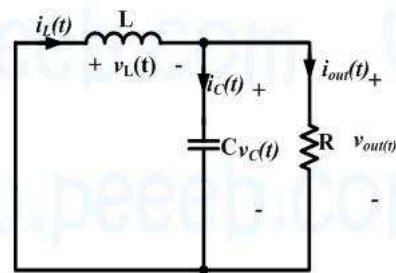
Buck Converter



When the switch is turned on, V_{in} appears across the diode and it makes it off.



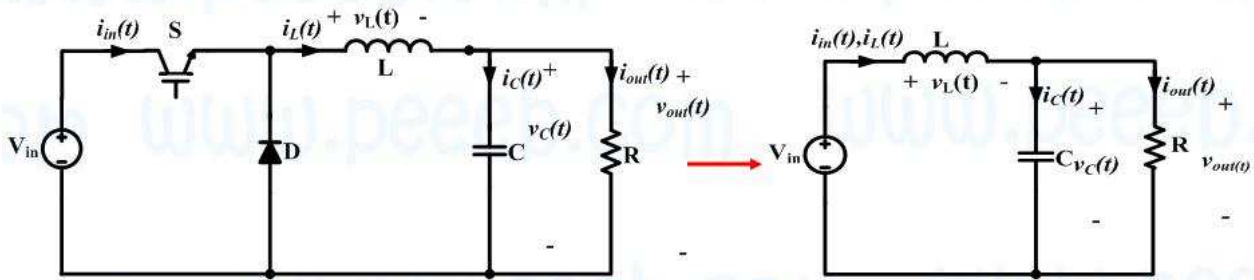
When the switch is turned off, due to inductor current, the diode conducts.



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Buck Converter



When the switch is turned on for t_{on} , $0 < t < t_{on}$

$$v_L(t) = V_{in} - v_{out}(t)$$

$$i_C(t) = i_L(t) - i_{out}(t)$$

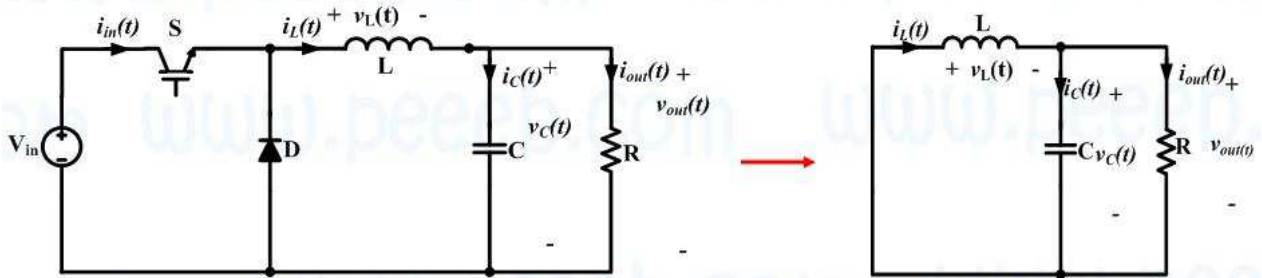
$$\left\{ \begin{array}{l} v_L(t) = V_{in} - V_{out} \\ i_C(t) = i_L(t) - I_{out} \end{array} \right.$$

Duty cycle (D) is defined as: $D = \frac{t_{on}}{T_{sw}}$

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Buck Converter



When the switch is turned off for the rest of cycle, $T_{sw} - t_{on}$. $t_{on} < t < T_{sw}$

$$v_L(t) = -v_{out}(t)$$

$$i_C(t) = i_L(t) - i_{out}(t)$$

$$\begin{cases} v_L(t) = -V_{out} \\ i_C(t) = i_L(t) - I_{out} \end{cases}$$

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Buck Converter

Steady State Equations

$$v_L(t) = \begin{cases} V_{in} - V_{out} & 0 < t < t_{on} \\ -V_{out} & t_{on} < t < T_{sw} \end{cases}$$

$$i_C(t) = i_L(t) - i_{out}(t) \quad 0 < t < T_{sw}$$

$$\overline{v_L(t)} = 0 \quad \frac{1}{T_{sw}} \int_0^{T_{sw}} v_L(t) dt = 0$$

$$\int_0^{DT_{sw}} (V_{in} - V_{out}) dt + \int_{DT_{sw}}^{T_{sw}} -V_{out} dt = 0$$

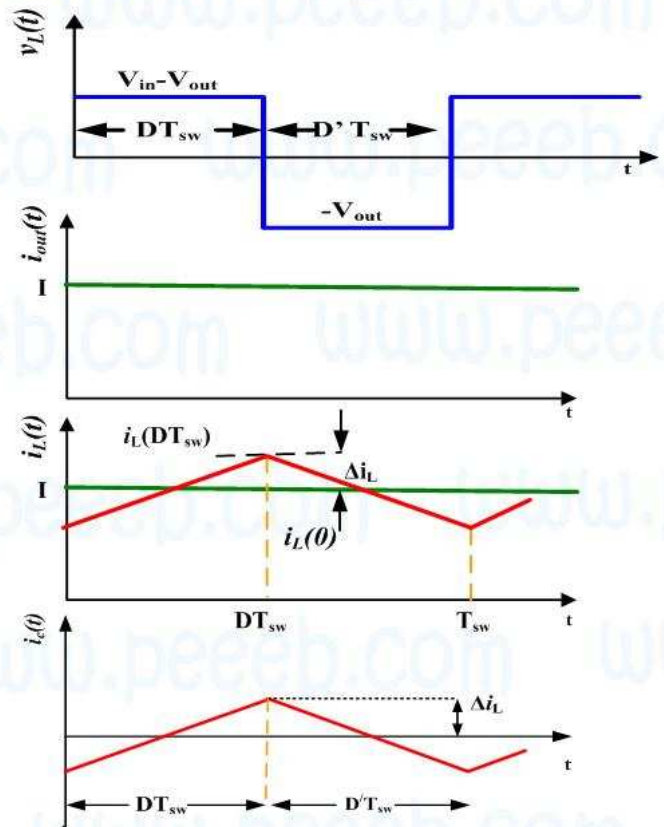
$$(V_{in} - V_{out})DT_{sw} + (-V_{out})(T_{sw} - DT_{sw}) = 0$$

$$V_{in}D - V_{out}(D + 1 - D) = 0$$

$$V_{in}D - V_{out} = 0$$

$$H(D) = \frac{V_{out}}{V_{in}} = D$$

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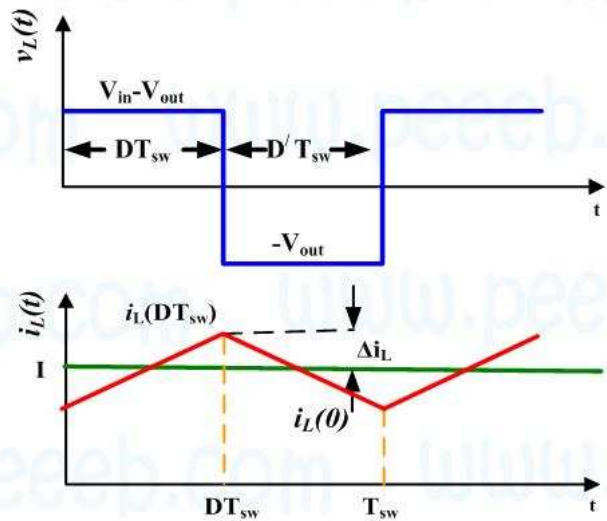


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Buck Converter

How to select L and C sizes?

Once we select the switching frequency based on other design issues such as switching losses and EMI. Then we can find the inductor and capacitor values.



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Buck Converter

How to select L and C sizes?

$$v_L(t) = L \frac{di_L(t)}{dt}$$

$$di_L(t) = \frac{v_L(t) dt}{L}$$

$$\int_{\theta}^{\theta} di_L(t) dt = \frac{1}{L} \int_{\theta}^{\theta} v_L(t) dt$$

$$\int_{\theta}^{DT_{sw}} di_L(t) = \frac{1}{L} \int_{\theta}^{DT_{sw}} v_L(t) dt$$

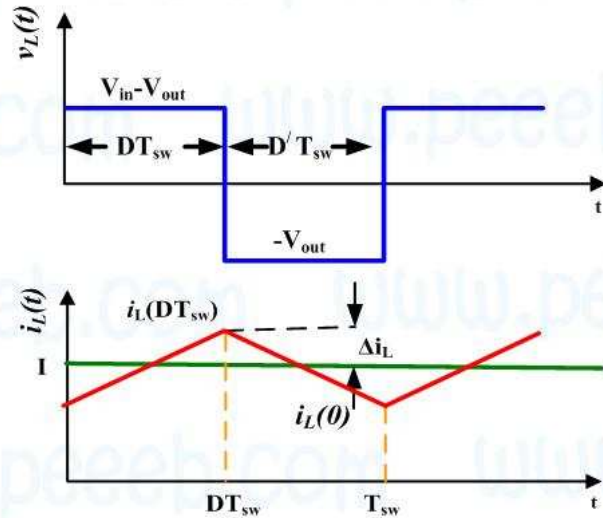
$$i_L(DT_{sw}) - i_L(\theta) = \frac{(V_{in} - V_{out})DT_{sw}}{L}$$

$$2\Delta i_L = \frac{DT_{sw}(V_{in} - V_{out})}{L} \quad T_{sw} = \frac{1}{f_{sw}}$$

$$\Delta i_L = \frac{D(V_{in} - V_{out})}{2Lf_{sw}}$$

$$\Delta i_L = \frac{D(V_{in} - V_{out})}{2Lf_{sw}} = \frac{DD'V_{in}}{2Lf_{sw}}$$

$$L = \frac{DD'V_{in}}{2f_{sw}\Delta i_L}$$



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Buck Converter

How to select L and C sizes?

When $i_c(t)$ is positive, the capacitor voltage $v_c(t)$ is charged.

$$C \frac{dv_c(t)}{dt} = i_c(t)$$

$$dv_c(t) = \frac{i_c(t)dt}{C}$$

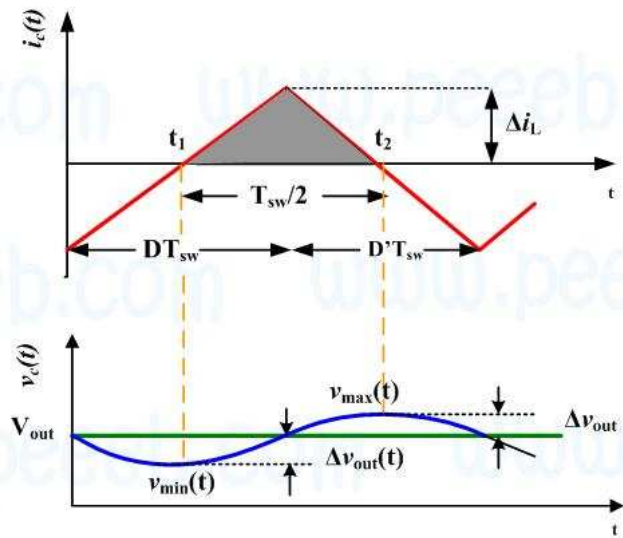
$$\int_{t_1}^{t_2} dv_c(t)dt = \frac{1}{C} \int_{t_1}^{t_2} i_c(t)dt$$

$$2\Delta v_c(t) = \frac{1}{C} \left(\frac{\Delta i_L}{2} \frac{T_{sw}}{2} \right)$$

$$\Delta v_c(t) = \frac{\Delta i_L}{8Cf_{sw}}$$

$$C = \frac{\Delta i_L}{8\Delta v_c f_{sw}}$$

$$C = \frac{DD'V_{in}}{16Lf_{sw}^2 \Delta v_c}$$

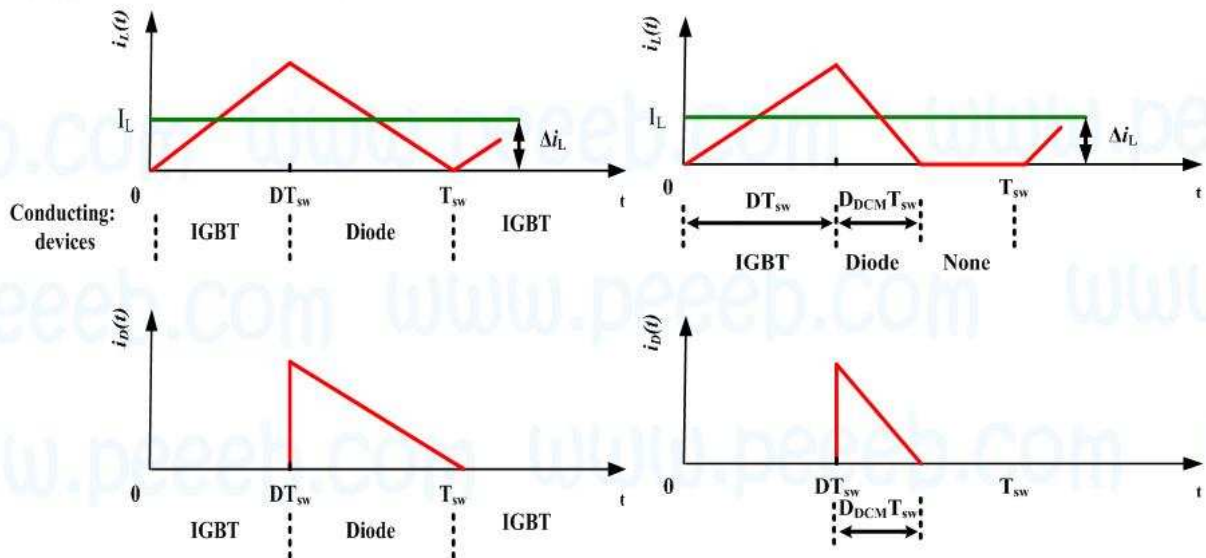


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Continues and Discontinues Conduction Modes CCM & DCM

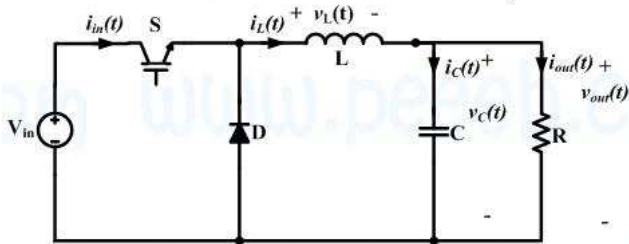


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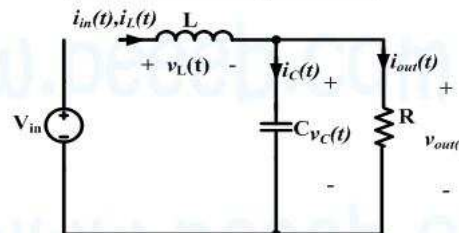
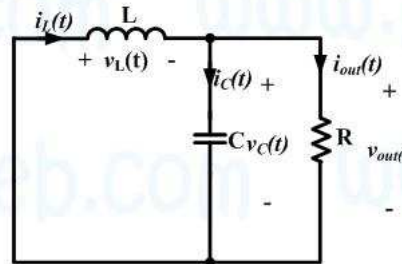
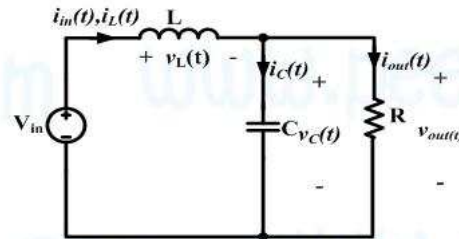
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Buck Converter

Discontinuous Conduction Modes, DCM



In the DCM, the inductor current is not continuous and the converter has three different states.



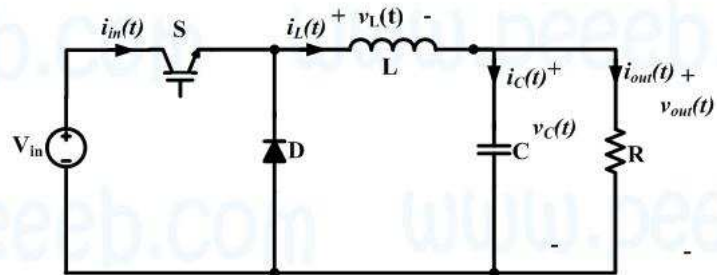
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Discontinuous Conduction Modes, DCM

The IGBT is on and the diode is off

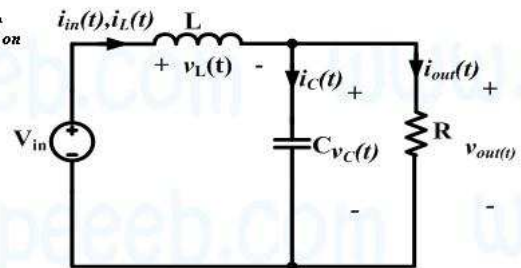


When the switch is turned on for t_{on} . $0 < t < t_{on}$

$$v_L(t) = V_{in} - v_{out}(t)$$

$$i_C(t) = i_L(t) - i_{out}(t)$$

$$\begin{cases} v_L(t) = V_{in} - V_{out} \\ i_C(t) = i_L(t) - I_{out} \end{cases}$$



Duty cycle (D) is defined as: $D = \frac{t_{on}}{T_{sw}}$

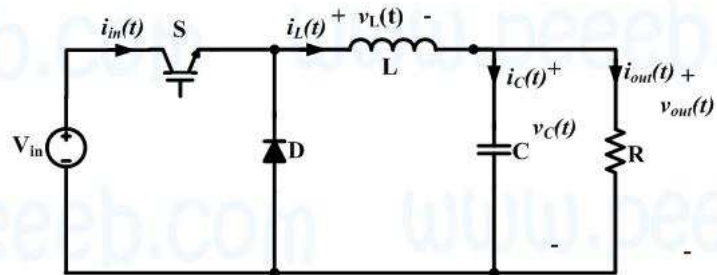
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Discontinuous Conduction Modes, DCM

The IGBT is off and the diode is on



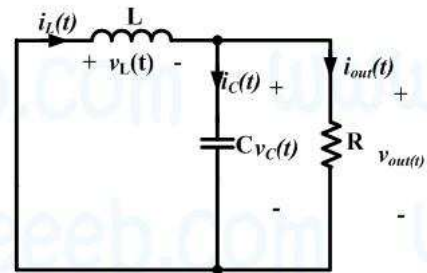
When the switch is turned off till the inductor current reaches to zero, $DT_{sw} < t < (D + D_{DCM})T_{sw}$

$$v_L(t) = -v_{out}(t)$$

$$i_C(t) = i_L(t) - i_{out}(t)$$

$$v_L(t) = -V_{out}$$

$$i_C(t) = i_L(t) - I_{out}$$



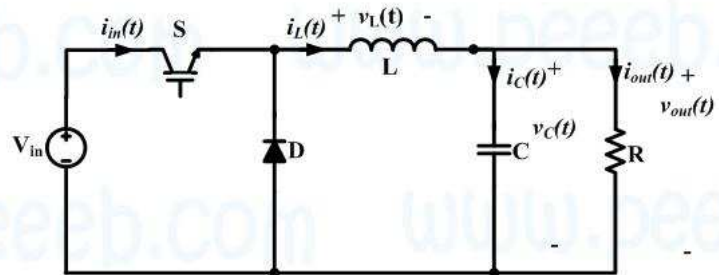
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Buck Converter

Discontinuous Conduction Modes, DCM

The diode and IGBT are off

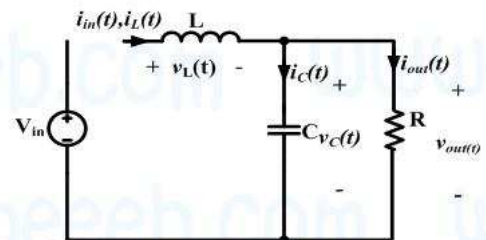


The inductor current is zero.

$$v_L(t) = 0$$

$$i_C(t) = -i_{out}(t)$$

$$\begin{cases} v_L(t) = 0 \\ i_C(t) = -I_{out} \end{cases}$$

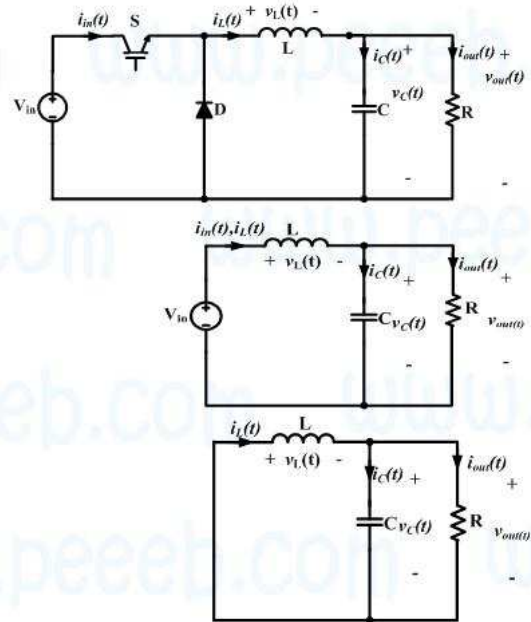
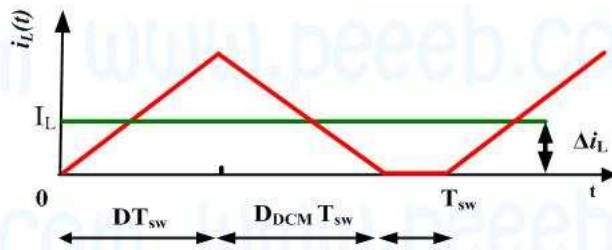


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Discontinuous Conduction Mode, DCM



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Buck Converter

$$V_{in} I_{in} = V_{out} I_{out}$$

$$I_{in} = \overline{i_{in}(t)} = \frac{1}{T_{sw}} \int_0^{DT_{sw}} i_L(t) dt = \frac{DT_{sw} \times 2\Delta i_L}{2T_{sw}} = D\Delta i_L$$

$$V_{in} D\Delta i_L = \frac{V_{out}^2}{R}$$

$$L \frac{di_L}{dt} = v_L(t) \quad L \left(\frac{2\Delta i_L}{DT_{sw}} \right) = V_{in} - V_{out} \quad 0 < t < t_{on}$$

$$\Delta i_L = \frac{(V_{in} - V_{out})DT_{sw}}{2L}$$

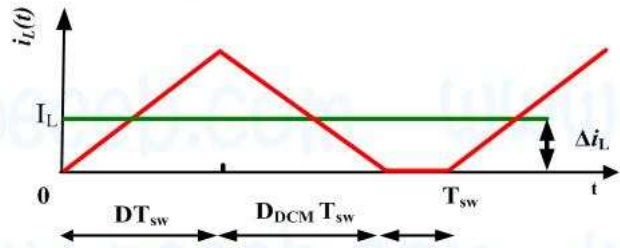
$$\frac{V_{in} D^2 (V_{in} - V_{out}) T_{sw}}{2L} = \frac{V_{out}^2}{R}$$

$$V_{in}^2 - V_{in} V_{out} = \frac{2LV_{out}^2}{RT_{sw} D^2}$$

$$\frac{V_{in}^2}{V_{out}^2} - \frac{V_{in}}{V_{out}} - K_1 = 0 \quad K_1 = \frac{2L}{RT_{sw} D^2}$$

$$\frac{V_{in}}{V_{out}} = \frac{+1 \pm \sqrt{1 + 4K_1}}{2}$$

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$$H(D) = \frac{V_{out}}{V_{in}} = \frac{2}{+1 + \sqrt{1 + \frac{8L}{RT_{sw} D^2}}}$$

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Buck Converter

Assuming that the converter operates in CCM:

$$\Delta i > I_L \quad DCM$$

$$\Delta i < I_L \quad CCM$$

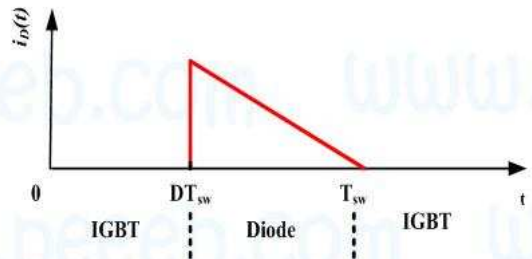
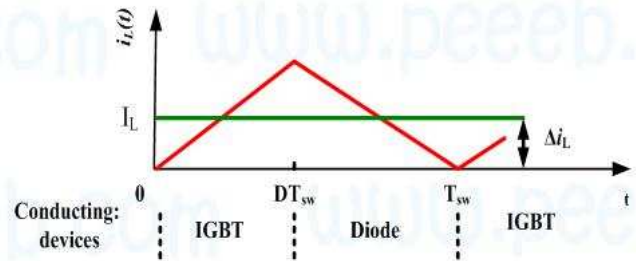
$$I_L = I_{out} \quad I_L = \frac{V_{out}}{R}$$

$$\Delta i = \frac{(V_{in} - V_{out})DT_{sw}}{2L} = \frac{(V_{in} - DV_{in})DT_{sw}}{2L}$$

$$\Delta i = \frac{V_{in}(1-D)DT_{sw}}{2L} = \frac{V_{in}DD'T_{sw}}{2L}$$

$$\Delta i < I_L \Rightarrow \frac{DD'V_{in}T_{sw}}{2L} < \frac{DV_{in}}{R}$$

$$\frac{D'T_{sw}}{2L} < \frac{1}{R} \Rightarrow D' < \frac{2L}{RT_{sw}}$$



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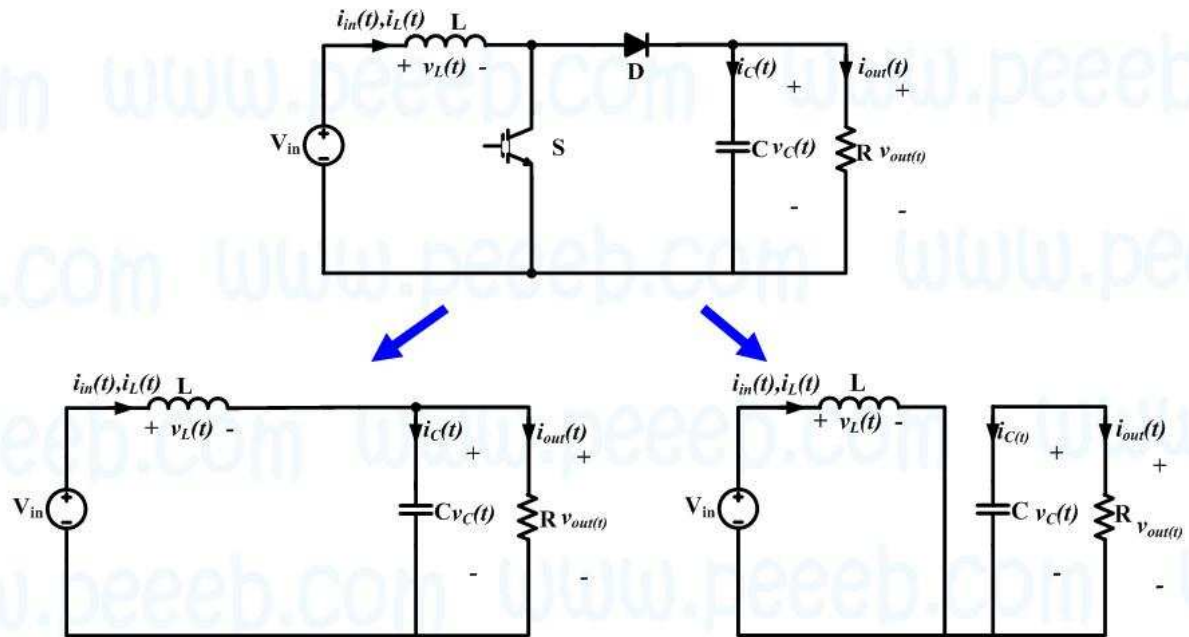
Boost Converter



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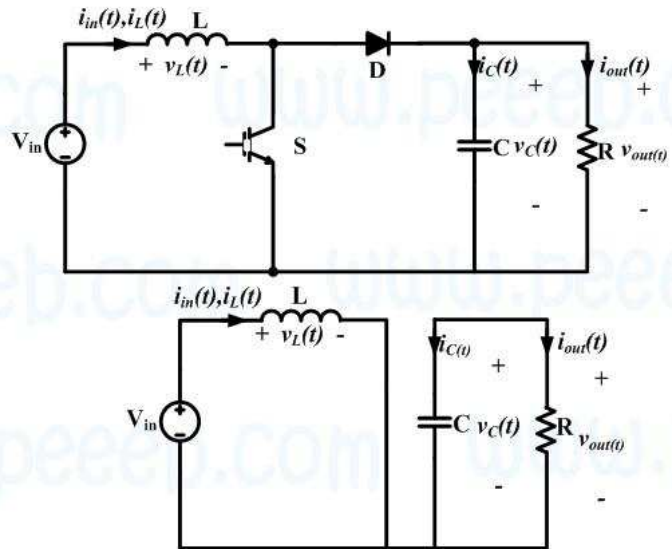
Boost Converter

$$v_L(t) = V_{in}$$

$$i_C(t) = -i_{out}(t)$$

$$\begin{cases} v_L(t) = V_{in} \\ i_C(t) = -I_{out} \end{cases}$$

$$0 < t < t_{on}$$



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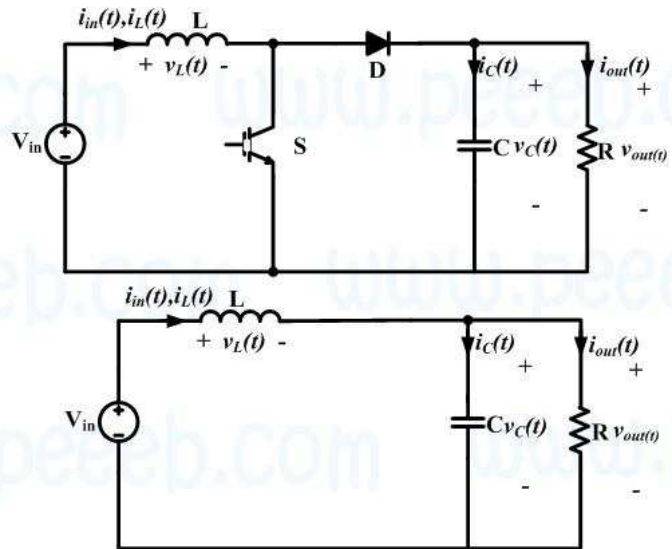
Boost Converter

$$v_L(t) = V_{in} - v_{out}(t)$$

$$i_C(t) = i_L(t) - i_{out}(t)$$

$$\begin{cases} v_L(t) = V_{in} - V_{out}(t) \\ i_C(t) = i_L(t) - I_{out} \end{cases}$$

$$t_{on} < t < T_{sw}$$



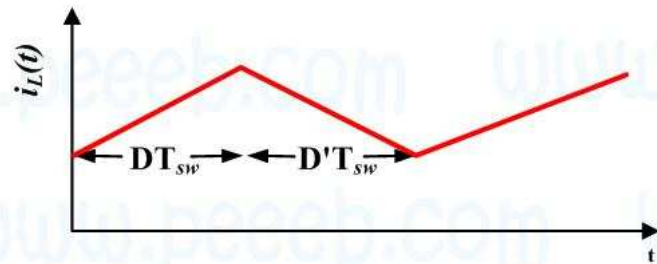
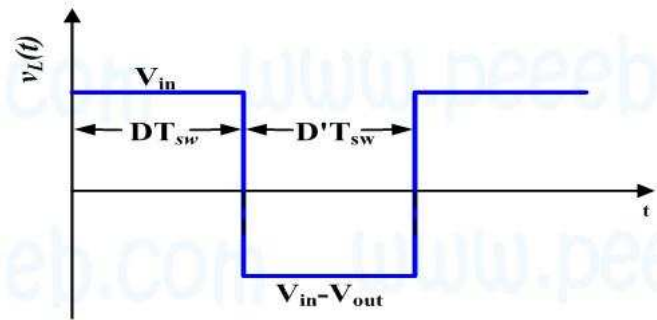
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Boost Converter

$$v_L(t) = \begin{cases} V_{in} & 0 < t < t_{on} \\ V_{in} - V_{out} & t_{on} < t < T_{sw} \end{cases}$$

$$i_c(t) = \begin{cases} -I_{out} & 0 < t < t_{on} \\ i_L(t) - I_{out} & t_{on} < t < T_{sw} \end{cases}$$



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Boost Converter

$$\overline{v_L(t)} = 0$$

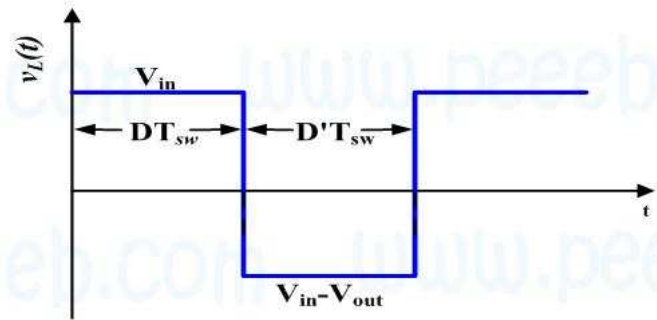
$$\int_0^{DT_{sw}} V_{in} dt + \int_{DT_{sw}}^{T_{sw}} (V_{in} - V_{out}) dt = 0$$

$$V_{in} DT_{sw} + (V_{in} - V_{out})(T_{sw} - DT_{sw}) = 0$$

$$V_{in} DT_{sw} + (V_{in} - V_{out})D'T_{sw} = 0$$

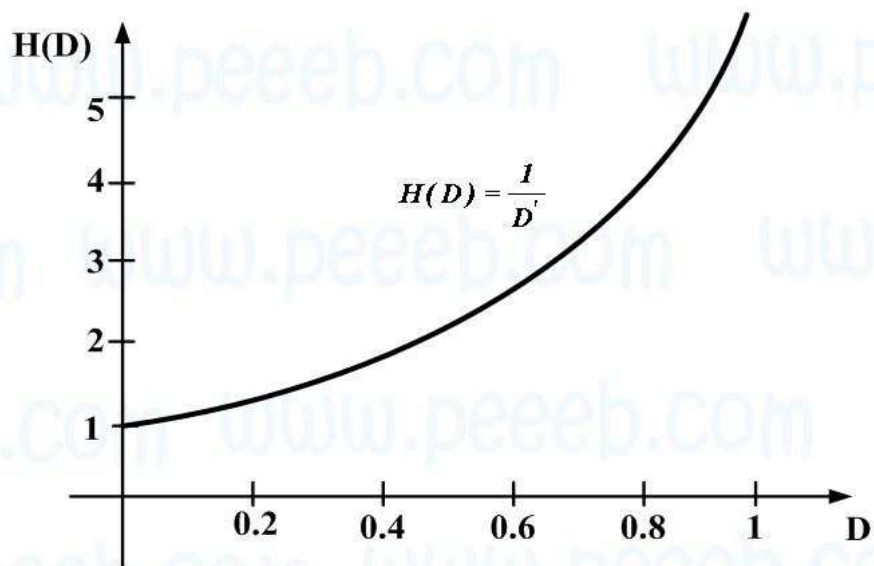
$$V_{in}(D + D') - V_{out}D' = 0$$

$$H(D) = \frac{V_{out}}{V_{in}} = \frac{1}{(1-D)} = \frac{1}{D'}$$



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Boost Converter

How to select L and C sizes?

$$L \frac{di_L(t)}{dt} = v_L(t)$$

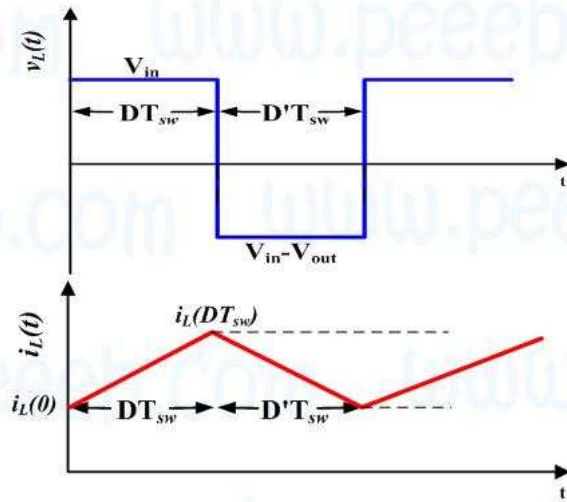
$$\int_0^{DT_{sw}} di_L(t) = \frac{1}{L} \int_0^{DT_{sw}} V_{in} dt$$

$$i_L(DT_{sw}) - i_L(0) = \frac{V_{in} DT_{sw}}{L}$$

$$2\Delta i_L = \frac{V_{in} DT_{sw}}{L} \Rightarrow$$

$$\Delta i_L = \frac{V_{in} D}{2Lf_{sw}}$$

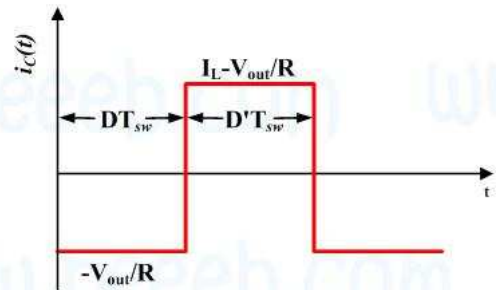
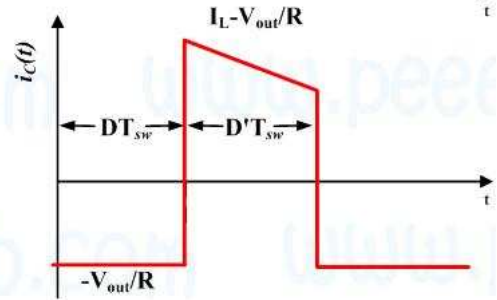
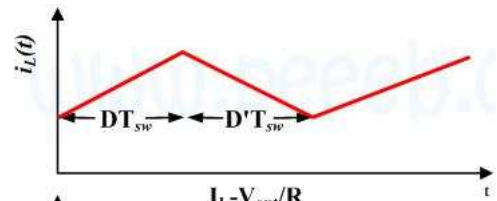
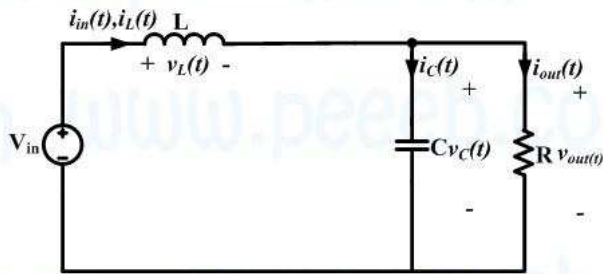
$$L = \frac{V_{in} D}{2f_{sw} \Delta i_L}$$



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Boost Converter

How to select L and C sizes?

$$C \frac{dv_c(t)}{dt} = i_c(t)$$

$$dv_c(t) = \frac{i_c(t) dt}{C}$$

$$\int_0^{DT_{sw}} dv_c(t) = \frac{1}{C} \int_0^{DT_{sw}} i_c(t) dt$$

$$-2\Delta v_c = \frac{-I_{out} DT_{sw}}{C}$$

$$\Delta v_c = \frac{I_{out} DT_{sw}}{2C}$$

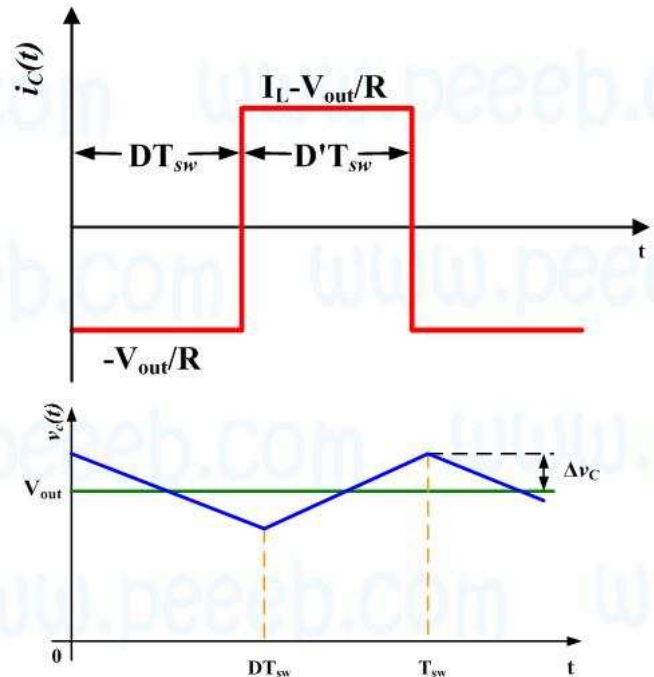
$$I_{out} = \frac{V_{out}}{R} \quad T_{sw} = \frac{1}{f_{sw}}$$

$$\Delta v_c = \frac{V_{out} D}{2RCf_{sw}}$$

$$C = \frac{V_{in} D}{2RD'f_{sw} \Delta v_c}$$

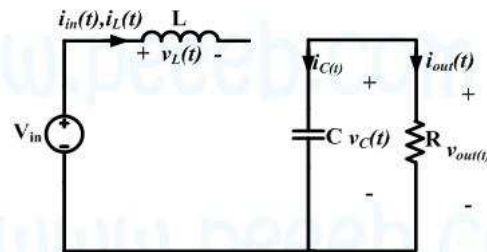
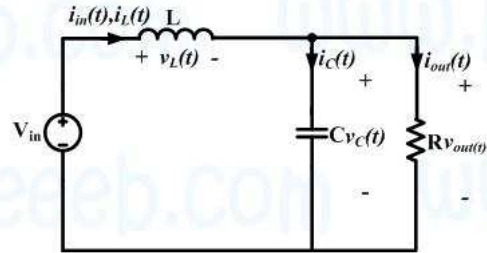
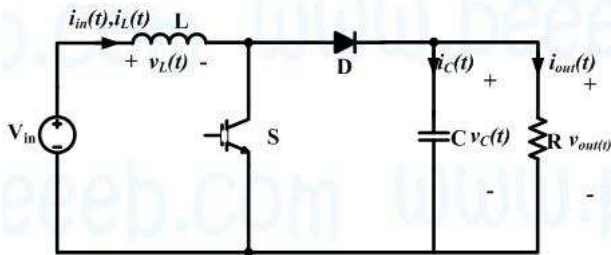
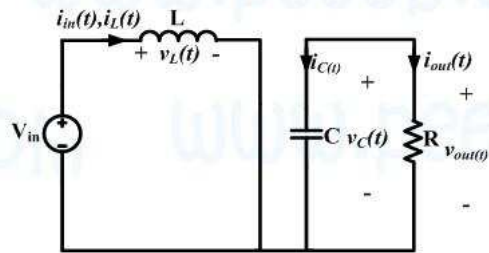
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Discontinuous Conduction Modes, DCM



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Discontinuous Conduction Modes, DCM

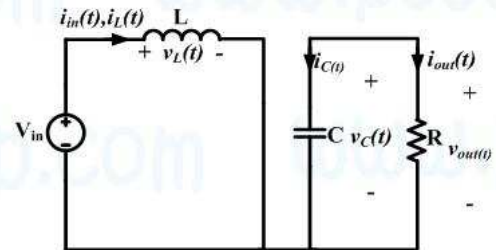
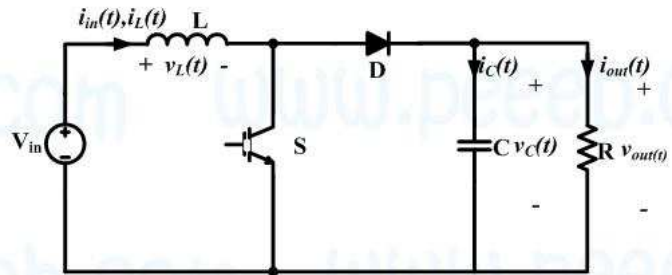
The IGBT is on and the diode is off.

$$v_L(t) = V_{in}$$

$$i_C(t) = -i_{out}(t)$$

$$\begin{cases} v_L(t) = V_{in} \\ i_C(t) = -I_{out} \end{cases}$$

$$0 < t < t_{on}$$



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Discontinuous Conduction Modes, DCM

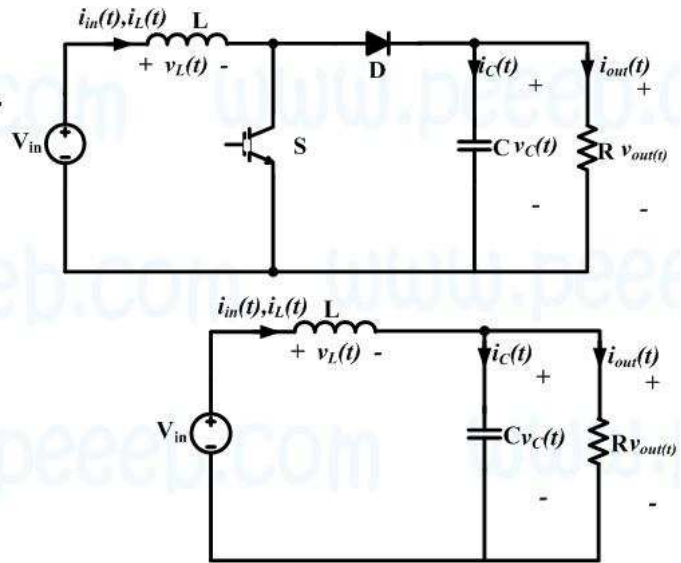
The IGBT is off and the diode is on.

$$v_L(t) = V_{in} - v_{out}(t)$$

$$t_{on} < t < D_{DCM} T_{sw}$$

$$i_C(t) = i_L(t) - i_{out}(t)$$

$$\begin{cases} v_L(t) = V_{in} - v_{out}(t) \\ i_C(t) = i_L(t) - I_{out} \end{cases}$$



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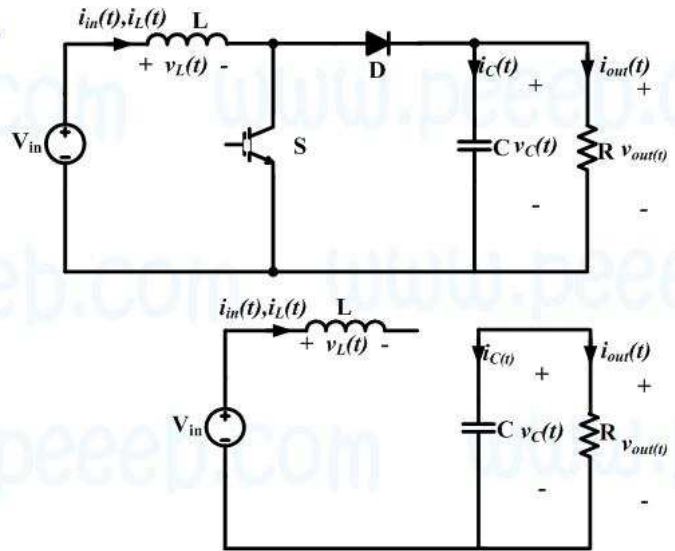
Discontinuous Conduction Modes, DCM

Both the IGBT and the diode are off.

$$v_L(t) = 0 \quad D_{DCM} T_{sw} < t < T_{sw}$$

$$i_C(t) = i_{out}(t)$$

$$\begin{cases} v_L(t) = 0 \\ i_C(t) = -I_{out} \end{cases}$$



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Discontinuous Conduction Modes, DCM



$$V_{in} I_{in} = V_{out} I_{out}$$

$$I_{in} = \overline{i_{in}(t)} = \frac{1}{T_{sw}} \int_0^{(D+D_{DCM})T_{sw}} i_L(t) dt = \frac{(D+D_{DCM})T_{sw} \times 2\Delta i_L}{2T_{sw}}$$

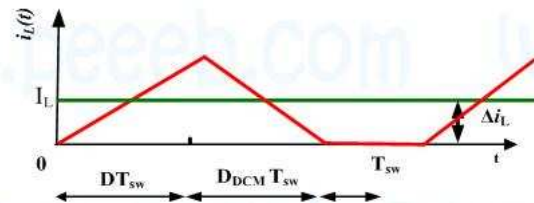
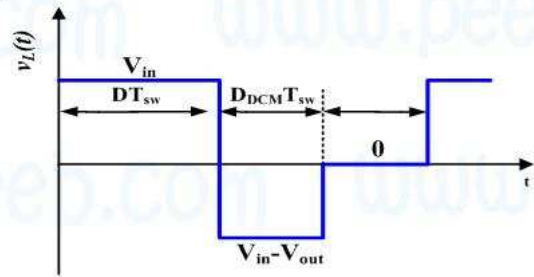
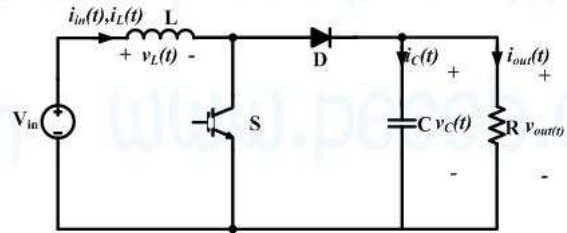
$$I_{in} = (D+D_{DCM})\Delta i_L$$

$$V_{in} \Delta i_L (D+D_{DCM}) = \frac{V_{out}^2}{R}$$

$$L \frac{di_L(t)}{dt} = v_L(t)$$

$$L \left(\frac{2\Delta i_L}{DT_{sw}} \right) = V_{in} \quad 0 < t < t_{on}$$

$$\Delta i_L = \frac{V_{in} DT_{sw}}{2L}$$



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Discontinuous Conduction Modes, DCM

$$V_{in} \left(\frac{V_{in} D T_{sw}}{2L} \right) (D + D_{DCM}) = \frac{V_{out}^2}{R}$$

$$V_{in}^2 \left(\frac{R T_{sw} D}{2L} \right) (D + D_{DCM}) = V_{out}^2$$

$$V_{in}^2 \times \left(\frac{R T_{sw} D}{2L} \right) \left(D + \frac{V_{in} D}{V_{out} - V_{in}} \right) = V_{out}^2$$

$$V_{in}^2 \left(\frac{R T_{sw} D}{2L} \right) (D V_{out} - D V_{in} + D V_{in}) = V_{out}^2 (V_{out} - V_{in})$$

$$V_{in}^2 \left(\frac{R T_{sw} D}{2L} \right) (D V_{out}) = V_{out}^2 (V_{out} - V_{in})$$

$$V_{in}^2 \left(\frac{R T_{sw} D}{2L} \right) D = V_{out}^2 - V_{out} V_{in}$$

$$\frac{V_{out}^2}{V_{in}^2} - \frac{V_{out}}{V_{in}} - \left(\frac{R D^2 T_{sw}}{2L} \right) = 0 \quad K_2 = \frac{R D^2 T_{sw}}{2L}$$

$$\frac{V_{out}^2}{V_{in}^2} - \frac{V_{out}}{V_{in}} - K_2 = 0$$

$$\frac{V_{out}}{V_{in}} = \frac{+1 \pm \sqrt{1 + 4K_2}}{2}$$

$$\frac{V_{out}}{V_{in}} = \frac{1 + \sqrt{1 + \frac{2 R D^2 T_{sw}}{L}}}{2}$$

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Boost Converter

Discontinuous Conduction Modes, DCM

$$\Delta i_L > I_L \quad DCM \quad \Delta i_L < I_L \quad CCM$$

$$\Delta i_L = \frac{V_{in} D T_{sw}}{2L} \quad \frac{V_{in} D T_{sw}}{2L} < I_L$$

$$\frac{V_{in} D T_{sw}}{2L} < \frac{V_{in}}{(1-D)^2 R}$$

$$D(1-D)^2 < \frac{2L}{R T_{sw}}$$

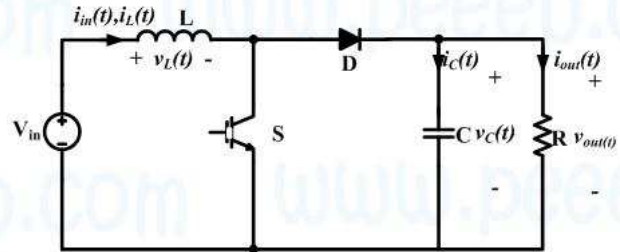
$$D(1-2D+D^2) < \frac{2L}{R T_{sw}}$$

$$D^3 - 2D^2 + D < \frac{2L}{R T_{sw}}$$

$$f(D) = D^3 - 2D^2 + D$$

$$\frac{df(D)}{dD} = 0 \Rightarrow 3D^2 - 4D + 1 = 0 \Rightarrow D = \frac{1}{3}$$

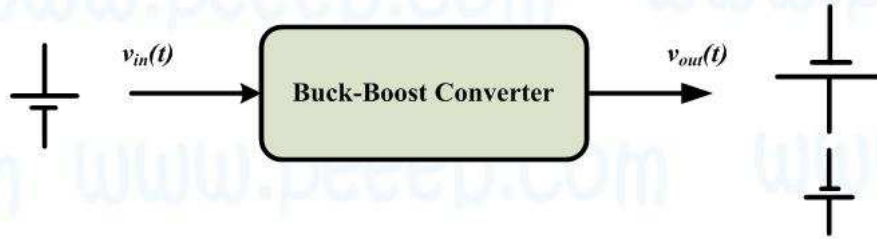
$$f\left(\frac{1}{3}\right)_{\max} = \frac{4}{27} < \frac{2L}{R T_{sw}}$$



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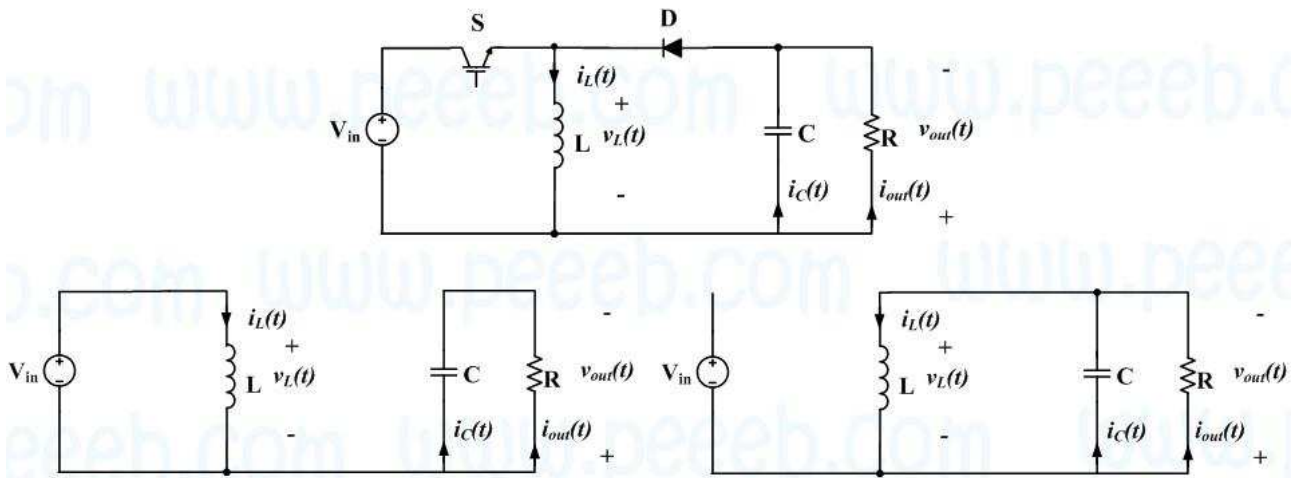
Buck-Boost Converter



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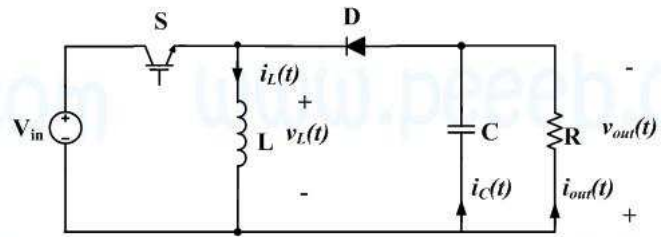
Buck-Boost Converter

$$v_L(t) = V_{in}$$

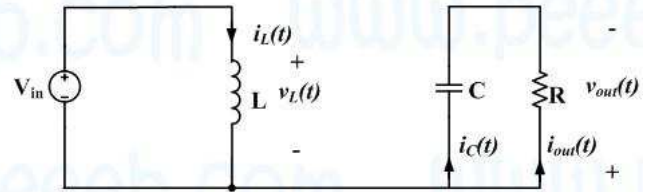
$$i_C(t) = -i_{out}(t)$$

$$\begin{cases} v_L(t) = V_{in} \\ i_C(t) = -I_{out} \end{cases}$$

$$0 < t < t_{on}$$



$$0 < t < t_{on}$$



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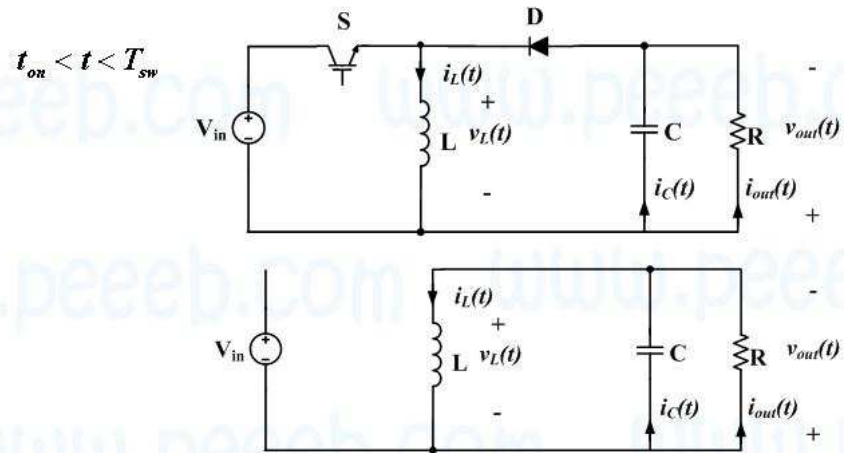
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$$v_L(t) = -v_{out}(t)$$

$$i_C(t) = i_L(t) - i_{out}(t)$$

$$\begin{cases} v_L(t) = -V_{out} \\ i_C(t) = i_L(t) - I_{out} \end{cases}$$



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Buck-Boost Converter

$$v_L(t) = \begin{cases} V_{in} & 0 < t < t_{on} \\ -V_{out} & t_{on} < t < T_{sw} \end{cases}$$

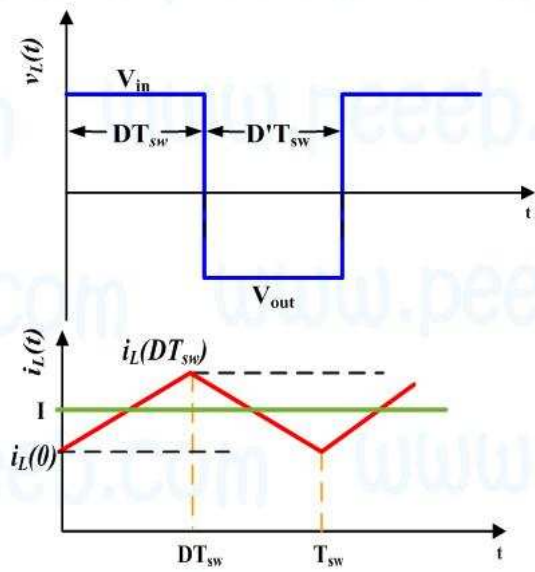
$$i_C(t) = \begin{cases} -I_{out} & 0 < t < t_{on} \\ i_L(t) - I_{out} & t_{on} < t < T_{sw} \end{cases}$$

$$0 < t < t_{on}$$

$$t_{on} < t < T_{sw}$$

$$0 < t < t_{on}$$

$$t_{on} < t < T_{sw}$$



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$$\overline{v_L(t)} = 0$$

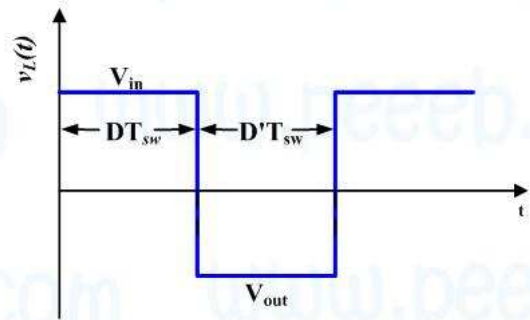
$$\int_0^{DT_{sw}} V_{in} dt + \int_{DT_{sw}}^{T_{sw}} V_{out} dt = 0$$

$$V_{in} DT_{sw} + (-V_{out})(T_{sw} - DT_{sw}) = 0$$

$$DV_{in} + (-V_{out})(1 - D) = 0$$

$$DV_{in} = (1 - D)V_{out}$$

$$H(D) = \frac{V_{out}}{V_{in}} = \frac{D}{(1 - D)} = \frac{D}{D'}$$



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Buck-Boost Converter

How to select L and C sizes?

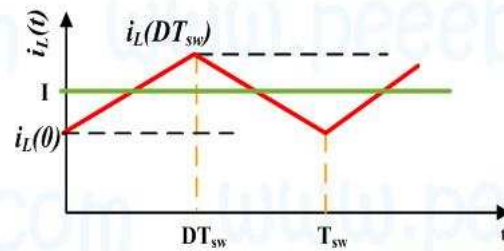
$$L \frac{di_L}{dt} = v_L(t)$$

$$\int_0^{DT_{sw}} di_L(t) = \frac{1}{L} \int_0^{DT_{sw}} V_{in} dt$$

$$i_L(DT_{sw}) - i_L(0) = \frac{V_{in} DT_{sw}}{L}$$

$$2\Delta i_L = \frac{V_{in} DT_{sw}}{L}$$

$$L = \frac{V_{in} D}{2 f_{sw} \Delta i_L}$$



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Buck-Boost Converter

How to select L and C sizes?

$$C \frac{dv_c(t)}{dt} = i_c(t)$$

$$\int_{\theta}^{DT_{sw}} dv_c(t) = \frac{1}{C} \int_{\theta}^{DT_{sw}} i_c(t) dt$$

$$-2\Delta v_c = \frac{(-I_{out})DT_{sw}}{C}$$

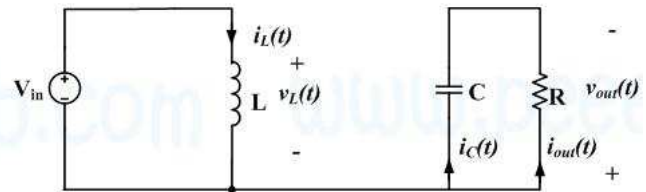
$$\Delta v_c = \frac{I_{out}DT_{sw}}{2C}$$

$$\Delta v_c = \frac{V_{out}D}{2RCf_{sw}} = \frac{V_{in}D^2}{2RCf_{sw}D'}$$

$$C = \frac{V_{in}D^2}{2Rf_{sw}D'\Delta v_c}$$

$$i_c(t) = -i_{out}(t) \quad \theta < t < t_{on}$$

$$i_c(t) = -I_{out}$$

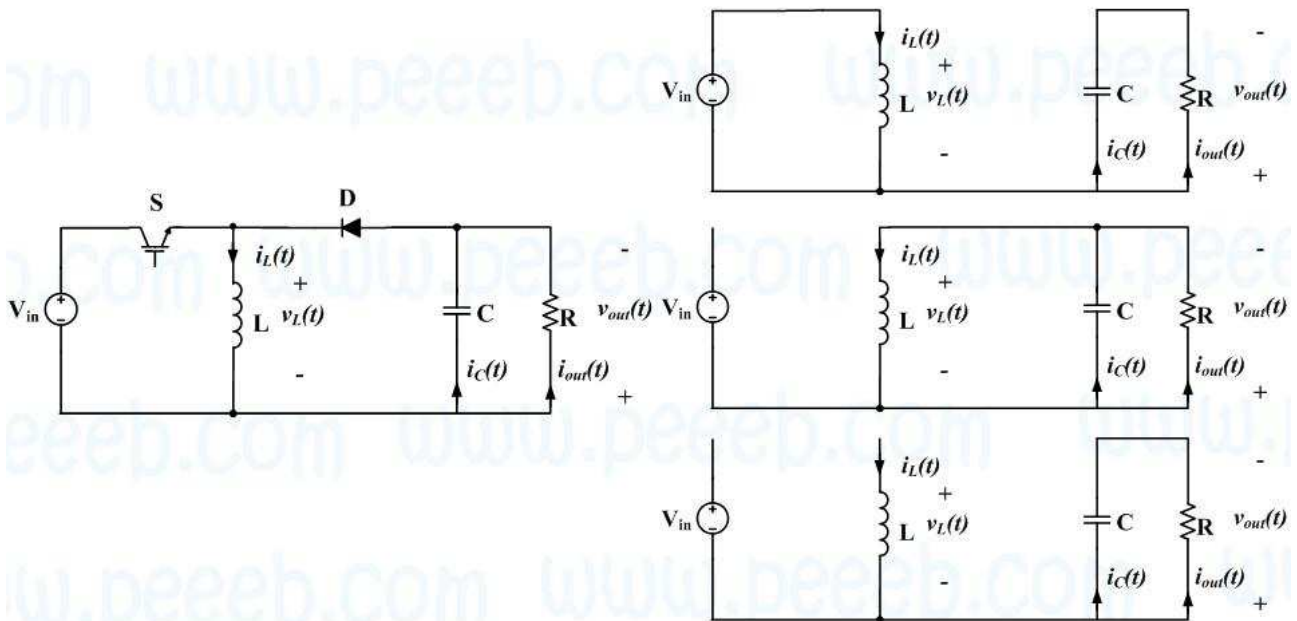


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Discontinuous Conduction Modes, DCM



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Discontinuous Conduction Modes, DCM

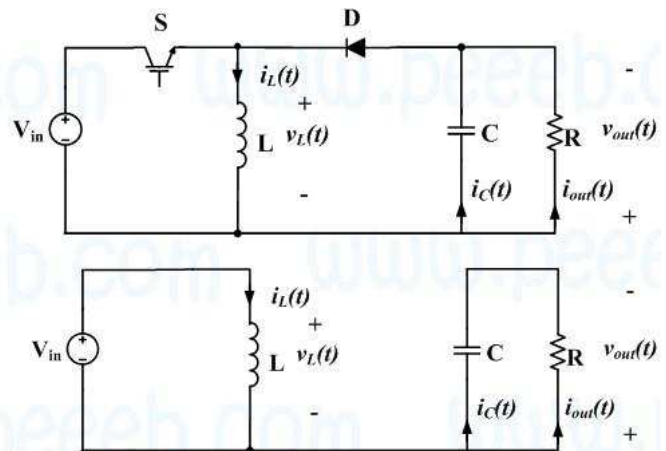
$$v_L(t) = V_{in}$$

$$i_C(t) = -i_{out}(t)$$

$$\begin{cases} v_L(t) = V_{in} \\ i_C(t) = -I_{out} \end{cases}$$

$$0 < t < t_{on}$$

$$0 < t < t_{on}$$



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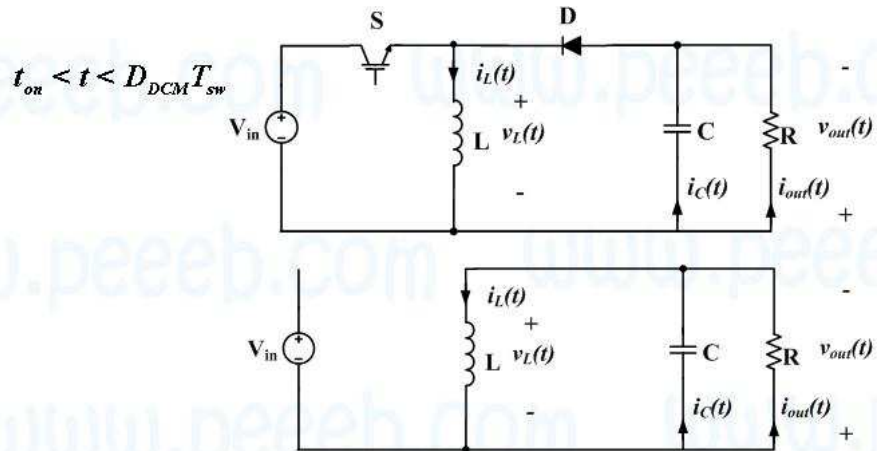
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Discontinuous Conduction Modes, DCM

$$v_L(t) = -v_{out}(t)$$

$$i_C(t) = i_L(t) - i_{out}(t)$$

$$\begin{cases} v_L(t) = -V_{out} \\ i_C(t) = i_L(t) - I_{out} \end{cases}$$



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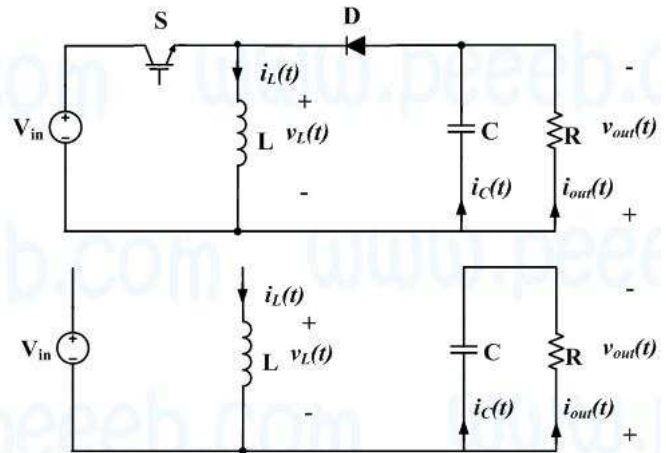
Discontinuous Conduction Modes, DCM

$$v_L(t) = 0$$

$$D_{DCM} T_{sw} < t < T_{sw}$$

$$i_C(t) = -i_{out}(t)$$

$$\begin{cases} v_L(t) = 0 \\ i_C(t) = -I_{out} \end{cases}$$



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Buck-Boost Converter

$$V_{in} I_{in} = V_{out} I_{out}$$

$$I_{in} = \overline{i_{in}(t)} = \frac{1}{T_{sw}} \int_{\theta}^{DT_{sw}} i_L(t) dt = \frac{DT_{sw} \times 2\Delta i_L}{2T_{sw}} = D\Delta i_L$$

$$V_{in} D\Delta i_L = V_{out} I_{out}$$

$$L \frac{di_L(t)}{dt} = v_L(t)$$

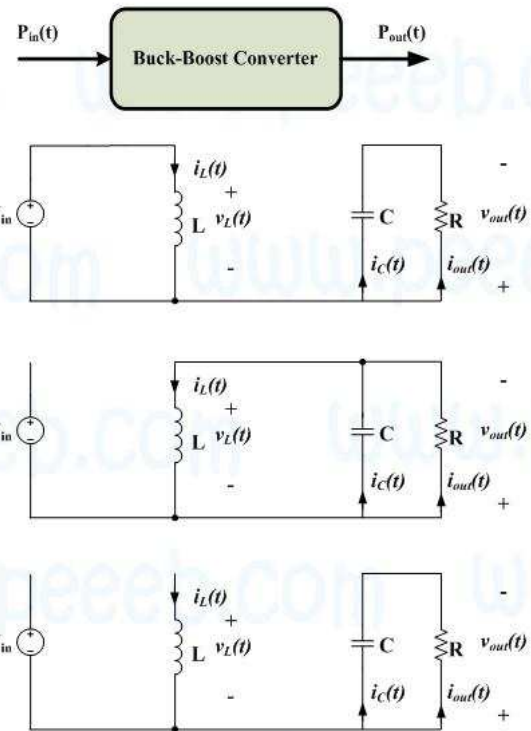
$$L \left(\frac{2\Delta i_L}{DT_{sw}} \right) = V_{in} \quad \Delta i_L = V_{in} \left(\frac{DT_{sw}}{2L} \right)$$

$$V_{in} D V_{in} \left(\frac{DT_{sw}}{2L} \right) = \frac{V_{out}^2}{R}$$

$$\left(\frac{V_{out}}{V_{in}} \right)^2 = D^2 \left(\frac{RT_{sw}}{2L} \right)$$

$$H(D) = \frac{V_{out}}{V_{in}} = D \sqrt{\frac{RT_{sw}}{2L}}$$

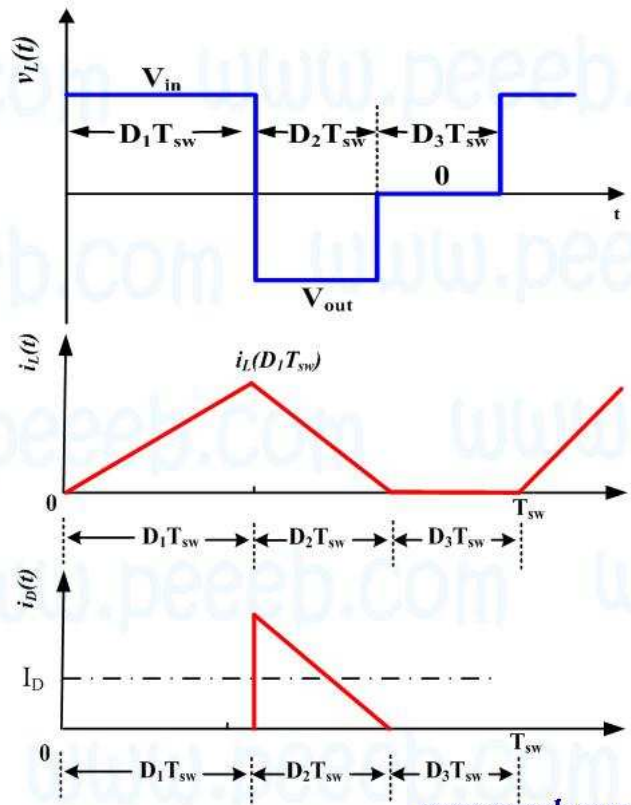
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Discontinuous Conduction Modes, DCM



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Assuming that the converter operates in CCM:

$$\Delta i_L > I_L \quad DCM$$

$$\Delta i_L < I_L \quad CCM$$

$$\Delta i_L < I_L \quad CCM$$

$$V_{in} \left(\frac{DT_{sw}}{2L} \right) < \frac{V_{out}}{RD'}$$

$$I_{out} = \frac{1}{T_{sw}} \int_{DT_{sw}}^{T_{sw}} i_L(t) dt$$

$$I_{out} = \frac{(T_{sw} - DT_{sw}) I_L}{T_{sw}}$$

$$I_{out} = (1 - D) I_L$$

$$V_{in} \left(\frac{DT_{sw}}{2L} \right) < \frac{V_{in} D}{D'} \frac{1}{RD'}$$

$$\frac{T_{sw}}{2L} < \frac{1}{R(D')^2}$$

$$I_L = \frac{I_{out}}{D'} = \frac{V_{out}}{RD'}$$

$$(D')^2 < \frac{2L}{RT_{sw}}$$

$$\Delta i_L = V_{in} \left(\frac{DT_{sw}}{2L} \right)$$

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