Chapter 1

Theory and Applications of Transmission Lines

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Contents

1. Introduction

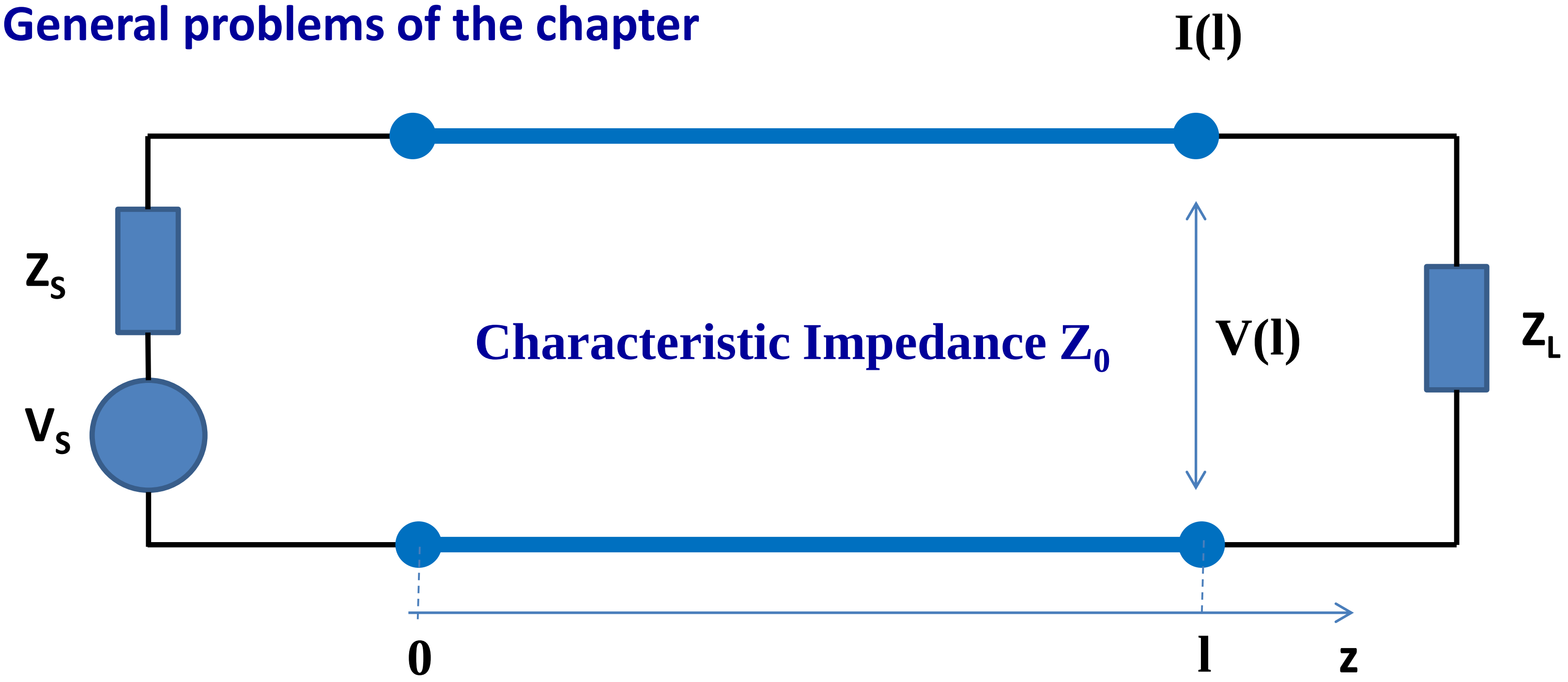
- The previous class provided the analysis of EM field and wave traveling in the free space. This chapter provides the analysis of wave propagations in the guided mediums : transmission lines.
- For efficient point-to-point transmission of power and information, the source energy must be directed or guided.
- The key difference between circuit theory and Transmission Line is electrical size.
- At low frequencies, an electrical circuit is completely characterized by the electrical parameters like resistance, inductance, capacitance etc. and the physical size of the electrical components plays no role in the circuit analysis.
- As the frequency increases however, the size of the components becomes important. The voltage and currents exist in the form of waves. Even a change in the length of a simple connecting wire may alter the behavior of the circuit.

1. Introduction

- The circuit approach then has to be re-investigated with inclusion of the space into the analysis. This approach is then called the **Transmission Line** approach.
- Although the primary objective of a transmission line is to carry electromagnetic energy efficiently from one location to other, they find wide applications in high frequency circuit design.
- Also at high frequencies, the transmit time of the signals can not be ignored. In the era of high speed computers, where data rates are approaching to few Gb/sec, the phenomena related to the electromagnetic waves, like the bit distortion, signal reflection, impedance matching play a vital role in high speed communication networks.

1. Introduction

General problems of the chapter

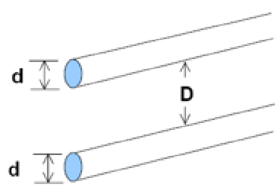


At a given location along the line, find:

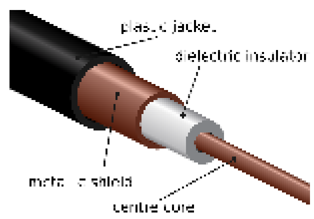
- **Current, voltage and power**
- **Reflection coefficient, impedance, VSWR**
- **Design real TLs, such as micro-strip lines, CPW lines**

2. Lumped-Element Circuit Model for Transmission Lines

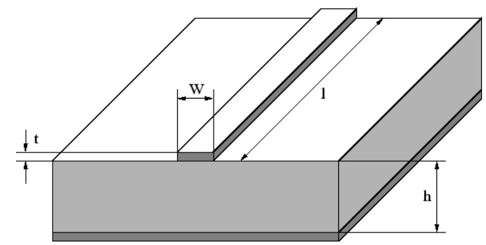
Examples of Transmission Lines:



Two-wire TL



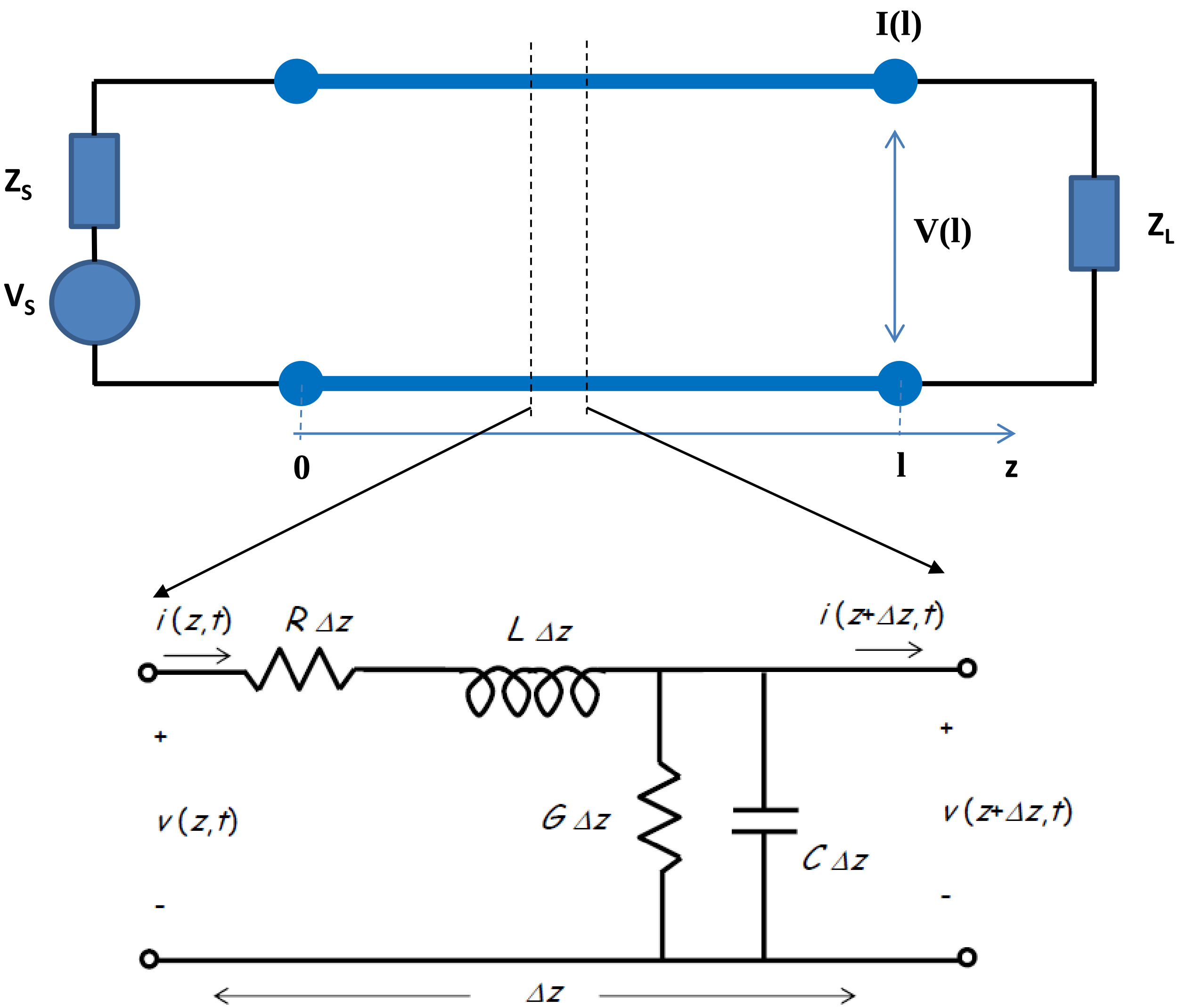
Coaxial TL



Microstrip TL

- Two-wire Transmission Line: consists of a pair of parallel conducting wires separated by a uniform distance. Examples: telephone line, cable connecting from roof-top antenna to TV receiver.
- Coaxial Transmission Line: consists of inner conductor and and a coaxial outer separated by a dielectric medium. Examples: TV Cable, etc.
- Microstrip Transmission Line: consists of two parallel conducting plates separated by a dielectric slab. It can be fabricated inexpensively on PCB.

2. Lumped-Element Circuit Model for Transmission Lines



- Current i and voltage v are a function of position z because a wire is never a "perfect" conductor. It will have:
  - Inductance (G)
  - Resistance (R)
  - Capacitance (C)
  - Conductance (L)

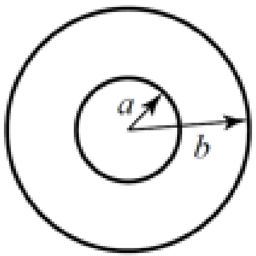
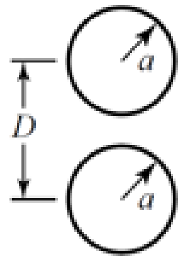
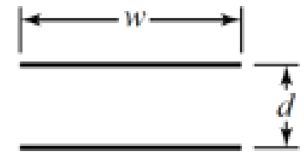
2. Lumped-Element Circuit Model for Transmission Lines

R, L, G, and C are per-unit-length quantities defined as follows:

- R = series resistance per unit length, for both conductors, in Ω/m.
- L = series inductance per unit length, for both conductors, in H/m.
- G = shunt conductance per unit length, in S/m.
- C = shunt capacitance per unit length, in F/m.
    - Series inductance L represents the total self-inductance of the two conductors.
    - Shunt capacitance C is due to the close proximity of the two conductors.
    - Series resistance R represents the resistance due to the finite conductivity of the individual conductors.
    - Shunt conductance G is due to dielectric loss in the material between the conductors.
    - R and G, therefore, represent loss.

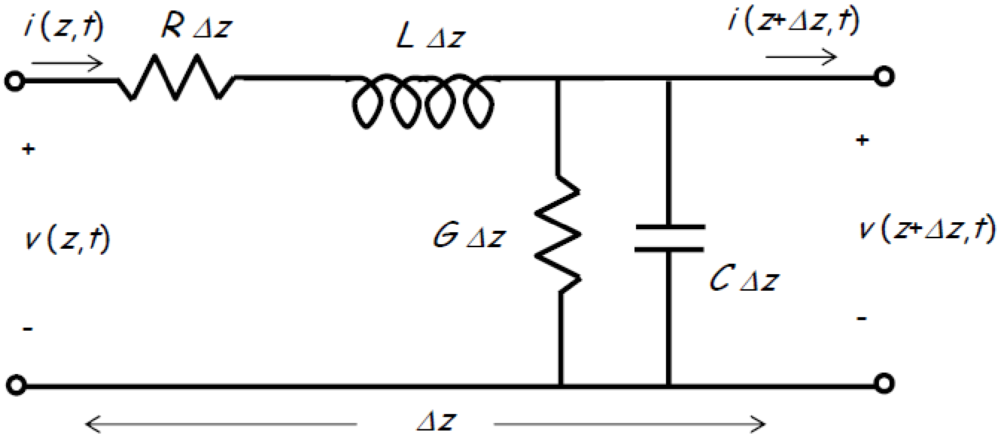
2. Lumped-Element Circuit Model for Transmission Lines

Table: Transmission Line Parameters of some common lines:

	COAX	TWO-WIRE	PARALLEL PLATE
			
L	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu d}{w}$
C	$\frac{2\pi\epsilon'}{\ln b/a}$	$\frac{\pi\epsilon'}{\cosh^{-1}(D/2a)}$	$\frac{\epsilon' w}{d}$
R	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$
G	$\frac{2\pi\omega\epsilon''}{\ln b/a}$	$\frac{\pi\omega\epsilon''}{\cosh^{-1}(D/2a)}$	$\frac{\omega\epsilon'' w}{d}$

Further reading: Kỹ thuật SCT, p.25-p.33

3. Transmission Line Equations and Solution



Applying Kirchoff's Voltage Law (KVL):

$$v(z+\Delta z,t)=v(z,t)-R\Delta z i(z,t)-L\Delta z\frac{\partial i(z,t)}{\partial t}$$

Applying Kirchoff's Current Law (KCL):

$$i(z+\Delta z,t)=i(z,t)-G\Delta z v(z,t)-C\Delta z\frac{\partial v(z,t)}{\partial t}$$

3. Transmission Line Equations and Solution

Then:
$$\frac{v(z+\Delta z,t)-v(z,t)}{\Delta z}=-Ri(z,t)-L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{i(z+\Delta z,t)-i(z,t)}{\Delta z}=-Gv(z,t)-C\frac{\partial v(z,t)}{\partial t}$$

When $\Delta z \to 0$:

$$\boxed{\begin{aligned}\frac{\partial v(z,t)}{\partial z}&=-Ri(z,t)-L\frac{\partial i(z,t)}{\partial t}\\ \frac{\partial i(z,t)}{\partial z}&=-Gv(z,t)-C\frac{\partial v(z,t)}{\partial t}\end{aligned}}$$

These equations are "telegrapher's equations". There are infinite number of solutions $v(z,t)$ and $v(z,t)$ for the "telegrapher's equations". The problem can be simplified by assuming that the function of time is "time harmonic" (sinusoidal).

3. Transmission Line Equations and Solution

- If a sinusoidal voltage source with frequency ω is used to excite a linear, time-invariant circuit then the voltage at every point with the circuit will likewise vary sinusoidal.
- The voltage along a transmission line when excited by a sinusoidal source must have the form:

$$v(z,t) = v(z)\cos\big(\omega t + \varphi(z)\big) = \Re e\left\{v(z)e^{j\omega t}e^{j\varphi(z)}\right\}$$

- The time harmonic voltage at every location z along a transmission line:

$$V(z) = v(z)e^{j\varphi(z)}$$

where: $v(z) = |V(z)|$ and $\varphi(z) = arg\{V(z)\}$

- There is no reason to explicitly write the complex function $e^{j\omega t}$ since the only unknown is the complex function $V(z)$. Once we determine $V(z)$, we can always recover the real function $v(z,t)$:

$$v(z,t) = \Re e\left\{V(z)e^{j\omega t}\right\}$$

3. Transmission Line Equations and Solution

- Let's assume that $\boldsymbol{v(z,t)}$ and $\boldsymbol{i(z,t)}$ each have the time harmonic form:

$$\boldsymbol{v(z,t) = \Re e\{V(z)e^{j\omega t}\}}$$

$$\boldsymbol{i(z,t) = \Re e\{I(z)e^{j\omega t}\}}$$

- Then time derivative of these functions are:

$$\boldsymbol{\frac{\partial v(z,t)}{\partial z} = \Re e\{j\omega V(z)e^{j\omega t}\}}$$

$$\boldsymbol{\frac{\partial i(z,t)}{\partial z} = \Re e\{j\omega I(z)e^{j\omega t}\}}$$

- The telegrapher's equations thus become:

$$\boxed{\begin{aligned}\boldsymbol{\Re e\left\{\frac{\partial V(z)}{\partial z}e^{j\omega t}\right\}} &= \boldsymbol{\Re e\{-(R + j\omega L)I(z)e^{j\omega t}\}}\\ \boldsymbol{\Re e\left\{\frac{\partial I(z)}{\partial z}e^{j\omega t}\right\}} &= \boldsymbol{\Re e\{-(G + j\omega C)V(z)e^{j\omega t}\}}\end{aligned}}$$

3. Transmission Line Equations and Solution

❖ Then the complex form of telegrapher's equations are:

$$\frac{\partial V(z)}{\partial z} = -(R + j\omega L)I(z)$$
$$\frac{\partial I(z)}{\partial z} = -(G + j\omega C)V(z)$$

Complex Value:

$v(z)e^{j\varphi(z)}$

Note that these functions are not a function of time t.

❖ Take the derivative with respect to z of the telegrapher's equations, lead to:

$$\frac{\partial^2 V(z)}{\partial z^2} = (R + j\omega L)(G + j\omega C)V(z)$$
$$\frac{\partial^2 I(z)}{\partial z^2} = (R + j\omega L)(G + j\omega C)I(z)$$

3. Transmission Line Equations and Solution

❖ These equations can be written as:

$$\boxed{\begin{gathered}\frac{\partial^2 V(z)}{\partial z^2} = \gamma^2(\omega)V(z)\\ \frac{\partial^2 I(z)}{\partial z^2} = \gamma^2(\omega)I(z)\end{gathered}}$$

where $\gamma(\omega) = \sqrt{(R + j\omega L)(G + j\omega C)}$ is propagation constant.

❖ Only special equations satisfy these equations. The solution of these equations can be found as:

$$\boxed{\begin{gathered}V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}\\ I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}\end{gathered}}$$

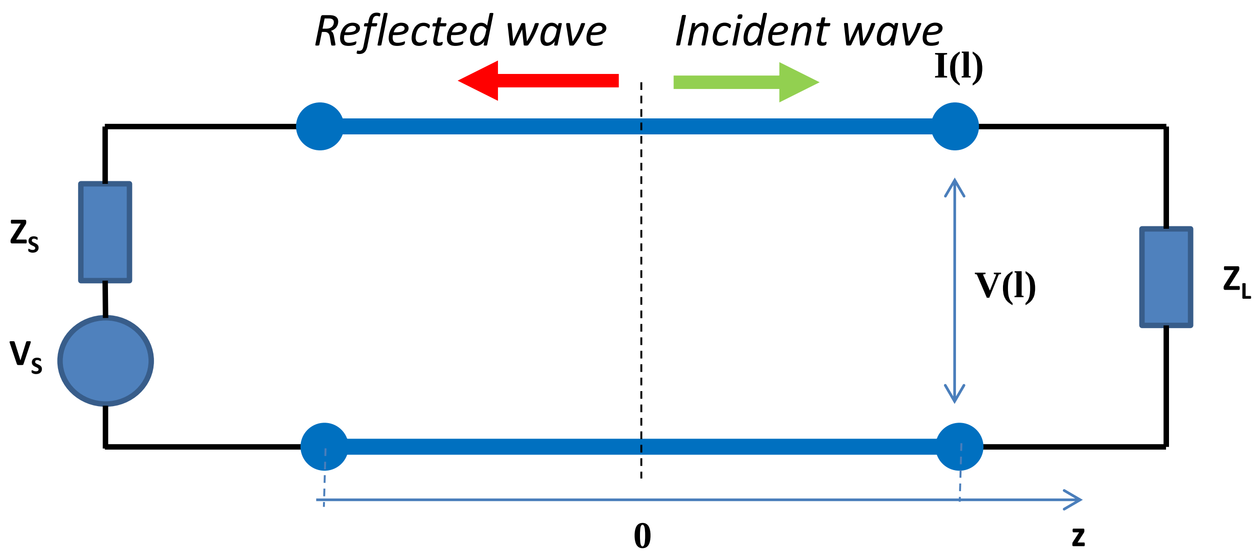
where $\gamma = \alpha + j\beta$.

3. Transmission Line Equations and Solution

❖ The current and voltage at a given point must have the form:

$$V(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{+\alpha z} e^{+j\beta z}$$
$$I(z) = I_0^+ e^{-\alpha z} e^{-j\beta z} + I_0^- e^{+\alpha z} e^{+j\beta z}$$

$$\frac{V_0^+}{I_0^+}$$



4. Characteristic Impedance of Transmission Line

- The terms in each equation describe two waves propagating in the transmission line, one propagating in one direction (+z) and the other wave propagating in the opposite direction (-z):

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$
$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

- Then: $$\frac{\partial V(z)}{\partial z} = -\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{+\gamma z} = -(R + j\omega L) I(z)$$

- After re-arranging, $I(z)$ must be:

$$I(z) = \frac{\gamma}{(R + j\omega L)} V_0^+ e^{-\gamma z} - \frac{\gamma}{(R + j\omega L)} V_0^- e^{+\gamma z} = I_0^+ e^{-\gamma z} + I_0^- e^{-\gamma z}$$

- For the equations to be true for all z, $\mathbf{I_0}$ and $\mathbf{V_0}$ must be related as:

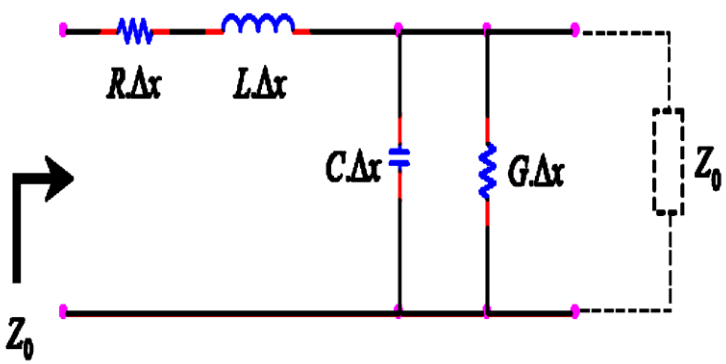
$$I_0^+ = \frac{V_0^+}{Z_0} \quad \text{and} \quad I_0^- = \frac{V_0^-}{Z} \quad \text{where: } Z_0 = \frac{(R + j\omega L)}{\gamma} = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

4. Characteristic Impedance of Transmission Line

❖ V_0^+ and I_0^+ are determined by the "boundary condition" (what is connected to either end of the transmission line) but the ratio $\frac{V_0^+}{I_0^+}$ is determined by the parameters of the transmission line only.

❖ Set $Z = R + j\omega L$ and $Y = G + j\omega C$. Then:

$$Z_0 = Z\Delta x + \left(\frac{1}{Y\Delta x} \parallel Z_0\right) \underset{x\to 0}{\Longrightarrow} \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$



❖ Lossless transmission line: $Z_0 = \sqrt{\frac{L}{C}}$

❖ In practice:

- ❖ Z_0 is always real.
- ❖ In communications system: $Z_0 = 50\Omega$. In telecommunications: : $Z_0 = 75\Omega$.

5a. Propagation Constant and Velocity

- Propagation constant: $\gamma(\omega) = \alpha(\omega) + j\beta(\omega) = \sqrt{(R + j\omega L)(G + j\omega C)}$

  α: attenuation constant [Np/m] or [dB/m].

  β: phase constant [rad/s].

$$\alpha[dB/m] = 20log_{10}e^{\alpha[Np/m]} = 8.68\alpha[Np/m]$$

- The "*wave velocity*" is described by its *"phase velocity"*. Since velocity is change in distance with respect to time, we need to first express the propagation wave in its real form:

$$V^{+}(z,t) = \Re e\{V^{+}(z)e^{-j\omega t}\} = |V_0^{+}|cos(\omega t - \beta z)$$

- Let's set the absolute phase to some arbitrary value: $\omega t - \beta z = \phi_c$. Then:

$$z = \frac{\omega t - \phi_c}{\beta} \quad \text{and} \quad v_p = \frac{\partial z}{\partial t} = \frac{\omega}{\beta}$$

5b. Line Impedance

- The Line Impedance is NOT the T.L Impedance Z_0. Recall that:

$$V(z) = V^+(z) + V^-(z)$$
$$I(z) = \frac{V^+(z) - V^-(z)}{Z_0}$$

- Therefore, the Line Impedance can be written as:

$$Z(z) = \frac{V(z)}{I(z)} = Z_0 \frac{V^+(z) + V^-(z)}{V^+(z) - V^-(z)}$$

- Or more specifically:

$$Z(z) = Z_0 \frac{V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}}$$

6. Lossless and Low-loss Transmission Line

- In practice, transmission lines have losses due to finite conductivity and/or lossy dielectric but these losses are usually small.
- In most practical microwave:
  - Losses may be neglected → Lossless Transmission Line.
  - Losses may be assumed to be very small → Low-loss Transmission Line.
- **Lossless Transmission Line:** $R = 0, G = 0$

$$\gamma(\omega) = \sqrt{(R + j\omega L)(G + j\omega C)} = j\omega\sqrt{LC}$$

$$\alpha(\omega) = 0$$

$$\beta(\omega) = \omega\sqrt{LC}$$

- **Low-loss Transmission Line:** both conductor and dielectric loss will be small, and we can assume that $R \ll \omega L$ and $G \ll \omega C$. Then: $RG \ll \omega^2 LC$. Then:

$$\gamma(\omega) \simeq j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)}$$

6. Lossless and Low-loss Transmission Line

- Using the Taylor series expansion* for:

$$\sqrt{1+x} \simeq 1 + {}^{x}/_{2} - {}^{x^2}/_{8} + {}^{x^3}/_{16} + \cdots$$

- Then: $$\gamma(\omega) \simeq j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)} \simeq j\omega\sqrt{LC}\left[1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right]$$

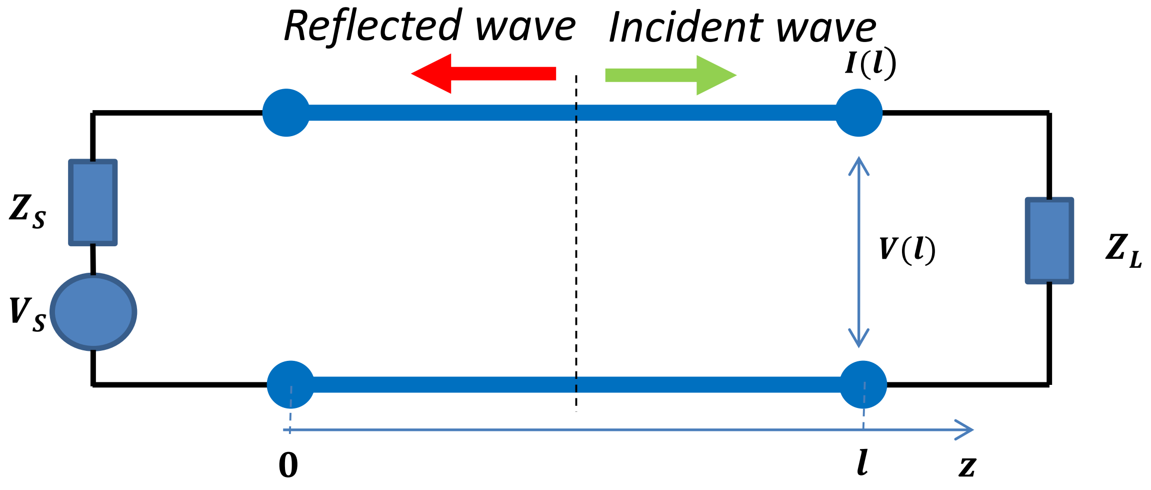
- Hence: $$\alpha \simeq \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right] = \frac{1}{2}\left[\frac{R}{Z_0} + GZ_0\right]$$

$$\beta \simeq \omega\sqrt{LC}$$

where: $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \simeq \sqrt{\frac{L}{C}}$

* *https://en.wikipedia.org/wiki/Taylor_series*

7. Reflection Coefficient



$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$
$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

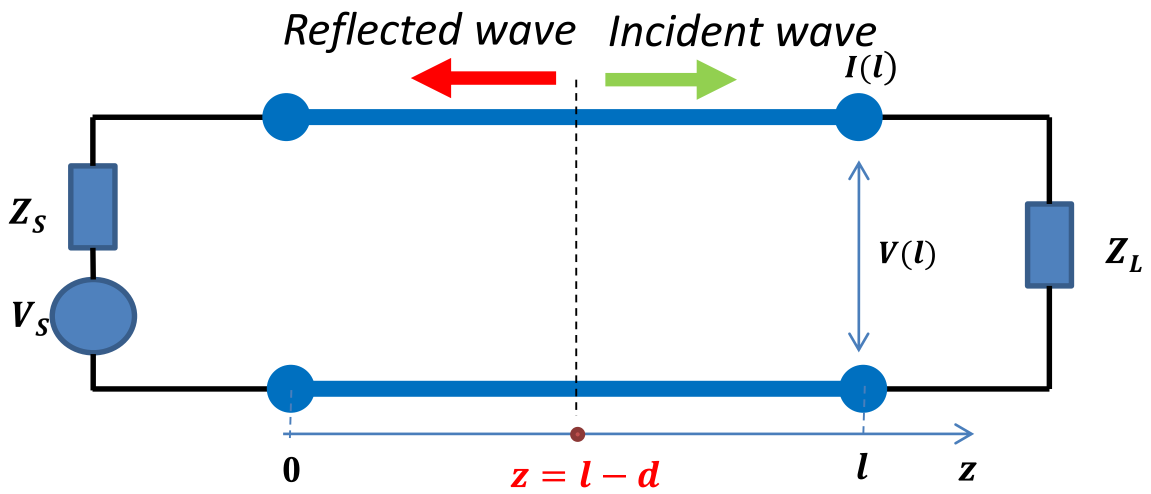
- Voltage Reflection Coefficient is defined as:

$$\Gamma_V(z) = \frac{Reflected\ Voltage}{Incident\ Voltage} = \frac{V_0^- e^{+\gamma z}}{V_0^+ e^{-\gamma z}} = \frac{V_0^-}{V_0^+} e^{2\gamma z}$$

- Current Reflection Coefficient is defined as:

$$\Gamma_I(z) = \frac{Reflected\ Current}{Incident\ Current} = \frac{I_0^- e^{+\gamma z}}{I_0^+ e^{-\gamma z}} = \frac{-V_0^-/Z_0}{V_0^+/Z_0} e^{2\gamma z} = -\Gamma_V(z)$$

7. Reflection Coefficient



$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$
$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

- **At load:** $\Gamma_L = \dfrac{V_0^-}{V_0^+} e^{2\gamma l}$

- Note that: $Z_L = \dfrac{V(l)}{I(l)} = Z_0 \dfrac{V_0^+ e^{-j\beta l} + V_0^- e^{j\beta l}}{V_0^+ e^{-j\beta l} - V_0^- e^{j\beta l}} = Z_0 \dfrac{1 + \Gamma_L}{1 - \Gamma_L}$

- Then: $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

- **At location z:** $\Gamma(z = l - d) = \dfrac{V_0^-}{V_0^+} e^{2\gamma z} = \dfrac{V_0^-}{V_0^+} e^{2\gamma (l-d)} = \dfrac{V_0^-}{V_0^+} e^{2\gamma l} e^{-2\gamma d} = \Gamma_L e^{-2\gamma d}$

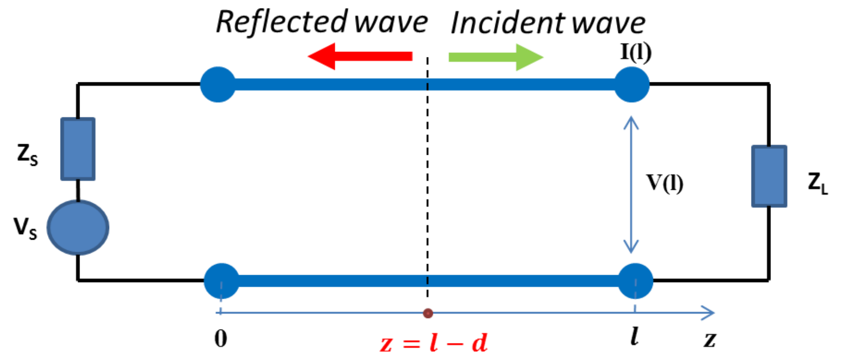
7. Reflection Coefficient - Representation on a complex plane

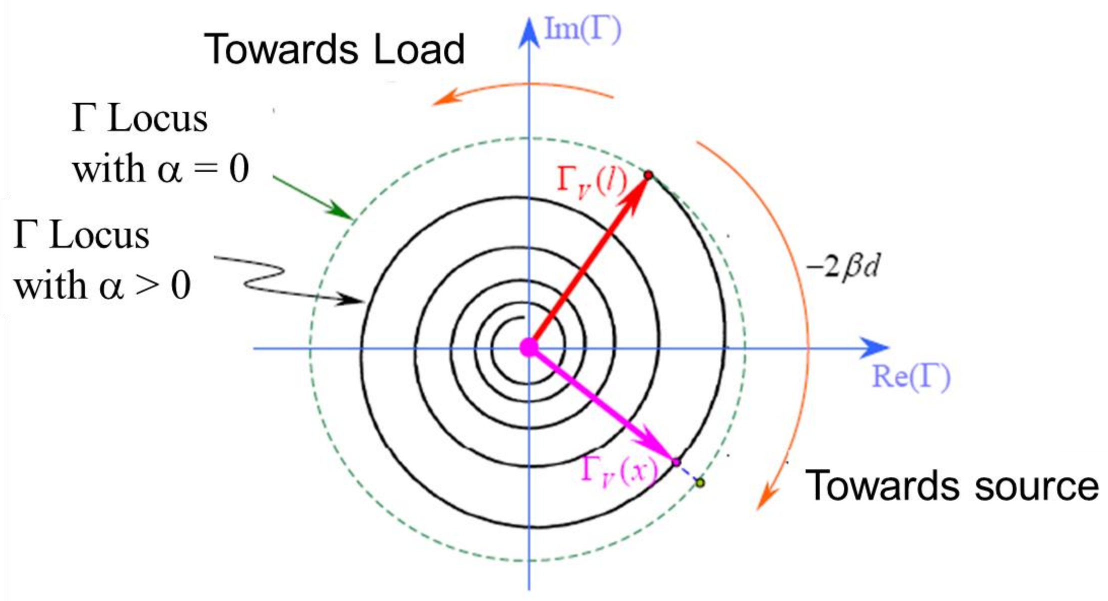
❖ Reflection Coefficient at $z = l - d$:

$$\Gamma(z = l - d) = \Gamma_L e^{-2\gamma d}$$

where: $\gamma = \alpha + j\beta$.

❖ Then: $\Gamma(z = l - d) = \Gamma_L e^{-2\alpha d} e^{-2j\beta d}$





$$d = \lambda / 2$$

$$2\beta d = 2\frac{2\pi}{\lambda} d$$

$$= 2\frac{2\pi}{\lambda}\frac{\lambda}{2} = 2\pi$$

Quiz 1: A 6-m section of 150Ω lossless line is driven by a source with

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30^0) \quad (V)$$

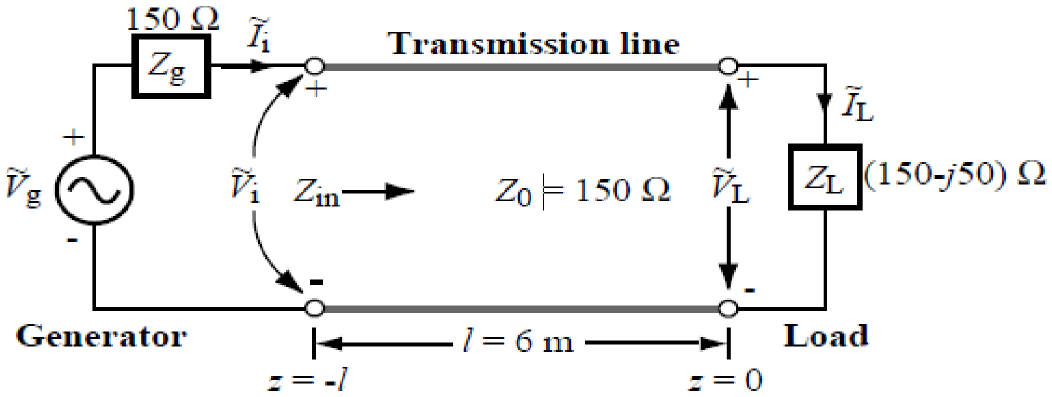
And $Z_g = 150\Omega$. If the line, which has a relative permittivity $\varepsilon_r = 2.25$ is terminated in a load $Z_L = (150 - j50)\Omega$, find:

a. λ on the line. Note that: $\lambda = {}^{v_P}/_f$ where $v_P = {}^{c}/_{\sqrt{\varepsilon_r}}$.

b. The reflection coefficient at the load.

c. The input impedance.

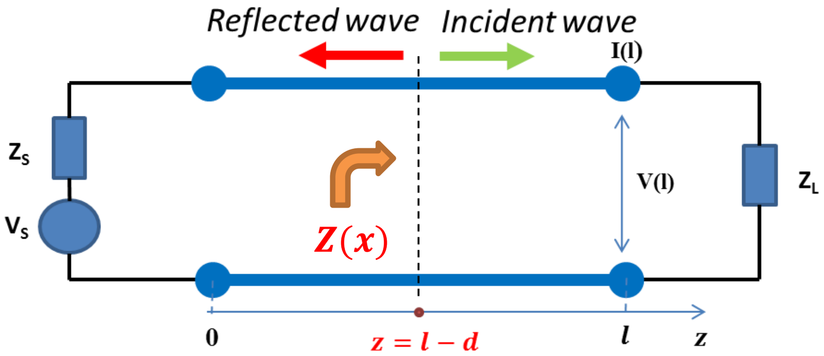
d. The input voltage V_i and time-domain voltage $v_i(t)$.



8. Transmission Line Impedance and Admittance

❖ The line impedance at $z = l - d$:

$$Z(z) = Z_0 \frac{V_0^+ e^{-j\gamma z} + V_0^- e^{j\gamma z}}{V_0^+ e^{-j\gamma z} - V_0^- e^{j\gamma z}}$$



❖ Note that:

$$\Gamma(z = l - d) = \frac{V_0^-}{V_0^+} e^{2\gamma l} e^{-2\gamma d} = \Gamma_L e^{-2\gamma d}$$

❖ Then the line impedance can be specified: $$Z(z) = Z_0 \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

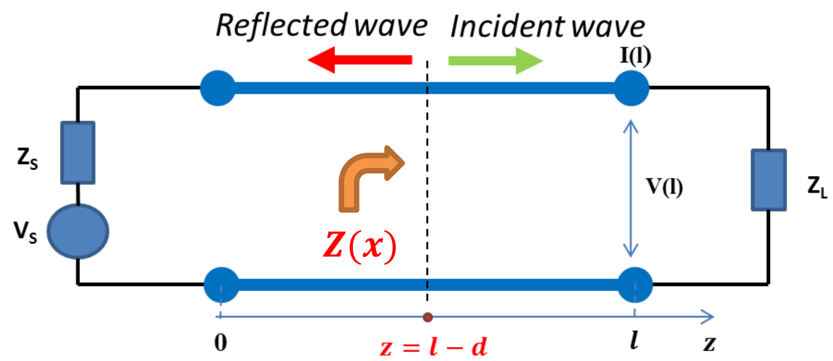
❖ More specifically: $$Z(z) = Z_0 \frac{(Z_L + Z_0)e^{\gamma d} + (Z_L - Z_0)e^{-\gamma d}}{(Z_L + Z_0)e^{\gamma d} - (Z_L - Z_0)e^{-\gamma d}}$$

$$= Z_0 \frac{Z_L\left(e^{\gamma d} + e^{-\gamma d}\right) + Z_0\left(e^{\gamma d} - e^{-\gamma d}\right)}{Z_L\left(e^{\gamma d} - e^{-\gamma d}\right) + Z_0\left(e^{\gamma d} + e^{-\gamma d}\right)}$$

$$= Z_0 \frac{Z_L \cosh(\gamma d) + Z_0 \sinh(\gamma d)}{Z_L \sinh(\gamma d) + Z_0 \cosh(\gamma d)} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma d)}{Z_0 + Z_L \tanh(\gamma d)}$$

8. Transmission Line Impedance and Admittance

❖ Lossless T.L ($\alpha = 0$):



$$Z(z) = Z_0 \frac{Z_L + jZ_0 tan(\beta d)}{Z_0 + jZ_L tan(\beta d)}$$

- $Z_L = Z_0$:

$$Z(z) = Z_0 \frac{Z_L + jZ_0 tan(\beta d)}{Z_0 + jZ_L tan(\beta d)} = Z_0$$

- $Z_L = jX_L$:

$$Z(z) = Z_0 \frac{jX_L + jZ_0 tan(\beta d)}{Z_0 - X_L tan(\beta d)}$$ **has imaginary part only**

- $Z_L = 0$:

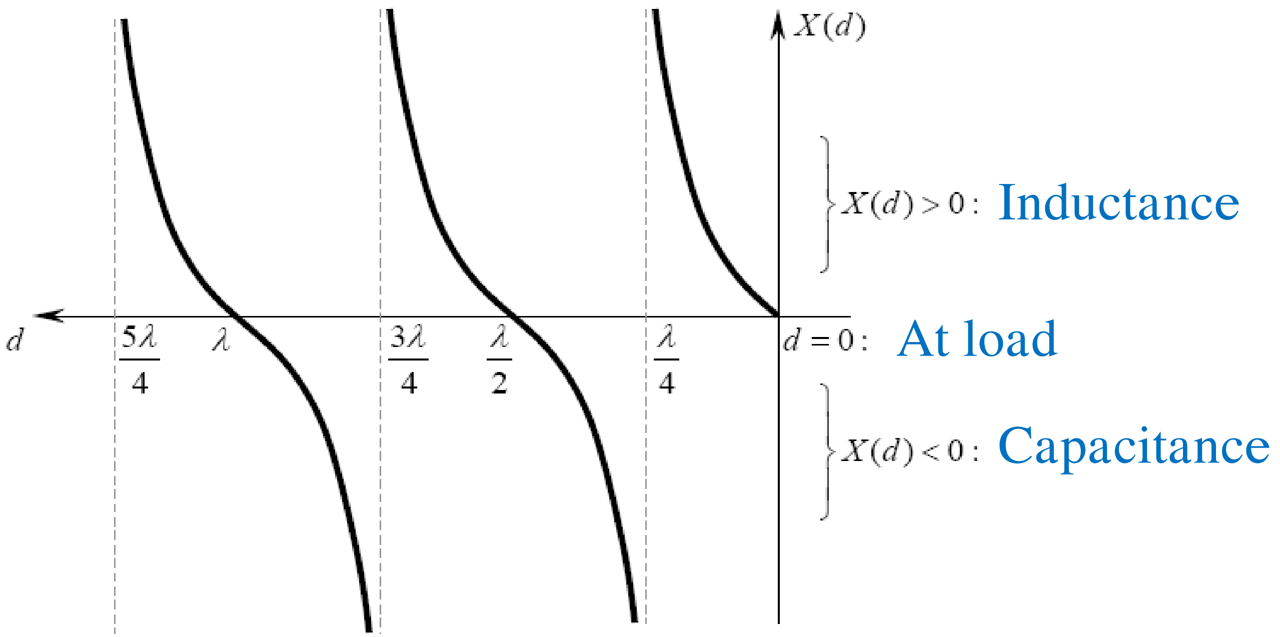
$$Z(z) = jZ_0 tan(\beta d)$$ **pure reactance**

- $Z_L = \infty$:

$$Z(z) = \frac{Z_0}{jtan(\beta d)} = -jZ_0 cotan(\beta d)$$ **pure reactance**

8. Transmission Line Impedance and Admittance

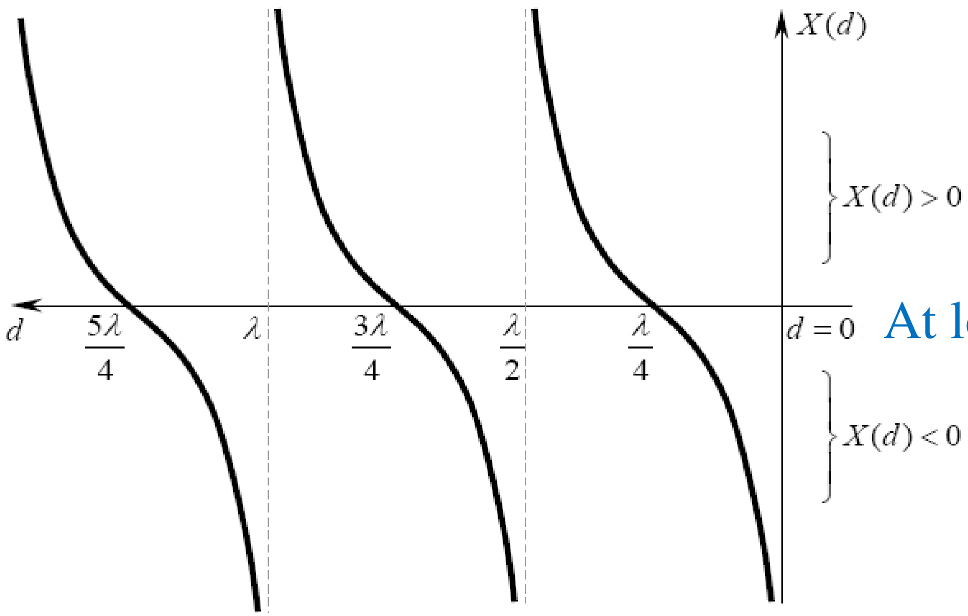
- $Z_L = 0$: $\boldsymbol{Z(z) = jZ_0 tan(\beta d) = jX(d)}$ **Pure reactance**



✓ Shorted-circuit T.L can be used to realize inductors or capacitors at specific frequencies → Distributed Components.

8. Transmission Line Impedance and Admittance

- $Z_L = \infty$: $\boldsymbol{Z(z) = -jZ_0 cotan(\beta d) = jX(d)}$ **Pure reactance**



Inductance

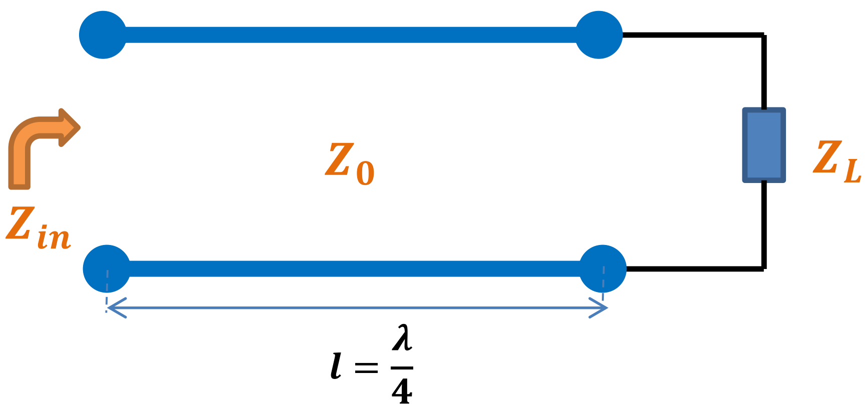
At load

Capacitance

✓ Open-circuit T.L can be used to realize inductors or capacitors at specific frequencies → Distributed Components.

8. Transmission Line Impedance and Admittance

❖ A quarter wavelength TL:

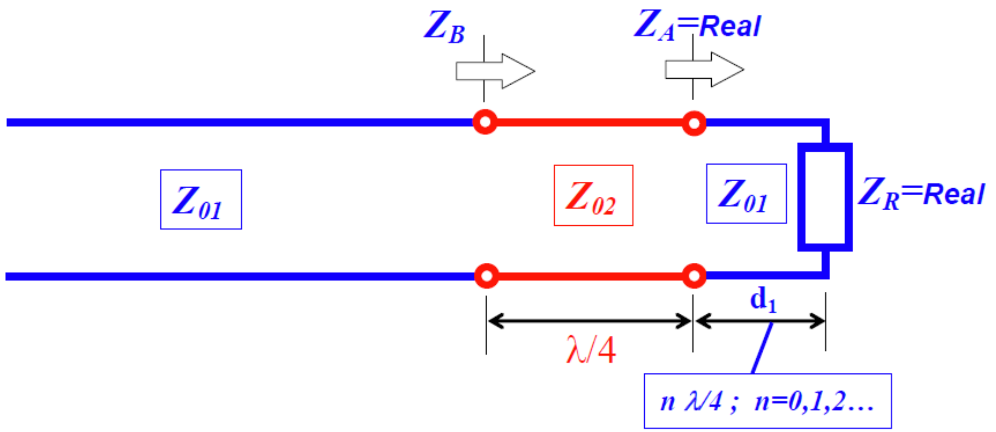


$$Z_{in} = Z_0 \frac{Z_L + jZ_0 tan(\beta d)}{Z_0 + jZ_L tan(\beta d)} = \frac{Z_0^2}{Z_L}$$

✓ If $Z_L \to \infty$: $Z_{in} = 0$.

✓ If $Z_L = 0$: $Z_{in} \to \infty$.

❖ Application for impedance transformation: $Z_{in} = \frac{Z_0^2}{Z_L} \to Z_0 = \sqrt{Z_{in} Z_L}$



8. Transmission Line Impedance and Admittance

Example 1: The open-circuit and short-circuit impedances measured at the input terminal of a very low-loss TL of length 1.5m which is less than a quarter wavelength, are respectively -54.6j (Ω) and 103j (Ω)

a. Find Z_0 and γ of the line.

b. Without changing the frequency, find the input impedance of a short-circuited TL that is twice the given length.

c. How long should the short-circuited TL be in order to appear as an open circuit at the input terminals?

8. Transmission Line Impedance and Admittance

Quiz 2: A voltage generator with

$$v_g(t) = 5\cos(2\pi \times 10^9 t) \quad (V)$$

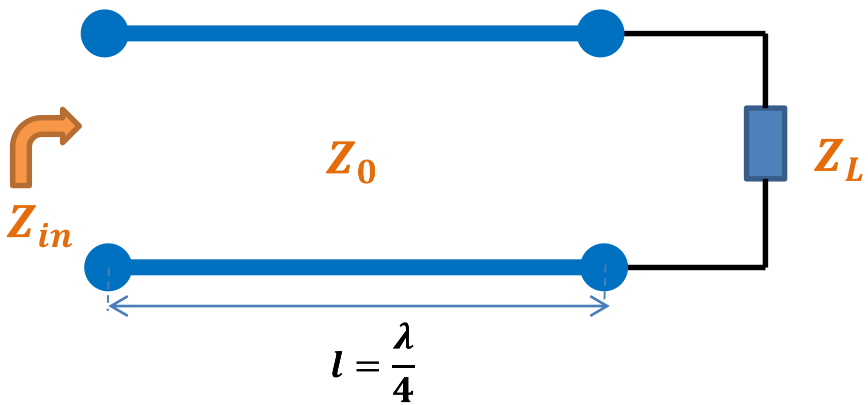
and internal impedance is $Z_g = 50\Omega$ is connected to a 50Ω lossless T.L. The line length is 5cm and the line is terminated in a load with impedance $Z_L = 100 - j100\Omega$. Determine:

a. Reflection coefficient at load Γ_L?

b. Z_{in} at the input of the T.L.

c. The input voltage $v_i(t)$ and input current $i_i(t)$?

9. Power Transmission on Transmission Lines



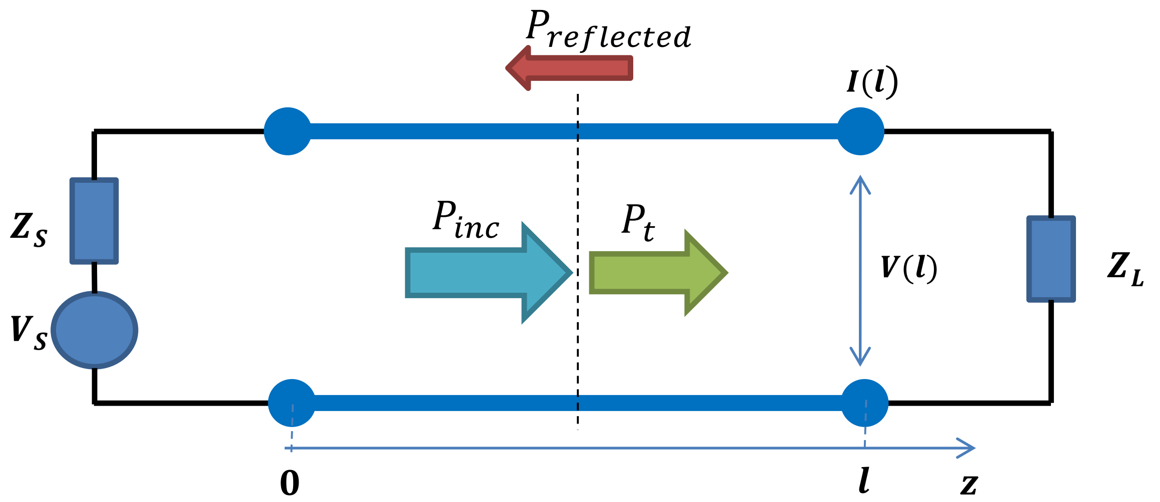
$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$
$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

- Steps to find V_0^+ and V_0^-:

1. $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$
2. $\Gamma_{in} = \Gamma_L e^{-2\gamma l}$
3. $Z_{in} = Z_0 \frac{1+\Gamma_{in}}{1-\Gamma_{in}}$
4. $V_{in} = V_S \frac{Z_{in}}{Z_{in}+Z_S}$
5. $V_{in} = V_0^+ + V_0^- = V_0^+(1 + \Gamma_{in})$
6. $V_0^+ = \frac{V_{in}}{1+\Gamma_{in}}$ $\quad V_0^- = \Gamma_{in} V_0^+$

- If $Z_L = Z_0$: $V_0^+ = {}^{V_S}/_{2}$

9. Power Transmission on Transmission Lines



❖ The time average power flows along a transmission line:

$$P_t = \frac{1}{2}\Re e\{V(z)I^*(z)\}$$

$$= \frac{1}{2Z_0}\Re e\{(V_0^+ e^{-\alpha z}e^{-j\beta z} + V_0^- e^{\alpha z}e^{j\beta z})(V_0^{+*} e^{-\alpha z}e^{j\beta z} - V_0^{-*} e^{\alpha z}e^{-j\beta z})\}$$

$$= \frac{1}{2Z_0}\Re e\{(|V_0^+|^2 e^{-2\alpha z} - V_0^+ V_0^{-*} e^{-j2\beta z} + V_0^{+*} V_0^- e^{j2\beta z} - |V_0^-|^2 e^{2\alpha z})\}$$

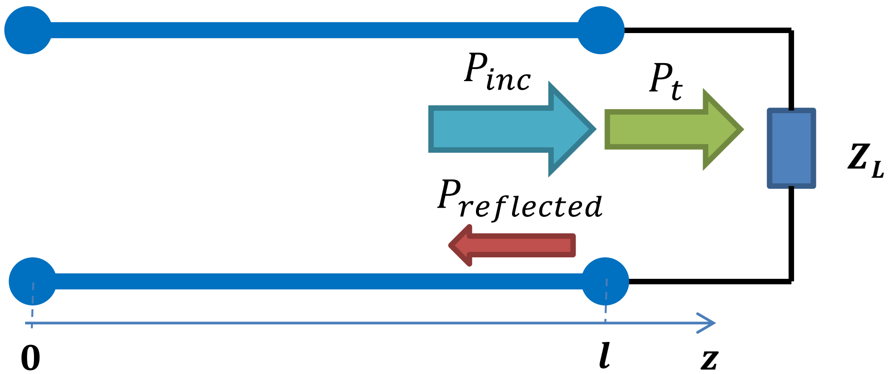
$$= \frac{1}{2Z_0}(|V_0^+|^2 e^{-2\alpha z} - |V_0^-|^2 e^{2\alpha z}) = \frac{|V_0^+|^2}{2Z_0} e^{-2\alpha z}(1 - |\Gamma_z|^2) = P_{inc} - P_{refl}$$

9. Power Transmission on Transmission Lines

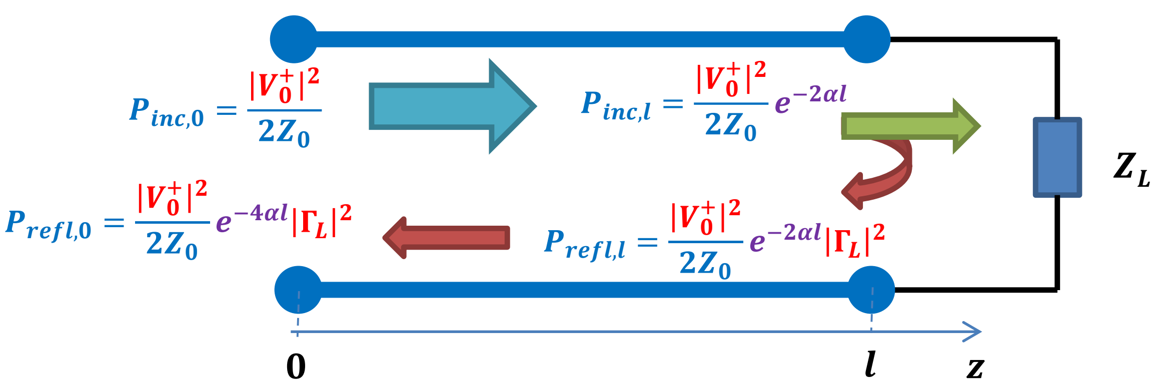
❖ The time average absorbed by load:

$$P_t = \frac{1}{2}\Re e\{V_L I_L^*\}$$

$$= \frac{|V_0^+|^2}{2Z_0} e^{-2\alpha l}\left(1 - |\Gamma_L|^2\right) = P_{inc} - P_{refl}$$



Power Flow:

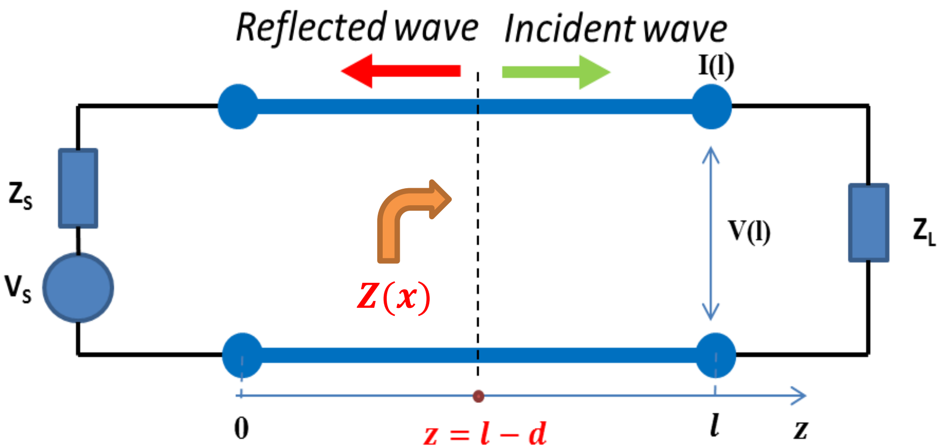


10. Standing Wave and Standing Wave Ratio

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$= V_0^+ e^{-\gamma z}\left(1 + \frac{V_0^-}{V_0^+} e^{+2\gamma z}\right)$$

$$= V_0^+ e^{-\gamma z}(1 + \Gamma(z))$$



$$\Gamma(z = l - d) = \Gamma_L e^{-2\gamma d}$$

- If $\alpha = 0$:

$$V(z) = V_0^+ e^{-j\beta z}\big(1 + \Gamma(z)\big) \quad \rightarrow \quad |V(z)| = |V_0^+||1 + \Gamma(z)|$$

- Then:

$$V(z)_{max} = |V_0^+|(1 + |\Gamma_L|) \qquad when\ \ \Gamma(z) = |\Gamma_L|$$

$$V(z)_{min} = |V_0^+|(1 - |\Gamma_L|) \qquad when\ \ \Gamma(z) = -|\Gamma_L|$$

$$VSWR = \frac{V(z)_{max}}{V(z)_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

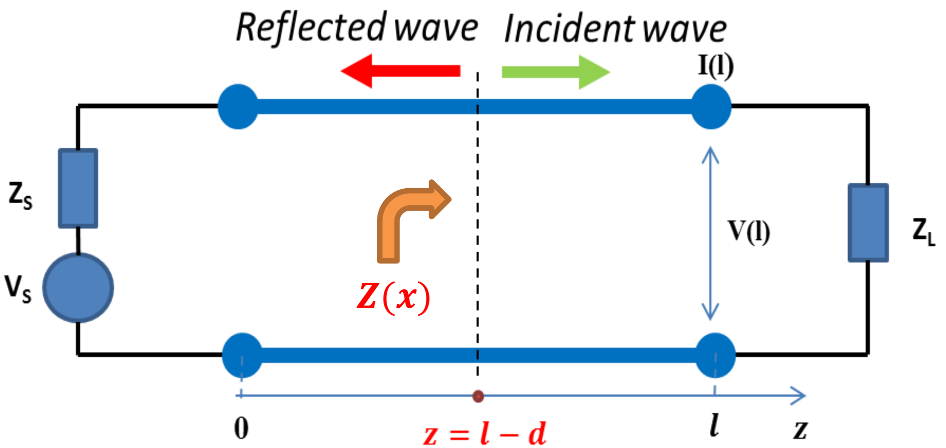
10. Standing Wave and Standing Wave Ratio

❖ We have:

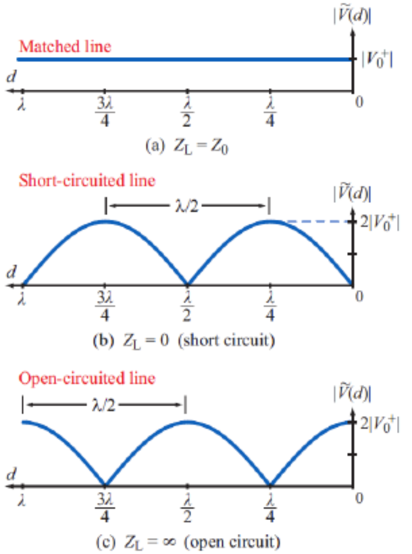
$$V(z) = V_0^+ e^{-j\beta z}\left(1 + \Gamma(z)\right)$$

where:

$$\Gamma(z = l - d) = \Gamma_L e^{-2\beta d} = |\Gamma_L| e^{j\theta_r} e^{-2j\beta d}$$



❖ Then: $|V(z)| = |V_0^+||1 + \Gamma(z)| = |V_0^+|\left[1 + |\Gamma_L|^2 + 2|\Gamma_L|\cos(2\beta d - \theta_r)\right]^{1/2}$

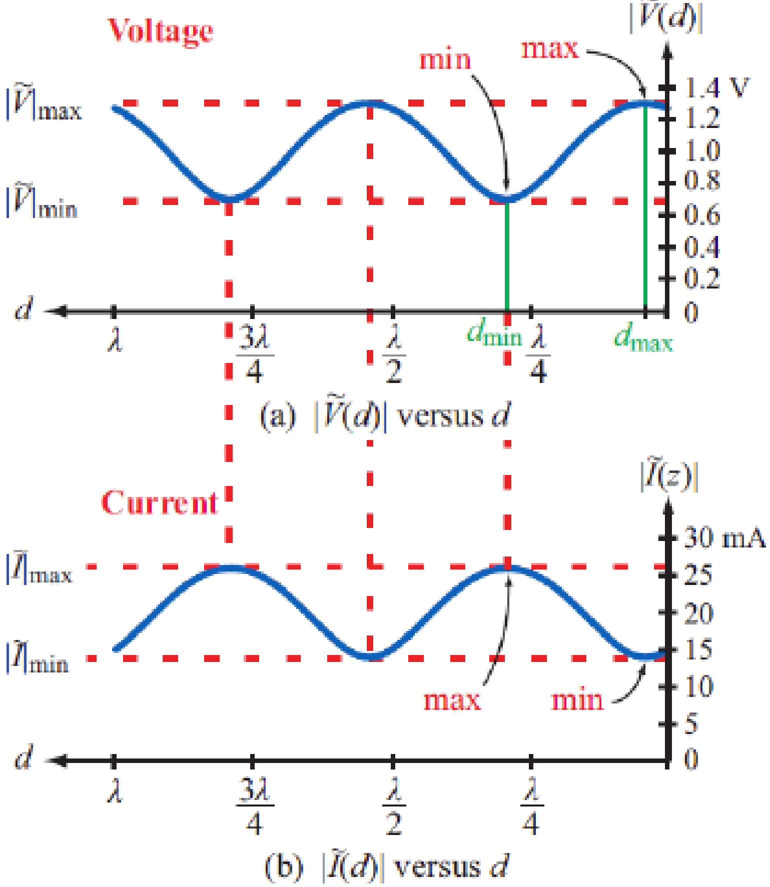


❖ Matched TL: $Z_L = Z_0 \rightarrow \Gamma = 0$

❖ Short circuit TL: $Z_L = 0 \rightarrow \Gamma = -1$

❖ Open circuit TL: $Z_L = \infty \rightarrow \Gamma = 1$

10. Standing Wave and Standing Wave Ratio



$$|V(z)| = |V_0^+|\left[1 + |\Gamma_L|^2 + 2|\Gamma_L|\cos(2\beta d - \theta_r)\right]^{1/2}$$

- $|V(z)| = |V(z)|_{min} = |V_0^+|(1 - |\Gamma_L|)$ *when*:

  $$\cos(2\beta d - \theta_r) = -1 \leftrightarrow 2\beta d - \theta_r = (2n+1)\pi$$

- $|V(z)| = |V(z)|_{max} = |V_0^+|(1 + |\Gamma_L|)$ *when*:

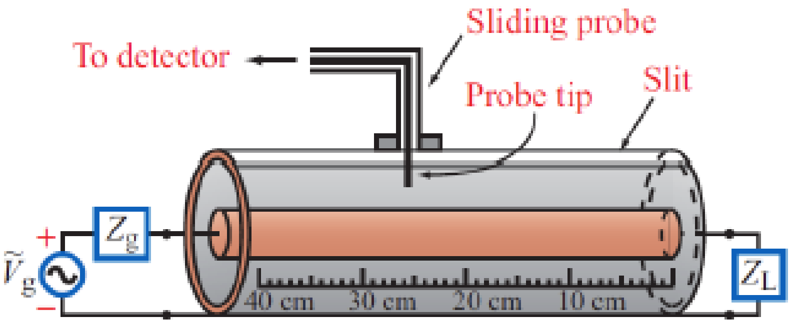
  $$\cos(2\beta d - \theta_r) = 1 \leftrightarrow 2\beta d - \theta_r = 2n\pi$$

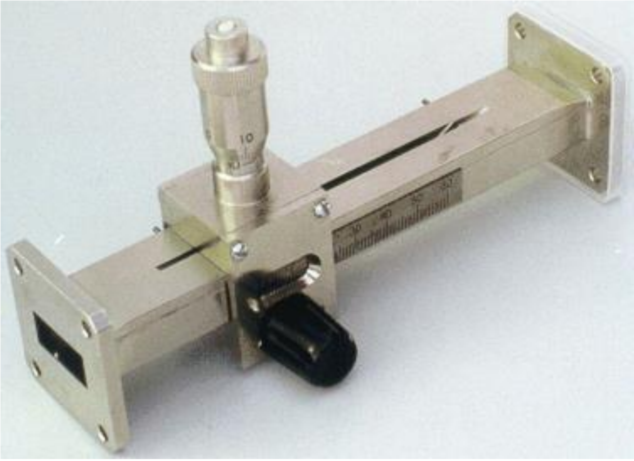
https://www.youtube.com/watch?v=yCZ1zFPvrIc

10. Standing Wave and Standing Wave Ratio

Example 2: in an unknown load impedance is found to be 3.0. The distance between successive voltage minima is 30cm and the first minimum is located at 12cm from the load. Determine:

a. The reflection coefficient Γ.
b. The load impedance Z_L



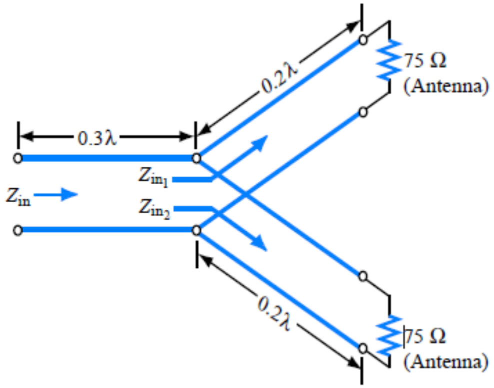


Exercises

Exercise 1: Two half-wave dipole antennas, each with impedance of 75Ω are connected in parallel through a pair of T.L. and the combination is connected to a feed T.L. as shown in the following figure. All lines are 50Ω lossless.

a. Calculate Z_{in1}

b. Calculate Z_{in} of the feed line.



Exercises

Exercise 2: A 50Ω lossless line of length $l = 0.15\lambda$ connects a 300MHz generator with $V_g = 300V$ and $Z_g = 50\Omega$ to a load $Z_L = 75\Omega$.

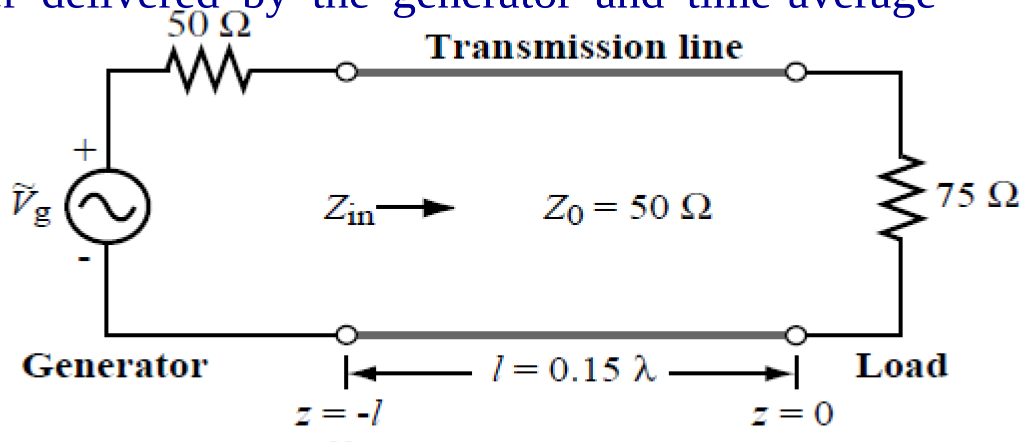
a. Compute Z_{in}

b. Compute V_i and I_i.

c. Compute the time-average power delivered to the line, $P_{in} = \frac{1}{2}\mathbb{R}e\{V_i I_i\}$.

d. Compute V_L, I_L and the time-average power delivered to the load, $P_L = \frac{1}{2}\mathbb{R}e\{V_L I_l\}$.

e. Compute the time-average power delivered by the generator and time-average power dissipated by in Z_g



Exercises

Exercise 3: In addition to not dissipating power, a lossless line has two important features:

(1) It is dispersionless (v_p is independent of frequency).

(2) Its characteristic impedance Z_0 is real.

Sometimes it is not possible to design a T.L. such that $R' \ll \omega L'$ and $G' \ll \omega C'$ but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition $R'C' = L'G'$ (distortionless line).

Such a line is called a distortionless line because despite the fact that it is not lossless, it nonetheless possesses the previous mentioned features of the lossless line. Show that for a distortionless line:

$$\alpha = R'\sqrt{\frac{C'}{L'}}$$

$$\beta = \omega\sqrt{L'C'}$$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

Exercises

Exercise 4: A 300Ω lossless line is connected to a complex load composed of a resistor $R = 600\Omega$ and an inductor with $L = 0.02mH$. At 10MHz, determine:

a. Reflection coefficient at load Γ_L?

b. Voltage Standing Wave Ratio (VSWR).

c. Location of voltage maximum nearest the load.

d. Location of current maximum nearest the load.

Exercise 5: On a 150Ω lossless line, the following observations were noted: distance of first voltage minimum from load is 3cm, distance of first voltage maximum from load is 9cm and VSWR=3. Find Z_L?

Exercise 6: A load with impedance $Z_L = 25 - j50\Omega$ is to be connected to a lossless T.L. with characteristic impedance Z_0 with chosen Z_0 such that the VSWR is the smallest possible. What should Z_0 be?

Exercises

Exercise 7: A 100MHz FM broadcast station uses a 300Ω T.L. between the transmitter and a tower-mounted half-wave dipole antenna. The antenna impedance is 73Ω. You are asked to design a quarter-wavelength transformer to match the antenna to the line.

a. Determine the length and characteristic impedance of the quarter-wavelength section?

b. If the quarter-wavelength is a two-wire line with $D = 2.5cm$ and the wires are embedded in polystyrene with $\varepsilon_r = 2.6$. Determine the physical length of the quarter-wave section and the radius of the two wire conductor.

Note that the characteristic parameters of T.Ls are given in the following table:

Exercises

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\epsilon_r}/c$	$u_p = c/\sqrt{\epsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\epsilon_r}/c$	$u_p = c/\sqrt{\epsilon_r}$	$Z_0 = \left(60/\sqrt{\epsilon_r}\right)\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\epsilon_r}/c$	$u_p = c/\sqrt{\epsilon_r}$	$Z_0 = \left(120/\sqrt{\epsilon_r}\right) \cdot \ln\left[(D/d) + \sqrt{(D/d)^2 - 1}\right]$ $Z_0 \approx \left(120/\sqrt{\epsilon_r}\right)\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\epsilon_r}/c$	$u_p = c/\sqrt{\epsilon_r}$	$Z_0 = \left(120\pi/\sqrt{\epsilon_r}\right)(h/w)$

Notes: (1) $\mu = \mu_0$, $\epsilon = \epsilon_r\epsilon_0$, $c = 1/\sqrt{\mu_0\epsilon_0}$, and $\sqrt{\mu_0/\epsilon_0} \approx (120\pi)\ \Omega$, where ϵ_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, d = wire diameter and D = separation between wire centers. (4) For parallel-plate line, w = width of plate and h = separation between the plates.

Exercises

Exercise 8: Consider the circuit below. A generator with $R_0 = 75\Omega$ is connected to a complex of $Z_L = 100 + j100\Omega$ through a T.L. of arbitrary length with $Z_0 = 75\Omega$ and $v_P = 0.8c$. Using the Smith Chart, evaluate the line for stub matching. The generator is operating at 100MHz. Find

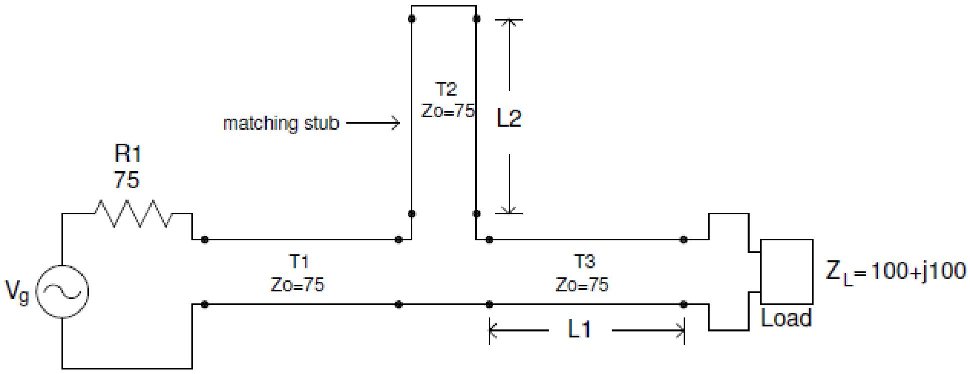
a. The electrical length of λ of the T.L.

b. The normalized load impedance.

c. The closest stub location as measured from the load.

d. The length of the stub at the closest location.

e. The lumped load element value that could take the place of the stub at the nearest location.



Exercises

Exercise 9: A Vector Network Analyzer (VNA) is attached to the end of a lossless, 15m long T.L. (50Ω, $\epsilon_r = 2.3$) operating at 220MHz. The VNA shows an input impedance of $Z_{in} = 75 - j35\Omega$. Using the Smith Chart:

a. Find the VSWR on the line.

b. Find the normalized, denormalized and equivalent circuit of the load impedance Z_L at the far end of the line. The equivalent circuit must show the correct schematic symbols (L and/or R and/or C) and the values of each symbol.

c. Find the normalized load admittance Y_L at the far end of the line. The length of the stub at the closest location.

d. Find the distance in meters from the load to the first matching point.

e. What is the normalized admittance at the first match point?

f. Find the shortest stub to match the susceptance found at the first match point. Give the length of the stub in meters.

g. If fabrication of a coaxial stub was not feasible but a lumped matching element was necessary, draw the component schematic symbol and give its value.

h. After the matching network is connected, where do standing waves exist and where do they not exist in this system? What is the SWR at the input to the line?

Q&A