**Week 9**

**Incremental Quantity Discount Example:**

* Assume now that pre-schedule is a follows
* **C(Q): total cost of ordering Q units**
  + 0.3Q for 0<= Q < 500
  + 150 + 0.29(Q-500) for 500 <= Q < 1000
    - the 150 is from the 500 units purchased at 0.3 (150 = 0.3•500)
  + 295 + 0.28(Q-1000) for Q >= 1000
    - the 295 is from the 500 units purchased at 0.3 plus the 500 units purchased at 0.29 (295 = 150+(.29•500)
* **rewrite the formulas from above in the form A + c•Q:**
* incremental Quantity Discount has associated fixed cost from ordering plus variable cost (A)
  + 0+0.3Q for 0<= Q < 500
  + 5 + 0.29Q for 500 <= Q < 1000
  + 15 + 0.28Q for Q >= 1000
* **Solve for = cost per unit = c**
  + 0.3
* **rewrite G(Q) (total cost equation) by replacing c with :**
  + Q\*=
* **Find fixed cost for each price schedule (recall that k = 8)**
* Add the k to the associated fixed cost from ordering found above (A + k)
  + K0 = 0 + 8 = 8
  + K1 = 5+ 8 = 13
  + K2 = 15 + 8 = 23
* **Calculate Q\* for each price schedule**
  + Recall that = 600, I= 0.2
  + = 400
    - Feasible because in range (0,500)
  + = 518.6
    - Feasible because in range (500,1000)
  + = 702.04
    - Infeasible because not in range (1000,∞)
* DO NOT NEED TO CHECK THE NEXT HIGHEST OUTCOME FOR INCREMENTAL QUANTITY DISCOUNT
* **Compare only feasible G(Q)**
  + G(Q0) vs. G(Q1)
  + = 204
    - K0 = 8, C(Q0)= 0+0.3Q, Q0 = 400
  + = 204.58
    - K0 = 13, C(Q0)= 5+0.29Q, Q0 = 519
  + is smaller

**Newsvendor Model**

* Consider the inventory control for a single planning period where our objective is to balance the cost of ordering too much (overage) vs. Ordering too little (underage)
* A single planning period is useful for highly fashioned items, short useful life, or perishable
* Consider a single product that is to be ordered at the beginning of the period and can only be used to satisfy demand during the period. All costs are determined on the basis of ending inventory
  + Salvage value: price you pay after the end of its useful life
* Variables:
  + Co= Cost of overage
  + Cu= Cost of underage
  + D = Random Variable on demand
  + fD and FD= pdf and CDF on demand
  + Q = lot size (number of unites I create)
  + G(Q,D) = total cost of ordering Q units when D units are demanded
* **Deriving G(Q,D):**
  + Total number of units over = Max[Q-D,0]
  + Total number of units under=Max[D-Q,0]
  + G(Q,D)= Co(Max[Q-D,0]) + Cu(Max[D-Q,0])
    - Need to get rid of random variable so find expected value
  + Expected value review:
    - * integrate over the entire set space
  + all values for D that are less than Q, when D < Q, you ordered to much= overage
  + all values for D that are greater than Q, when D > Q, you ordered to little = shortage
  + Take the derivative of G(Q,D) using Leibniz rule
  + set equal to zero and solve for F(Q)
    - F(Q\*)= critical ratio
    - is the numerator because F(Q\*) represents the probability that demand is less than Q\*

TO DO:

* Look up how to use Leibniz rule
* What happens when switch to 1 –F(Q)
* HW
* How does this value change with salvage value

**(Q,R) Model**

* Consider a continuous review inventory system. The system has constant lead time (τ) shortage is backordered and incurs a stockout/penalty cost of $p/unit. Demand is random and distributed randomly ~ N(. The demand lead time is ~ N(.
  + **Continuous review:** always checking amount of inventory in system
  + **Backordered:** when all out of inventory still take orders. Allow inventory to go negative, and allow subsequent orders to meet past demand



* Because of randomness (stochastic) need to safety stock
  + S= = safety stock
  + more inventory held, the more likely to meet all of the demand, so the more standard deviations I go away from the mean, the more mass being covered
* **The Cost function:**
  + Penalty, holding and setup



* + **Setup cost:**
  + **Holding Cost:**
  + **Penalty Cost:** 
    - Only pay a penalty for experiencing demand during lead time
  + **Number of shortages when the reorder point is R:**
    - Expected Value = Max{D-R,0}
    - 1-F(R)= Probability that Demand is greater than R = Probability of a shortage
      * use to find area under normal curve to the right of R
      * Shift normal curve to be standard normal (0,1)
        + 1 -φ(z) = area under the curve to the right of Z
        + n(R)=L(Z)σ

L(Z) looked up in table on T-square based on F(R)

* + **G(Q,R) = expected annual cost per unit**
    - in order to find minimal value need to take derivative, But since there are two unknowns (Q and R) need to take partial derivative of each
      * #1: partial Derivative of G(Q ,R) in terms of Q:
      * #2: partial Derivative of G(Q ,R) in terms of R:
    - **steps for finding minimum Q and R values:**
      * Q0=Q\* using base EOQ model
      * Solve for R using equation #1.
        + When solving for R calculate the values of F(R), and from there look up L(Z) in table, once you have L(Z) calculate n(R)
        + Calculate R using F-1(R) to get R of use R =
      * Once you have Ro and n(R0) can calculate Q1 in equation
      * Use Q1 in equation #2 to find R1
      * Continue with this process and until Q values start to converge and R values begin to converge
        + The values that Q and R converge to are the optimal points