1. $SSE = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{N} (y_i - b_0 - b_1 z_{i1} - b_2 z_{i2})^2$

2. Let $e_i = y_i - b_0 - b_1 z_{i1} - b_2 z_{i2}$

$$\frac{\partial SSE}{\partial b_0} = -2 \sum_{i=1}^{N} e_i = 0$$

$$\frac{\partial SSE}{\partial b_1} = -2 \sum_{i=1}^{N} e_i z_{i1} = 0$$

$$\frac{\partial SSE}{\partial b_2} = -2 \sum_{i=1}^{N} e_i z_{i2} = 0$$

$$\sum_{i=1}^{N} e_i = 0$$

$$\sum_{i=1}^{N} e_i z_{i1} = 0$$

$$\sum_{i=1}^{N} e_i z_{i2} = 0$$

3. The first equation says $\sum_i e_i = 0$, so the average error is 0:

$$\sum_{i=1}^{N} e_i = 0$$

The second two equations say the residual vector e is orthogonal to each predictor vector z_1 and z_2:

$$\sum_{i=1}^{N} e_i z_{i1} = 0$$

$$\sum_{i=1}^{N} e_i z_{i2} = 0$$

4. Optimal intercept is $b_0^* = \bar{y}$

4. Optimal intercept is $b_0 = \bar{y}$

Using:

$$\sum_{i=1}^{N} e_i = 0$$

Expand to:

$$\sum_{i=1}^{N} (y_i - b_0 - b_1 z_{i1} - b_2 z_{i2}) = 0$$

Because $\sum_i z_{i1} = \sum_i z_{i2} = 0$

$$\sum_{i=1}^{N} y_i - N b_0 = 0 \Rightarrow b_0 = \bar{y}$$

Eliminate b_0 by subbing $b_0 = \bar{y}$

Let $\bar{y}_i = y_i - \bar{y}$

$$\sum_{i=1}^{N} z_{i1} (y_i - b_1 z_{i1} - b_2 z_{i2}) = 0$$

$$\sum_{i=1}^{N} z_{i2} (y_i - b_1 z_{i1} - b_2 z_{i2}) = 0$$

Rearrange:

$$b_1 \sum_i z_{i1}^2 + b_2 \sum_i z_{i1} z_{i2} = \sum_i z_{i1} \bar{y}_i$$

$$b_1 \sum_i z_{i1} z_{i2} + b_2 \sum_i z_{i2}^2 = \sum_i z_{i2} \bar{y}_i$$

5. $A b = c$

$$\begin{array}{ccc} A & b & c \\ \begin{pmatrix} \sum_i z_{i1}^2 & \sum_i z_{i1} z_{i2} \\ \sum_i z_{i1} z_{i2} & \sum_i z_{i2}^2 \end{pmatrix} & \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = & \begin{pmatrix} \sum_i z_{i1} \bar{y}_i \\ \sum_i z_{i12} \bar{y}_i \end{pmatrix} \end{array}$$

A b C

6. $$\frac{1}{N}\begin{pmatrix} \sum_i z_{i1}^2 & \sum_i z_{i1} z_{i2} \\ \sum_i z_{i1} z_{i2} & \sum_i z_{i2}^2 \end{pmatrix}\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \frac{1}{N}\begin{pmatrix} \sum_i z_{i1} \bar{y}_i \\ \sum_i z_{i2} \bar{y}_i \end{pmatrix}$$

$$\frac{1}{N} \sum_i z_{i1}^2 = Var(z_1)$$

$$\frac{1}{N} \sum_i z_{i1} z_{i2} = cov(z_1, z_2)$$

$$\frac{1}{N} \sum_i z_{i1} \bar{y}_i = cov(z_1, y)$$

matrix A

$$A/N = \begin{pmatrix} \frac{1}{N} \sum_i (x_{i1} - m_1)^2 \\ \frac{1}{N} \sum_i (x_{i1} - m_1)^2 (x_{i2} - m_2) \\ \frac{1}{N} \sum_i (x_{i1} - m_1)(x_{i2} - m_2) \\ \frac{1}{N} \sum_i (x_{i1} - m_1)^2 \end{pmatrix}$$

$$= \begin{pmatrix} Var(x_1) & cov(x_1, x_2) \\ cov(x_1, x_2) & Var(x_2) \end{pmatrix}$$

Vector C/N

$$\frac{1}{N} C = \begin{pmatrix} \frac{1}{N} \sum_i (x_{i1} - m_1)(y_i - \bar{y}) \\ \frac{1}{N} \sum_i (x_{i2} - m_2)(y_i - \bar{y}) \end{pmatrix} = \begin{pmatrix} Cov(x_1, y) \\ Cov(x_2, y) \end{pmatrix}$$

$$\begin{pmatrix} Var(x_1) & Cov(x_1, x_2) \\ Cov(x_1, x_2) & Var(x_2) \end{pmatrix}\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} Cov(x_1, y) \\ Cov(x_2, y) \end{pmatrix}$$