Q1

1. $m(a+6X) = a + 6 * m(X)$

$$m(a+6X) = \frac{1}{N}\sum_{i=1}^{N}(a+6X)$$

$$z = a + 6X \Rightarrow z_i = a + 6x_i$$

$$= \frac{1}{N}\left(\sum_{i=1}^{N} a + \sum_{i=1}^{N} 6x_i\right)$$

$$= \frac{1}{N}\left(Na + 6\sum_{i=1}^{N} x_i\right)$$

$$= \frac{Na}{N} + (6)\cdot\frac{1}{N}\sum_{i=1}^{N} x_i$$

$$= a + 6 \cdot m(x)$$

$$m(x) = \frac{1}{N}\sum_{i=1}^{N} x_i$$

2. $cov(X, a+6Y) = 6 * cov(X, Y)$

$$z = a + 6Y \Rightarrow z_i = a + 6y_i$$

$$cov(X, a+6Y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+6y_i) - m(a+6(Y)))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(6(y_i - m(Y)))$$

$$= 6\,\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(Y))$$

$$= 6 * cov(x, y)$$

$$cov(X,Y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(Y))$$

3. $cov(a+6X, a+6X) = 6^2\, cov(X, X)$ $\quad cov(x, X) = s^2$

$$cov(a+6X, a+6X) = 6 * \underbrace{cov(a+6X, X)}$$

$$cov(a+6X, X) = 6\, cov(X, X)$$

$cov(a + bX, a + bX) = b^2 \, cov(X, X)$

4. Yes it is the median; the median doesn't change after the variable is transformed with a non decreasing transformation. The same argument holds for any quantile. For the IQR and range, you have to transform the endpoints then subtract; you can't just apply g as the IQR and range are spread measures.

5. No, it's not always true because taking the average before or after the functions takes the average of different numbers as g() is a transformation.