Chapter 2

Smith Chart and Impedance Matching

Trinh Xuan Dung, PhD
dung.trinh@hcmut.edu.vn

Department of Telecommunications
Faculty of Electrical and Electronics Engineering
Ho Chi Minh city University of Technology

Contents

1. **Introduction**
2. **Smith Chart**
3. **Smith Chart Applications**
4. **Y Smith Chart**
5. **Impedance Matching**

Problems

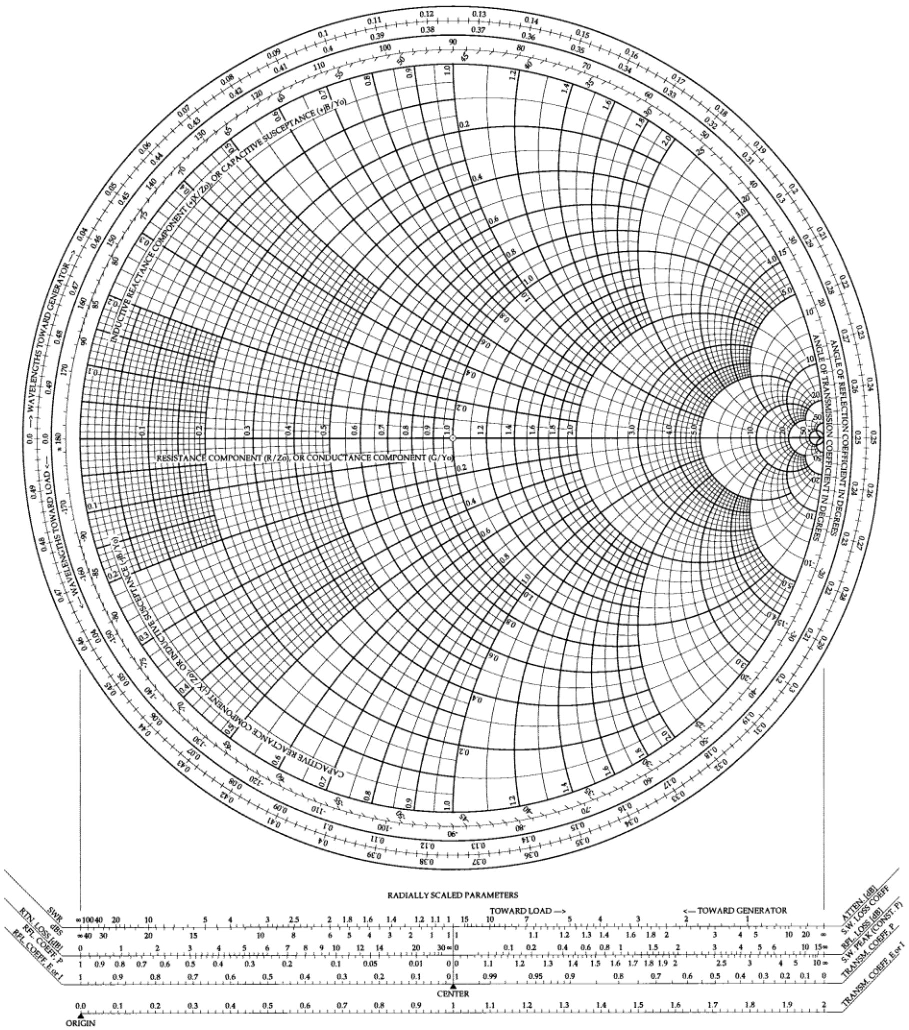
1. Introduction

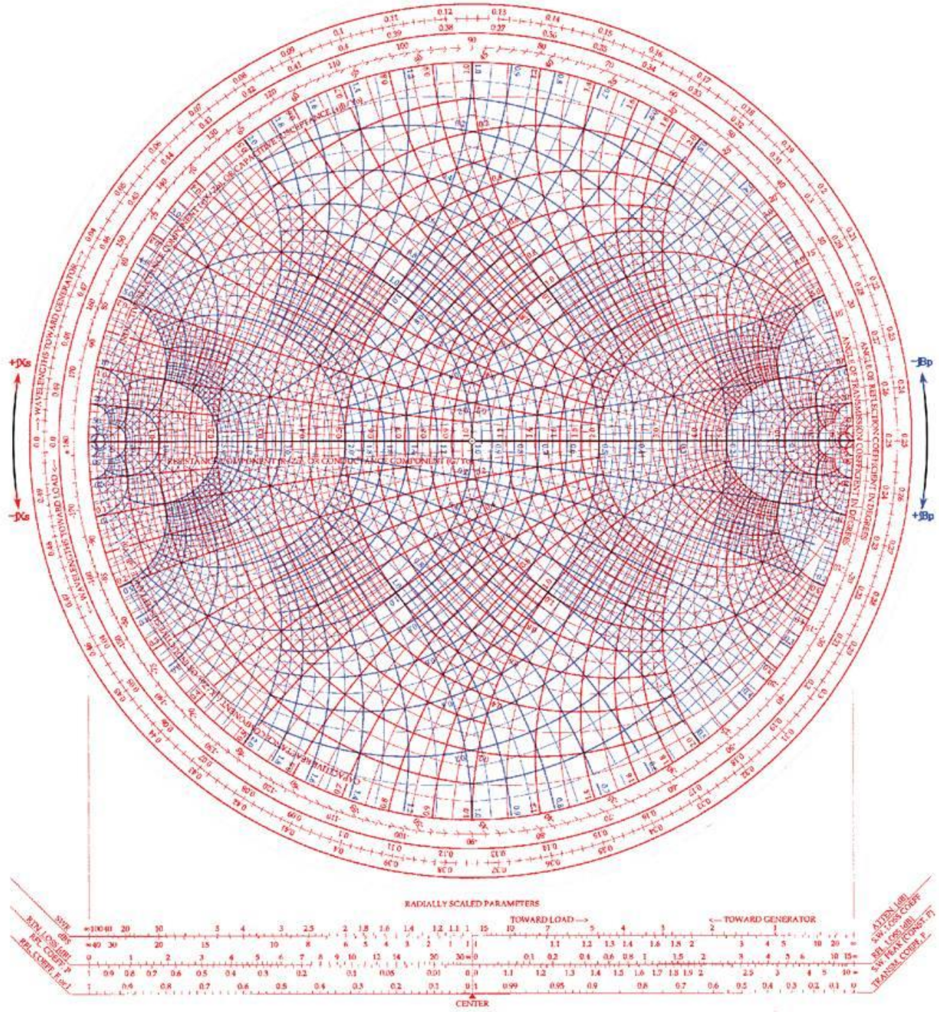
- Many of calculations required to solve T.L. problems involve the use of complicated equations.
- Smith Chart, developed by Phillip H. Smith in 1939, is a graphical aid that can be very useful for solving T.L. problems.
- The Smith chart, however, is more than just a graphical technique as it provides a useful way of visualizing transmission line phenomenon without the need for detailed numerical calculations.
- A microwave engineer can develop a good intuition about transmission line and impedance-matching problems by learning to think in terms of the Smith chart.
- From a mathematical point of view, the Smith chart is simply a representation of all possible complex impedances with respect to coordinates defined by the reflection coefficient.
- The domain of definition of the reflection coefficient is a circle of radius 1 in the complex plane. This is also the domain of the Smith chart.

1. Introduction

The Complete Smith Chart

Black Magic Design

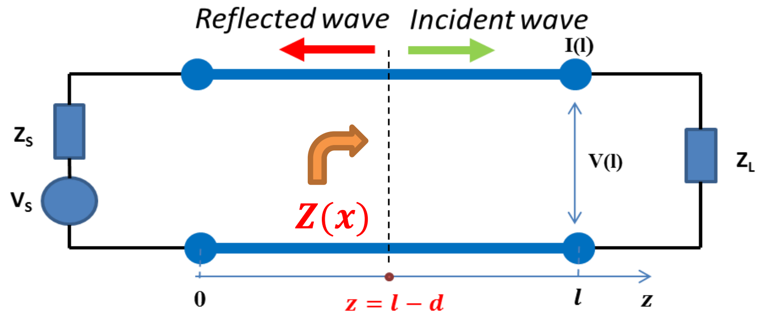




2. Smith Chart

- We start from the general definition of reflection coefficient:

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{Z/Z_0 - 1}{Z/Z_0 + 1} = \frac{z-1}{z+1}$$



- Now z can be written as:

$$z = \frac{1+\Gamma}{1-\Gamma} = \frac{1 + \mathrm{Re}(\Gamma) + j\,\mathrm{Im}(\Gamma)}{1 - \mathrm{Re}(\Gamma) - j\,\mathrm{Im}(\Gamma)} = \frac{1 - \mathrm{Re}^2(\Gamma) - \mathrm{Im}^2(\Gamma) + 2j\,\mathrm{Im}(\Gamma)}{[1 - \mathrm{Re}(\Gamma)]^2 + \mathrm{Im}^2(\Gamma)}$$

where: $z = r + jx$. Then: $r = \dfrac{1 - \mathrm{Re}^2(\Gamma) - \mathrm{Im}^2(\Gamma)}{[1 - \mathrm{Re}(\Gamma)]^2 + \mathrm{Im}^2(\Gamma)}$ $\quad x = \dfrac{2\,\mathrm{Im}(\Gamma)}{[1 - \mathrm{Re}(\Gamma)]^2 + \mathrm{Im}^2(\Gamma)}$

- These equations can be re-arranged into:

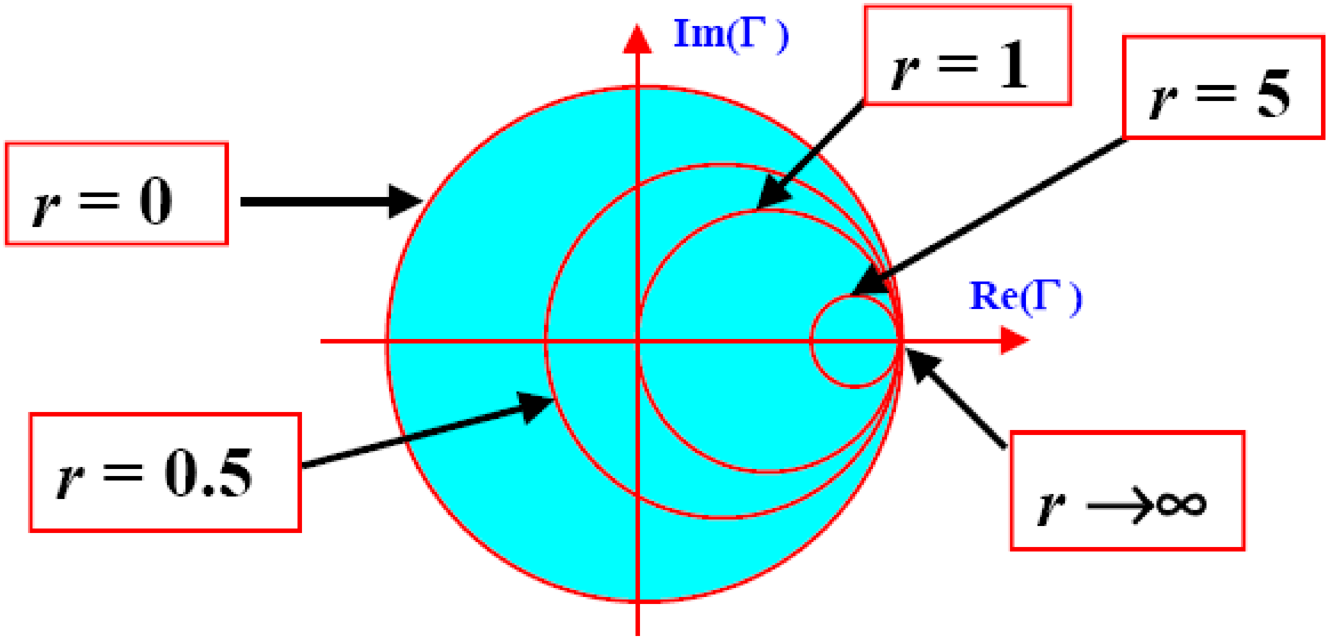
$$\left(\mathrm{Re}(\Gamma) - \frac{r}{1+r}\right)^2 + \mathrm{Im}^2(\Gamma) = \left(\frac{1}{1+r}\right)^2$$

$$(\mathrm{Re}(\Gamma) - 1)^2 + \left(\mathrm{Im}(\Gamma) - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

2. Smith Chart

$$\left(\mathrm{Re}(\Gamma) - \frac{r}{1+r}\right)^2 + \mathrm{Im}^2(\Gamma) = \left(\frac{1}{1+r}\right)^2$$: Resistance circles

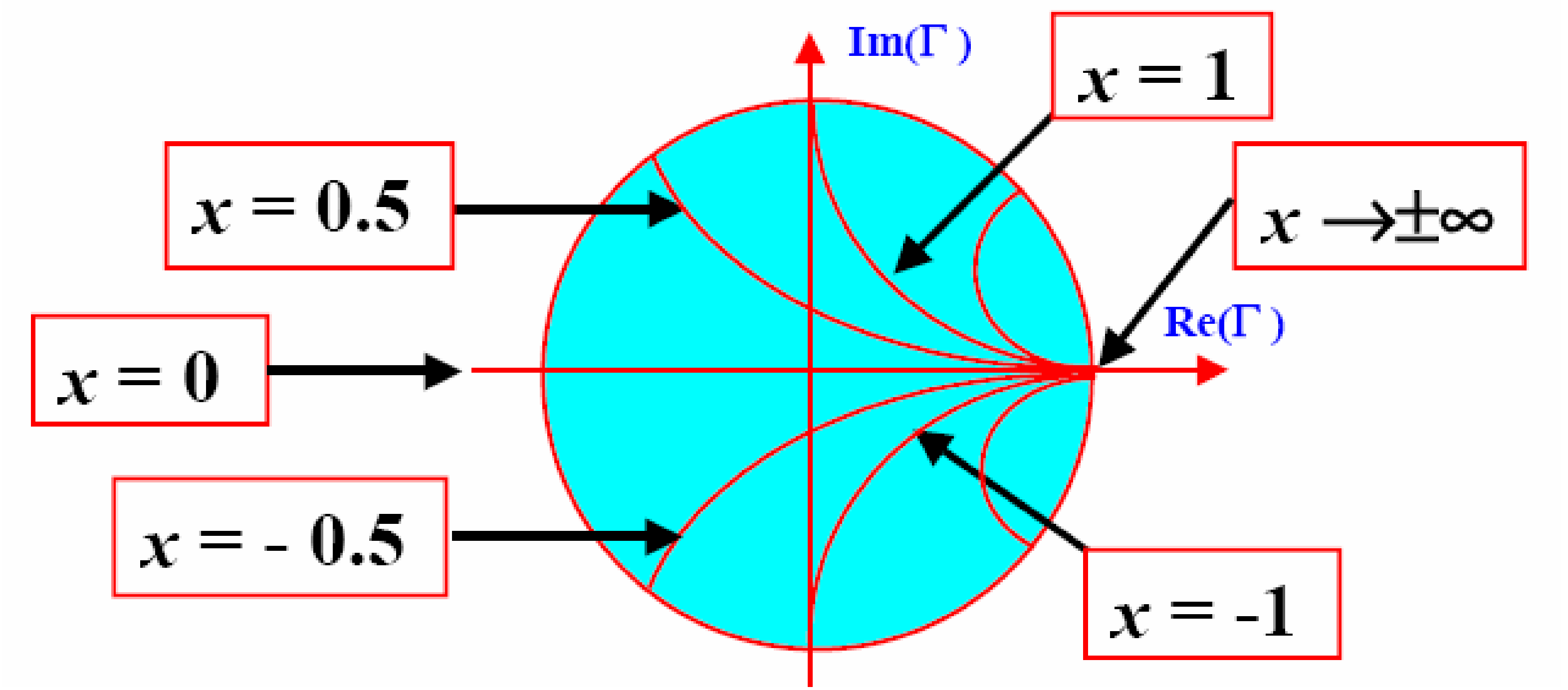
- Center: $\left(\frac{r}{1+r}, 0\right)$
- Radius: $\frac{1}{1+r}$



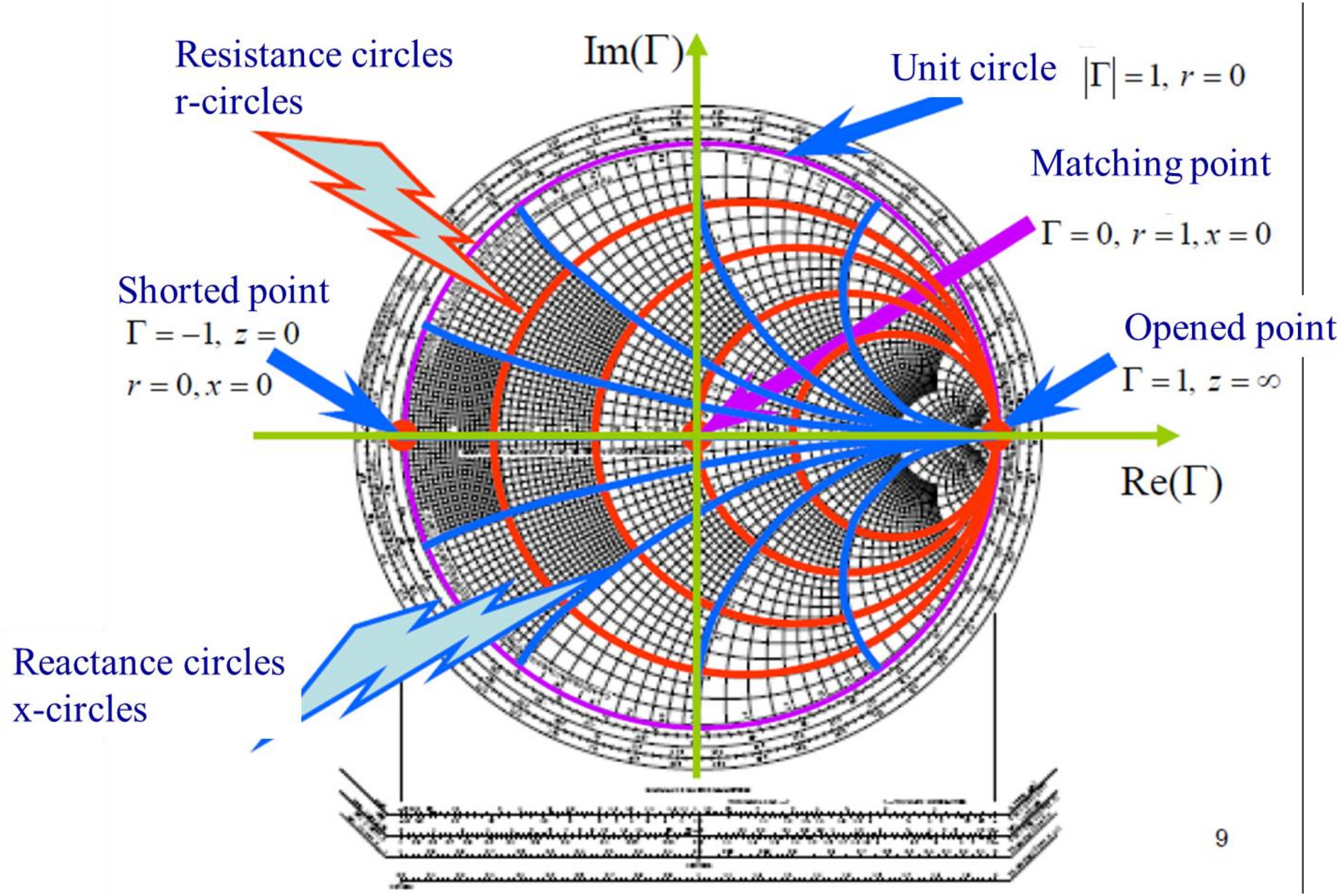
2. Smith Chart

$$(\mathrm{Re}(\Gamma)-1)^2+\left(\mathrm{Im}(\Gamma)-\frac{1}{x}\right)^2=\left(\frac{1}{x}\right)^2$$: Reactance circles

- Center: $\left(1,\frac{1}{x}\right)$
- Radius: $\frac{1}{x}$



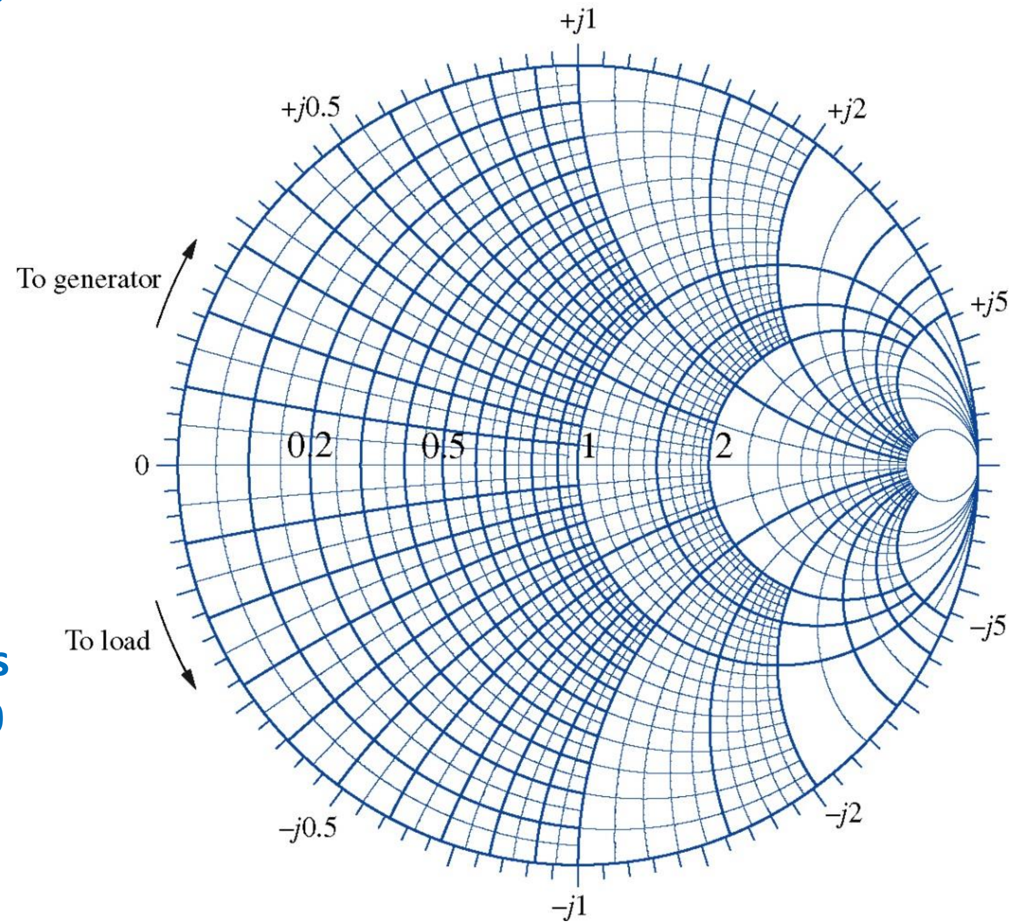
2. Smith Chart



2. Smith Chart

For the constant *r* circles:

1. The centers of all the constant *r* circles are on the horizontal axis – real part of the reflection coefficient.
2. The radius of circles decreases when *r* increases.
3. All constant *r* circles pass through the point $\Gamma_r = 1, \Gamma_i = 0$.
4. The normalized resistance $r = \infty$ is at the point $\Gamma_r = 1, \Gamma_i = 0$.



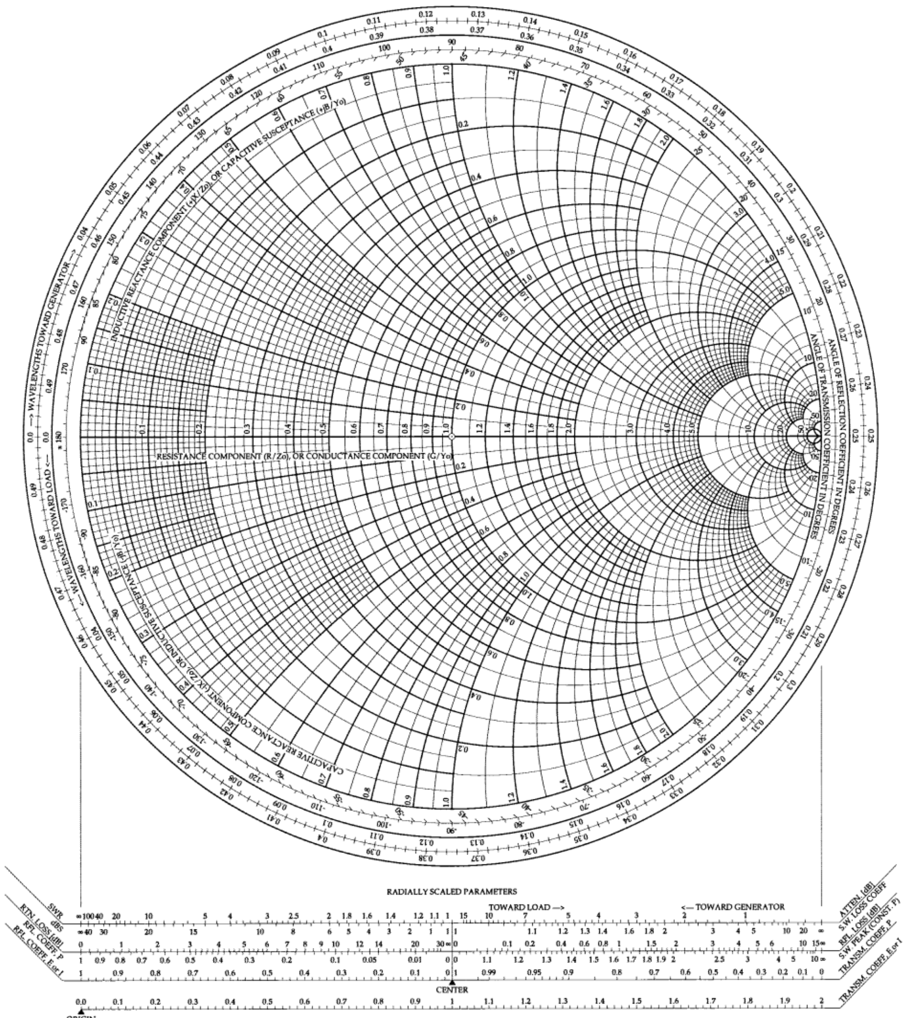
For the constant *x* (partial) circles:

1. The centers of all the constant *x* circles are on the $\Gamma_r = 1$ line. The circles with $x > 0$ (inductive reactance) are above the Γ_r axis; the circles with $x < 0$ (capacitive) are below the Γ_r axis.
2. The radius of circles decreases when absolute value of *x* increases.
3. The normalized reactances $x = \pm\infty$ are at the point $\Gamma_r = 1, \Gamma_i = 0$

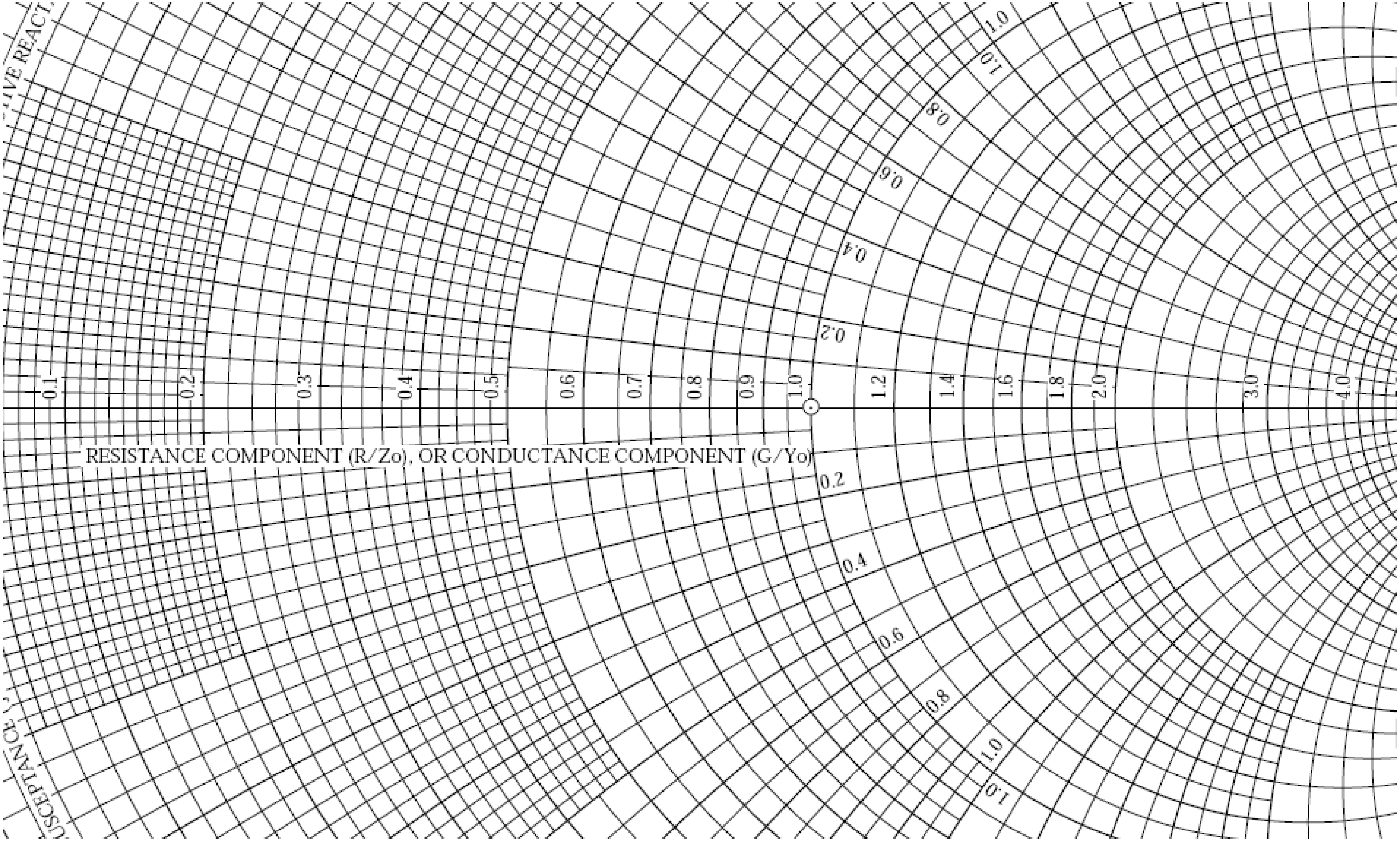
2. Smith Chart

The Complete Smith Chart

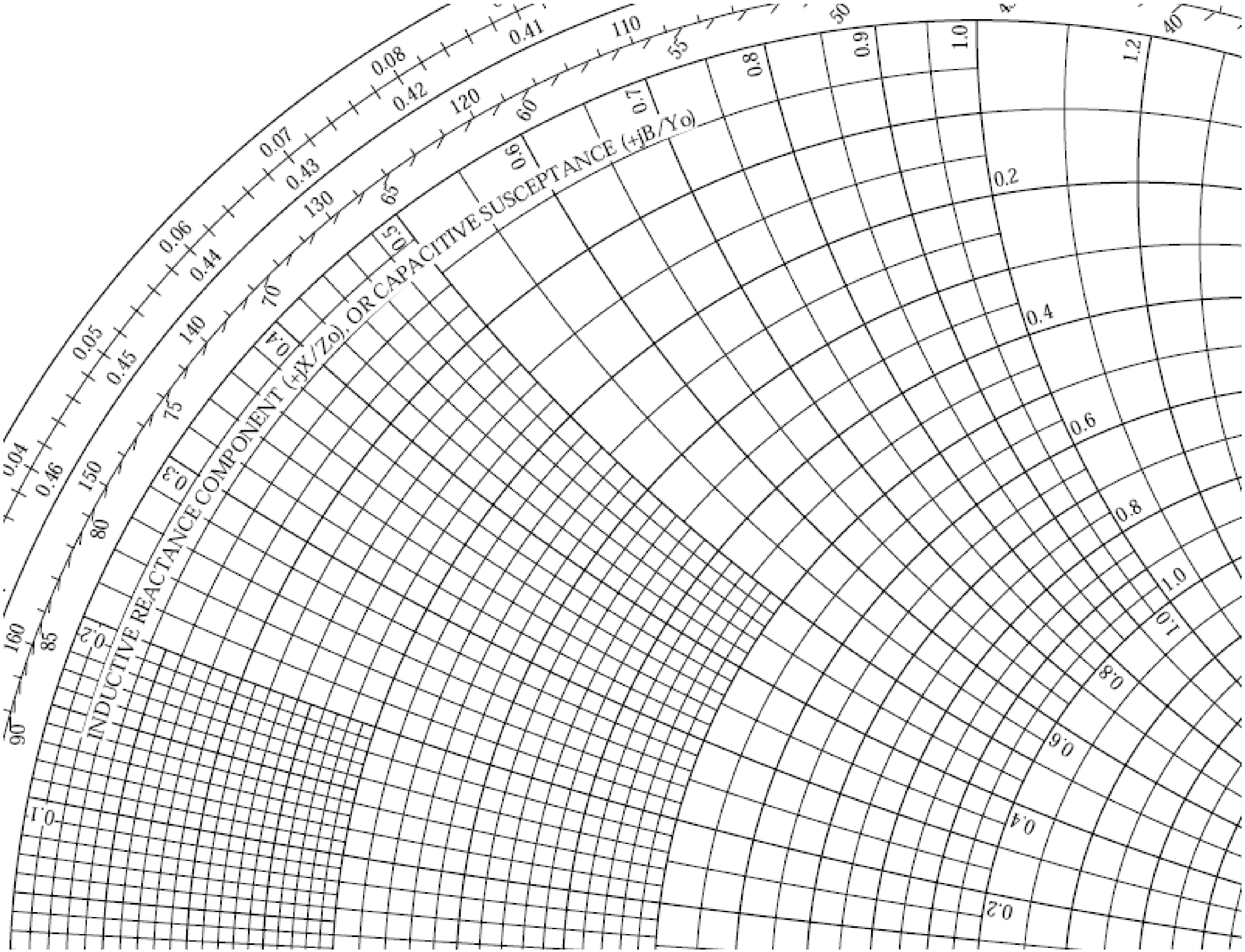
Black Magic Design



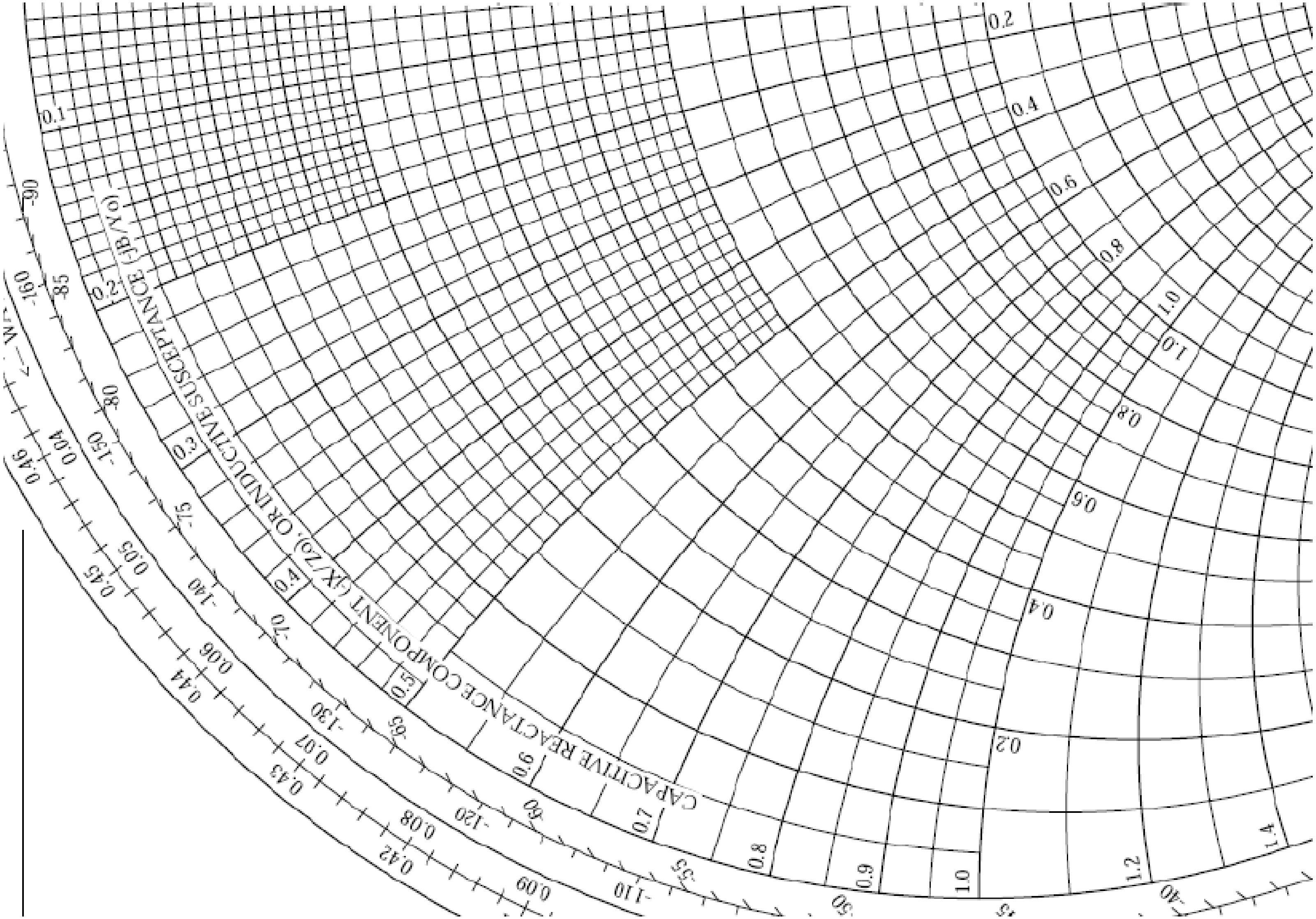
2. Smith Chart



2. Smith Chart

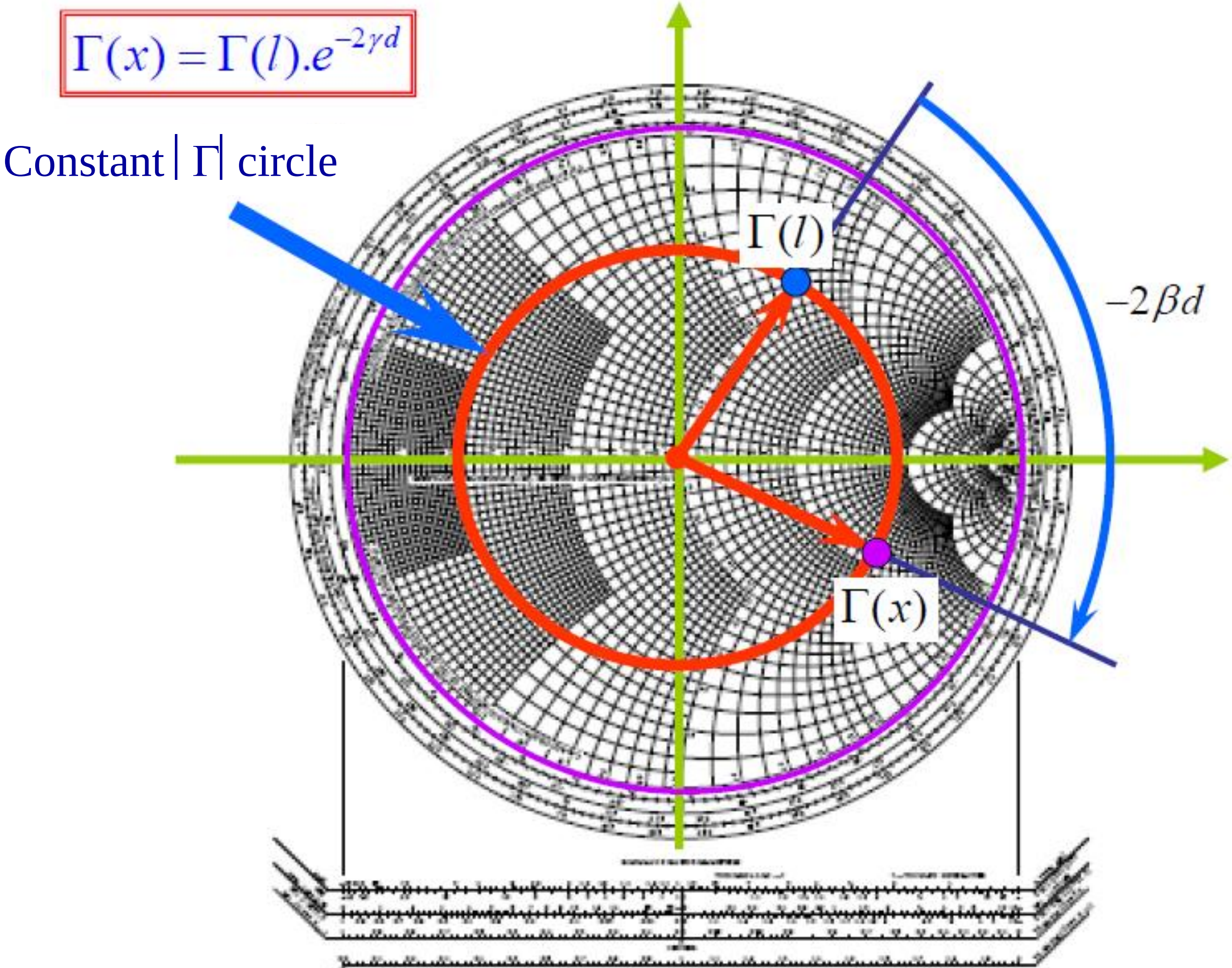


2. Smith Chart

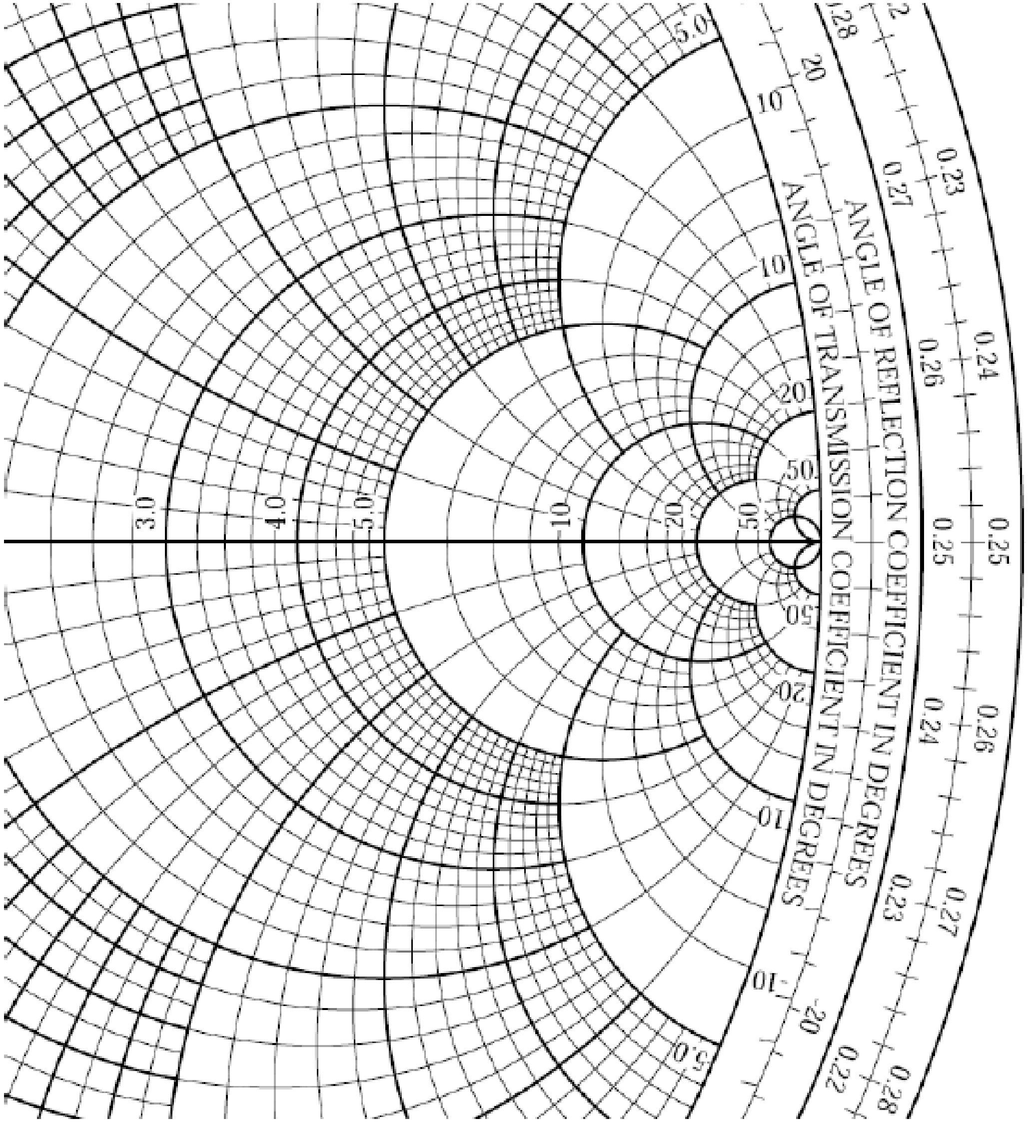


2. Smith Chart

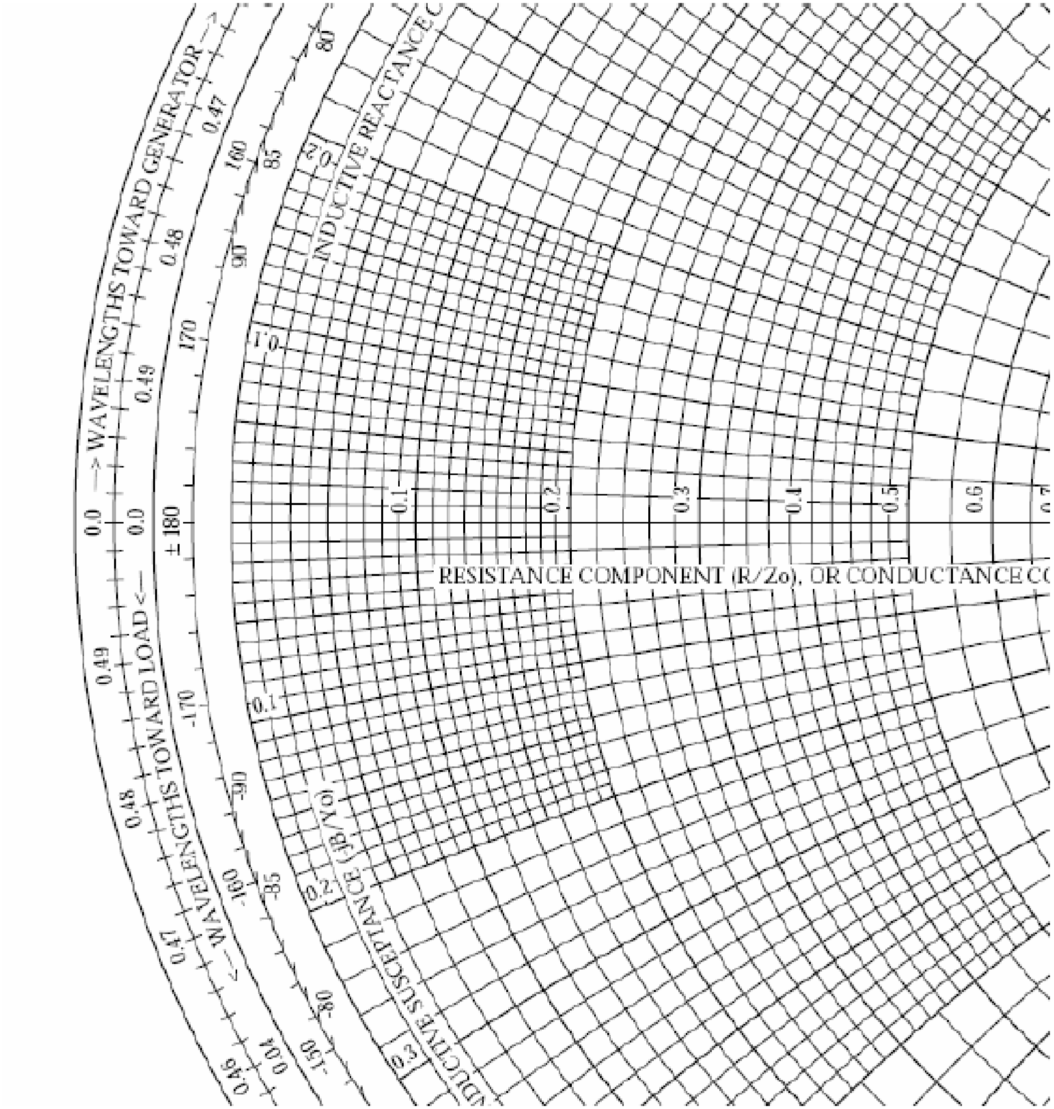
$$\boxed{\Gamma(x) = \Gamma(l).e^{-2\gamma d}}$$



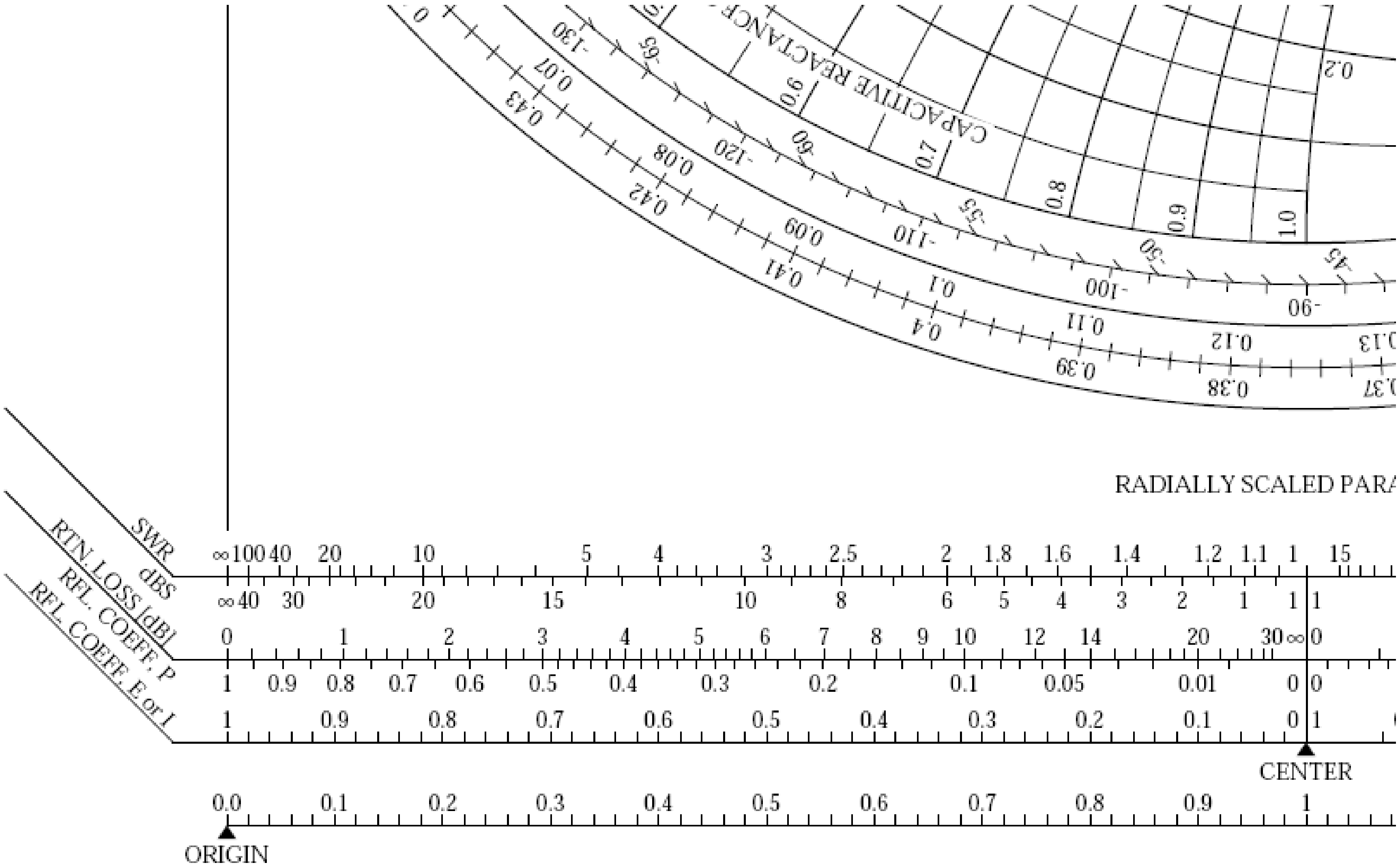
2. Smith Chart



2. Smith Chart



2. Smith Chart



3. Smith Chart Applications

A. Given $Z(d)$, find $\Gamma(d)$ or Given $\Gamma(d)$, find $Z(d)$.

B. Given Γ_L *and* Z_L, find $\Gamma(d)$ and $Z(d)$.
Given $\Gamma(d)$ and $Z(d)$, find Γ_R *and* Z_R.

C. Find d_{max} and d_{min} (maximum and minimum locations for the VSW pattern).

D. Find the VSWR.

E. Given $Z(d)$, find $Y(d)$ or Given $Y(d)$, find $Z(d)$.

3. Smith Chart Applications

A. Given $Z(d)$, find $\Gamma(d)$

1. Normalize the impedance:

$$z(d) = \frac{Z(d)}{Z_0} = \frac{R}{Z_0} + j\frac{X}{Z_0} = r + jx$$

2. Find the circle of constant normalized resistance r.
3. Find the circle of constant normalized reactance x.
4. Find the interaction of the two curves indicates the reflection coefficient in the complex plane. The chart provides directly magnitude and the phase angle of $\Gamma(d)$.

Example 1: Find $\Gamma(d)$ given $Z(d) = 25 + j100\Omega$ and $Z_0 = 50\Omega$

3. Smith Chart Applications

1. Normalization

$$z(d) = (25 + j\,100)/50 = 0.5 + j\,2.0$$

2. Find normalized resistance circle

$$r = 0.5$$

3. Find normalized reactance arc

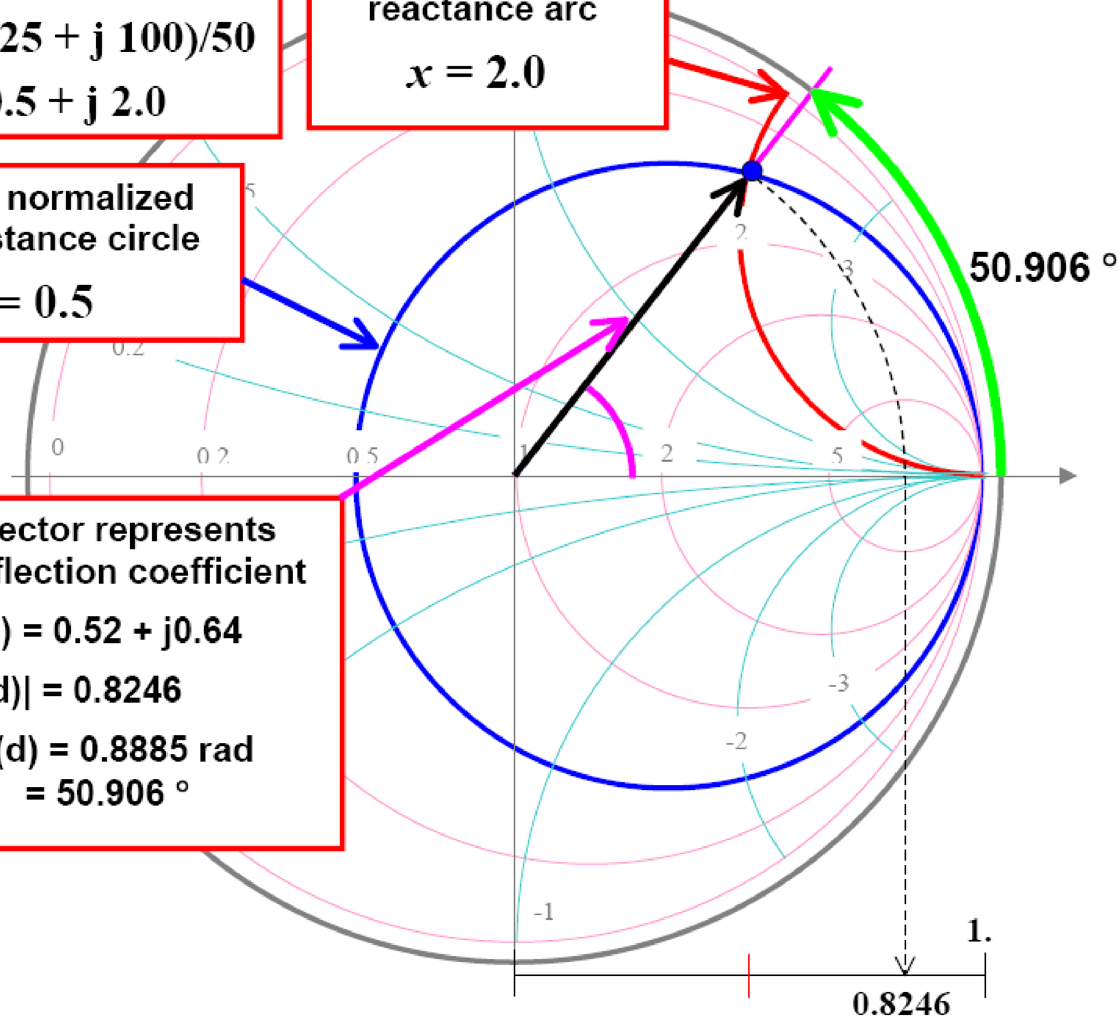
$$x = 2.0$$

4. This vector represents the reflection coefficient

$$\Gamma(d) = 0.52 + j0.64$$

$$|\Gamma(d)| = 0.8246$$

$$\angle\Gamma(d) = 0.8885 \text{ rad} = 50.906\,°$$



3. Smith Chart Applications

A. Given $\Gamma(d)$, find $Z(d)$

1. Determine the complex point representing the given reflection coefficient $\Gamma(d)$ on the chart.
2. Read the value of normalized resistance r and the normalized reactance x that correspond to the reflection coefficient point.
3. The normalized impedance is: $z(d) = r + jx$
4. The actual impedance is: $Z(d) = z(d)Z_0$

3. Smith Chart Applications

B. Given Γ_L *and* Z_L, find $\Gamma(d)$ and $Z(d)$

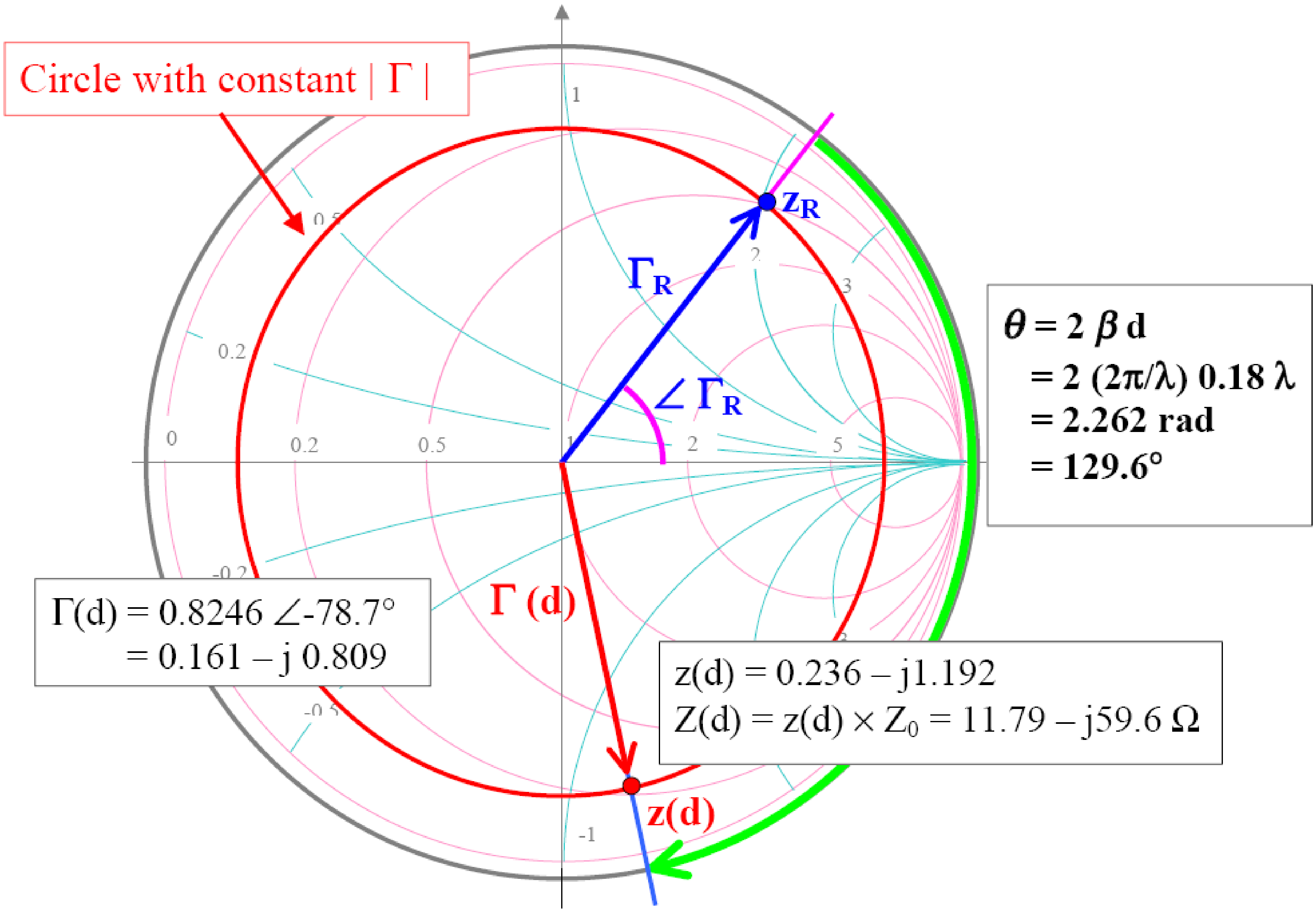
The magnitude of the reflection coefficient is constant along a lossless T.L. terminated by a specific load, since:

$$|\Gamma(d)| = \left|\Gamma_L e^{-j2\beta d}\right| = |\Gamma_L|$$

1. Identify the load reflection coefficient Γ_L and the normalized load impedance Z_L on the Smith Chart.
2. Draw the circle of constant coefficient amplitude $|\Gamma(d)| = |\Gamma_L|$
3. Starting from the point representing the load, travel on the circle in the *clockwise* direction by an angle $\theta = 2\beta d$.
4. The new location on the chart corresponds to location d on the T.L. Here the value of $\Gamma(d)$ and $Z(d)$ can be read from the chart.

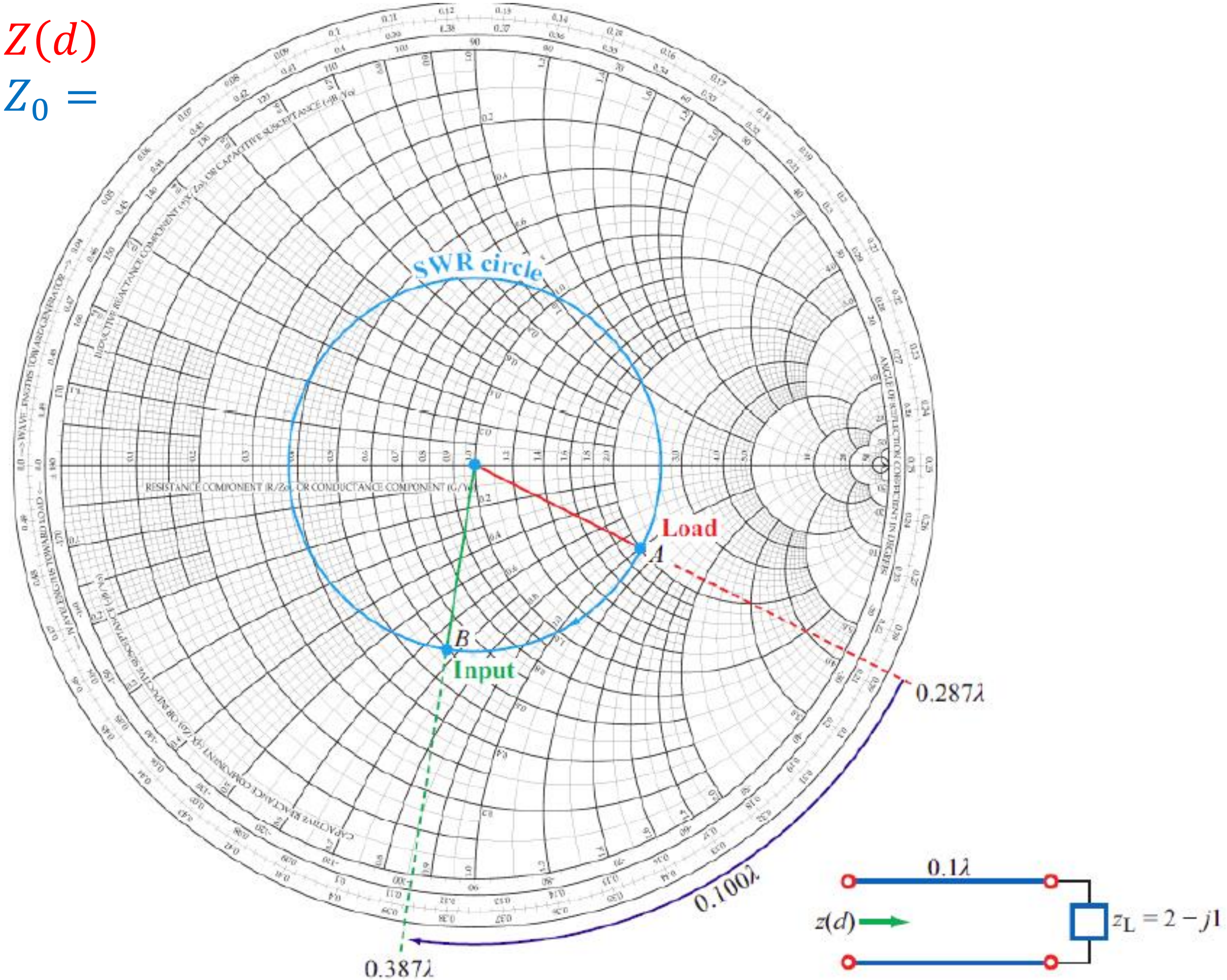
Example: Find $\Gamma(d)$ and $Z(d)$ given $Z_L = 25 + j100\Omega$, $Z_0 = 50\Omega$ and $d = 0.18\lambda$

3. Smith Chart Applications



3. Smith Chart Applications

Example 3: Find $\Gamma(d)$ and $Z(d)$ given $Z_R = 100 - j50\Omega$, $Z_0 = 50\Omega$ and $d = 0.1\lambda$



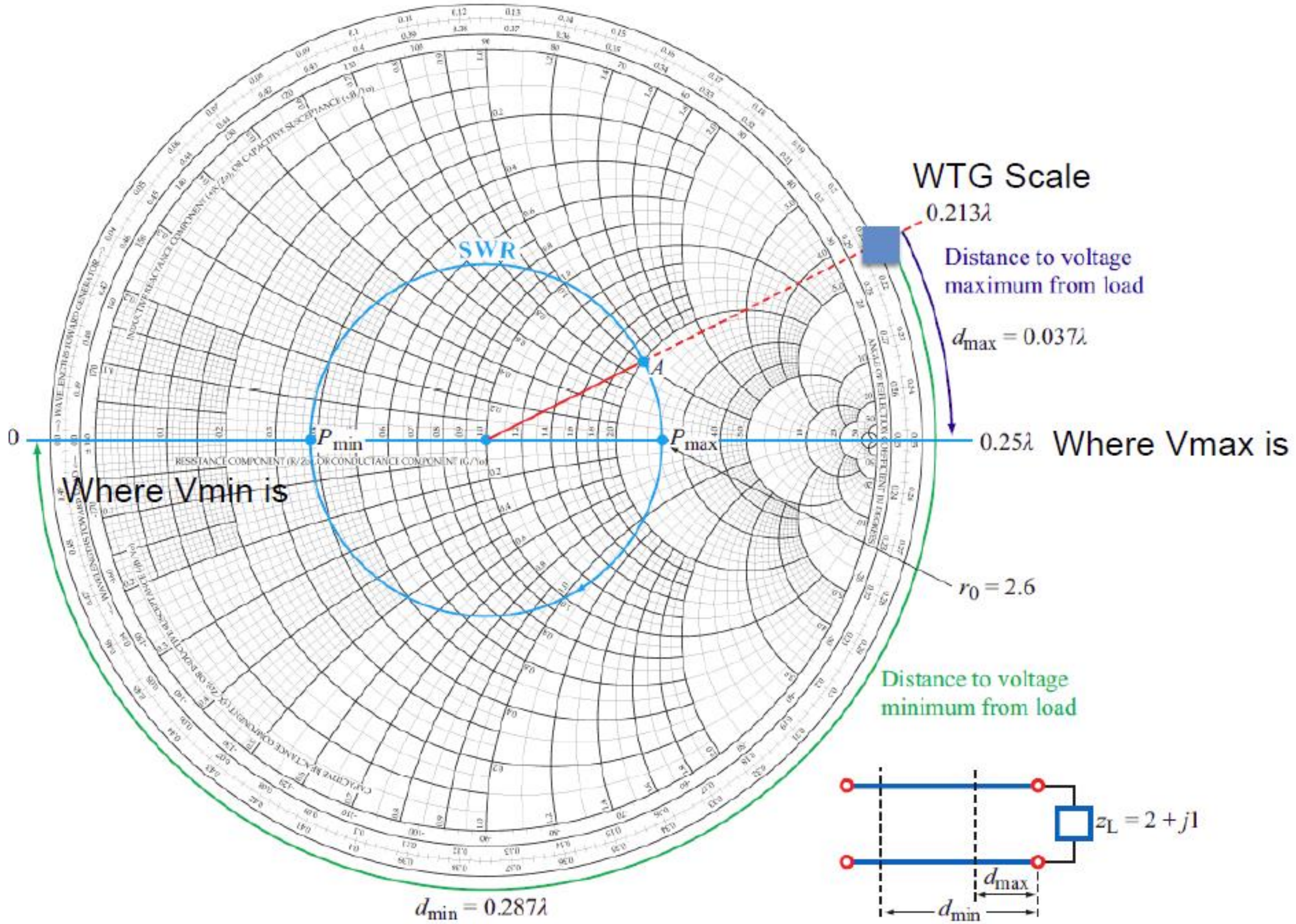
3. Smith Chart Applications

C. Given Γ_L *and* Z_L, find d_{max} and d_{min}

1. Identify the load reflection coefficient Γ_L and the normalized load impedance Z_L on the Smith Chart.
2. Draw the circle of constant coefficient amplitude $|\Gamma(d)| = |\Gamma_L|$
3. The circle intersects the real axis of the reflection coefficient at two points which identify d_{max} (when $\Gamma(d)$ = real positive) and d_{min} (when $\Gamma(d)$ = real negative).
4. The Smith chart provides an outer graduation where the distances normalized to the wavelength can be read directly.

Example 4: Find d_{max} and d_{min} for $Z_L = 100 + j50\Omega$, $Z_0 = 50\Omega$ and $d = 0.18\lambda$

3. Smith Chart Applications



3. Smith Chart Applications

D. Given Γ_L *and* Z_L, find $VSWR$

The VSWR is defined as: $$VSWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

The normalized impedance at the maximum location of the SW pattern is given by:

$$z(d_{max}) = \frac{1+\Gamma(d_{max})}{1-\Gamma(d_{max})} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = VSWR$$

This quantity is always real and greater than 1. The VSWR is simply obtained on the Smith Chart by reading the value of real normalized impedance at the location d_{max} where Γ is real and positive.

Example 5: Find VSWR for $Z_L = 25 \pm j100\Omega$, $Z_0 = 50\Omega$.

3. Smith Chart Applications

E. Given $Z(d)$, find $Y(d)$

❖ The normalized impedance and admittance are defined as:

$$z(d) = \frac{1+\Gamma(d)}{1-\Gamma(d)} \qquad y(d) = \frac{1-\Gamma(d)}{1+\Gamma(d)}$$

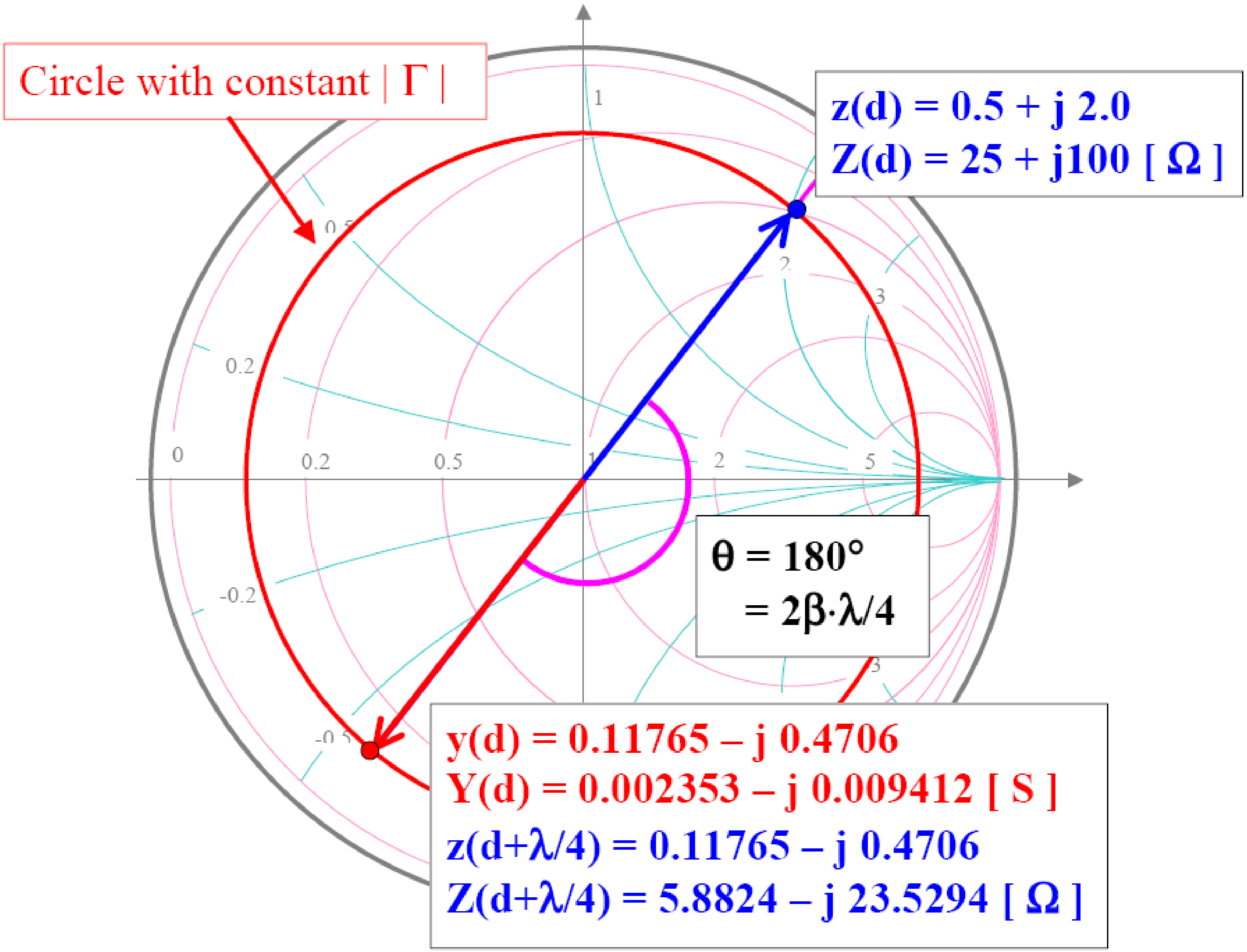
❖ Since: $\Gamma\left(d+\frac{\lambda}{4}\right) = -\Gamma(d) \rightarrow z\left(d+\frac{\lambda}{4}\right) = y(d)$

❖ The actual values are given by:

$$Z\left(d+\frac{\lambda}{4}\right) = Z_0 z\left(d+\frac{\lambda}{4}\right) \qquad Y\left(d+\frac{\lambda}{4}\right) = Y_0 y\left(d+\frac{\lambda}{4}\right) = \frac{y\left(d+\frac{\lambda}{4}\right)}{Z_0}$$

Example 6: Find Y_L given $Z_L = 25 \pm j100\Omega$, $Z_0 = 50\Omega$.

3. Smith Chart Applications

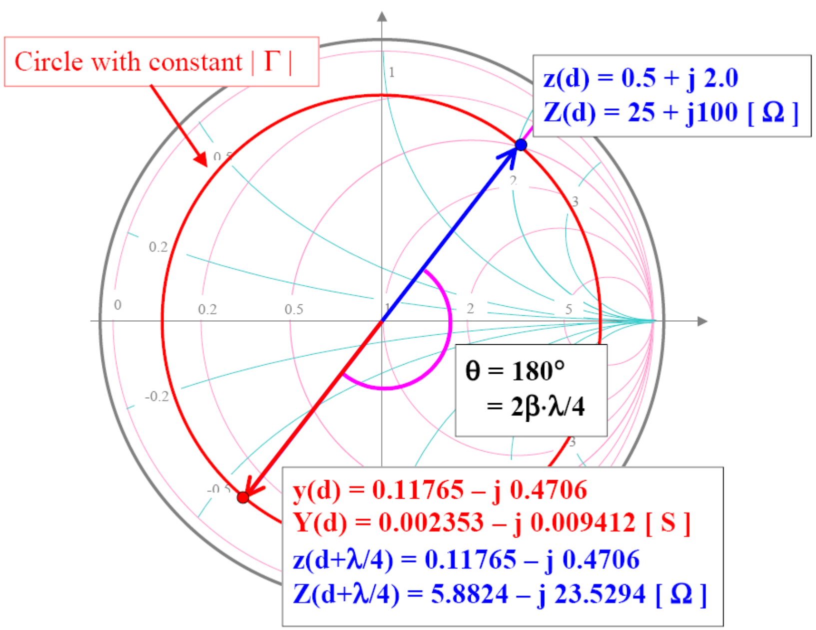


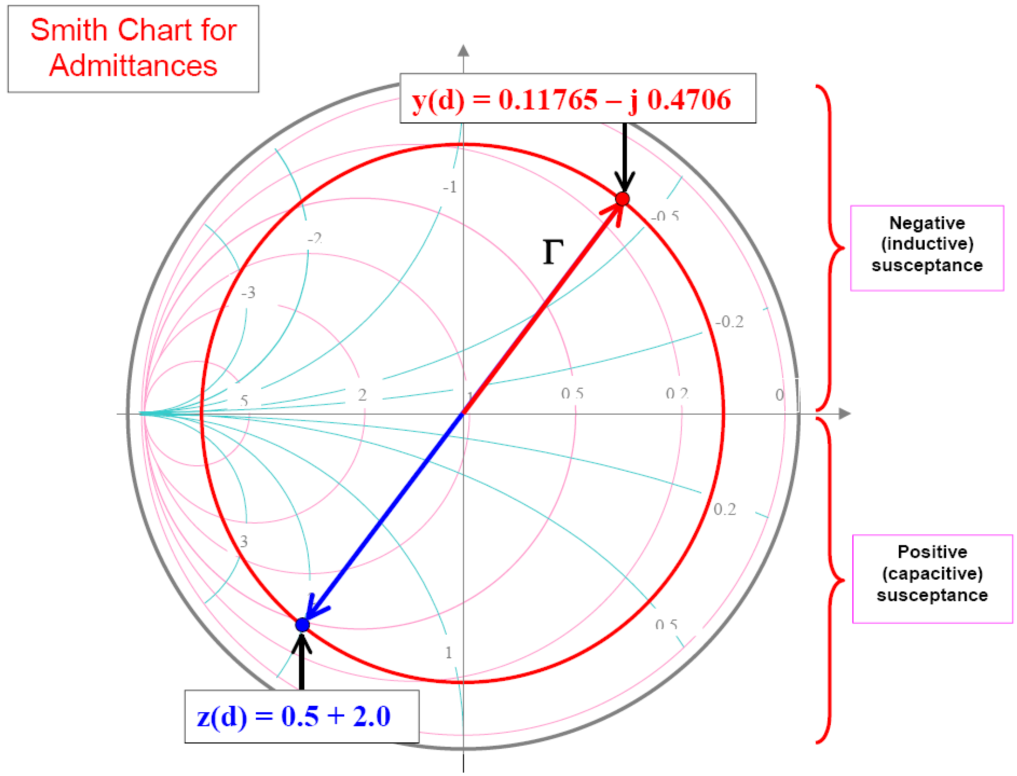
3. Smith Chart: Y Smith Chart

❖ The reflection coefficient is written as: $\Gamma = \dfrac{z-1}{z+1} = \dfrac{\frac{1}{y}-1}{\frac{1}{y}+1} = -\dfrac{y-1}{y+1}$

$$\Gamma = \frac{z-1}{z+1}$$: Z Smith Chart

$$-\Gamma = \frac{y-1}{y+1}$$: Y Smith Chart





3. Smith Chart: Y Smith Chart

$$\Gamma = \frac{z-1}{z+1}$$: Z Smith Chart

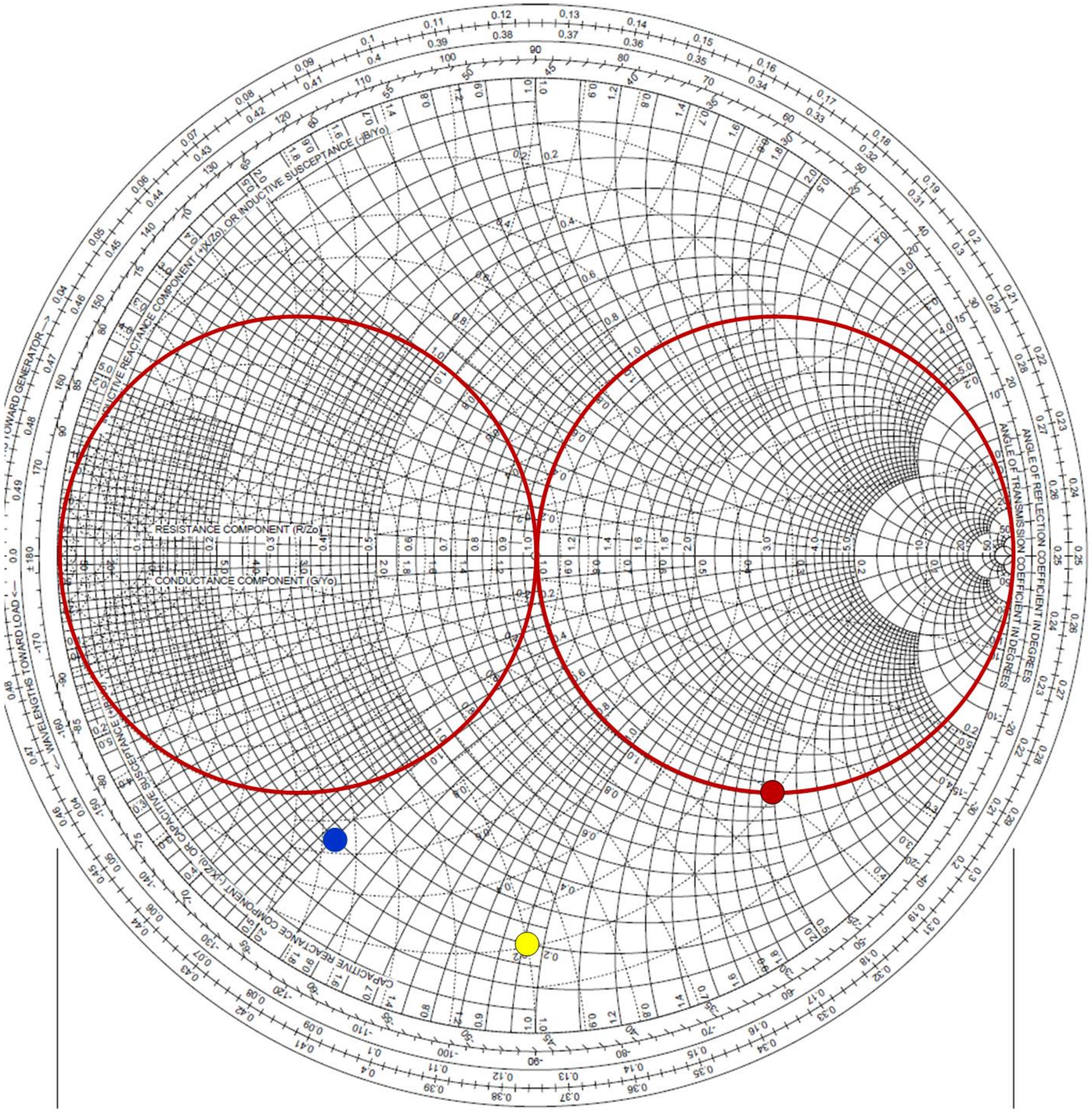
$$-\Gamma = \frac{y-1}{y+1}$$: Y Smith Chart

❖ Since related impedance and admittance are on opposite sides of the same Smith Chart, the imaginary parts always have different sign. Numerically we have:

$$z = r + jx \qquad y = g + jb = \frac{1}{z}$$

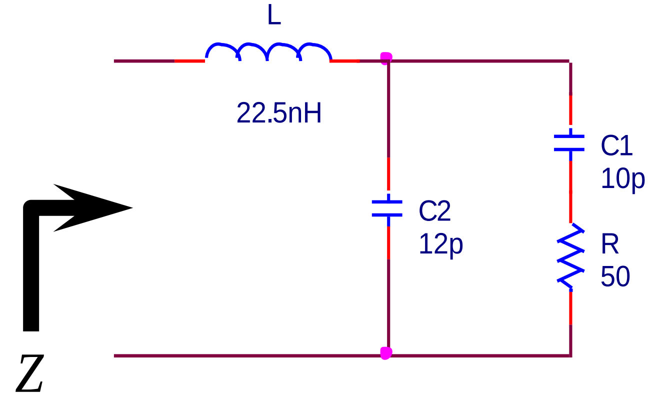
$$g = \frac{r}{r^2 + x^2}$$

$$b = -\frac{x}{r^2 + x^2}$$

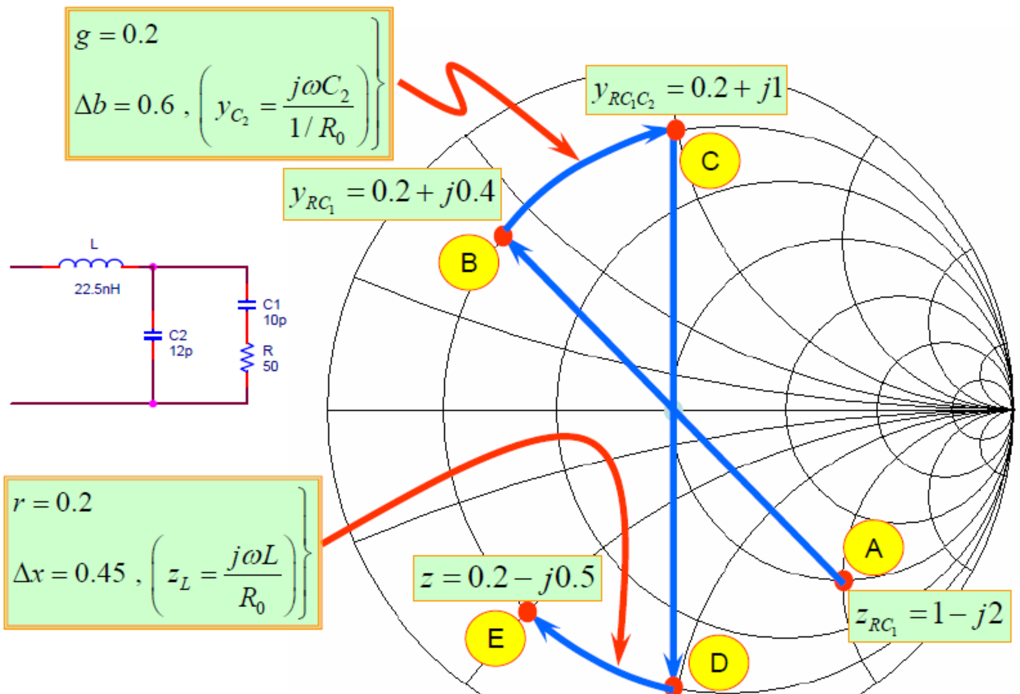


3. Smith Chart Applications

Example 7: Find impedance of a complex circuit using Smith Chart where $R_0 = 50\Omega$ and $\varpi = 10^9\ rad/s$.

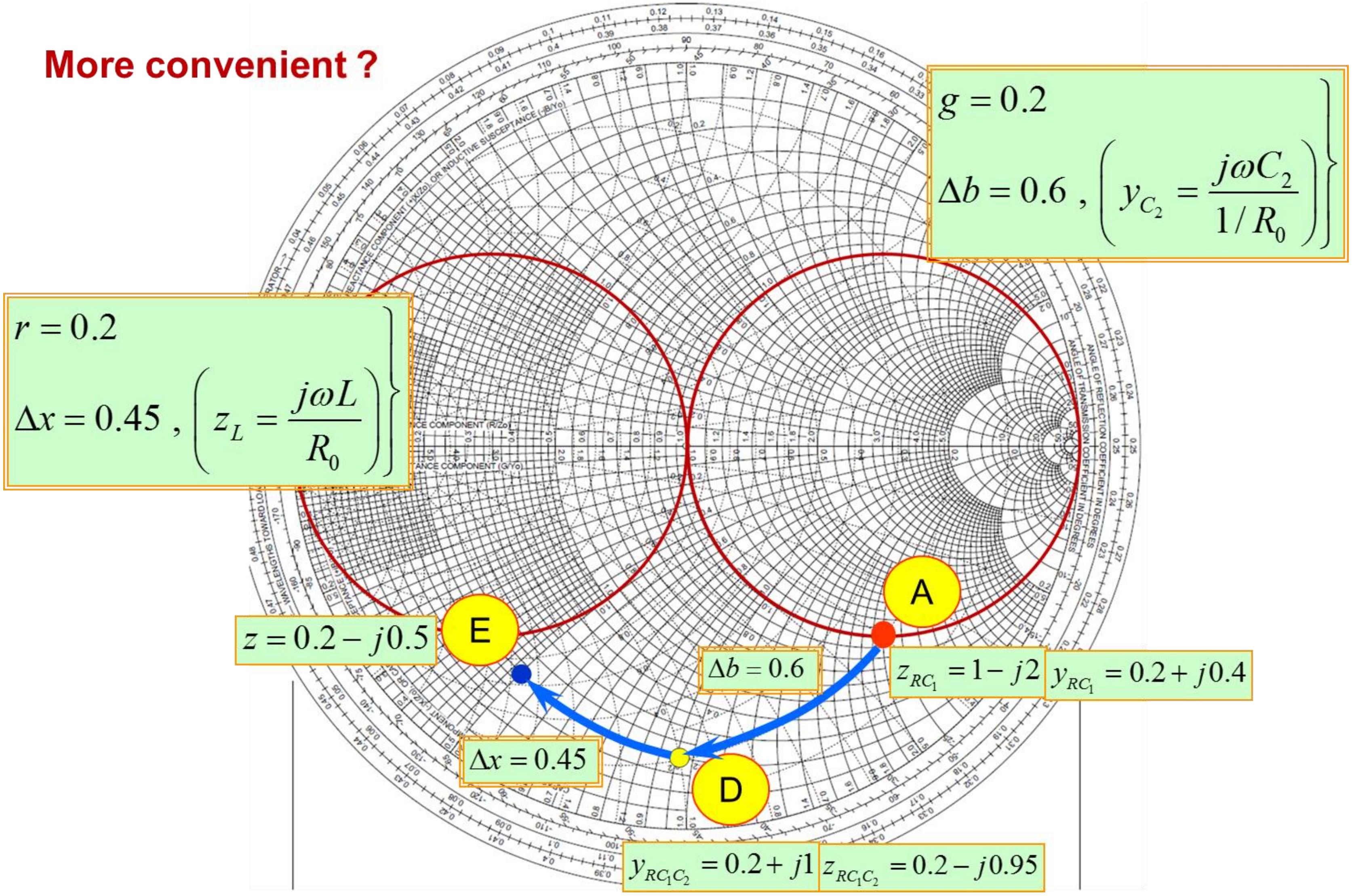


$$z_{RC_1} = \frac{R + 1/\,j\varpi C_1}{R_0} = 1 - j2$$



$$\boxed{\Rightarrow Z = R_0.z = 10 - j25\,(\Omega)}$$

3. Smith Chart Applications



3. Smith Chart Applications

Example 8: A 50Ω lossless T.L. of length 3.3λ is terminated by a load impedance $Z_L = (25 + j50)\Omega$.

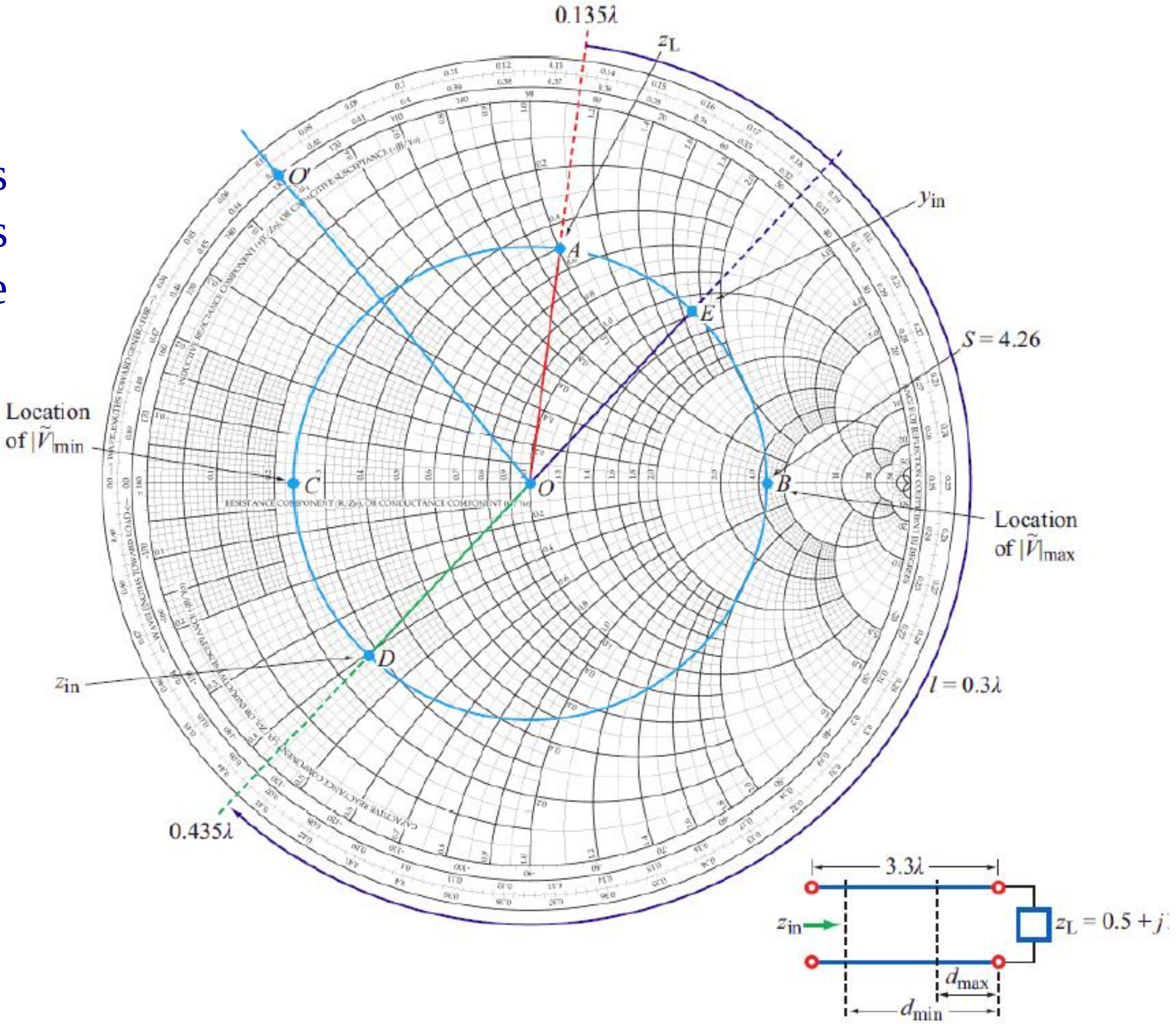
a. Find Γ.

b. Find VSWR.

c. Find d_{max} and d_{min}.

d. Find Z_{in} of T.L.

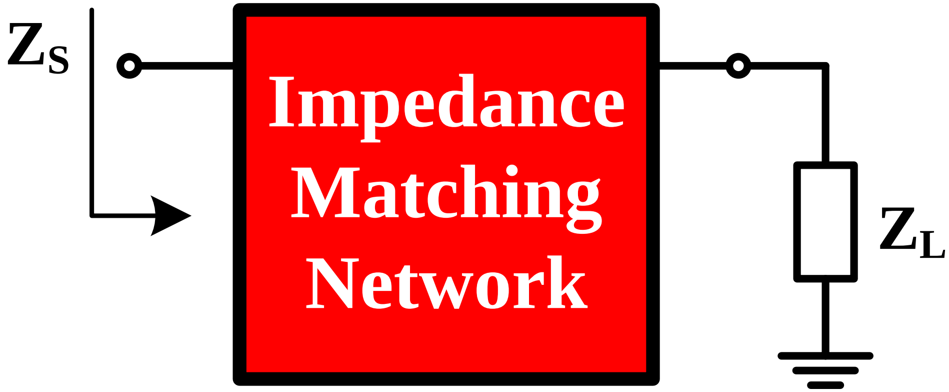
e. Find Y_{in}.



4. Impedance Matching

Maximum power transfer

Impedance Matching



- Using lump elements
- Using transmission lines
- ADS Smith Chart tool

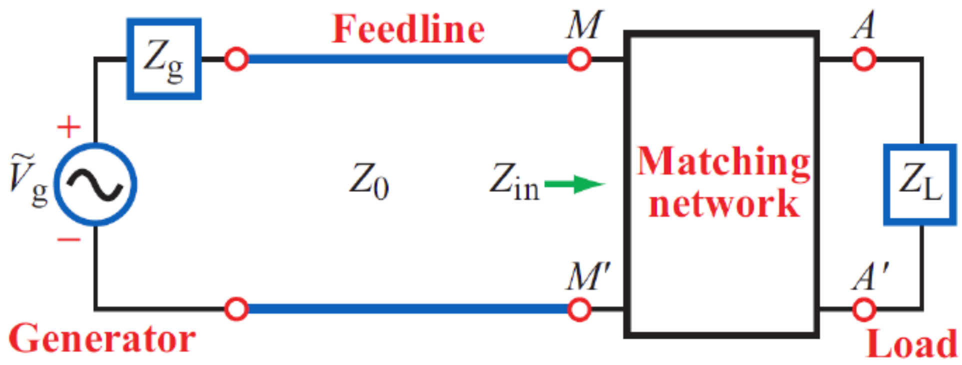
What are Applications ?

- T.L.
- Amplifier Design PA, LNA
- Component Design
- Equipment Interfaces

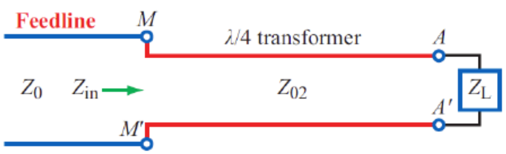
- Matching with Lumped Elements
- Single-Stub Matching Networks
- Quarter-wave Transformer

4. Impedance Matching

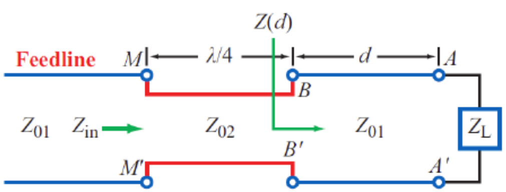
- The purpose of the matching network is to eliminate reflections at terminal MM’ for wave incident from the source. Even though multiple reflections may occur between AA’ and MM’, only a forward travelling wave exists on the feedline.



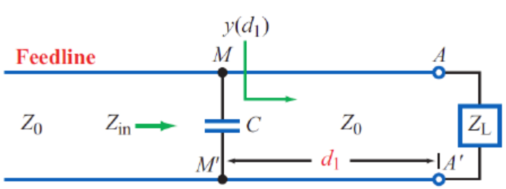
Zin = Z02 bình / ZL = Z0



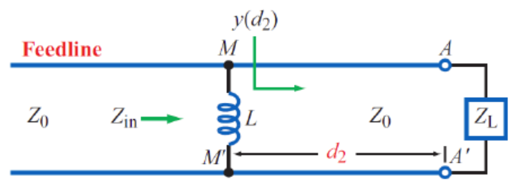
(a) In-series $\lambda/4$ transformer inserted at AA'



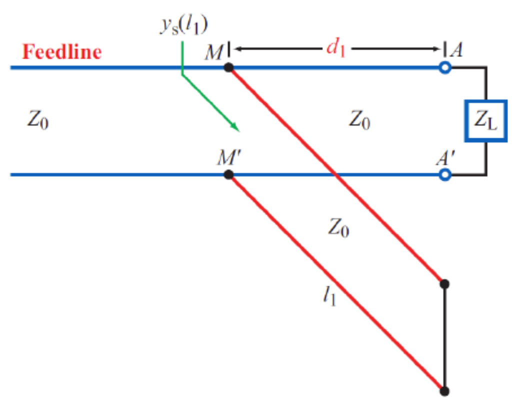
(b) In-series $\lambda/4$ transformer inserted at $d = d_{max}$ or $d = d_{min}$



(c) In-parallel insertion of capacitor at distance d_1



(d) In-parallel insertion of inductor at distance d_2

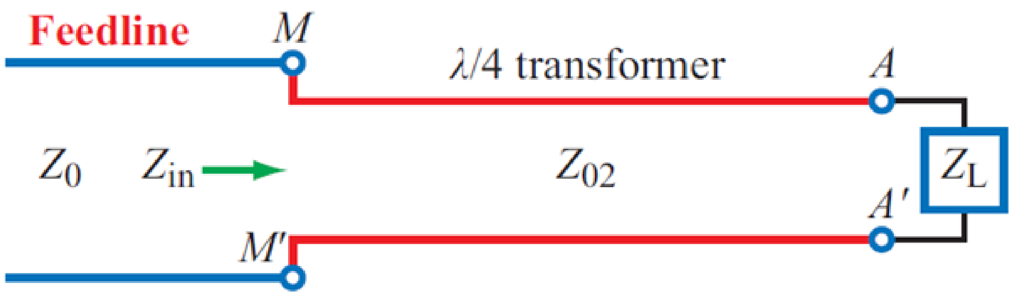


(e) In-parallel insertion of a short-circuited stub

4. Impedance Matching

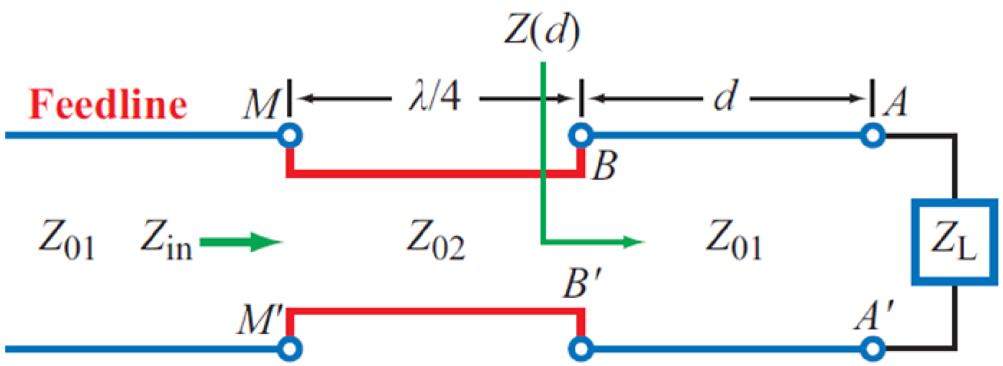
A. Quarter wavelength Transformer Matching:

$Z_0 = 50$



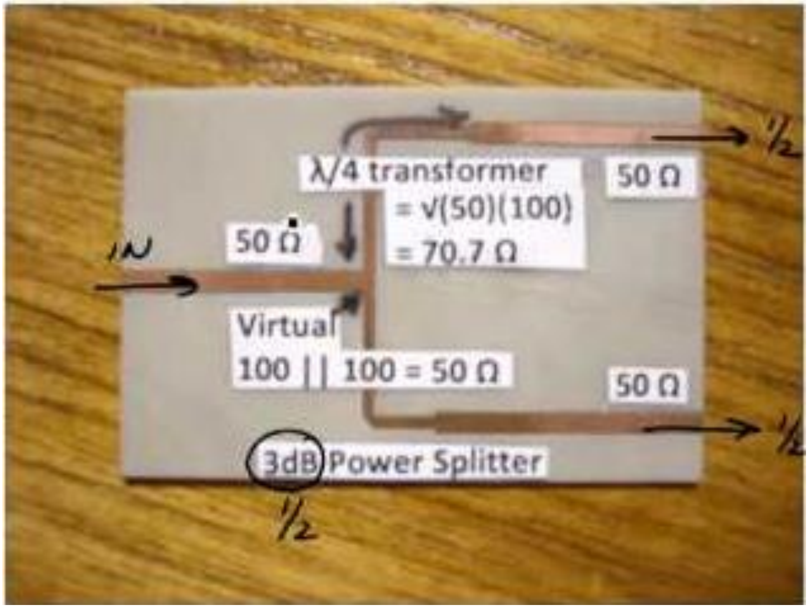
$Z_L = 40\ \Omega$

❖ In case of complex impedance:



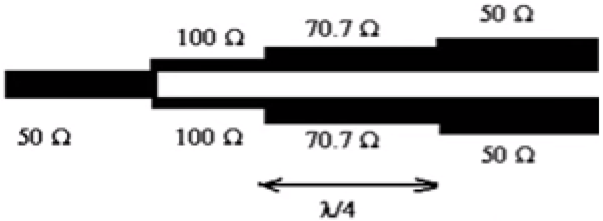
4. Impedance Matching

Example:



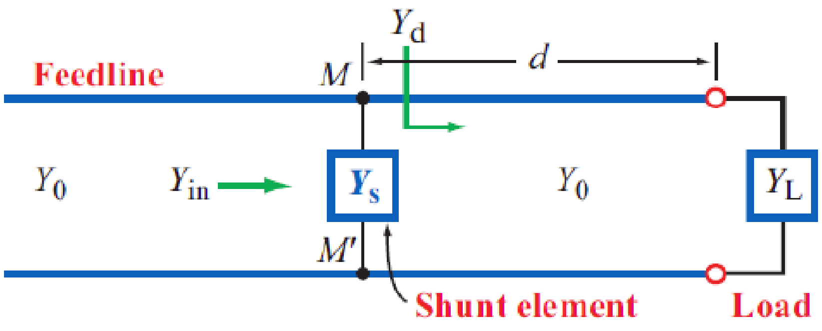
- A simple but accurate equation for Microstrip Characteristic Impedance:

$$Z_0 = \frac{60}{\sqrt{\varepsilon}} \ln\left(\frac{8h}{W} + \frac{W}{4h}\right) \qquad for\, W \le h$$

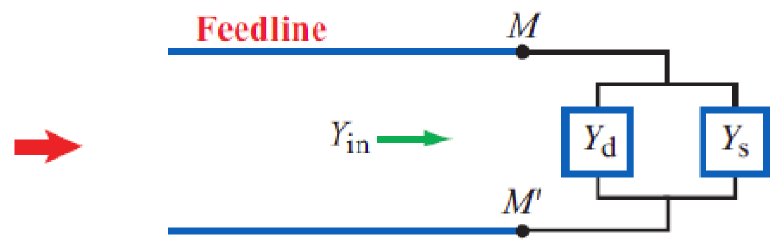


4. Impedance Matching

B. Lumped-Element Matching: choose d and Y_S to achieve a match at MM'.



(a) Transmission-line circuit



(b) Equivalent circuit

- The input admittance at MM' can be written as:

$$Y_{in} = Y_d + Y_s = (G_d + jB_d) + jB_s$$

- To achieve a matched condition at MM', it is necessary that $y_{in} = 1$, which translates into two specific conditions, namely:

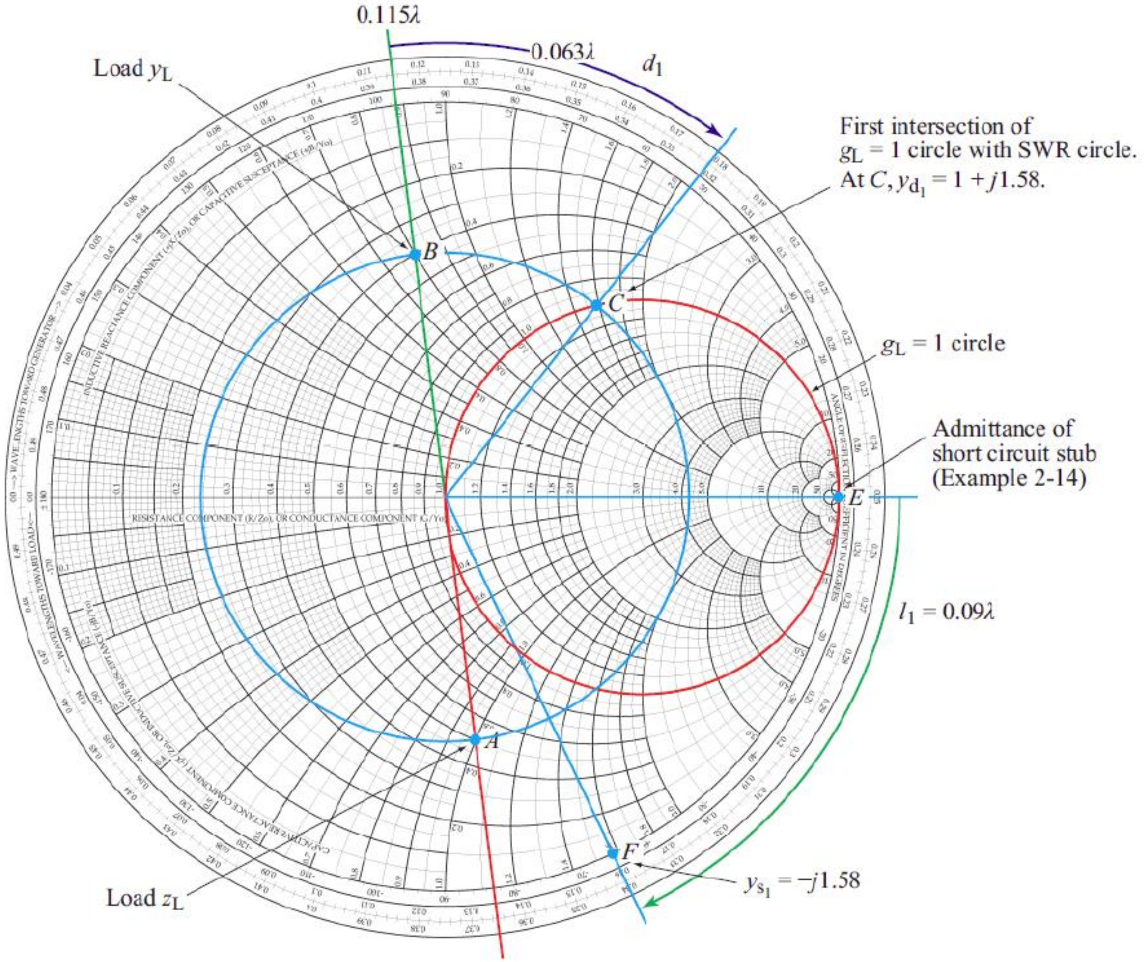
$$g_d = 1$$

$$b_s = -b_d$$

4. Impedance Matching

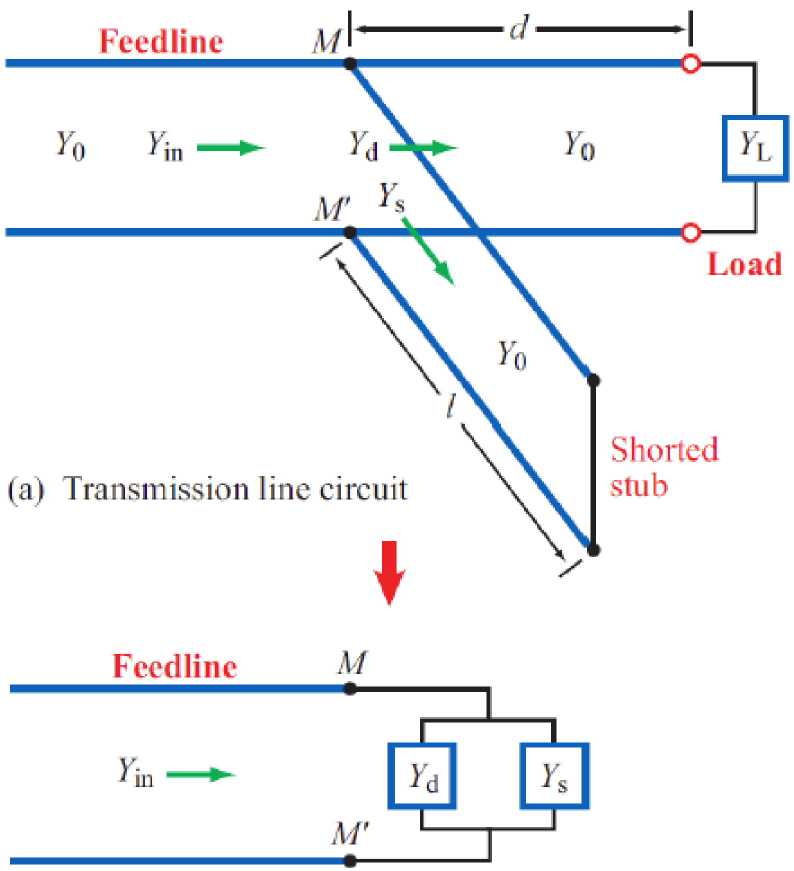
B. Lumped-Element Matching: choose d and Y_S to achieve a match at MM'.

Example 9: A load impedance $Z_L = 25 - j50\Omega$ is connected to a 50Ω T.L. Insert a shunt element to eliminate reflections towards the sending end of the line. Specify the insert location d (in wavelengths), the type of element and its value, given that $f = 100MHz$.



4. Impedance Matching

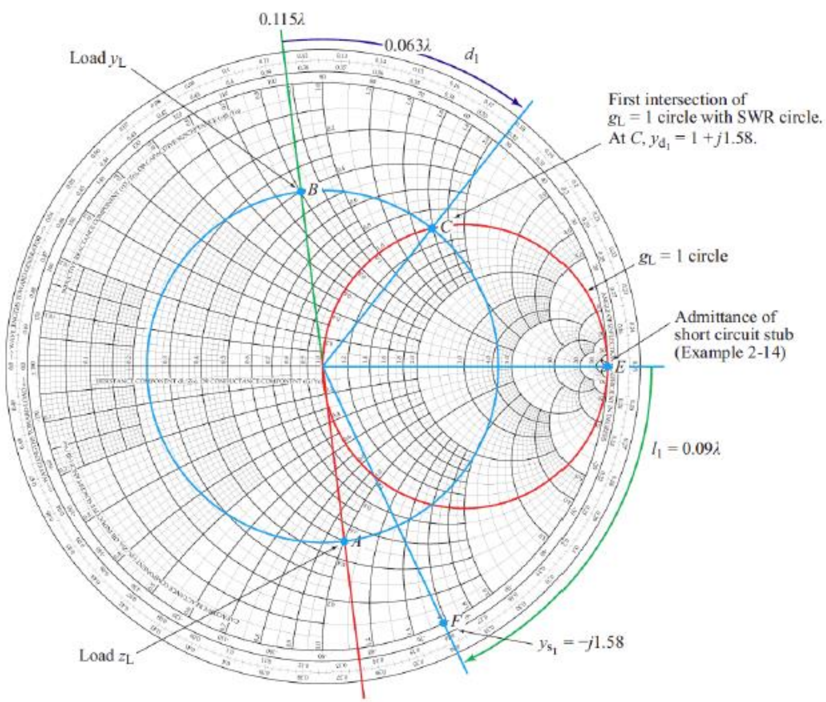
C. Single Stub Matching: choose d and length of stub l to achieve a match at MM'.



(a) Transmission line circuit

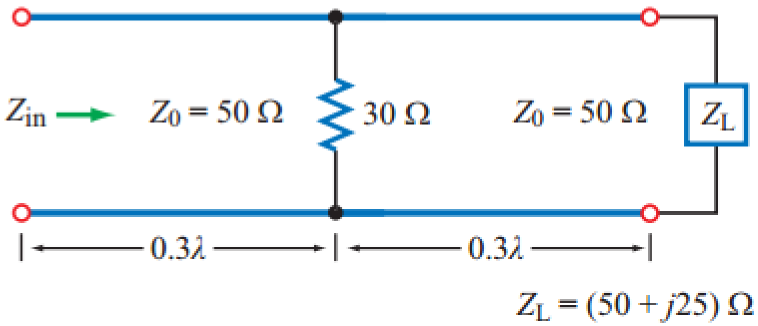
(b) Equivalent circuit

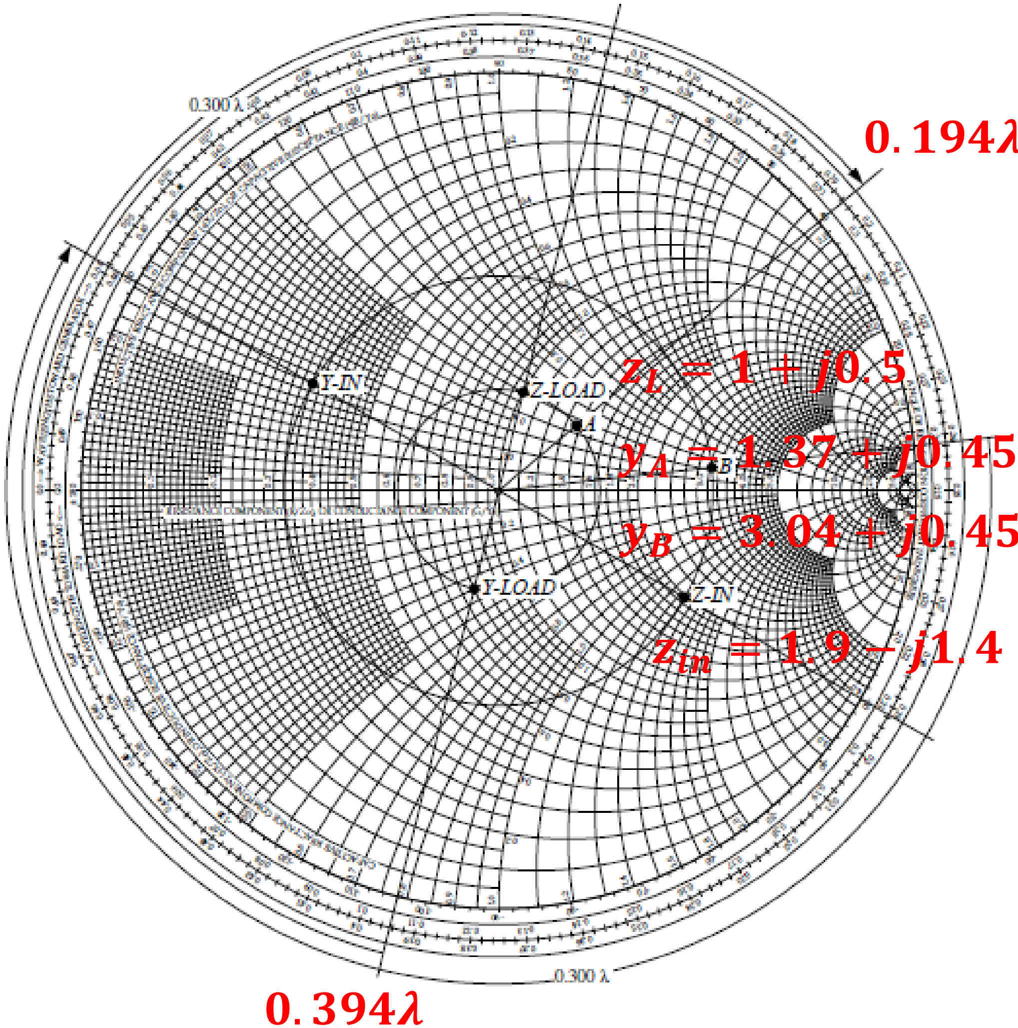
Example 10: Repeat Example 9 but use a shorted stub to match the load impedance.



More Examples

Example 11: A 50Ω lossless line 0.6λ long is terminated in a load with $Z_L = (50 + j25)\Omega$. At 0.3λ from load, a resistor with resistance $R = 30\Omega$ is connected as shown in following figure. Use the Smith Chart to find Z_{in}.

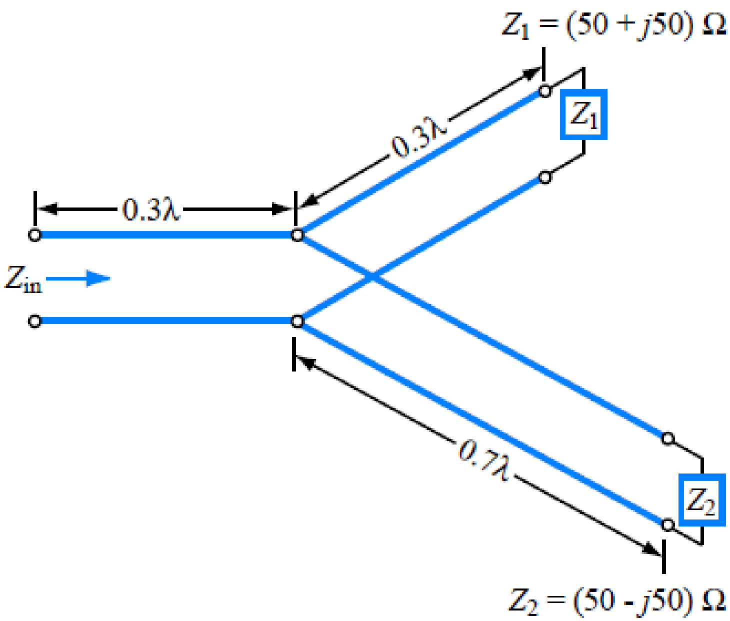


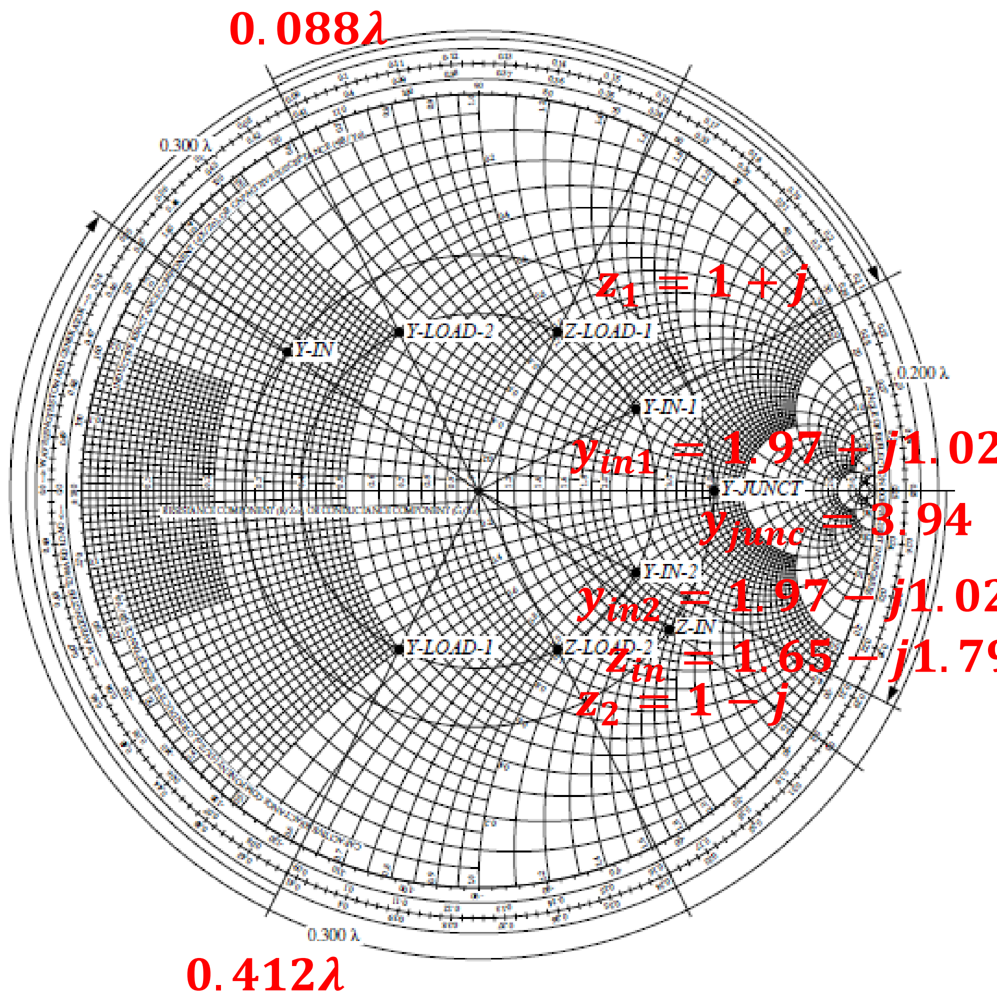


$$Z_{in} = (95 - j70)\Omega$$

More Examples

Example 12: Use the Smith Chart to find Z_{in} of the 50Ω feedline shown in following figure.

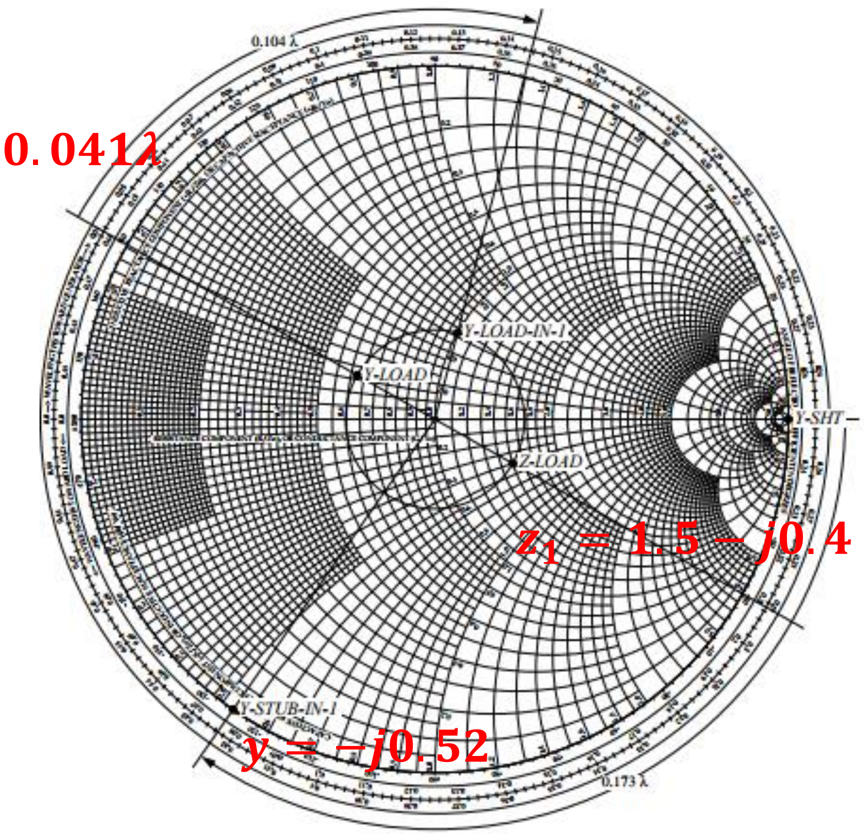


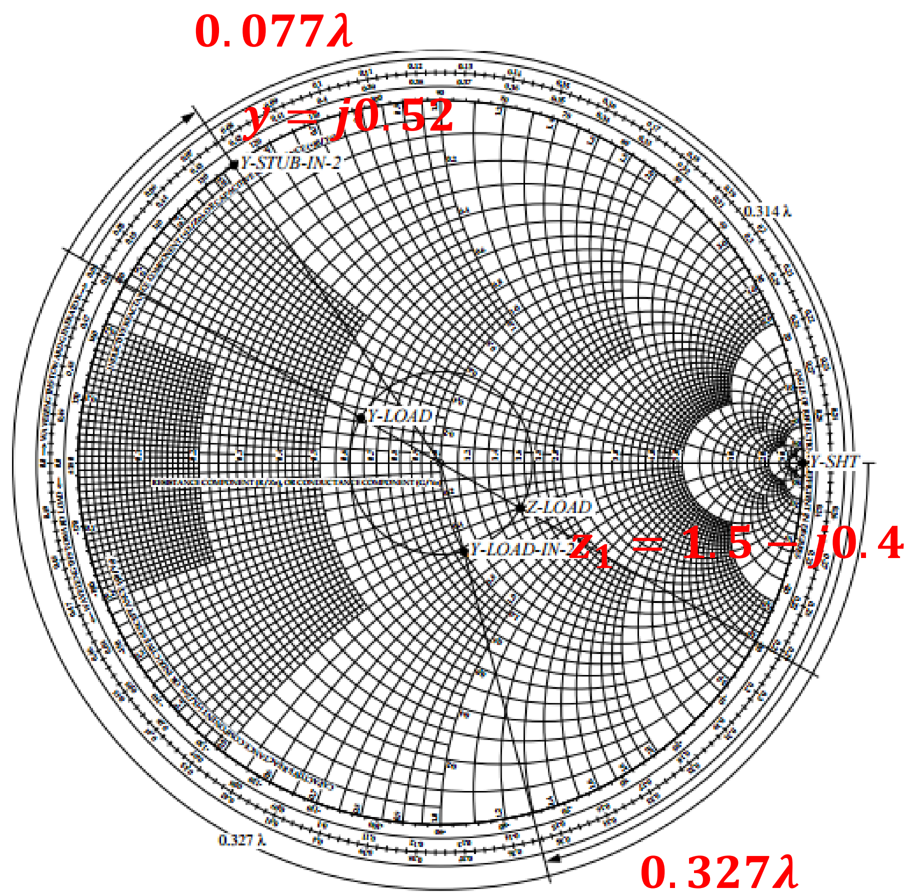


$$Z_{in} = (82.5 - j89.5)\Omega$$

More Examples

Example 13: A 50Ω lossless line is to be matched to an antenna with $Z_L = (75 - j20)\Omega$ using a shorted stub. Use the Smith Chart to determine the stub length and distance between the antenna and stub.





$d_1 = 0.104\lambda, l_1 = 0.173\lambda$ $d_2 = 0.314\lambda, l_2 = 0.327\lambda$

Q&A