HW 6

4.5 7, 8, 9ac, 10, 14 T/F bcd    4.7 14, 15, 24

4.7 9, 10 T/F d

7. a) $2y = 3x + 5z$    dim = 2

$y = 3/2\,x + 5/2\,z$

$x = 1,\ z = 0,\ y = 3/2$

$x = 0,\ z = 1,\ y = 5/2$

$\left\{ \begin{bmatrix} 1 \\ 3/2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5/2 \\ 1 \end{bmatrix} \right\}$

b) $x - y = 0$   $x = y$

$x = 1,\ y = 1,\ z = 0$

$x = 0,\ y = 0,\ z = 1$

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ dim = 2

c) $x = 2t,\ y = -t,\ z = 4t$

$\left\{ \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \right\}$ dim = 1

d) $b = a + c$

$a = 1,\ b = 1,\ c = 0$

$a = 0,\ b = 1,\ c = 1$

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ dim = 2

8. a) $(a, b, c, 0)$  $a = 1,\ b = 0,\ c = 0,\ d = 0$

$a = 0,\ b = 1,\ c = 0,\ d = 0$

$a = 0,\ b = 0,\ c = 1,\ d = 0$    dim = 3

$\{(1,0,0,0), (0,1,0,0), (0,0,1,0)\}$

b) $a = 1,\ b = 0,\ c = 1,\ d = 1$

$a = 0,\ b = 1,\ c = -1,\ d = 1$

$\{(1,0,1,1), (0,1,-1,1)\}$ dim = 2

c) $\{(1,1,1,1)\}$ dim = 1

9. a) $n$

b) $n + \frac{n(n-1)}{2} = \frac{n(n+1)}{2}$

c) $\frac{n(n+1)}{2}$

10. $a_0 = 0,\ a_1 = 1,\ a_2 = 0,\ a_3 = 0$

$a_0 = 0,\ a_1 = 0,\ a_2 = 1,\ a_3 = 0$

$a_0 = 0,\ a_1 = 0,\ a_2 = 0,\ a_3 = 1$

$\{(0,1,0,0), (0,0,1,0), (0,0,0,1)\}$

11. dim = 3

14. $c_1\vec{u}_1 + c_2\vec{u}_2 + c_3\vec{u}_3 = 0$

$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = 0$

$(c_1 + c_2 + c_3)\vec{v}_1 + (c_2 + c_3)\vec{v}_2 + c_3\vec{v}_3 = 0$

$c_1 = c_2 = c_3 = 0$

$\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is linearly indep.

∴ has dim = 3

T/F b) T,

c) F, not enough for all dim.

d) T

9. a) $\{(1\ 0\ -16), (0\ 1\ -19/2)\}$

b) $\{(1\ 0\ -1/2)\}$

10. a) $\{(1\ 0\ 1\ -2/7), (0\ 1\ 1\ 4/7)\}$   dim = 2

b) $\{(1\ 0\ 1\ 2\ 1), (0\ 1\ 1\ 1\ 2)\}$

T/F d) F, A needs to be in row echelon form

14. $\begin{bmatrix} 1 & 1 & -4 & -3 \\ 2 & 0 & 2 & -2 \\ 2 & -1 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1/2 \\ 0 & 1 & 0 & -9/2 \\ 0 & 0 & 1 & -1/2 \end{bmatrix}$

$\{(1\ 0\ 0\ -1/2), (0\ 1\ 0\ -9/2), (0\ 0\ 1\ -1/2)\}$

15. $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ -2 & 0 & 2 & 2 \\ 0 & -3 & 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\{(1000), (0100), (0010), (0001)\}$

24. a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

4.8 1, 5ab, 15, 17, 21a

5.1 3, 5ad, 7, 9, 11, 13, 19ab, 25 T/F bef

1. a) $\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Rank: 1 Nullity: 3

b) $\begin{bmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ Rank: 2 Nullity: 1

5. a) Rank = 1 Nullity: 2

b) $2+1=3=n$

15. Rank 1: no

Rank 2: $r=2$, $s=1$

17. No, has to be planes since nullity = 1

21. a) $7-4=3$

3. $A\vec{x} = \begin{bmatrix} 5 \\ 10 \\ 5 \end{bmatrix}$ is $5 \times \vec{x}$

eigenvalue: 5

5. a) $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ $\det(\lambda I - A) = \begin{bmatrix} \lambda-1 & -4 \\ -2 & \lambda-3 \end{bmatrix}$

$= (\lambda-1)(\lambda-3) - 8$ $\boxed{0 = (\lambda-5)(\lambda+1)}$

$= (\lambda^2 - 4\lambda + 3) - 8$

$= (\lambda-5)(\lambda+1)$

$\boxed{\lambda = 5, -1}$

$\left[\begin{array}{cc|c} 4 & -4 & 0 \\ -2 & 2 & 0 \end{array}\right] \rightarrow \begin{array}{cc} x & y \\ \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \end{array}$ $x = 1y$ $y = 1y$

$\boxed{\left\{\begin{bmatrix} +1 \\ 1 \end{bmatrix}\right\}}$

$\left[\begin{array}{cc|c} -2 & -4 & 0 \\ -2 & -4 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $x = -2y$ $y = 1y$

$\boxed{\left\{\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right\}}$

~~b) $\begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$ $\det(\lambda I - A) = \begin{bmatrix} \lambda+2 & 7 \\ -1 & \lambda-2 \end{bmatrix}$~~

~~$= (\lambda-2)(\lambda+2) + 7 = (\lambda^2 - 4) + 7$~~

~~$\boxed{0 = \lambda^2 + 3}$ $\boxed{\lambda = \pm\sqrt{3}}$~~

~~15. $T(x, y) = (x + 4y, 2x + 3y)$~~

~~$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$~~

~~$\lambda = -5, 1$ $\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$ $\left\{\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right\}$~~

d) $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ $\det(\lambda I - A) = \begin{bmatrix} \lambda-1 & 2 \\ 0 & \lambda-1 \end{bmatrix}$

$\boxed{0 = (\lambda-1)^2}$ $\boxed{\lambda = 1}$

$\left[\begin{array}{cc|c} 0 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\begin{array}{cc} x & y \\ \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \end{array}$

$x = 1x$ $y = 0x$

$\boxed{\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}}$

7. $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ $\det(\lambda I - A) = \begin{bmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{bmatrix}$

$= +(\lambda-1)[(\lambda-1)(\lambda-4) + 2]$

$= (\lambda-1)(\lambda^2 - 5\lambda + 6)$

$\boxed{0 = (\lambda-1)(\lambda-2)(\lambda-3)}$

$\boxed{\lambda = 1, 2, 3}$

$\left[\begin{array}{ccc|c} -3 & 0 & -1 & 0 \\ 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{array}\right] \rightarrow \begin{array}{ccc} x & y & z \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$ $x = 0y$ $y = 1y$ $z = 0y$

$\boxed{\{(0, 1, 0)\}}$

$\left[\begin{array}{ccc|c} -2 & 0 & -1 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \end{array}\right] \rightarrow \begin{array}{ccc} x & y & z \\ \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$

$x = -1/2 z$ $y = z$ $z = z$ $\boxed{\{(-1/2, 1, 1)\}}$

19ab, 25, T/F bef

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} x = -z \\ y = z \\ z = z \end{array}$$

$\boxed{\{(-1, 1, 1)\}}$

25. a) degree of $p(\lambda)$ is 6, 6x6

b) yes, $p(0) \neq 0$

c) 3, since 3 val. for $\lambda$

T/F b) F, if $\lambda$ is eigenval, then $~~bA~~$ $(\lambda I - A)\vec{x} = \vec{0}$ has nontriv. sol.

e) F, eigenval of $2I$ is 2, but $I$ is 1

f) F, linearly dependent

9. $\begin{bmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$ $\det(\lambda I - A) = \begin{bmatrix} \lambda-6 & -3 & 8 \\ 0 & \lambda+2 & 0 \\ -1 & 0 & \lambda+3 \end{bmatrix}$

$= (\lambda+2)[(\lambda-6)(\lambda+3)+8]$

$= (\lambda+2)[\lambda^2 - 3\lambda - 10)$ $\boxed{\lambda = -2, 5}$

$\boxed{0 = (\lambda+2)(\lambda+2)(\lambda-5)}$

$$\left[\begin{array}{ccc|c} -8 & -3 & 8 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} x = z \\ y = 0z \\ z = z \end{array}$$

$\boxed{\{(1, 0, 1)\}}$ $\boxed{\{(8, 0, 1)\}}$

$$\left[\begin{array}{ccc|c} -1 & -3 & 8 & 0 \\ 0 & 7 & 0 & 0 \\ -1 & 0 & 8 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -8 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} x = 8z \\ y = 0z \\ z = z \end{array}$$

11. $\begin{bmatrix} 4 & 0 & -1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ $\det(\lambda I - A) = \begin{bmatrix} \lambda-4 & 0 & 1 \\ 0 & \lambda-3 & 0 \\ -1 & 0 & \lambda-2 \end{bmatrix}$

$= (\lambda-3)[(\lambda-4)(\lambda-2)+1]$

$= (\lambda-3)[\lambda^2 - 6\lambda + 9)$ $\boxed{\lambda = 3}$

$= \boxed{(\lambda-3)(\lambda-3)(\lambda-3) = 0}$

$$\left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} x = 0y + z \\ y = 1y + 0z \\ z = 0y + 1z \end{array}$$

$\boxed{\{(0, 1, 0), (1, 0, 1)\}}$

13. $(\lambda-3)(\lambda-7)(\lambda-1) = 0$

19. a) $\lambda = 1, -1$

eigen vector: $\{(1, 1)\}, \{(-1, 1)\}$

b) $\lambda = 1$, $\lambda = 0$

$\{(1, 1)\}, \{(0, 1)\}$