$\frac{3}{2}$

HW 5

4.2 11ab, 12abc, 14a, T/F cef

11. a) $\left[\begin{array}{ccc|c} 2 & 0 & 0 & b_1 \\ 2 & 0 & 1 & b_2 \\ 2 & 3 & 1 & b_3 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 2 & 3 & 1 & b_3 \\ 2 & 0 & 1 & b_2 \\ 2 & 0 & 0 & b_1 \end{array}\right]$

$\frac{1}{2}R_1 \left[\begin{array}{ccc|c} 1 & 3/2 & 1/2 & b_3/2 \\ 2 & 0 & 1 & b_2 \\ 2 & 0 & 0 & b_1 \end{array}\right]$ $-2R_1+R_2$, $R_3 \to -2R_1+R_3$

$\left[\begin{array}{ccc|c} 1 & 3/2 & 1/2 & b_3/2 \\ 0 & -3 & -1 & -b_3+b_2 \\ 0 & -3 & -2 & -b_3+b_1 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 3/2 & 1/2 & \frac{b_3}{2} \\ 0 & 1 & 1/3 & \frac{b_3+b_2}{3} \\ 0 & -3 & -2 & -b_3+b_1 \end{array}\right]$

$R_3 \to 3R_2+R_3 \left[\begin{array}{ccc|c} 1 & 3/2 & 1/2 & b_3/2 \\ 0 & 1 & 1/3 & \frac{-b_3+b_2}{3} \\ 0 & 0 & -1 & -2b_3+b_1+b_2 \end{array}\right]$

yes, $\vec{v}_1, \vec{v}_2$ & $\vec{v}_3$ span $\mathbb{R}$

b) $\left[\begin{array}{ccc|c} 2 & 4 & 8 & b_1 \\ -1 & 1 & -1 & b_2 \\ 3 & 2 & 8 & b_3 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 2 & 4 & b_1/2 \\ -1 & 1 & -1 & b_2 \\ 3 & 2 & 8 & b_3 \end{array}\right]$

$R_2 \to R_1+R_2$, $R_3 \to -3R_1+R_3$ $\left[\begin{array}{ccc|c} 1 & 2 & 4 & b_1/2 \\ 0 & 3 & 3 & \frac{b_1}{2}+b_2 \\ 0 & -4 & -4 & \frac{3b_1}{2}+b_3 \end{array}\right] \xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 2 & 4 & \frac{b_1}{2} \\ 0 & 0 & 0 & \frac{b_1}{2}+b_2 \\ 0 & 4 & -4 & \frac{3b_1}{2}+b_3 \end{array}\right]$

No, $\vec{v}_1, \vec{v}_2$, & $\vec{v}_3$ do not span $\mathbb{R}$

12. a) $\left[\begin{array}{ccc|c} 2 & 3 & -1 & 2 \\ 1 & -1 & 0 & 3 \\ 0 & 5 & 2 & -7 \\ 3 & 2 & 1 & 3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Yes, $2\vec{v}_1 - 1\vec{v}_2 - 1\vec{v}_3 = (2, 3, -7, 3)$

b) $\left[\begin{array}{ccc|c} 2 & 3 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 5 & 2 & 0 \\ 3 & 2 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

~~No, (0,0,0,0) is not in $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$~~

Yes, $(0,0,0,0)$ is in $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$

4.3 2a, 3a, 4b, 5, 6, 7, 8, 9, 15, 16af
T/F bcf

c) $\left[\begin{array}{ccc|c} 2 & 3 & -1 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & 5 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

No, $(1,1,1,1)$ is not in span$(\{\vec{v}_1, \vec{v}_2, \vec{v}_3\})$

14. a) $a\cos^2 x + b\sin^2 x = \cos 2x$

$\cos 2x = \cos^2 x - \sin^2 x$

Yes, $\cos 2x$ is in the span of f & g

T/F c) F, set of non-neg ints contains 0, not closed under mul.

e) F, $A\vec{x} = \vec{b}$, $A\vec{y} = \vec{b}$ does not mean $A(\vec{x}+\vec{y}) = \vec{b}$

f) T, def. of vector space

2. a) $\left[\begin{array}{ccc|c} -3 & 5 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 4 & 2 & 3 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ yes

3. a) $\left[\begin{array}{cccc|c} 3 & 1 & 2 & 4 & 0 \\ 8 & 5 & -1 & 2 & 0 \\ 7 & 3 & 2 & 6 & 0 \\ -3 & -1 & 6 & 4 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

no

4. b) $\left[\begin{array}{cccc|c} 3 & 4 & 3 & -1 & 0 \\ 3 & 1 & 6 & 2 & 0 \\ 1 & 0 & 5 & 7 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 0 & -17/4 & 0 \\ 0 & 1 & 0 & 5/4 & 0 \\ 0 & 0 & 1 & 9/4 & 0 \end{array}\right]$

no

5. a) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 1 & 2 & 2 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ yes

4.3 6-9, 15, 16a), T/F bcf

4.4 3, 5, 7a, 10, 15, 19, T/F abc

b) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ yes

6. $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & k & 1 & 0 \\ k & 1 & 3 & 0 \end{array}\right]$ $R_3 \to -R_1 + R_3$, $R_4 \to -kR_1 + R_4$

$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & k+1 & -1 & 0 \\ 0 & k+1 & -2k+3 & 0 \end{array}\right]$

7. a) $\left[\begin{array}{ccc|c} 2 & 6 & 2 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 4 & -4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ no

b) $\left[\begin{array}{ccc|c} -6 & 3 & 4 & 0 \\ 7 & 2 & -1 & 0 \\ 2 & 4 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -1/3 & 0 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ yes

8. a) $\left[\begin{array}{ccc|c} -1 & 2 & -3 & 0 \\ 2 & -4 & 6 & 0 \\ 3 & -6 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ no

b) $\left[\begin{array}{ccc|c} 2 & 4 & 2 & 0 \\ -1 & 2 & 7 & 0 \\ 4 & 3 & -6 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ no

c) $\left[\begin{array}{ccc|c} 4 & 2 & -2 & 0 \\ 6 & 3 & -3 & 0 \\ 8 & 4 & -4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0.5 & -0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ yes

9. a) $\left[\begin{array}{ccc|c} 0 & 6 & 4 & 0 \\ 3 & 0 & -7 & 0 \\ 1 & 5 & 1 & 0 \\ -1 & 1 & 3 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -7/3 & 0 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

a) $a = 7/3c$, $b = -2/3c$, c is free

b) $c = 1$, $a = 7/3$, $b = -2/3$

$\vec{v}_3 = -7/3\vec{v}_1 + 2/3\vec{v}_2$

$\vec{v}_1 = 2/7\vec{v}_2 - 3/7\vec{v}_3$

$\vec{v}_2 = 7/2\vec{v}_1 + 3/2\vec{v}_3$

15. a) yes, not linearly dependent at origin

b) yes, also not linearly dependent at origin

16. a) $\sin^2 x + \cos^2 x = 1$

$(-1)(6) + 2(3\sin^2 x) + 3(2\cos^2 x) = 0$

yes, linearly dependent

b) yes, linearly dependent

b) T, definition of linear dependency

c) F, can have set of 2 linearly dependent vectors w/o (0, 0)

f) F

3. $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & -1 & -1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

yes

5. $\left[\begin{array}{cccc|c} 3 & 0 & 0 & 1 & 0 \\ 6 & 1 & -8 & 0 & 0 \\ 3 & -1 & -12 & -1 & 0 \\ -6 & 0 & -4 & 2 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$

~~yes~~ only trivial sol.

7. a) $\left[\begin{array}{ccc|c} 2 & 4 & 0 & 0 \\ -3 & 1 & -7 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

not trivial sol, linearly dependent

4.4 10, 15, 19, T/F abc

4.5 1, 3, 5

4.7 9, 10

10. a) $\cos^2 x - \sin^2 x = \cos 2x$, so vectors are not linearly independent

b) $\{\cos^2 x, \sin^2 x\}$

15. $\begin{array}{cccc|c} a & b & c & d & \\ \hline \end{array}$ $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$

$a = 1 \quad a+b=0$

$a+b+c=1 \quad a+b+c+d=0 \quad 1+b=0$

$1+(-1)+c=1 \quad 1+(-1)+1+d=0 \quad b=-1$

$c=1 \quad d=-1$

$A = A_1 - A_2 + A_3 - A_4$

19. a) $\vec{u}_3 = \vec{u}_1 + \vec{u}_2$, linear dependence

b) span plane, not all of $\mathbb{R}^3$

c) cannot express things like $p=1$

d) all 0s on top right, can't express every number

T/F a) F, also needs to be linearly independent

b) F, also needs to span V

c) T, def. of basis

1. $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -2 & -1 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 - x_3 = 0 \quad x_2 = 0$

$x_1 = x_3$

dim = 1

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

3. $\left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 0 & 5 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

no basis, dim = 0

5. $\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 2 & -6 & 2 & 0 \\ 3 & -9 & 3 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 = 3x_2 - x_3$

dim = 2

$\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

9. a) $\left[\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 5 & -4 & -4 & 0 \\ 7 & -6 & 2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -16 & 0 \\ 0 & 1 & -19 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

null space: $\left\{ \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix} \right\}$

row space: $\{(1\ 0\ -16)\ (0\ 1\ -19)\}$

b) $\left[\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 4 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

null space: $\left\{ \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

row space: $\{(1\ 0\ -1/2)\}$

10. a) $\left[\begin{array}{cccc|c} 1 & 4 & 5 & 2 & 0 \\ 2 & 1 & 3 & 0 & 0 \\ -1 & 3 & 2 & 2 & 0 \end{array}\right] \rightarrow \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2/7 & 0 \\ 0 & 1 & 1 & 4/7 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$

$a + c - 2/7\, d = 0 \quad b + c + 4/7\, d = 0$

$a = -c + 2/7\, d \quad b = -c - 4/7\, d$

$\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2/7 \\ -4/7 \\ 0 \\ 1 \end{bmatrix} \right\}$

$$\text{b)} \left[\begin{array}{ccccc|c} 1 & 4 & 5 & 6 & 9 & 0 \\ 3 & -2 & 1 & 4 & -1 & 0 \\ -1 & 0 & -1 & -2 & -1 & 0 \\ 2 & 3 & 5 & 7 & 8 & 0 \end{array}\right] \to \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$a + c + 2d + e = 0$ $\qquad$ $b + c + d + 2e = 0$

$a = -c - 2d - e$ $\qquad$ $b = -c - d - 2e$

$$\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$