HW 7

5.2 5, 7, 15, 17, 20a, 27, T/F dh

5. $\det(\lambda I - A) = \begin{bmatrix} \lambda-1 & 0 \\ -6 & \lambda+1 \end{bmatrix} = (\lambda-1)(\lambda+1)$ $\lambda = 1, -1$

$\lambda = 1$ $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ -6 & 2 & 0 \end{array}\right] \rightarrow \begin{array}{c} \begin{array}{ccc} x & y & \end{array} \\ \left[\begin{array}{cc|c} 1 & -1/3 & 0 \\ 0 & 0 & 0 \end{array}\right] \end{array}$ $x = \frac{1}{3}y$, $y = 1y$

$\lambda = -1$ $\left[\begin{array}{cc|c} -2 & 0 & 0 \\ -6 & 0 & 0 \end{array}\right] \rightarrow \begin{array}{c} \begin{array}{ccc} x & y & \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \end{array}$ $x = 0y$, $y = 1y$

$P = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ $P^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \cdot \frac{1}{1}$

$P^{-1}AP = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

7. $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $\det(\lambda I - A) = \begin{bmatrix} \lambda-2 & 0 & +2 \\ 0 & \lambda-3 & 0 \\ 0 & 0 & \lambda-3 \end{bmatrix}$ $\lambda = 2, 3$

$\lambda = 2$ $\left[\begin{array}{ccc|c} 0 & 0 & 2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right] \rightarrow \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$ $x = 1x$, $y = 0x$, $z = 0x$

$\lambda = 3$ $\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$ $x = 0y + 2z$, $y = 1y + 0z$, $z = 0y + 1z$

$P = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $P^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$P^{-1}AP = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

15. a) 3x3, dim = 1

b) 6x6, dim: $\lambda = 0$ 1, 2

$\lambda = 1$ 1

$\lambda = 2$ 1, 2, 3

17. $A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$ $\det(\lambda I - A) = \begin{bmatrix} \lambda-0 & -3 \\ -2 & \lambda+1 \end{bmatrix}$

$= \lambda(\lambda+1) - 6 = \lambda^2 + \lambda - 6 = (\lambda+3)(\lambda-2)$

$\lambda = 2, -3$

$\lambda = 2$ $\left[\begin{array}{cc|c} 2 & -3 & 0 \\ -2 & 3 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -3/2 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $x = \frac{3}{2}y$, $y = 1y$

$\lambda = -3$ $\left[\begin{array}{cc|c} -3 & -3 & 0 \\ -2 & -2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $x = -y$, $y = 1y$

$P = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ $P^{-1} = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix} \frac{1}{3+2} = \begin{bmatrix} 1/5 & 1/5 \\ -2/5 & 3/5 \end{bmatrix}$

$P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} = D$

$A^{10} = PD^{10}P^{-1} = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2^{10} & 0 \\ 0 & (-3)^{10} \end{bmatrix}\begin{bmatrix} 1/5 & 1/5 \\ -2/5 & 3/5 \end{bmatrix}$

$= \begin{bmatrix} 24234 & -34815 \\ -23210 & 35839 \end{bmatrix}$

20. a) $P^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}$

$P^{-1}AP = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^{1000} = PD^{1000}P^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

27. a) $\lambda = 1$: 1, $\lambda = 3$: 1, 2 $\lambda = 4$: 1, 2, 3
since degree

b) max (1, 2, 3) bc it needs to = 6

c) must be $\lambda = 4$, since others have dim < 3

6.1 1acde, 17, 19, 33, 34, T/F bcd

T/F d) F, ~~P does not~~ is not unique, can switch order of cols.

h) T, geom. multiplicity = alg. multiplicity

1. a) $\langle \vec{u}, \vec{v} \rangle = 2(1)(3) + 3(1)(2) = 12$

c) $\langle \vec{u}+\vec{v}, \vec{w} \rangle = 2(4)(0) + 3(4)(-1)$

$= -12$

d) $\|\vec{v}\| = \sqrt{2(3)(3) + 3(2)(2)} = \sqrt{30}$

e) $d(\vec{u}, \vec{v}) = \|\vec{u} - \vec{v}\|$

$= \sqrt{\langle (-2,-1), (-2,-1) \rangle}$

$= \sqrt{2(-2)(-2) + 3(-1)(-1)}$

$= \sqrt{11}$

17. $\|\vec{u}\| = \sqrt{2(-3)(-3) + 3(2)(2)}$

$= \sqrt{30}$

$d(\vec{u}, \vec{v}) = \sqrt{\langle (-4,-5), (-4,-5) \rangle}$

$= \sqrt{2(-4)(-4) + 3(-5)(-5)}$

$= \sqrt{107}$

19. $\|\vec{p}\| = \sqrt{(-2)^2 + 1^2 + 3^2} = \sqrt{14}$

$d(\vec{p}, \vec{q}) = \sqrt{(-2-4)^2 + 1^2 + (3+7)^2} = \sqrt{137}$

33. 1) $\langle \vec{u}, \vec{v} \rangle = \langle \vec{v}, \vec{u} \rangle$

$\langle \vec{u}, \vec{v} \rangle = u_1^2 v_1^2 + u_2^2 v_2^2 + u_3^2 v_3^2$ ✓

$\langle \vec{v}, \vec{u} \rangle = v_1^2 u_1^2 + v_2^2 u_2^2 + v_3^2 u_3^2$

2) $\langle \vec{u}+\vec{v}, \vec{w} \rangle = \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$

$\langle \vec{u}+\vec{v}, \vec{w} \rangle = (u_1+v_1)^2 w_1^2 + (u_2+v_2)^2 w_2^2 + (u_3+v_3)^2 w_3^2$ ✗

$\langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle = (u_1^2 w_1^2 + u_2^2 w_2^2 + u_3^2 w_3^2) + (v_1^2 w_1^2 + v_2^2 w_2^2 + v_3^2 w_3^2)$

3) $\langle k\vec{u}, \vec{v} \rangle = k \langle \vec{u}, \vec{v} \rangle$ ✗ ~~✗~~

$\langle k\vec{u}, \vec{v} \rangle = (ku_1^2) v_1^2 + (ku_2^2) v_2^2 + (ku_3^2) v_3^2$

$k \langle \vec{u}, \vec{v} \rangle = k(u_1^2 v_1^2 + u_2^2 v_2^2 + u_3^2 v_3^2)$

4) $\langle \vec{v}, \vec{v} \rangle \geq 0$ ✓

$\langle \vec{v}, \vec{v} \rangle = v_1^2 v_1^2 + v_2^2 v_2^2 + v_3^2 v_3^2$

34. 1) $\langle \vec{u}, \vec{v} \rangle = \langle \vec{v}, \vec{u} \rangle$ ✓

$\langle \vec{u}, \vec{v} \rangle = u_1 v_1 - u_2 v_2 + u_3 v_3$

$\langle \vec{v}, \vec{u} \rangle = v_1 u_1 - v_2 u_2 + v_3 u_3$

2) $\langle \vec{u}+\vec{v}, \vec{w} \rangle = \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$ ✓ ~~✗~~

$\langle \vec{u}+\vec{v}, \vec{w} \rangle = (u_1+v_1) w_1 - (u_2+v_2) w_2 + (u_3+v_3) w_3$

$\langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle = (u_1 w_1 - u_2 w_2 + u_3 w_3) + (v_1 w_1 - v_2 w_2 + v_3 w_3)$

3) $\langle k\vec{u}, \vec{v} \rangle = k \langle \vec{u}, \vec{v} \rangle$ ✗

$\langle k\vec{u}, \vec{v} \rangle = (ku_1) v_1 - (ku_2) v_2 + (ku_3) v_3$

$k \langle \vec{u}, \vec{v} \rangle = k(u_1 v_1) - k(u_2 v_2) + k(u_3 v_3)$

4) $\langle \vec{v}, \vec{v} \rangle \geq 0$ ✗

$\langle \vec{v}, \vec{v} \rangle = v_1 v_1 - v_2 v_2 + v_3 v_3$

T/F b) F, vectors can have neg. components → neg. inner products

c) T, like axiom 2

d) T, like axiom 3