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HW5

1. $T(N) = 2T(N-1) + 1$

$T(N-1) = 2T(N-2) + 1$

$T(N-2) = 2T(N-3) + 1$

$T(N) = 2(2T(N-2)+1) + 1 = 4T(N-2) + 2 + 1$

$T(N) = 2(2(2T(N-3)+1)+1) + 1 = 8T(N-3) + 2^2 + 2 + 1$

$T(N) = 2^k T(N-k) + 2^{k-1} + 2^{k-2} + \dots 2^2 + 2 + 1$

Assume $n-k = 0$ $\quad T(0) = 1$

$n = k$

$T(N) = 2^n T(0) + 2^{n-1} + 2^{n-2} + \dots + 1$

$= 2^n + 2^k - 1 = 2^n + 2^n - 1$

$= 2^{n+1} - 1$

order of growth of $f(n) = 2^{n+1} - 1$

time complexity of $\Theta(2^n)$

2. $T(N) = 3T(N-1) + n$

$a = 3 \quad b = 1 \quad f(n) = O(n) \quad k = 1$

$a > 1 \rightarrow T(n) = O(n^k a^{n/b})$

$T(n) = O(n \cdot 3^{n/1})$

$\boxed{T(n) = O(n \cdot 3^n)}$ by Master theorem

3. $T(N) = 9T(N/2) + n^2$

$a = 9 \quad b = 2 \quad n^{\log_2 9} > n^2$

$f(n) = O(n^{\log_2 9 - \epsilon}) \rightarrow \Theta(n^{\log_2 9}) = \Theta(n^{3.17})$

$\boxed{T(N) = \Theta(n^{\log_2 9}) = \Theta(n^{3.17})}$

4. $T(N) = 100T(N/2) + n^{\log_2 cn+1} = 100T(N/2) + n^{\log_2 c + \log n + 1}$

$a = 100 \quad b = 2 \quad n^{\log_2 100}$

let $e = \log_2 c + 1$:

$T(N) = 100T(N/2) + n^{\log_2 n + e}$

$f(n) = \Omega(n^{\log_2 (n) + e})$

$100 \cdot f(\frac{n}{b}) \leq c \cdot f(n), \quad c > 1, \quad n > n_0$

$100 \cdot (\frac{n}{2})^{\log_2 (\frac{n}{2}) + e} \leq c \cdot n^{\log_2 n + e}$

$100 \cdot (\frac{n}{2})^{\log_2 (\frac{n}{2})} \cdot (\frac{n}{2})^{e} \leq c \cdot n^{\log_2 n} \cdot n^{e}$

let $100 = c$

$c \cdot (\frac{n}{2})^{\log_2 (\frac{n}{2})} \cdot (\frac{n}{2})^{e} \leq c \cdot n^{\log_2 n} \cdot n^{e}$

case 2 ✓

$\boxed{T(n) = \Theta(n^{\log_2 n + e}) = \Theta(n^{\log n})}$

5. $T(N) = 4T(n/2) + n^2 \log n$

$a = 4 \quad b = 2 \quad \log_2 4 = 2$

$f(n) = \Theta(n^{\log_2 4} \log^k n) = \Theta(n^2 \log n)$

$\boxed{T(N) = \Theta(n^2 \log n)}$

6. $T(N) = 5T(N/2) + \frac{n^2}{\log n}$

$a = 5 \quad b = 2 \quad \log_b a = \log_2 5 > 2$

$n^{\log_2 5 - \epsilon} = \frac{n^{\log_2 5}}{n^\epsilon}$ let $\epsilon > 4$

$\frac{n^{\log_2 5}}{n^\epsilon} \rightarrow \frac{n^2}{\log n}$

$f(n) = O(n^{\log_2 5 - \epsilon}) \rightarrow \boxed{T(n) = \Theta(n^{\log_2 5})}$

Problem 2:

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func(n):                                  T(N)
  if n > 1:                                |
    for (i=0; i<10n; i++)                  10n+1
      do something                         10n
    func(n/2)                              T(n/2)
    func(n/2)                              T(n/2)
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$$T(N) = \begin{cases} n & n \leq 1 \\ 2T(n/2) + n & n > 1 \end{cases}$$

$a = 2 \quad b = 2 \qquad n^{\log_b a} = n^{\log_2 2} = n^1 = n$

$f(n) = \Theta(n^{\log_b a - \epsilon} \log n)$

let $\epsilon = 0$

$$\boxed{f(n) = \Theta(n \log n)}$$