



Welcome to PEEEB



Lecture 4: Controlled Rectifiers

Presenter: Dr. Firuz Zare

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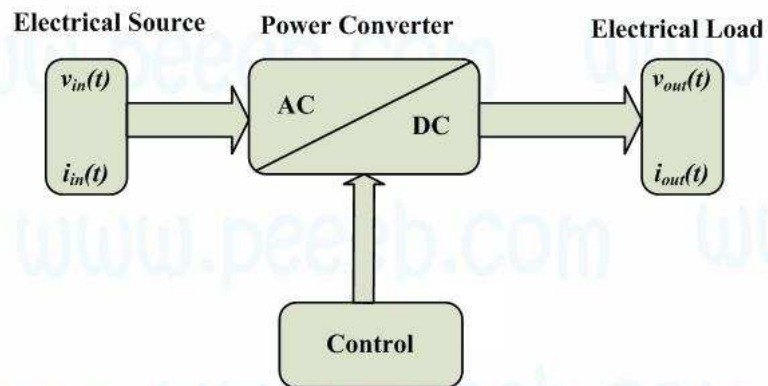
Contents:

- 1: Control circuit**
- 2: Single phase half wave controlled rectifier**
- 3: Single phase full wave controlled rectifier**
- 4: Three phase half wave controlled rectifier**
- 5: Three phase full wave controlled rectifier**
- 6: Single phase full wave half controlled rectifier**
- 7: Three phase full wave half controlled rectifier**

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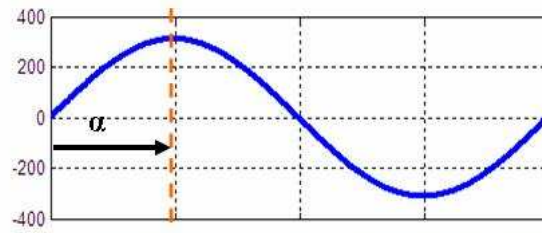
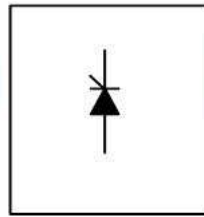
Block Diagram of a Controlled Rectifier



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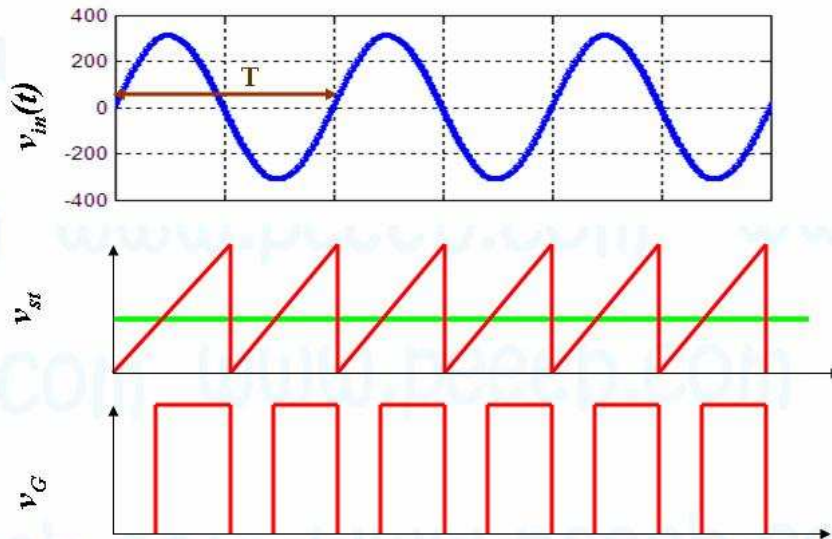
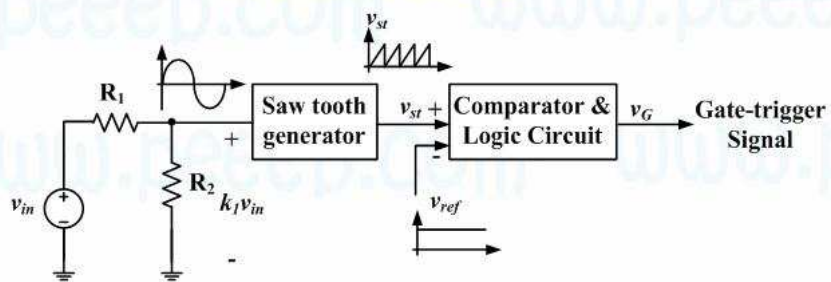
Control Circuit



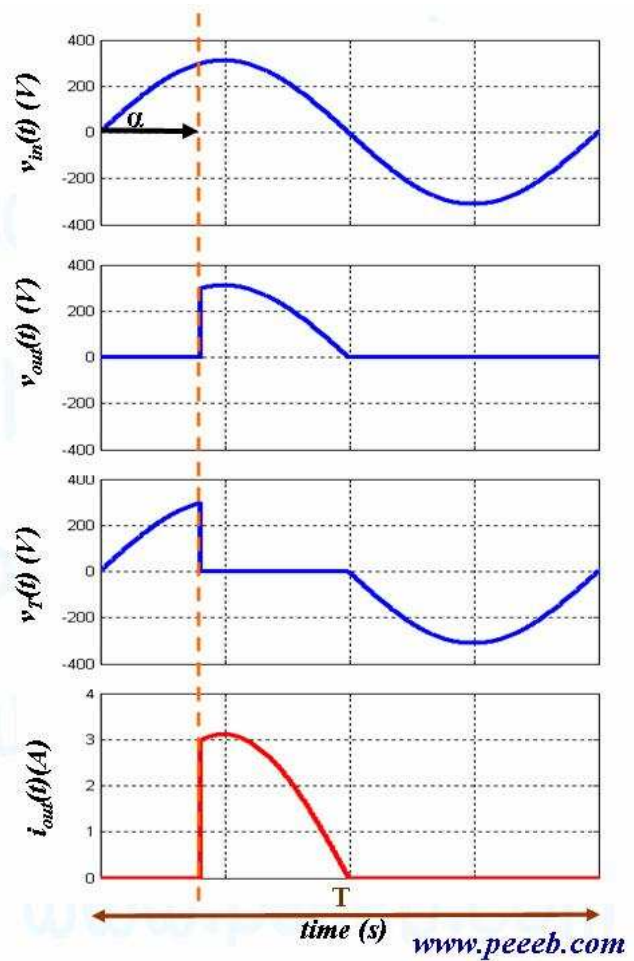
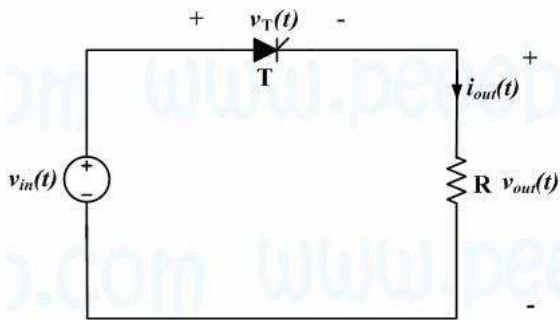
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Control Circuit



Single phase half wave rectifier (Resistive load)



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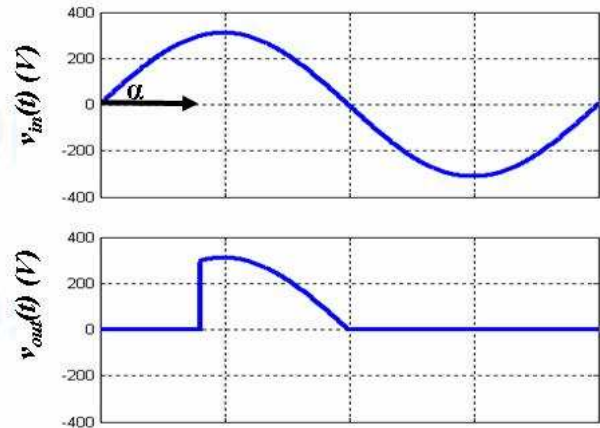
Single phase half wave rectifier (Resistive load)

$$v_{out}(t) = \begin{cases} 0 & 0 < t \leq t_1 \\ V_m \sin\left(\frac{2\pi t}{T}\right) & t_1 < t \leq \frac{T}{2} \\ 0 & \frac{T}{2} < t \leq T \end{cases}$$

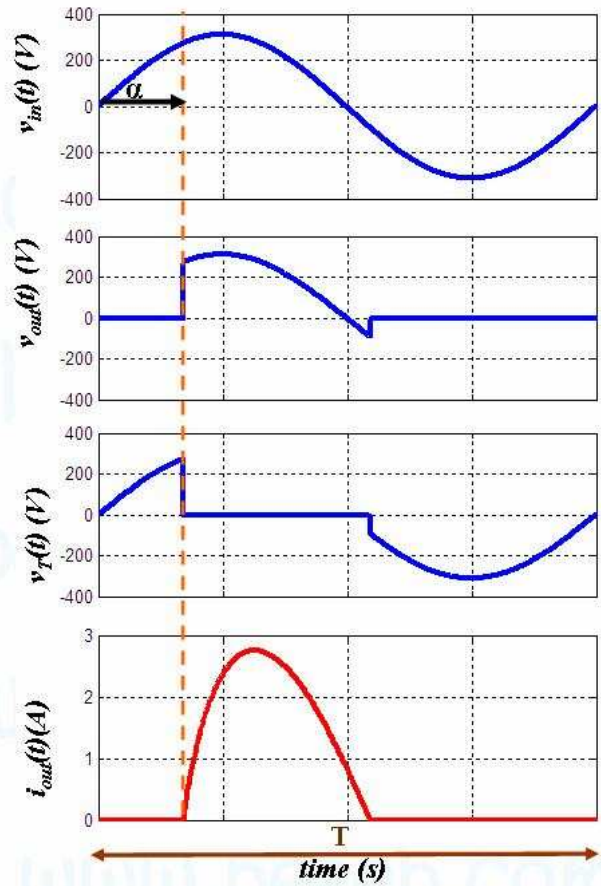
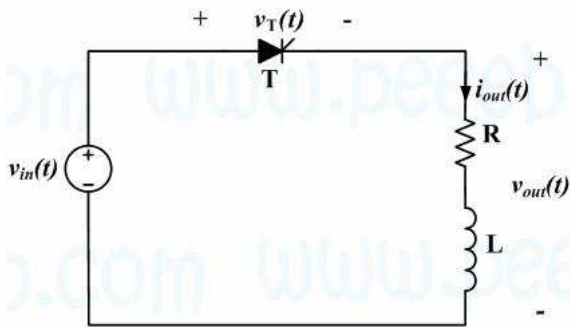
$$\begin{aligned} V_{out} &= \frac{1}{T} \int_0^T v_{out}(t) dt \\ &= \frac{1}{T} \int_{t_1}^{\frac{T}{2}} V_m \sin\left(\frac{2\pi t}{T}\right) dt \\ &= \frac{V_m}{T} \left(\frac{T}{2\pi} \right) \left[-\cos\left(\frac{2\pi t}{T}\right) \right]_{t_1}^{\frac{T}{2}} \\ &= \frac{V_m}{2\pi} \left[-\cos(\pi) + \cos\left(\frac{2\pi t_1}{T}\right) \right] \\ &= \frac{V_m}{2\pi} \left[1 + \cos\left(\frac{2\pi t_1}{T}\right) \right] \\ &= \frac{V_m}{2\pi} [1 + \cos(\alpha)] \end{aligned}$$

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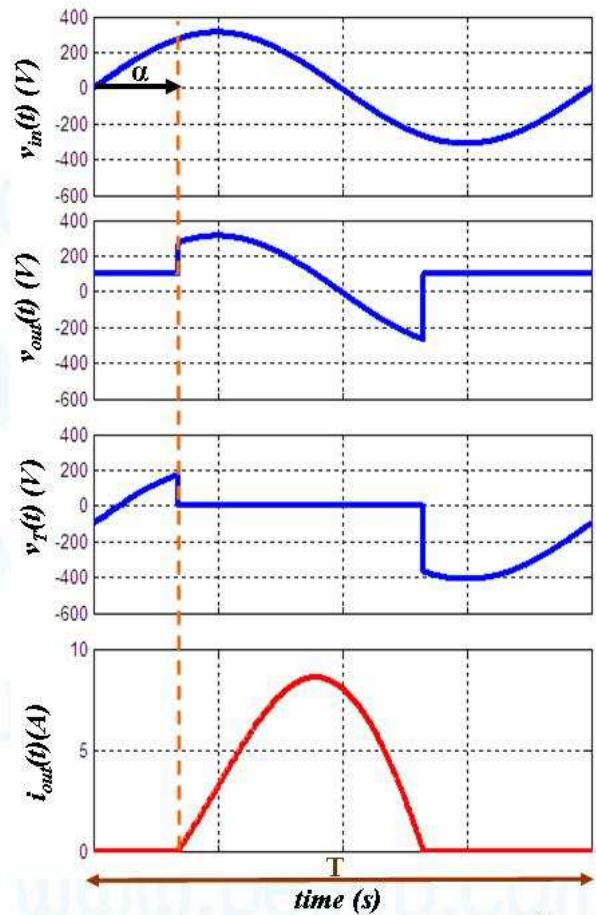
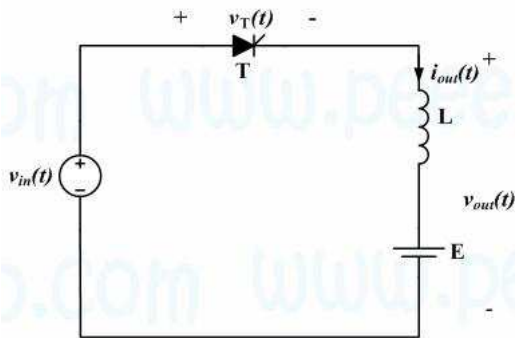
Single phase half wave rectifier (R-L load)



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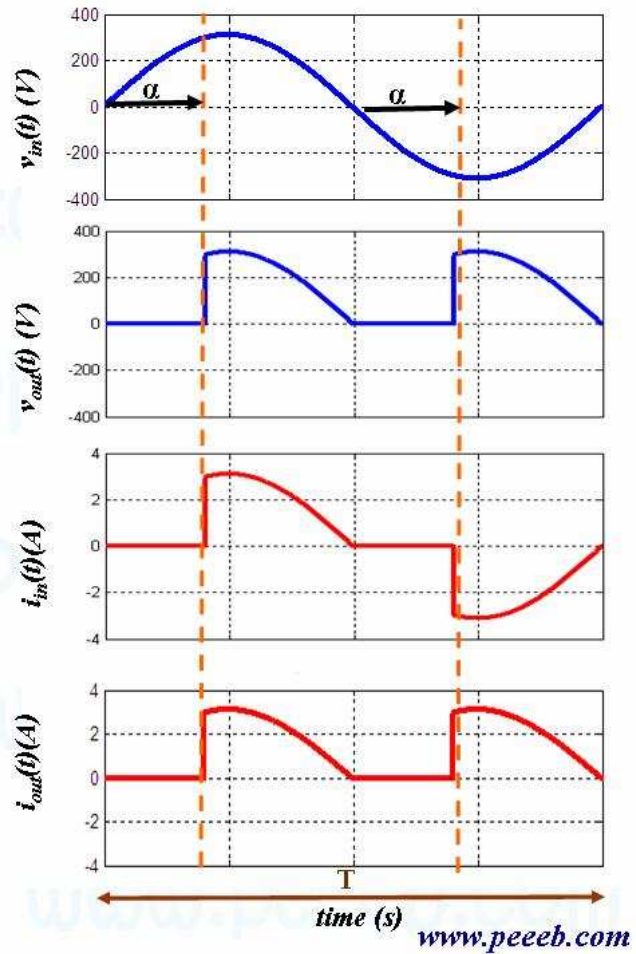
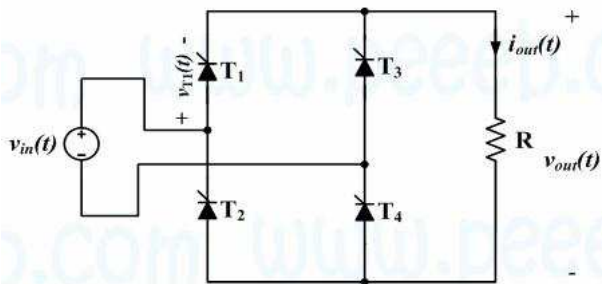
Single phase half wave rectifier (L-E load)



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Single phase full wave bridge rectifier (Resistive load)

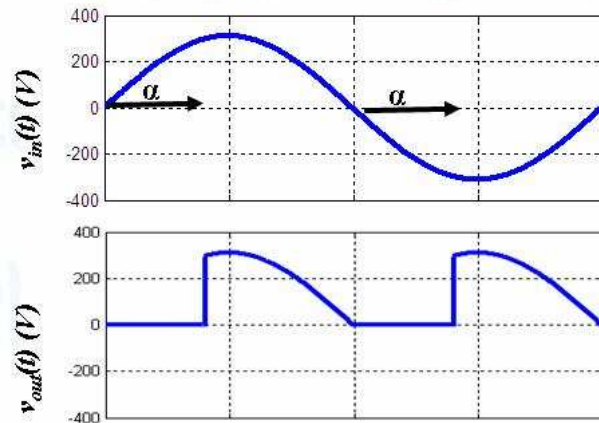


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Single phase full wave bridge rectifier (Resistive load)

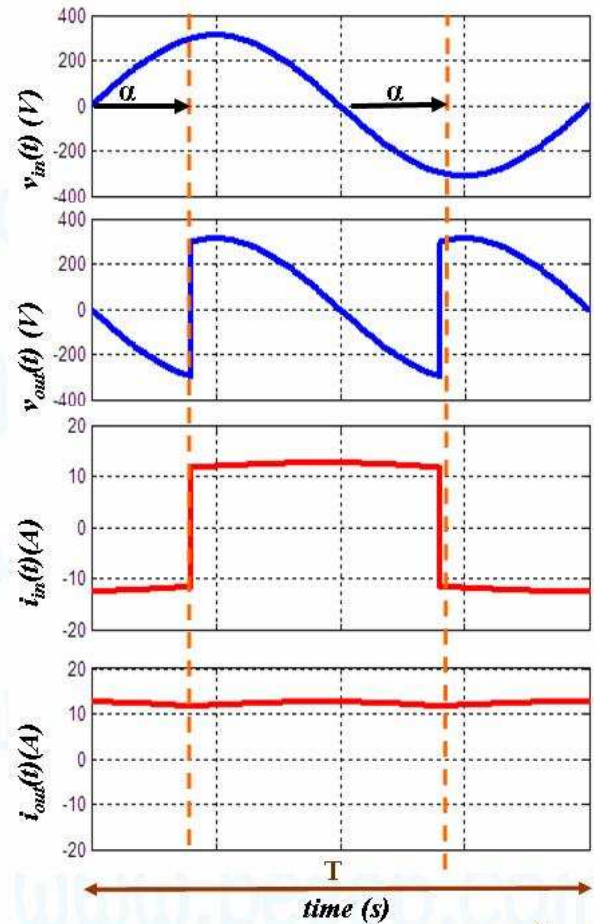
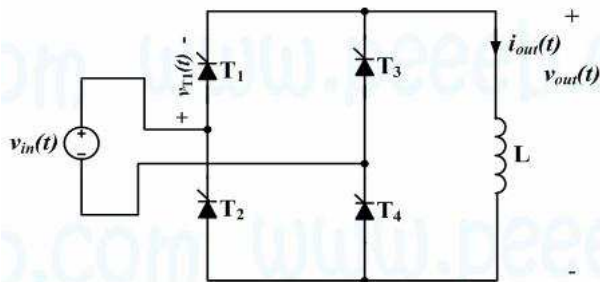
$$\begin{aligned}
 V_{out} &= \frac{1}{T} \int_0^T v_{out}(t) dt \\
 V_{out} &= 2 \left(\frac{1}{T} \int_{t_1}^T V_m \sin\left(\frac{2\pi t}{T}\right) dt \right) \\
 &= 2 \left[\frac{V_m}{2\pi} \left(1 + \cos\left(\frac{2\pi t_1}{T}\right) \right) \right] \\
 &= \frac{V_m}{\pi} \left[1 + \cos\left(\frac{2\pi t_1}{T}\right) \right] \\
 &= \frac{V_m}{\pi} [1 + \cos(\alpha)]
 \end{aligned}$$



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Single phase full wave bridge rectifier (Inductive load)



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Single phase full wave bridge rectifier (Inductive load)

$$v_{out}(t) = V_m \sin\left(\frac{2\pi t}{T}\right) \quad t_1 < t \leq \frac{T}{2} + t_1$$

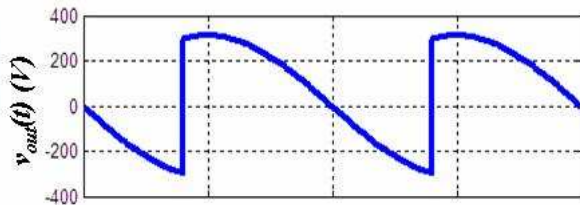
$$V_{out} = \frac{1}{T} \int_{t_1}^{\frac{T}{2} + t_1} V_m \sin\left(\frac{2\pi t}{T}\right) dt$$

$$= \frac{2V_m}{T} \left(\frac{T}{2\pi} \right) \left[-\cos\left(\frac{2\pi t}{T}\right) \right]_{t_1}^{\frac{T}{2} + t_1}$$

$$= \frac{V_m}{\pi} \left[-\cos\left(\pi + \frac{2\pi t_1}{T}\right) + \cos\left(\frac{2\pi t_1}{T}\right) \right]$$

$$\begin{aligned} 2\pi &\rightarrow T \\ \alpha &\rightarrow t_1 \end{aligned} \quad \frac{\alpha T}{2\pi} = t_1$$

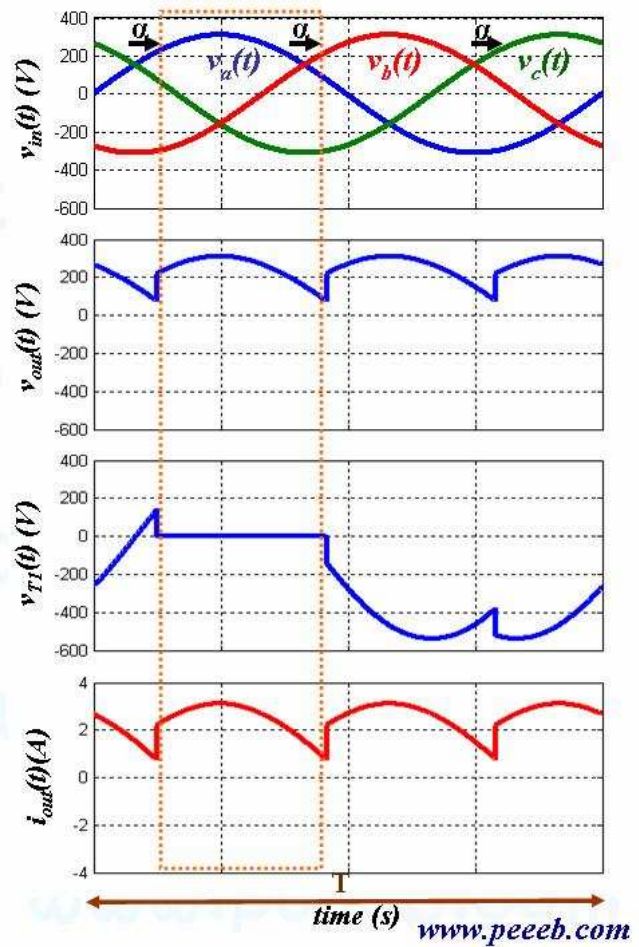
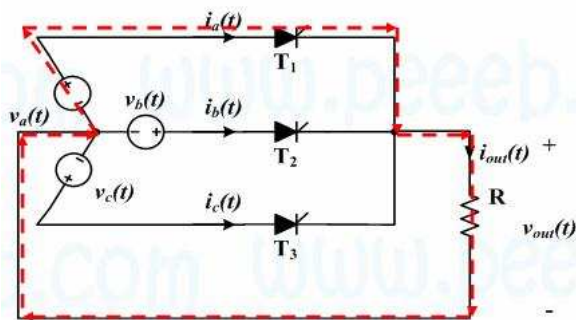
$$= \frac{V_m}{\pi} \left[2 \cos\left(\frac{2\pi t_1}{T}\right) \right] = \frac{2V_m}{\pi} \cos\left(\frac{2\pi t_1}{T}\right) = \frac{2V_m}{\pi} \cos(\alpha)$$



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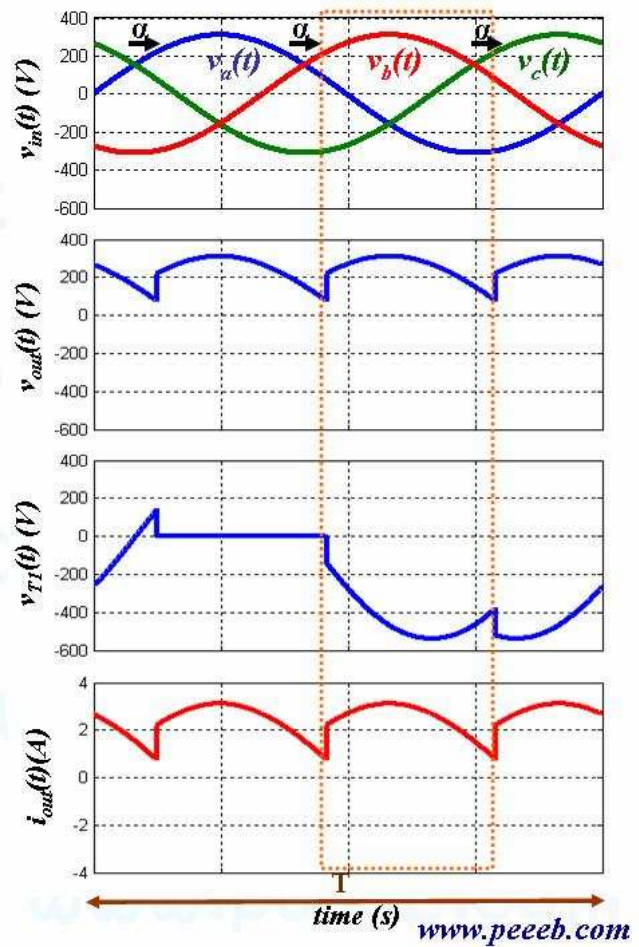
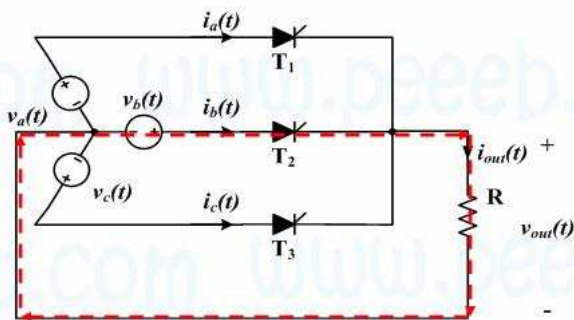
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Three phase half wave rectifier (Resistive load and firing angle of 15°)



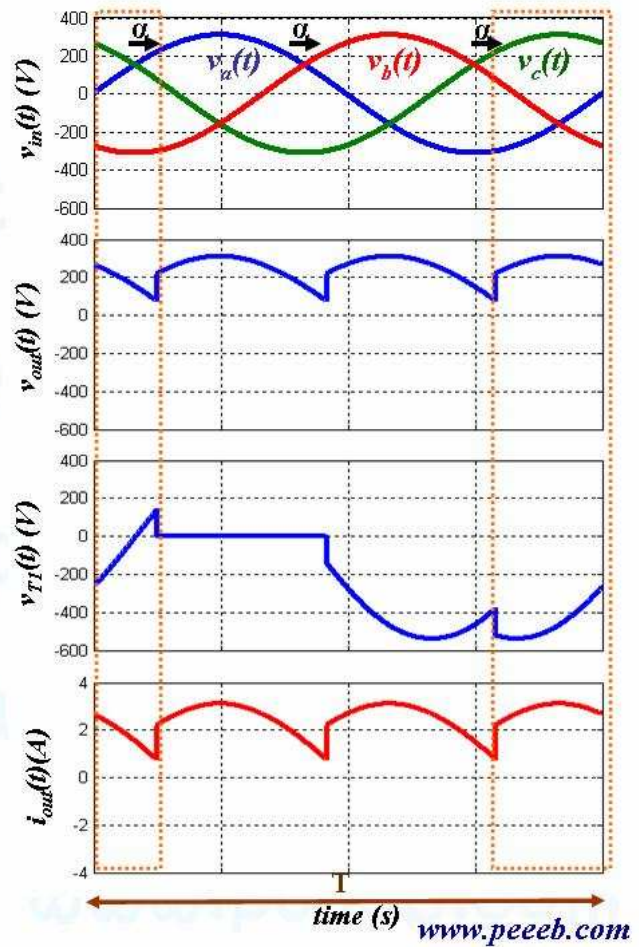
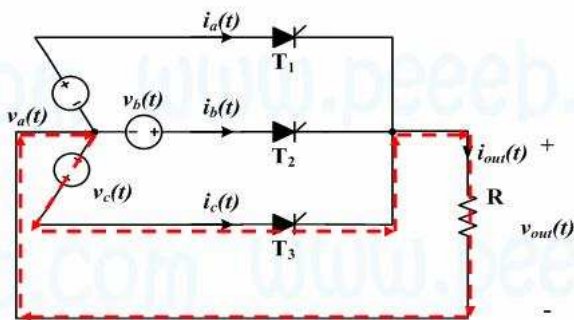
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Three phase half wave rectifier (Resistive load and firing angle of 15°)



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Three phase half wave rectifier (Resistive load and firing angle of 15°)



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Three phase half wave rectifier

$$v_{out}(t) = V_m \cos\left(\frac{2\pi t}{T}\right) \quad -\frac{T}{6} + \Delta t < t \leq \frac{T}{6} + \Delta t$$

$$V_{out} = \frac{1}{T} \int_{-\frac{T}{6} + \Delta t}^{\frac{T}{6} + \Delta t} V_m \cos\left(\frac{2\pi t}{T}\right) dt$$

$$= \frac{3V_m}{T} \left(\frac{T}{2\pi}\right) \left[\sin\left(\frac{2\pi t}{T}\right) \right]_{-\frac{T}{6} + \Delta t}^{\frac{T}{6} + \Delta t}$$

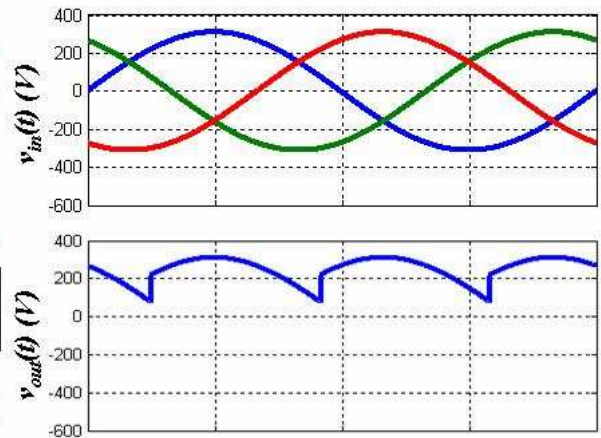
$$= \frac{3V_m}{2\pi} \left[\sin\left(\frac{2\pi}{T} \left(\frac{T}{6} + \Delta t\right)\right) - \sin\left(\frac{2\pi}{T} \left(-\frac{T}{6} + \Delta t\right)\right) \right]$$

$$= \frac{3V_m}{2\pi} \left[2 \sin\left(\frac{2\pi}{T} \left(\frac{T}{6}\right)\right) \right] \cos\left(\frac{2\pi}{T} (\Delta t)\right)$$

$$= \frac{3V_m}{\pi} \sin\left(\frac{\pi}{3}\right) \cos\left(\frac{2\pi \Delta t}{T}\right)$$

$$\frac{\Delta t}{T} = \frac{\alpha}{2\pi} \Rightarrow \Delta t = \frac{T\alpha}{2\pi}$$

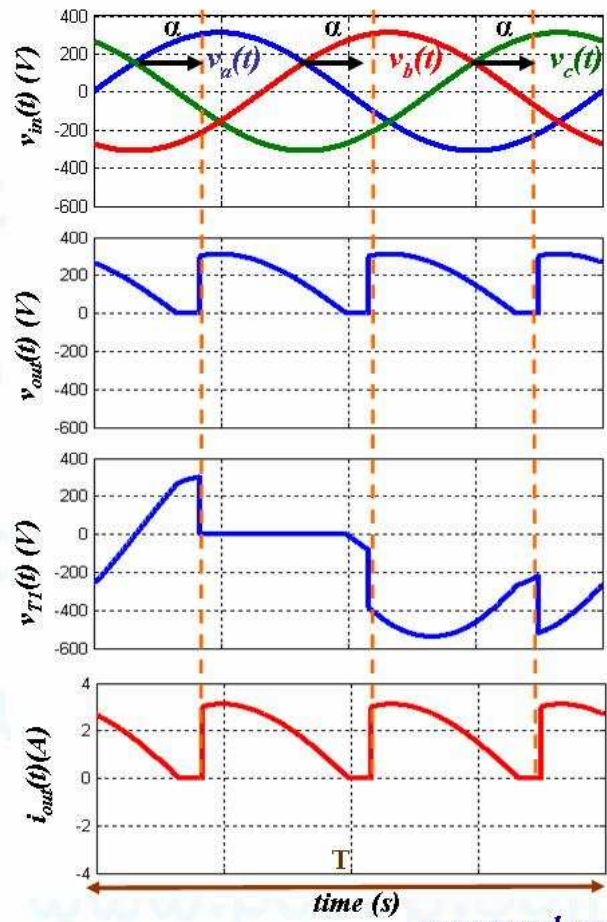
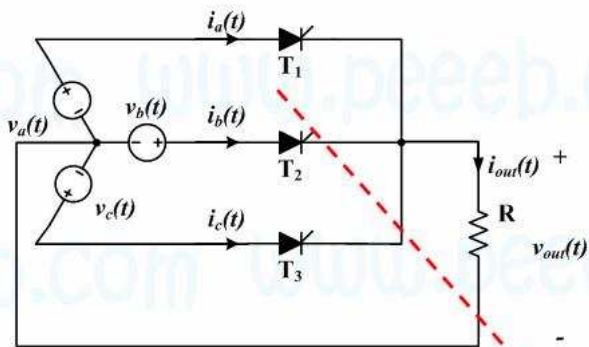
$$= \frac{3\sqrt{3}V_m}{2\pi} \cos(\alpha)$$



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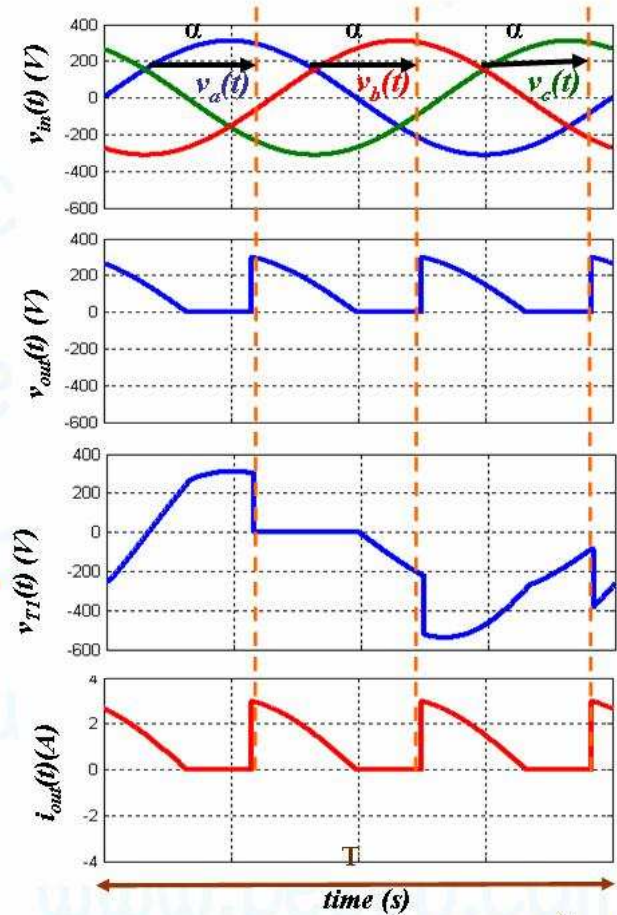
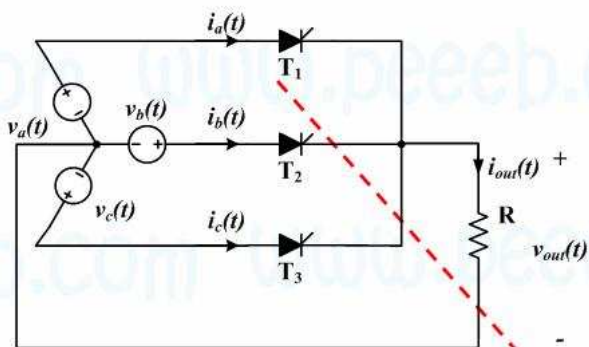
Three phase half wave rectifier (Resistive load and firing angle of 45°)



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Three phase half wave rectifier (Resistive load and firing angle of 75°)



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Three phase half wave rectifier

$$V_{out} = \frac{1}{T} \int_{-\frac{T}{6} + \Delta t}^{\frac{T}{6} + \Delta t} v_{out}(t) dt$$

$$= \frac{3}{T} \int_{-\frac{T}{6} + \Delta t}^{\frac{T}{6} + \Delta t} V_m \cos\left(\frac{2\pi t}{T}\right) dt$$

$$= \frac{3V_m}{T} \left(\frac{T}{2\pi}\right) \left[\sin\left(\frac{2\pi t}{T}\right) \right]_{-\frac{T}{6} + \Delta t}^{\frac{T}{6} + \Delta t}$$

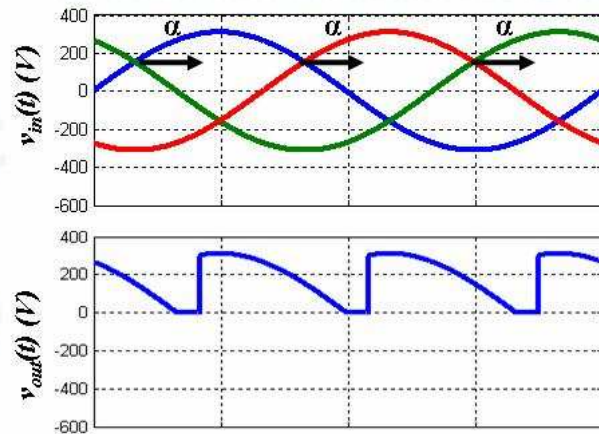
$$= \frac{3V_m}{2\pi} \left[\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{2\pi}{T} \left(-\frac{T}{6} + \Delta t\right)\right) \right]$$

$$= \frac{3V_m}{2\pi} \left[1 - \sin\left(-\frac{\pi}{3} + \frac{2\pi\Delta t}{T}\right) \right]$$

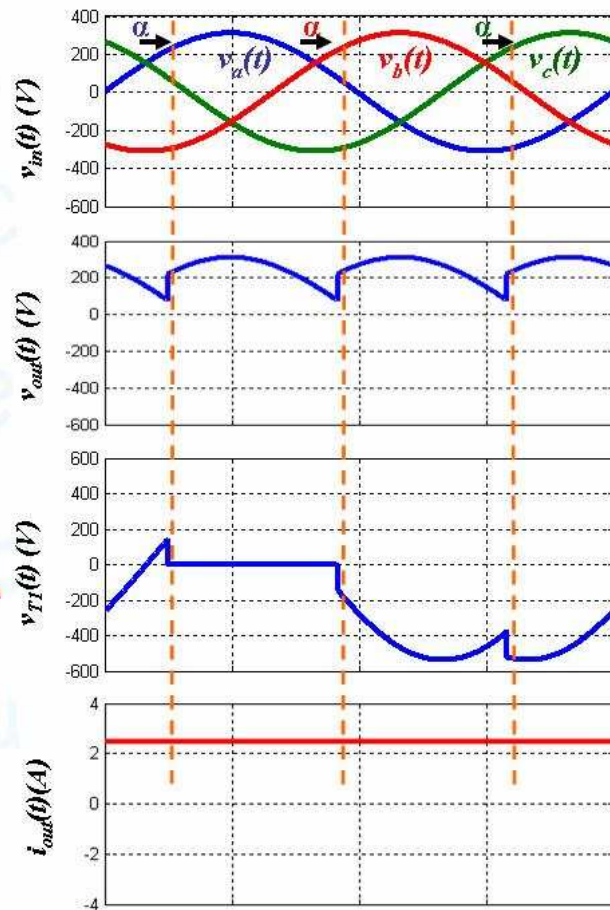
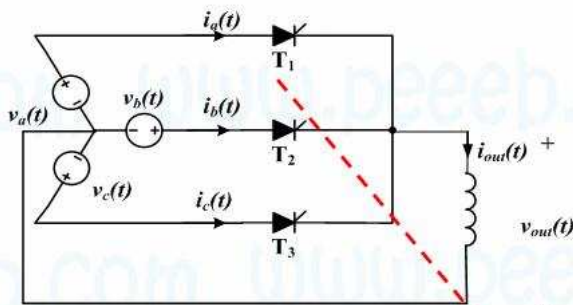
$$= \frac{3V_m}{2\pi} \left[1 + \sin\left(\frac{\pi}{3} - \frac{2\pi\Delta t}{T}\right) \right] = \frac{3V_m}{2\pi} \left[1 + \sin\left(\frac{\pi}{3} - \alpha\right) \right]$$

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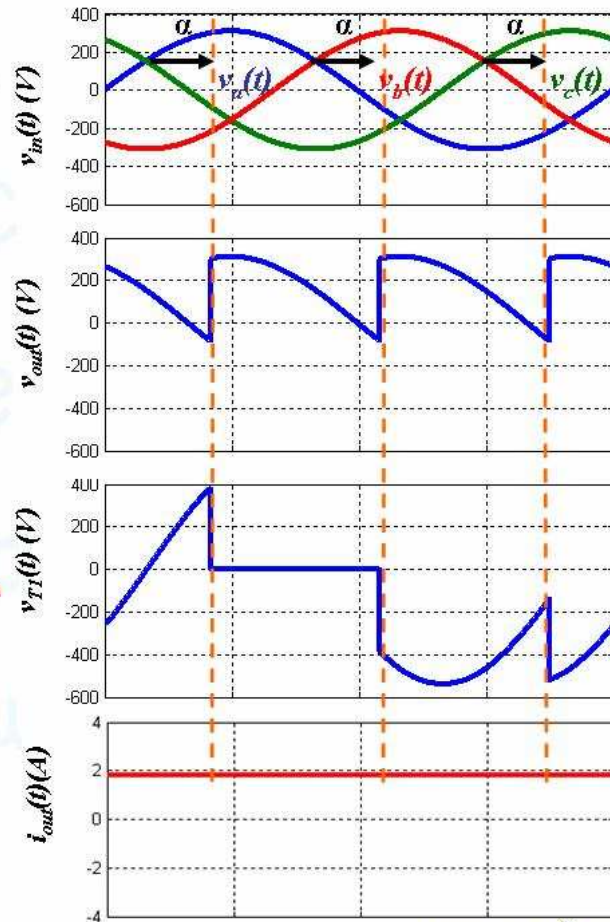
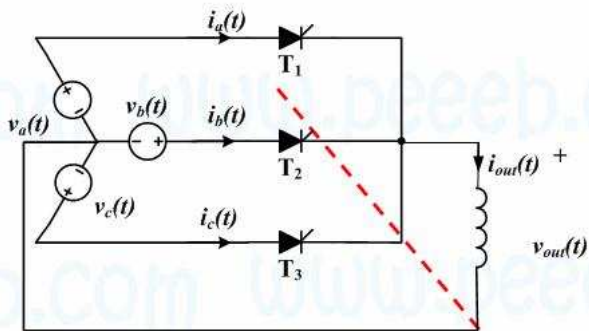
Three phase half wave rectifier (Inductive load and firing angle of 15°)



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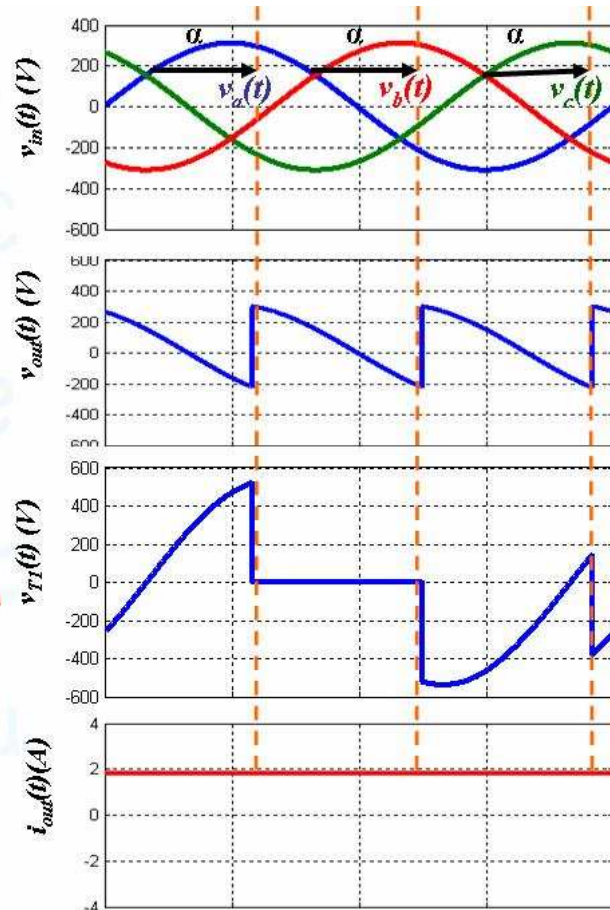
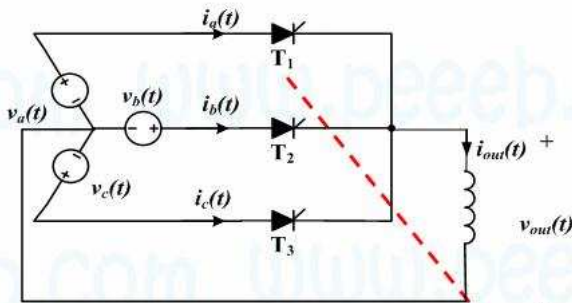
Three phase half wave rectifier (Inductive load and firing angle of 45°)



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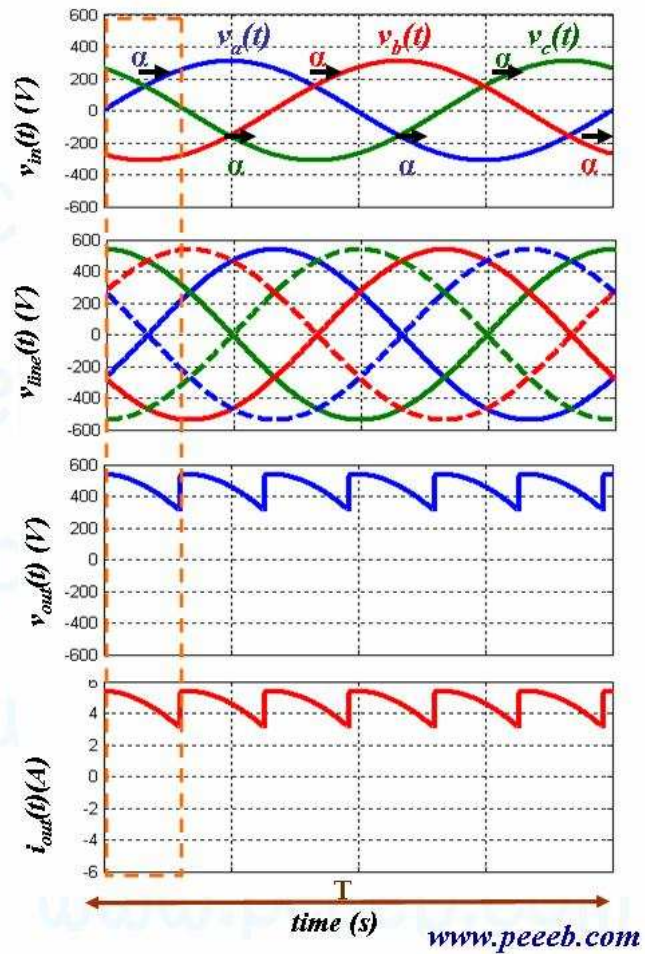
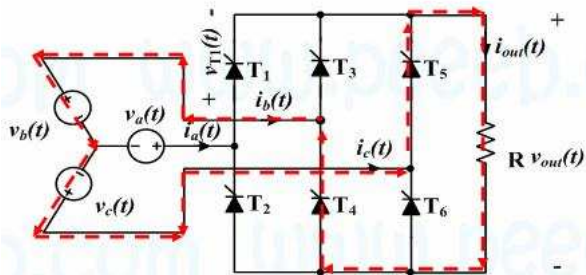
Three phase half wave rectifier (Inductive load and firing angle of 75°)



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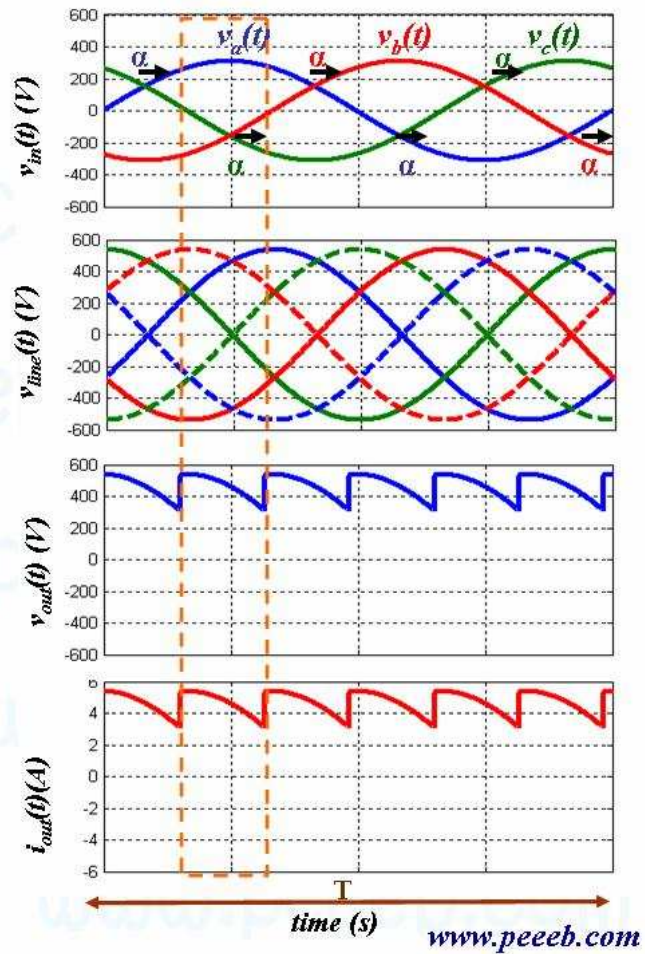
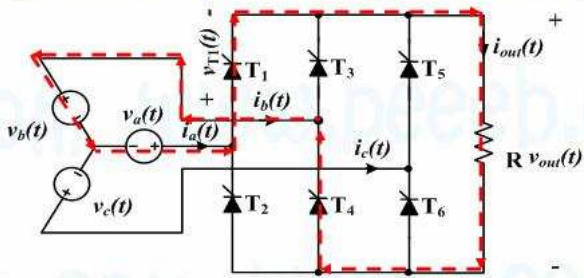
Three phase full wave rectifier (Resistive load and firing angle of 15°)



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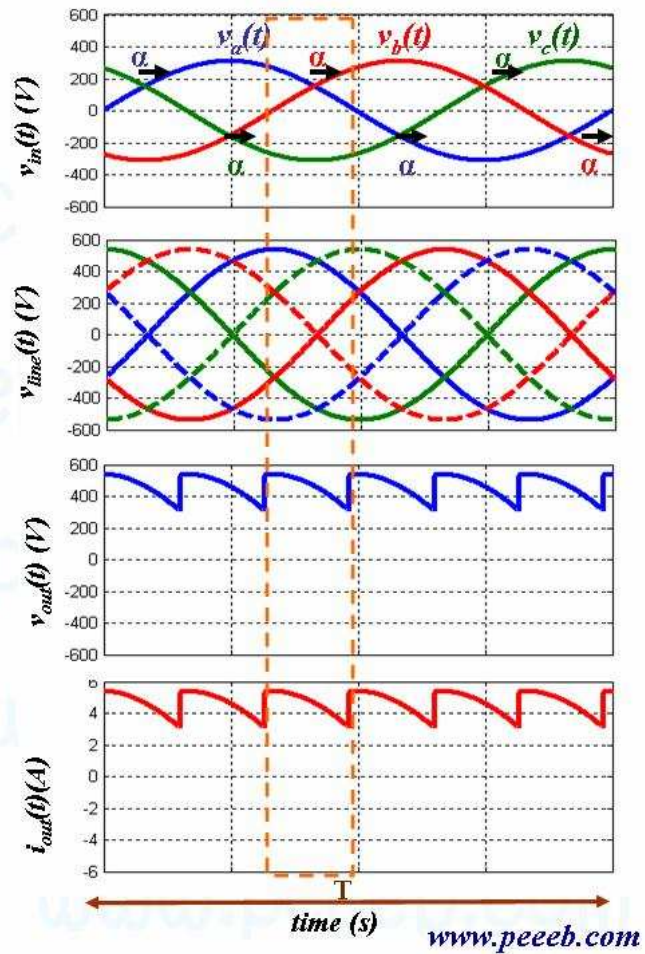
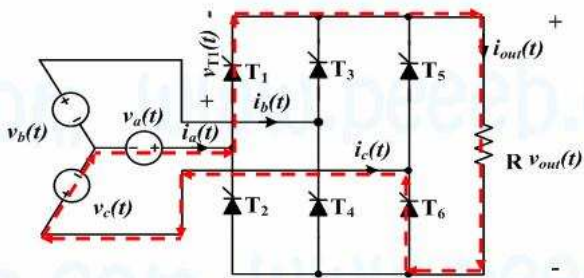
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Three phase full wave rectifier (Resistive load and firing angle of 15°)



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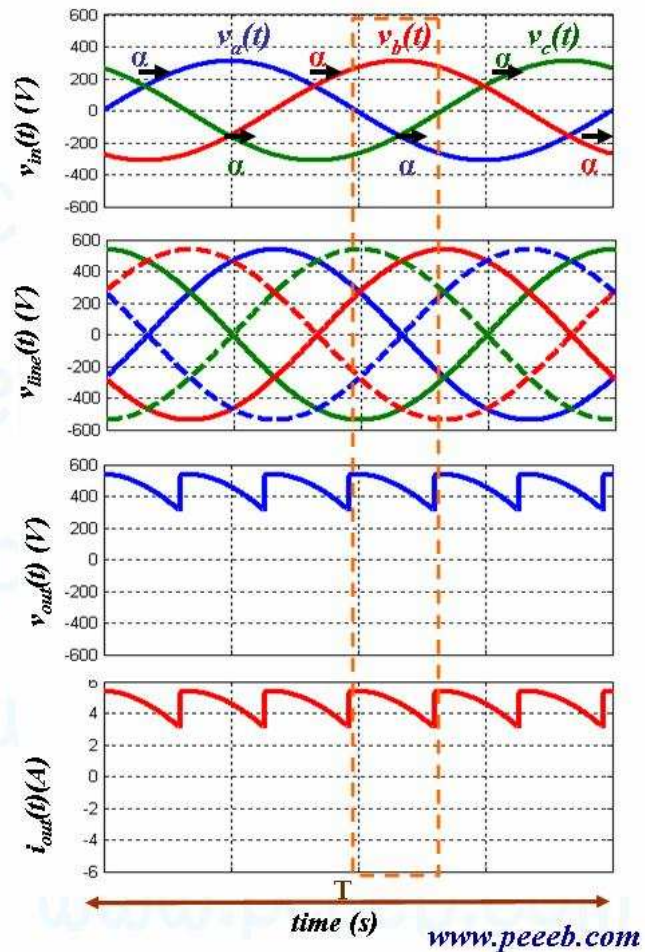
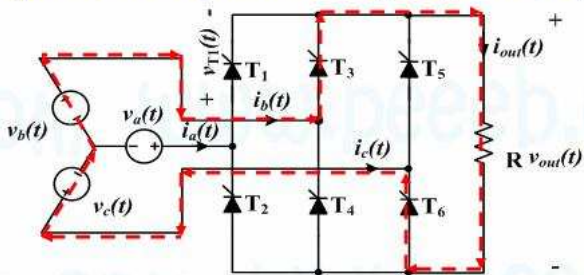
Three phase full wave rectifier (Resistive load and firing angle of 15°)



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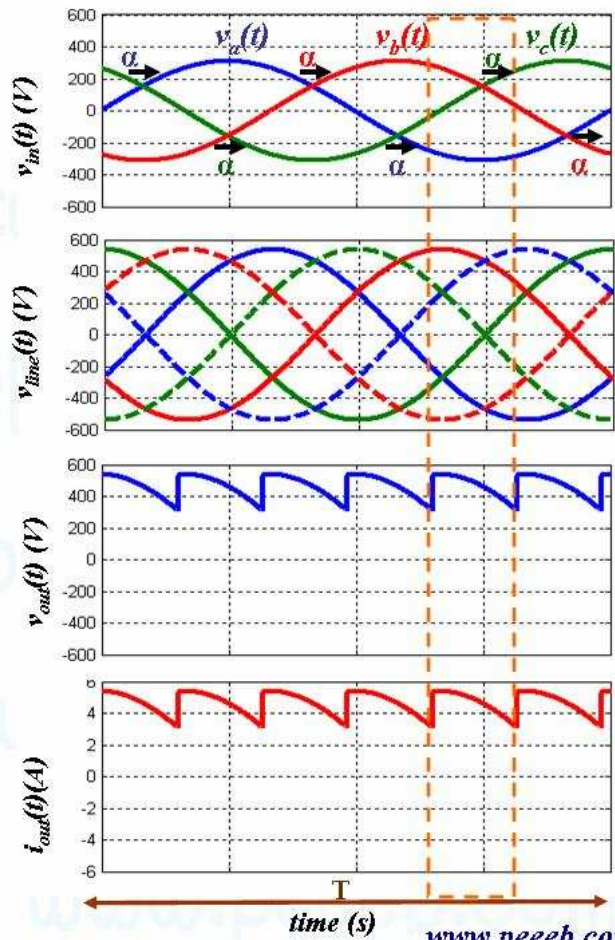
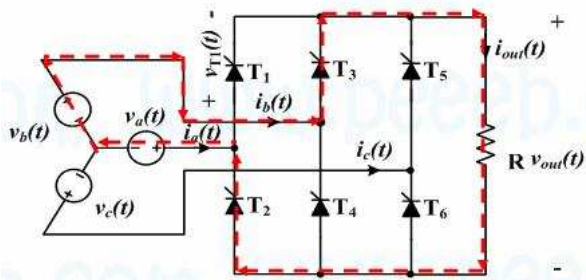
Three phase full wave rectifier (Resistive load and firing angle of 15°)



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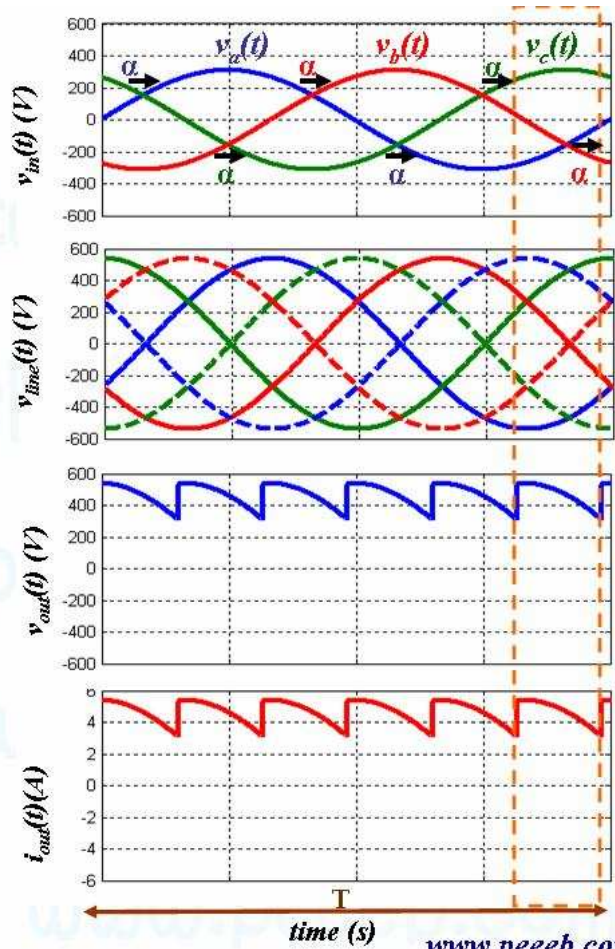
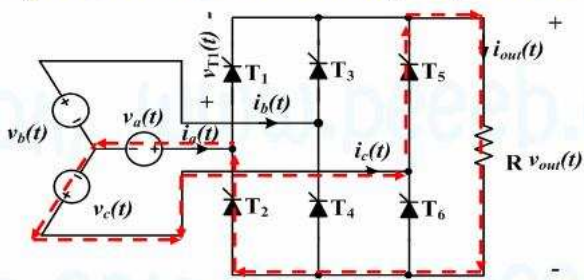
Three phase full wave rectifier (Resistive load and firing angle of 15°)



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Three phase full wave rectifier (Resistive load and firing angle of 15°)

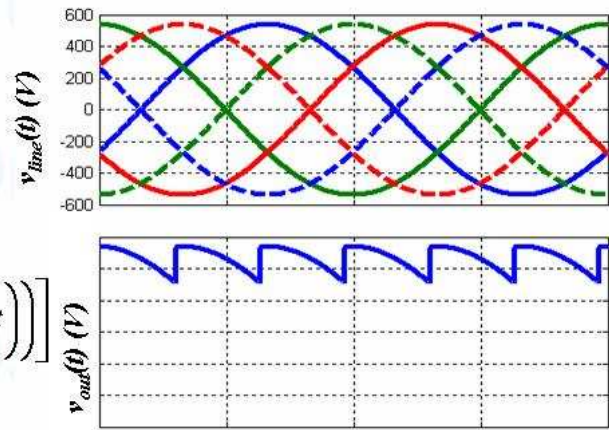


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Three phase full wave rectifier

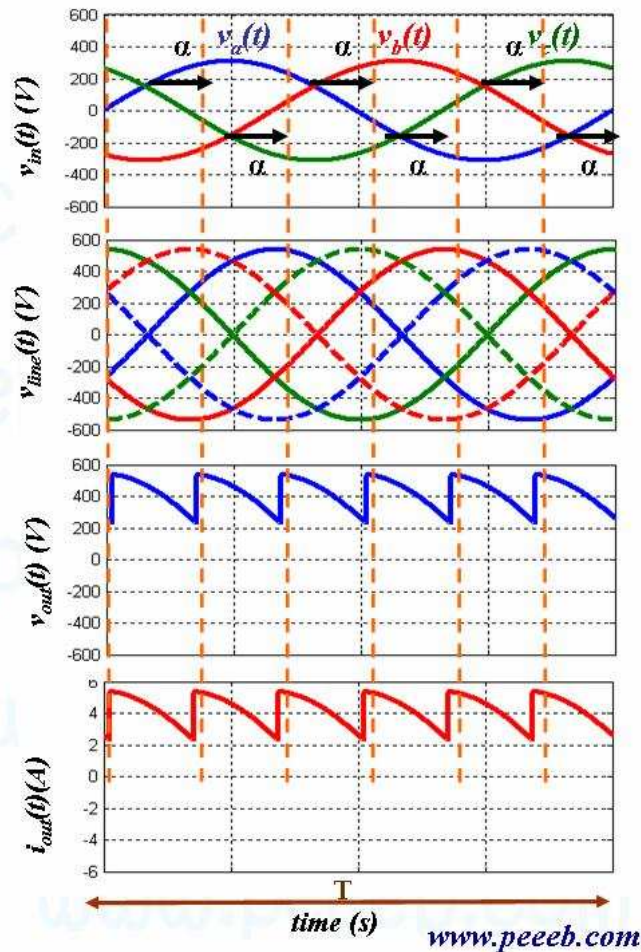
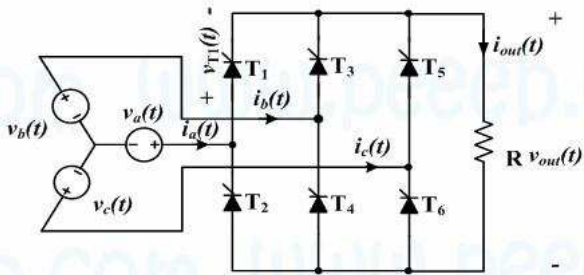
$$\begin{aligned}
 V_{out} &= \frac{1}{T} \int_{-\frac{T}{12} + \Delta t}^{\frac{T}{12} + \Delta t} \sqrt{3} V_m \cos\left(\frac{2\pi t}{T}\right) dt \\
 &= \frac{6}{T} \sqrt{3} V_m \left(\frac{T}{2\pi} \right) \left[\sin\left(\frac{2\pi t}{T}\right) \right]_{-\frac{T}{12} + \Delta t}^{\frac{T}{12} + \Delta t} \\
 &= \frac{3\sqrt{3}V_m}{\pi} \left[\sin\left(\frac{2\pi}{T} \left(\frac{T}{12} + \Delta t \right)\right) - \sin\left(\frac{2\pi}{T} \left(-\frac{T}{12} + \Delta t \right)\right) \right] \\
 &= \frac{3\sqrt{3}V_m}{\pi} \left[\sin\left(\frac{\pi}{6} + \frac{2\pi\Delta t}{T}\right) - \sin\left(-\frac{\pi}{6} + \frac{2\pi\Delta t}{T}\right) \right] \\
 &= \frac{3\sqrt{3}V_m}{\pi} \left[2 \sin\left(\frac{\pi}{6}\right) \cos\left(\frac{2\pi\Delta t}{T}\right) \right] \\
 &= \frac{3\sqrt{3}V_m}{\pi} \left[2 \times \frac{1}{2} \times \cos\left(\frac{2\pi\Delta t}{T}\right) \right] = \frac{3\sqrt{3}V_m}{\pi} \cos\left(\frac{2\pi\Delta t}{T}\right) \\
 &= \frac{3\sqrt{3}V_m}{\pi} \cos(\alpha)
 \end{aligned}$$



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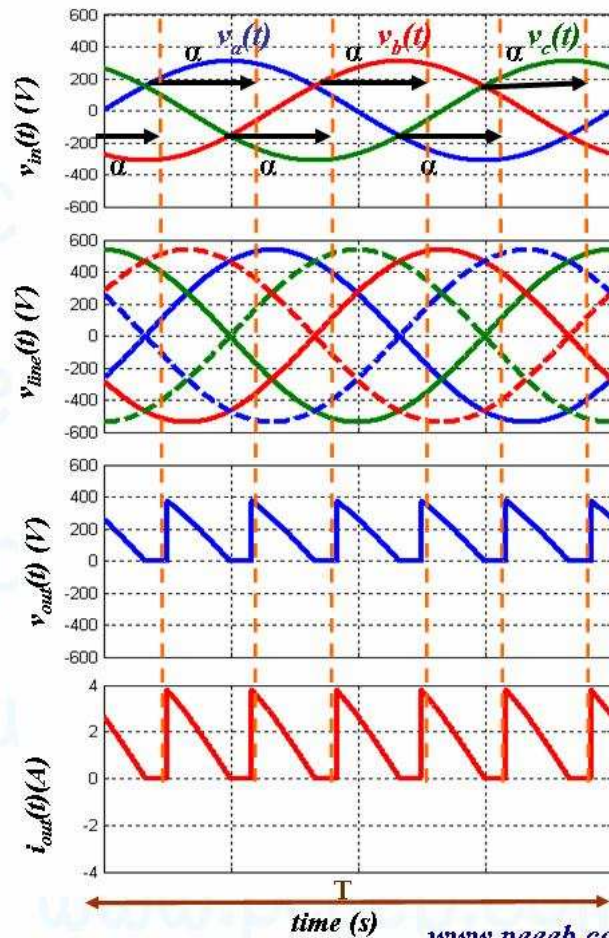
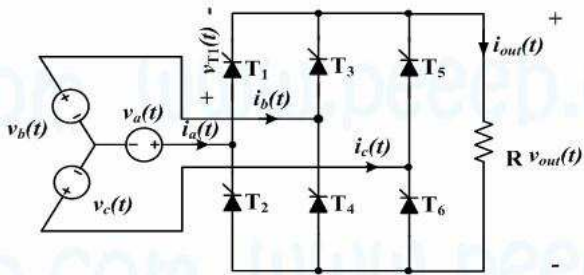
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Three phase full wave rectifier (Resistive load and firing angle of 45°)



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Three phase full wave rectifier (Resistive load and firing angle of 75°)

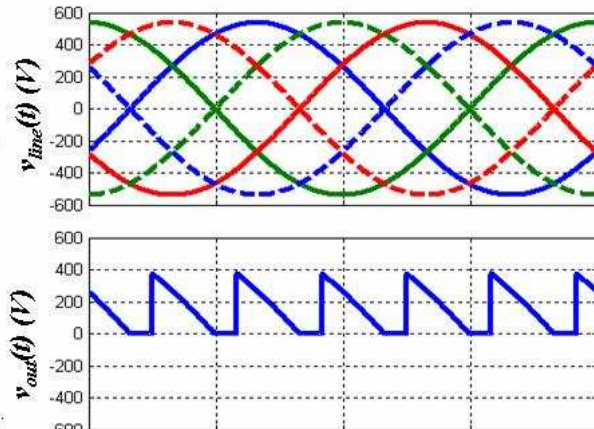


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Three phase full wave rectifier

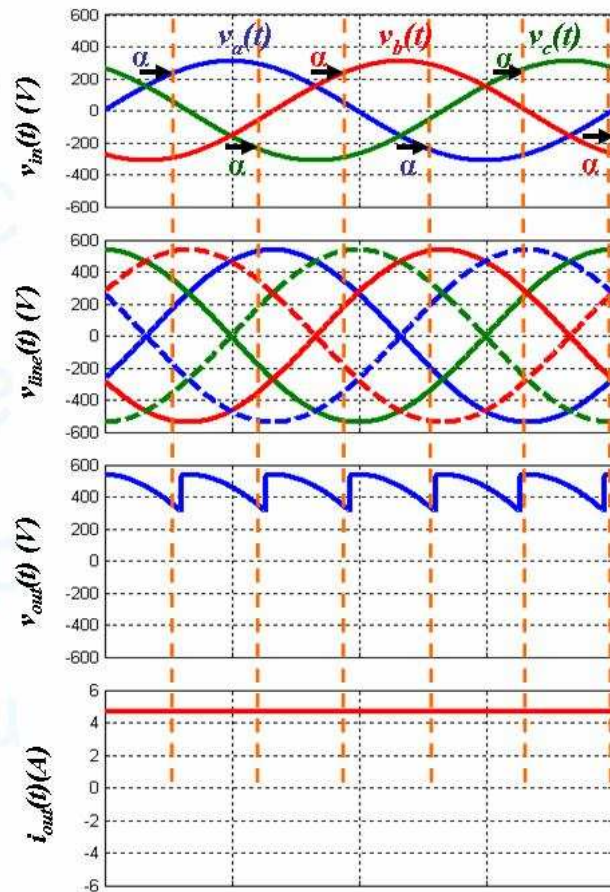
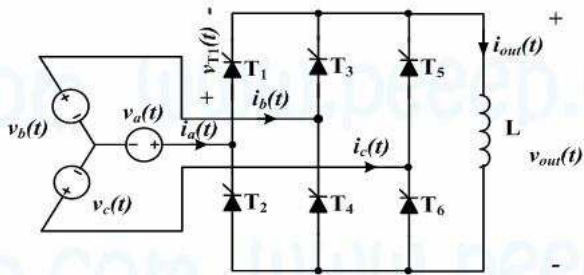
$$\begin{aligned}
 V_{out} &= \frac{1}{T} \int_{-\frac{T}{12} + \Delta t}^{\frac{T}{12} + \Delta t} v_{out}(t) dt \\
 V_{out} &= \frac{1}{T} \int_{-\frac{T}{12} + \Delta t}^{\frac{T}{12}} \sqrt{3} V_m \cos\left(\frac{2\pi t}{T}\right) dt \\
 &= \frac{6\sqrt{3}V_m}{T} \left(\frac{T}{2\pi}\right) \left[\sin\left(\frac{2\pi t}{T}\right) \right]_{-\frac{T}{12} + \Delta t}^{\frac{T}{12}} \\
 &= \frac{3\sqrt{3}V_m}{\pi} \left[\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{2\pi}{T}\left(-\frac{T}{12} + \Delta t\right)\right) \right] \\
 &= \frac{3\sqrt{3}V_m}{\pi} \left[1 - \sin\left(-\frac{\pi}{6} + \frac{2\pi\Delta t}{T}\right) \right] \\
 &= \frac{3\sqrt{3}V_m}{\pi} \left[1 + \sin\left(\frac{\pi}{6} - \frac{2\pi\Delta t}{T}\right) \right] \quad \Delta t = \frac{T\alpha}{2\pi} \\
 &= \frac{3\sqrt{3}V_m}{\pi} \left[1 + \sin\left(\frac{\pi}{6} - \alpha\right) \right]
 \end{aligned}$$



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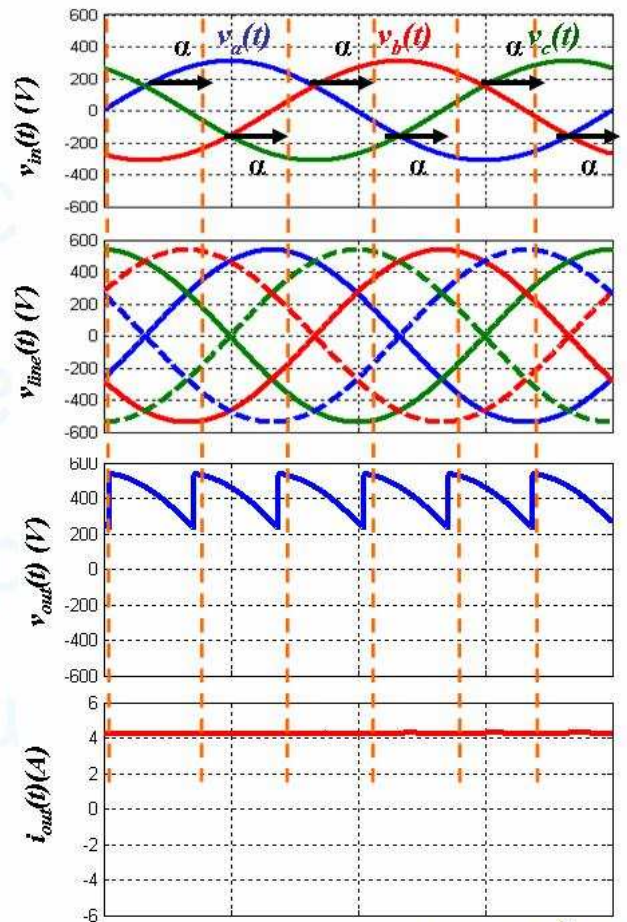
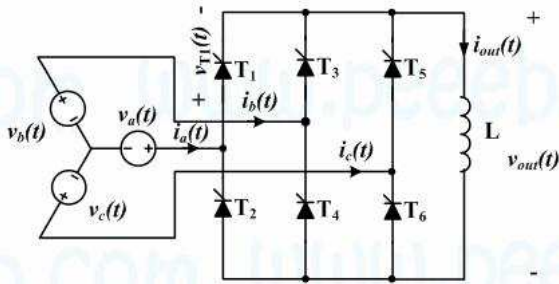
Three phase full wave rectifier (Inductive load and firing angle of 15°)



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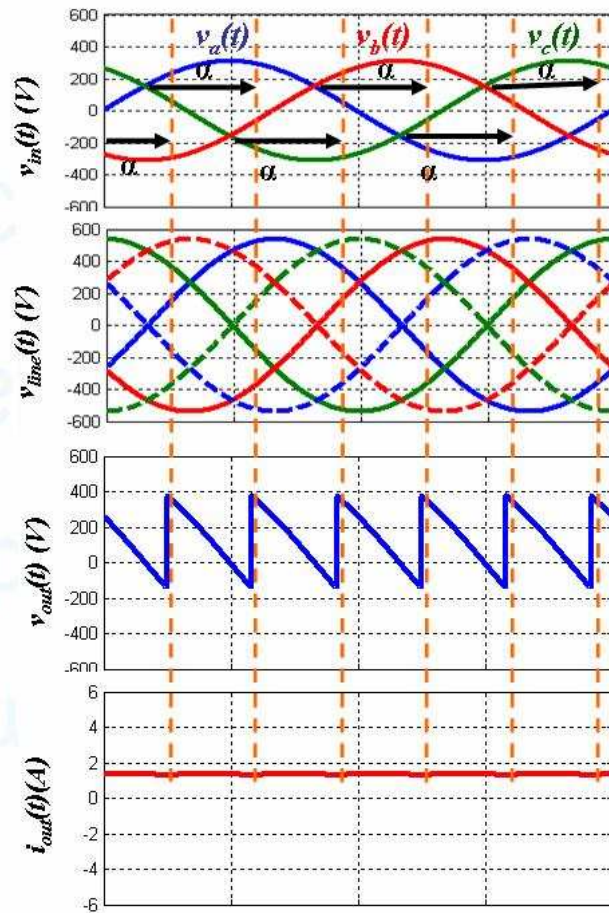
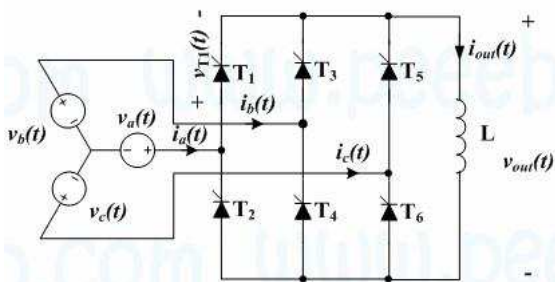
Three phase full wave rectifier (Inductive load and firing angle of 45°)



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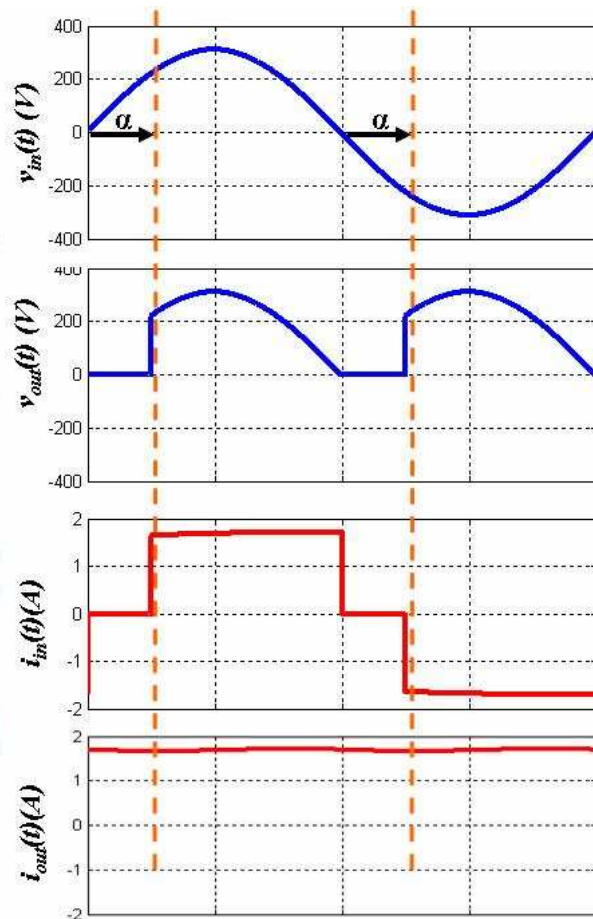
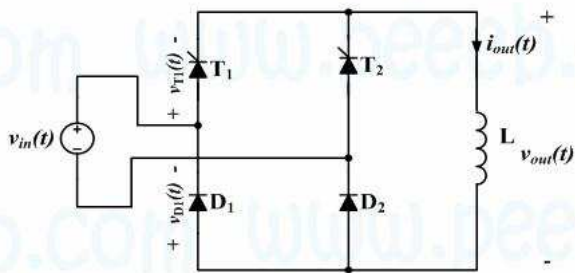
Three phase full wave rectifier (Inductive load and firing angle of 75°)



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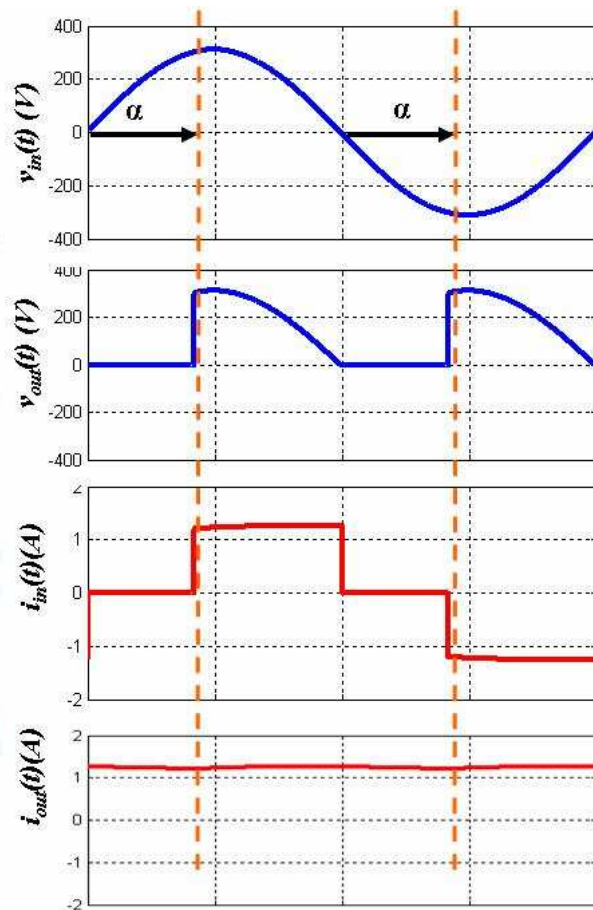
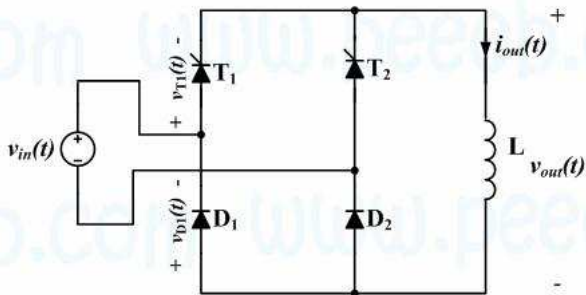
Single phase full wave half controlled rectifier
(Inductive load, firing angle of 45°)



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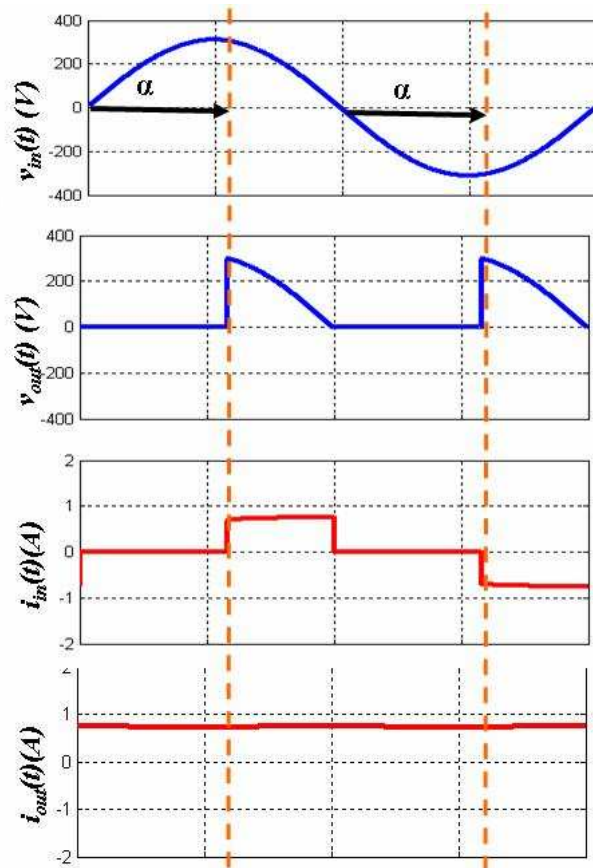
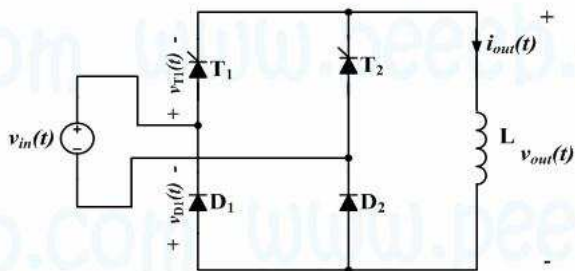
Single phase full wave half controlled rectifier
(Inductive load, firing angle of 75°)



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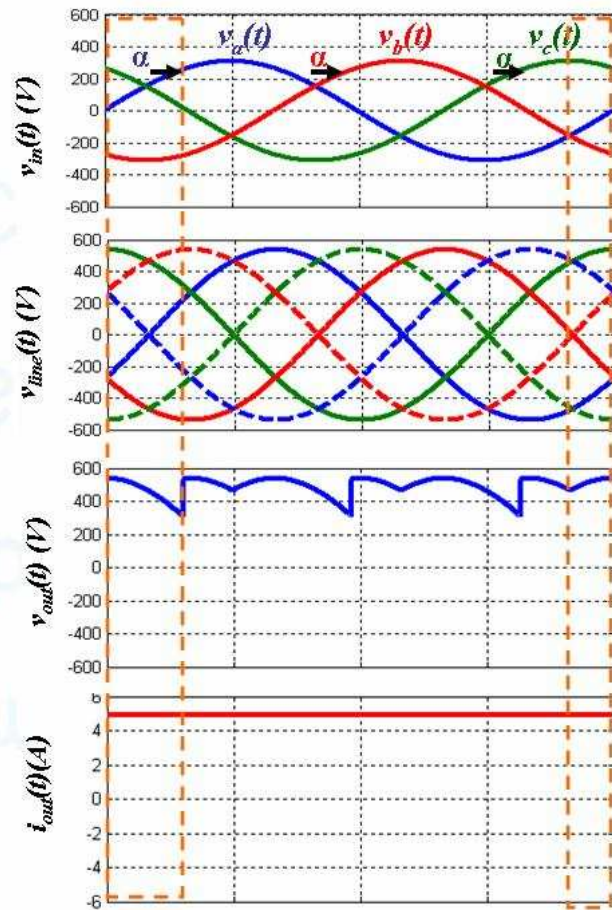
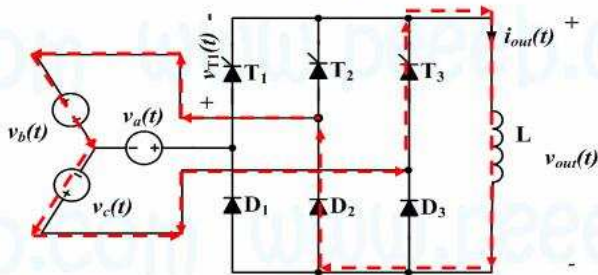
Single phase full wave half controlled rectifier
(Inductive load, firing angle of 105°)



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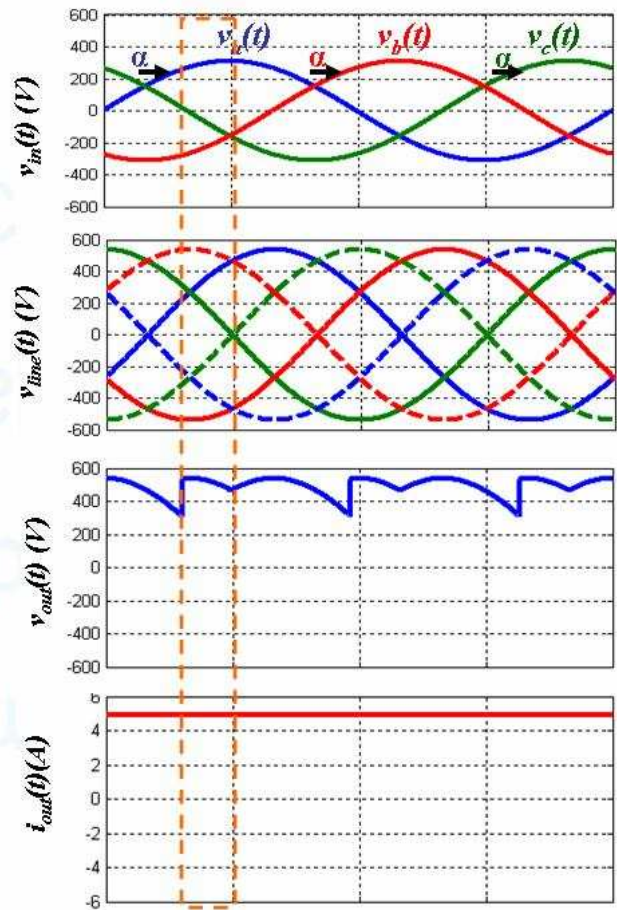
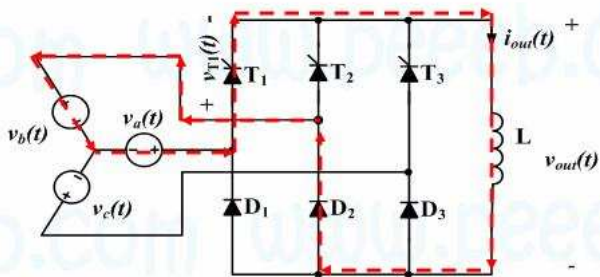
Three phase full wave half controlled rectifier (Inductive load, firing angle of 15°)



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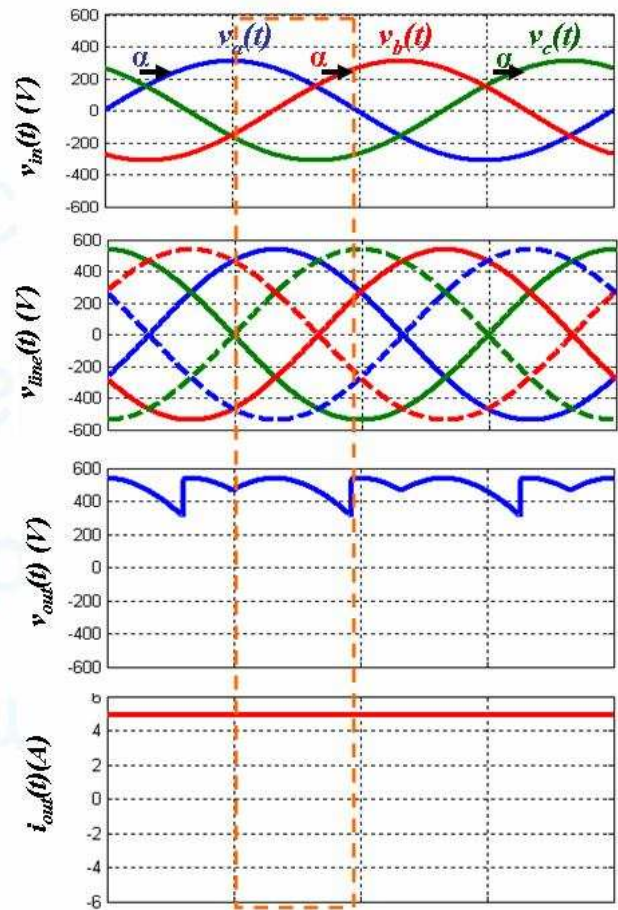
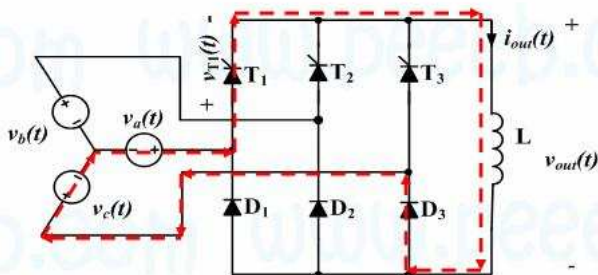
Three phase full wave half controlled rectifier (Inductive load, firing angle of 15°)



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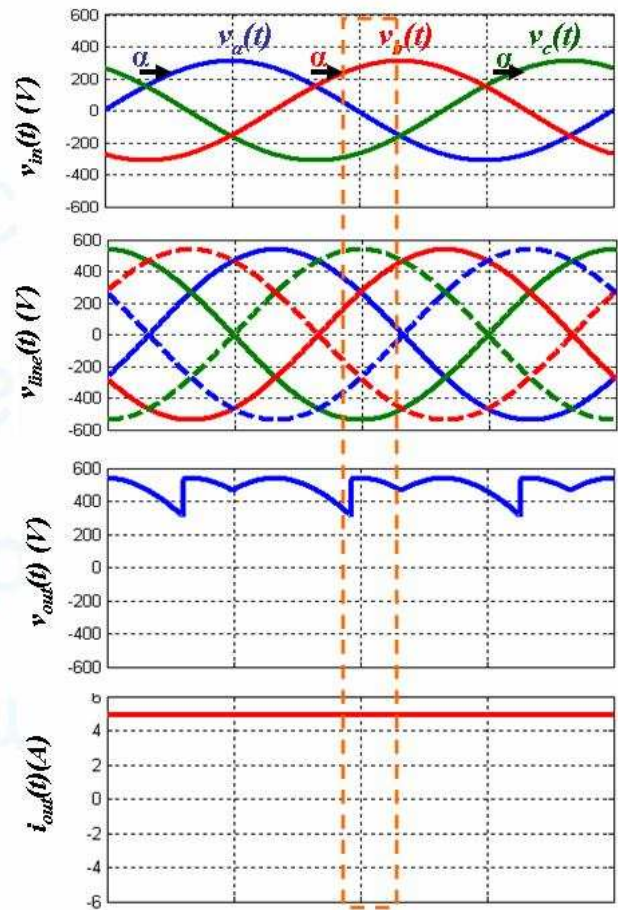
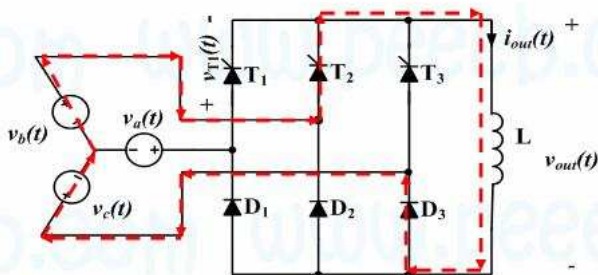
Three phase full wave half controlled rectifier (Inductive load, firing angle of 15°)



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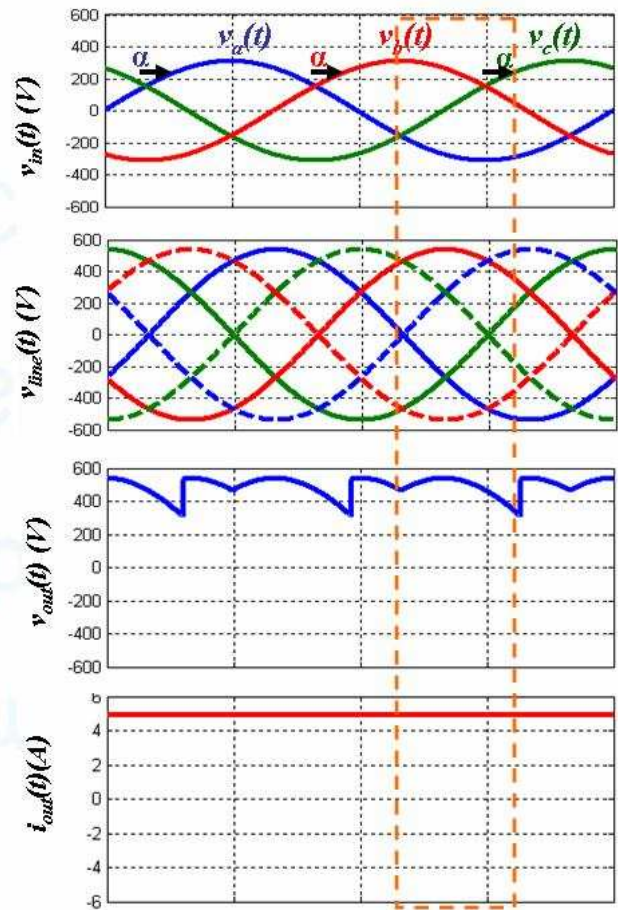
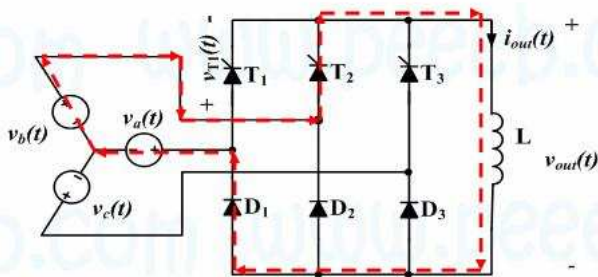
Three phase full wave half controlled rectifier (Inductive load, firing angle of 15°)



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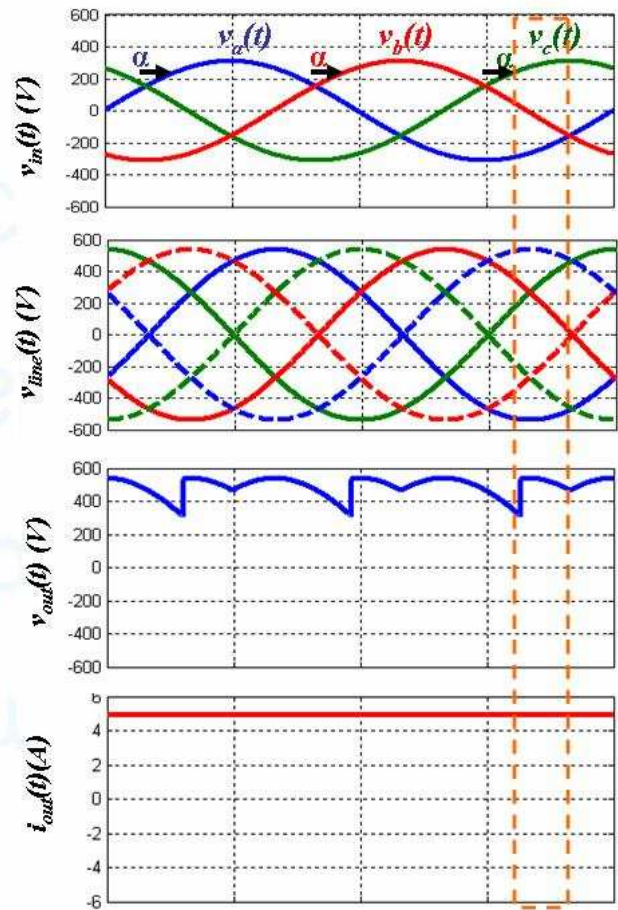
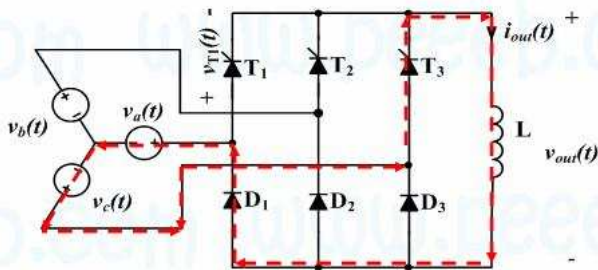
Three phase full wave half controlled rectifier (Inductive load, firing angle of 15°)



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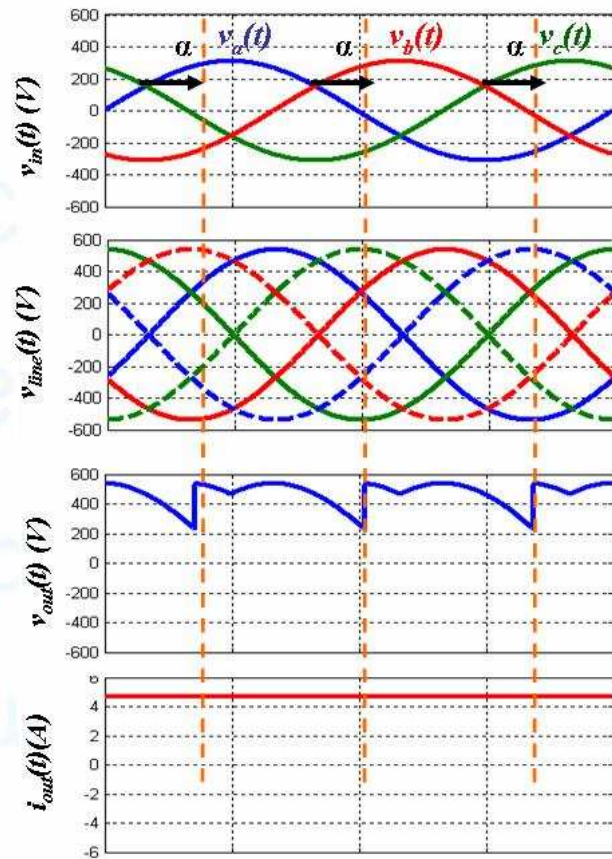
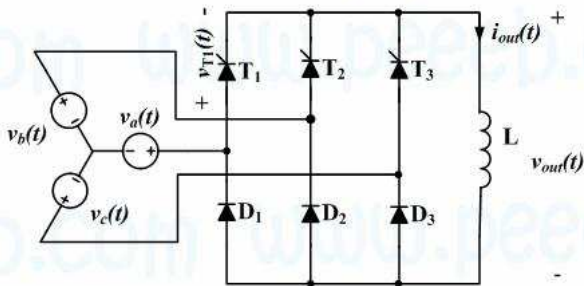
Three phase full wave half controlled rectifier (Inductive load, firing angle of 15°)



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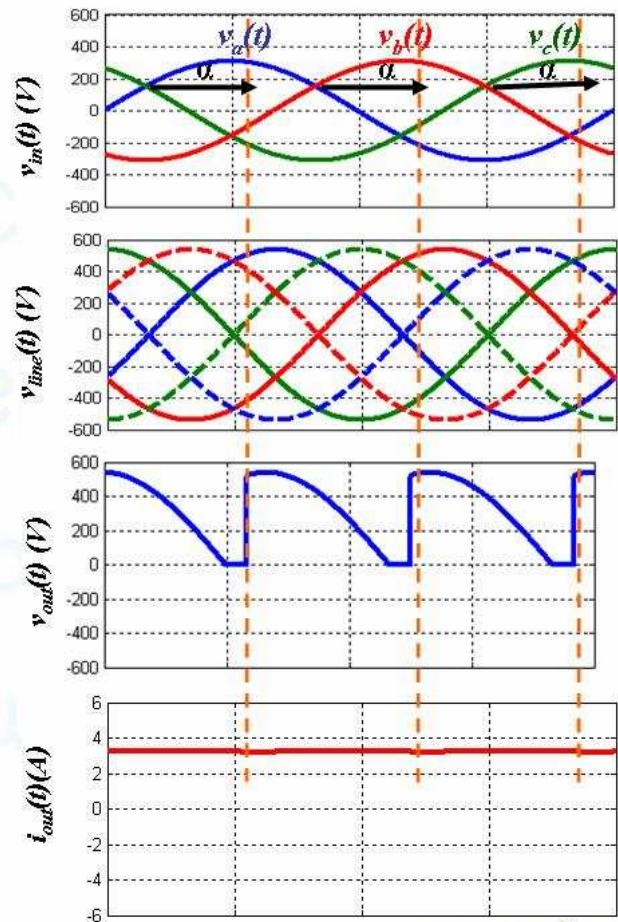
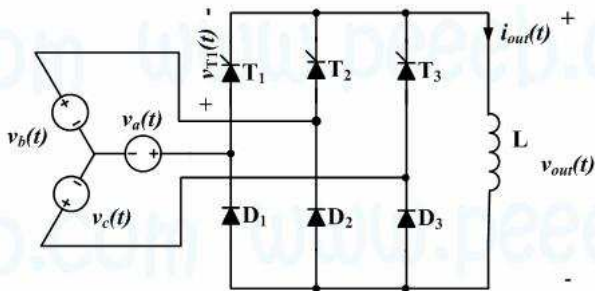
Three phase full wave half controlled rectifier (Inductive load, firing angle of 45°)



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Three phase full wave half controlled rectifier (Inductive load, firing angle of 75°)



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