THE 11th INTERNATIONAL SYMPOSIUM ON ADVANCED TOPICS IN ELECTRICAL ENGINEERING
March 28-30, 2019
Bucharest, Romania

UWB Indoor Positioning Application Based on Kalman Filter and 3-D TOA Localization Algorithm

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***Abstract*- In recent years, with the continuous development of short-range wireless communication and mobile technology, location-based services in indoor environments have paid more and more attention several solutions being reported in the literature. Ultra-Wide Band positioning technology has become one of frequently selected solution due to its low power consumption, anti-multipath capabilities, high security, low system complexity, and high precision. In this paper, 3D positioning algorithms were discussed, and a new one 3D time of arrival (TOA) positioning algorithm was proposed. The main idea of the proposed algorithm is to replace the quadratic term in the positioning estimation with a new variable and the usage of the weighted least squares linear estimation followed by the combination with Kalman filter to reduce the interference error in the transmission process.**

***Keywords*: UWB, indoor positioning, 3D, TOA, Kalman filter**

I. INTRODUCTION

The continuous development of wireless sensor networks (WSN) the indoor positioning technology has been also developed. It has a wide range of applications in smart buildings, shopping malls, smart healthcare and other. Some of the common used indoor localization technologies are based on communication protocols such as Wi-Fi, Bluetooth, ZigBee as so as RFID positioning technology, infrared positioning technology, and Ultra-Wide Band (UWB) positioning technology [1]. However, existing indoor positioning technology cannot meet the needs of people, especially for: high-precision positioning, strong adaptability in complex environments, low power consumption and low cost. As one of indoor positioning technologies the UWB one however is still necessary new studies for complex environments and multiple dynamic targets.

UWB signals have a large bandwidth and good temporal resolution, so they are widely used in high-rate and short-range wireless communication [2]. UWB positioning technology can be roughly divided into three categories: Time of Arrival (TOA), Angle of Arrival (AOA), and Received Signal strength (RSS)[3]. The AOA and RSSI methods cannot effectively guarantee the positioning accuracy in a complex indoor environment when used alone. Generally, they are used as an auxiliary method for TOA and TDOA [4][5][6], and the TOA/TDOA technology can make full use of the good temporal resolution of the UWB signal. It has been widely used in the localization of traditional 2D scene, but the localization in the 3D scene has not been popularized. Xue et. al proposed a minimum error algorithm based on TOA measurement, which achieved good 3D positioning accuracy [7]. Additional optimization algorithm is described in [8] that focuses on TOA optimization model algorithm that permits to get location information for indoor 3D positioning of wireless communication stations. The paper [9] the iterative time-of-arrival and multivariate model are used to eliminate time synchronization providing accurate indoor localization.

Hence, this paper aims to improve the traditional TOA positioning algorithm to 3D space for higher precision requirements combining TOA with Kalman filtering algorithm.

II. UWB LOCALIZATION ALGORITHM

A. TOA Ranging Principle

Using two ways-time of flight (TW-TOF) technique can be achieving good accuracy on estimating the point-to-point distance between two wireless transceivers. In the UWB case TOF ranging method is a two-way ranging technology, which mainly use the time of flight of a signal between two asynchronous transceivers to measure the distance between transceivers. The two-way ranging principle is presented in Fig 1. The Device A transmits the signal at time T_{a1}, and the Device B receives the signal at time T_{a2}. After waiting for the TW time, Device B forwards the signal to Device A at time T_{b1} and Device A receives the signal at time T_{b2}. Thus, the time of flight of the signal between Device A and B can be calculated to determine the flight distance [10], where c – is the light velocity.

$$S = c * \left[(T_{a2} - T_{a1}) - (T_{b2} - T_{b1}) \right] \qquad (1)$$

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Fig 1.Two-way ranging principle

B. 3D TOA Localization Principle

If the coordinates of the nodes to be positioned are (xi, yi, zi) in a 2D space, only three anchor nodes are needed for positioning comparing with 3D space, where at least four anchor nodes are required for positioning. Considering the work objective the coordinates of four anchor nodes are assumed to be $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$, $D(x_4, y_4, z_4)$ respectively. d is the distance between the tag (with unknown position) and each base station, as shown in Fig 2[11][12].

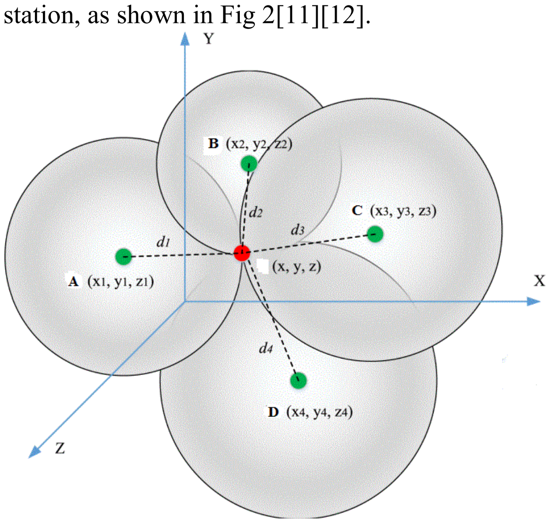

Fig 2. 3D TOA Localization Principle

The distances d_i from nodes to the target X are given by:

$$\begin{cases} d_1^2 = (x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 \\ d_2^2 = (x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2 \\ d_3^2 = (x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2 \\ d_4^2 = (x_4 - x)^2 + (y_4 - y)^2 + (z_4 - z)^2 \end{cases} \quad (2)$$

By calculation, the node location can be determined using the following equation:

$$X = \left(A^T A\right)^{-1} A^T b \quad (3)$$

Where:

$$A = \begin{bmatrix} 2(x_1 - x_4) & 2(y_1 - y_4) & 2(z_1 - z_4) \\ 2(x_2 - x_4) & 2(y_2 - y_4) & 2(z_2 - z_4) \\ 2(x_3 - x_4) & 2(y_3 - y_4) & 2(z_3 - z_4) \end{bmatrix}$$

$$b = \begin{bmatrix} (x_1^2 + y_1^2 + z_1^2) - (x_4^2 + y_4^2 + z_4^2) - (d_1^2 - d_4^2) \\ (x_2^2 + y_2^2 + z_2^2) - (x_4^2 + y_4^2 + z_4^2) - (d_2^2 - d_4^2) \\ (x_3^2 + y_3^2 + z_3^2) - (x_4^2 + y_4^2 + z_4^2) - (d_3^2 - d_4^2) \end{bmatrix}$$

C. The modified 3D TOA localization

When TOA technology is used to locate our tag using UWB positioning system the TOA value can be measured to get the distance between tag and four base stations. Multiple TOA measurements can form a set of circular equations about the position of the tag in 3D coordinates. We assumed that the tag coordinate are (x, y, z) in 3D space, the known positions of the base stations are (X_p, Y_i, Z_i), and the distances between base stations and the tag are R_i where Ri is given by :

$$R_i^2 = (X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2$$
$$= K_i - 2X_i x - 2Y_i y - 2Z_i z + R = (c\tau)^2$$

(4)

where τ_i is the time of arrival and:

$$K_i = X_i^2 + Y_i^2 + Z_i^2, i = 1,2,3\cdots, R = x^2 + y^2 + z^2,$$

Assuming $z_a = \left[z_p^T, R\right]^T$ as an unknown vector, and $z_p = [x, y, z]^T$, so linear equations with z_a as the variable are established from formula(4):

$$h = G_a z_a \quad (5)$$

Then the error vector corresponding to the estimated position of the tag is given by:

$$\psi = h - G_a z_a^0 \quad (6)$$

Where: z_a^0 is the value of z_a corresponding to the actual position of tag.

$$h = \begin{bmatrix} R_1^2 - K_1 \\ R_2^2 - K_2 \\ \vdots \\ R_M^2 - K_M \end{bmatrix}, G_a = \begin{bmatrix} -2X_1 & -2Y_1 & -2Z_1 & 1 \\ -2X_2 & -2Y_2 & -2Z_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -2X_M & -2Y_M & -2Y_M & 1 \end{bmatrix}$$

Weighted least squares (WLS) is used to replace the covariance matrix of error ψ approximately by the covariance matrix Q of the measured value of TOA:

$$z_a = \left(G_a^T Q^{-1} G_a\right)^{-1} \left(G_a^T Q^{-1} h\right) \quad (7)$$

Considering:(x,y,z) as the estimated position of the tag, Q as TOA covariance matrix and each TOA measurement is independent, the Q matrix is a diagonal matrix:

$$Q = diag\left\{\sigma_1^2, \sigma_2^2, \cdots, \sigma_M^2\right\} \quad (8)$$

In formula (6), replacing the covariance matrix of the error vector ψ by the Q matrix approximation will also bring some errors. Therefore, when the TOA error is small, the error vectors corresponding to the TOA measurements of M are:

$$\psi = 2Bn + n \odot n \approx 2Bn \tag{9}$$
$$B = diag\{R_1^0, R_2^0, \cdots, R_M^0\}$$

Set: R_i^0 is the actual distance between tag and base station of i,n is TOA measurement error in distance(approximate normal distribution).

The covariance matrix of the error vector ψ constructed from TOA measurements in formula (6):

$$\Psi = E[\psi\psi^T] = 4BQB \tag{10}$$

In order to get the matrix B, the measured R_i can be substituted for R_i^0, then the first WLS estimate of z_a is:

$$z_a = \left(G_a^T \Psi^{-1} G_a\right)^{-1}\left(G_a^T \Psi^{-1} h\right) \tag{11}$$

New matrix B can be obtained by using the value of z_a. Using the above process to perform another WLS calculation can result in a modified estimated position. The above calculation assumes that the elements of z_a are independent of each other, but R is related to the position of the tag(x, y, z), which we can use to get a more accurate position estimate. We first calculate the covariance matrix of the estimated position in the presence of noise:

$$R_i = R_i^0 + cn_i, G_a = G_a^0 + \Delta G_a, h = h^0 + \Delta h \tag{12}$$

Where $G_a^0 z_a^0 = h^0$, formula (5) indicates:

$$\psi = \Delta h - \Delta G_a z_a^0 \tag{13}$$

We set $z_a = z_a^0 + \Delta z_a$,reusing formula (9) and (13), Δz_a and its covariance matrix are:

$$\Delta z_a = c\left(G_a^T \Psi^{-1} G_a\right)^{-1} G_a^T \Psi^{-1} Bn$$
$$\operatorname{cov}(z_a) = E\left[\Delta z_a \Delta z_z^T\right] = \left(G_a^T \Psi^{-1} G_a\right)^{-1} \tag{14}$$

The vector z_a is the actual value, the covariance matrix is determined by the random variable of formula (14), so the elements of z_a can be expressed as:

$$z_{a,1} = x^0 + e_1, z_{a,2} = x^0 + e_2$$
$$z_{a,3} = x^0 + e_3, z_{a,4} = x^0 + e_4 \tag{15}$$

Of which: e_1, e_2, e_3, e_4 is estimation error of z_a. According to the correlation between (x, y, z) and R, a new error vector can be constructed as follows:

$$\psi' = h' - G_a' z_a' \tag{16}$$

Of which: $h' = \begin{bmatrix} z_{a,1}^2 \\ z_{a,2}^2 \\ z_{a,3}^2 \\ z_{a,4}^2 \end{bmatrix}, G_a' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, z_a' = \begin{bmatrix} x^2 \\ y^2 \\ z^2 \end{bmatrix}$

ψ' is defined as the estimated error of z_a, and the formula 15 is substituted into the formula 6 to get:

$$\psi_1' = 2x^0 e_1 + e_1^2 \approx 2x^0 e_1; \psi_2' = 2y^0 e_2 + e_2 \approx 2y^0 e_2 \tag{17}$$
$$\psi_1' = 2x^0 e_1 + e_1^2 \approx 2x^0 e_1; \psi_2' = 2y^0 e_2 + e_2 \approx 2y^0 e_2$$

The covariance matrix of the error is:

$$\Psi' = E[\psi'\psi'] = 4B' \operatorname{cov}(z_a) B'$$
$$B' = diag\left\{x^0, y^0, z^0, \frac{1}{2}\right\} \tag{18}$$

In the matrix B', (x^0, y^0, z^0) can be approximately replaced by the value of the formula z_a, and WLS of z_a' is estimated as:

$$z_a' = \left(G_a' \Psi'^{-1} G_a'\right)^{-1}\left(G_a'^T \Psi'^{-1} h\right) \tag{19}$$

The final positioning calculation result of tag is:

$$z_p = \pm\sqrt{z_a'} \tag{20}$$

In the matrix z_p, (x, y, z) selection of plus-minus sign should be consistent with corresponding elements in z_a of formula 11, so as to eliminate the ambiguity of positioning results.

D. Kalman Filtering

In the complex indoor environment, it is inevitable to encounter non-line-of-sight obstacles in the process of signal transmission. For this problem, the Kalman filter error detection algorithm is used to eliminate the transmission error as was presented in [13]. The calculation of Kalman filtering is based on the following assumption: all the measured results are based on real signals and additive Gaussian noise. If the above hypotheses are true, Kalman filtering can effectively obtain signal information from noise-containing measurement results.

Suppose the state equation of a discrete control process system is:

$$x_{k+1} = Ax_k + Bu_{k+1} + \omega_{k+1} \tag{21}$$

Of where: x_{k+1} is the system state at time k+1; ω_{k+1} is Gaussian random variable of process noise; u_{k+1} is the amount of control of the system at time k+1;A and B are system parameters, they are matrices for this system.

The corresponding observation matrix is:

$$Z_{k+1} = \begin{bmatrix}\theta_{s,k+1} & \gamma_{s,k+1}\end{bmatrix}^T = H_{k+1}x_{k+1} + v_{k+1} \tag{22}$$

Of where: Z_{k+1} is a measurement at time k+1; H_{k+1} is unit matrix; v_{k+1} is Gaussian random variable that measures noise.

The model of the process system is used to predict the next state. The iterative process of the Kalman filter principle described in [14] [15] includes:

Estimate the current state based on the previous state of the system:

$$\hat{x}_{k+1|k} = Ax_{k|k} + Bu_{k+1} \tag{23}$$

Of where: $\hat{x}_{k+1|k}$ is predicted result of previous state; $x_{k|k}$ is optimal result of previous state; u_{k+1} is control of current state.

The minimum mean square error matrix is:

$$P_{k+1|k} = AP_{k|k}A^T + Q \tag{24}$$

where: $P_{k|k}$ is minimum mean square error matrix for estimated value $x_{k|k}$; $P_{k+1|k}$ is covariance of $\hat{x}_{k+1|k}$; Q is covariance of the system process.

Kalman gain is:

$$K_{k+1} = P_{k+1|k}H_{k+1}^T\left(R_{k+1} + H_{k+1}P_{k+1|k}H_{k+1}^T\right)^{-1} \tag{25}$$

Of where: R_{k+1} is covariance matrix of observed noise v_{k+1} .

The optimization estimates for K+1 state is:

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\left(Z_{k+1} - H_{k+1}\hat{x}_{k+1|k}\right) \tag{26}$$

Keep the continuity of the system until the end, update the $x_{k+1|k+1}$ covariance of k+1 state.

$$P_{k+1|k+1} = \left(I - K_{k+1}\right)HP_{k+1|k} \tag{27}$$

Through the above-mentioned process, the interference caused by the transmission can be eliminated.

III. EXPERIMENTAL TESTING AND VERIFICATION

A. *Experimental Setup*

This experiment was carried out in an indoor environment with a size of 4m * 2m. It was used for 4 UWB positioning base stations, which are placed in 4 positions, and their position coordinates are $(0,0,2)$、$(4,0,2)$、$(4,2,2)$、$(0,2,0.8)$, the unit is meter. The experimental arrangement is presented in Fig 3. In this experiment, the modified positioning algorithm combined with this paper will be compared with the standard 3D TOA positioning method.

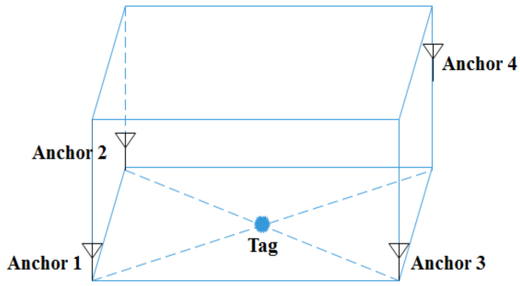

Fig 3.3D Positioning Arrangement of anchors and tag

The positioning server in this article uses a 4-core 4-thread Intel i5-4770 processor with 8GB of RAM. The positioning program is developed on the platform of CoIDE. It is used to program windows form control and C++ language. The GUI is shown in Fig 4. Before the positioning starting, the coordinates of the base stations are applied on the algorithm inputs. The 3D coordinate information (X, Y, Z) of the tag is displayed directly above, the distance between tag and each base station is displayed at the top right, and the movement track of the tag can be displayed at the bottom left, which can be displayed the geographic location of the tag visually.

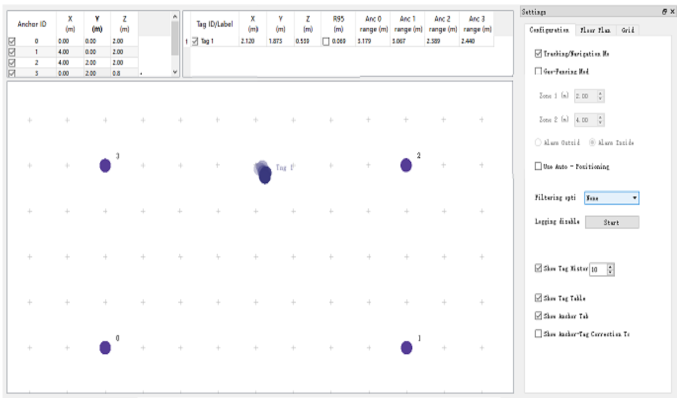

Fig 4. The GUI of application system

B. *The comparison of RMSE*

This paper compared the standard 3D TOA positioning algorithm with the modified 3D TOA positioning algorithm. The RMSE is selected as the measurement index of the positioning accuracy, and then the difference of the RMSE value between two methods under different mean square errors is explored.

$$MSE = E\left[(x-\hat{x})^2 + (y-\hat{y})^2 + (z-\hat{z})^2\right] \tag{28}$$

Of which: (x, y, z) is the actual position of tag; $(\hat{x}, \hat{y}, \hat{z})$ is the measured position of the tag. RMSE is often used to estimate the positioning accuracy and its calculation formula:

$$RMSE = \sqrt{E\left[(x-\hat{x})^2 + (y-\hat{y})^2 + (z-\hat{z})^2\right]} \tag{29}$$

RMSE comparison of standard and modified 3D TOA positioning algorithms is shown in Fig 5.

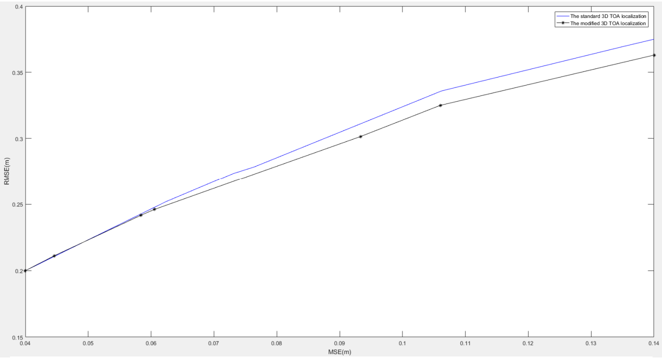

Fig 5. RMSE comparison between standard 3D TOA and modified 3D TOA localization methods

It can be judged from Figure 4 that the performance of the standard 3D TOA and modified 3D TOA positioning method is very close when the mean square error is 0.06 /m or less. When the mean square error gradually increasing, RMSE of the two localization methods also increases gradually. We can see that RMSE values of the standard and modified TOA algorithm are basically the same. However, compared with the standard TOA algorithm, the modified TOA algorithm has higher positioning accuracy.

C. Simulation of joint location algorithm

Through the simulation by MATLAB, the performance of the standard and modified 3D TOA positioning in X, Y, and Z coordinates are analyzed and compared. The abscissa represents the number of measurement point, and the ordinate represents each simulated object coordinate. As can be seen from the figure6, 7, 8, the estimated values of X coordinate errors is relatively stable, while Y and Z estimation are less stable. Y and Z coordinate maximum error are about 15/cm and positioning accuracy are within the allowable range.

In the ideal conditions, the accuracy of standard 3D TOA positioning combined with the Kalman filtering algorithm can reach about 15~25/cm. As shown in the figure, compared with the standard 3D TOA joint Kalman filter algorithm, simulation results show that the modified 3D TOA positioning combined with the Kalman filter algorithm is stable at 5~10/cm. This result proves that the modified 3D TOA positioning algorithm combined with Kalman filtering is effective.

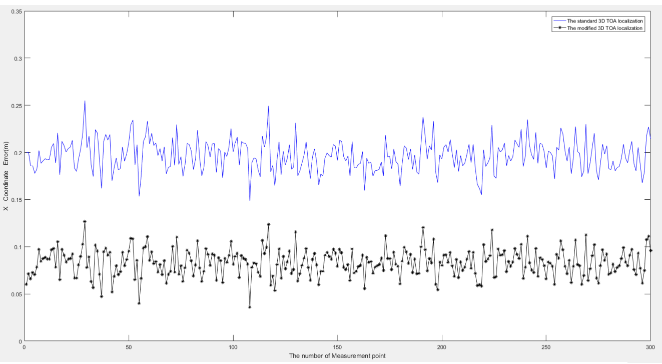

Fig 6. X coordinate comparison for two 3D localization methods

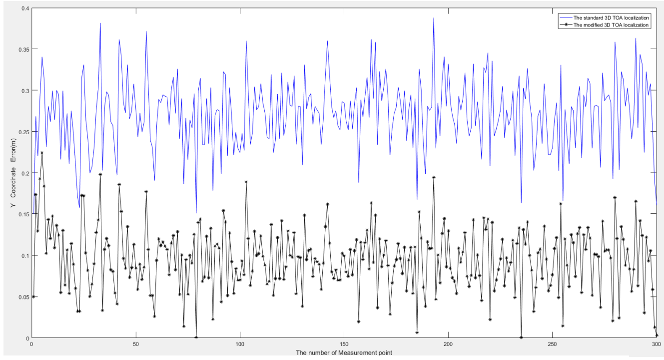

Fig 7. Y coordinate comparison for two 3D localization methods

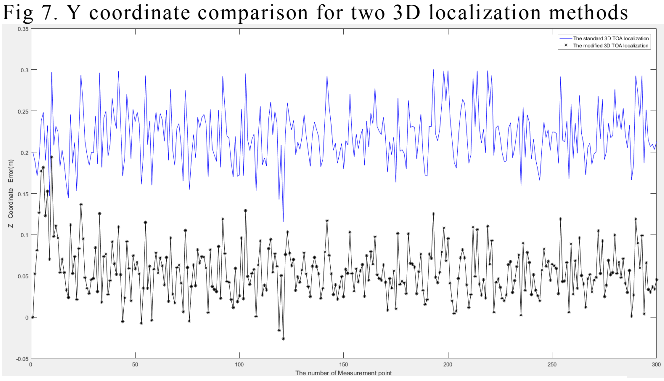

Fig 8. Z coordinate comparison for two 3D localization methods

D. Experimental Results

The experiment measures positioning information of the tags at six locations in the field. The information of tag localization was obtained by application of the standard and modified 3D TOA methods. Several results are presented in the table. The modified 3D TOA positioning algorithm results compared with the standard 3D TOA positioning method results are more accurate taking into account the real position of the tag.

Table 1. Experimental data of tag localization

Actual Coordinates(m)	Measured Coordinates (standard 3D TOA)	Measured Coordinates (modified 3D TOA)
(1,0,2)	(0.8645,0.1517,1.8506)	(0.9785,0.0895,2.0531)
(1,1,1.2)	(0.8486,1.2141,1.4075)	(0.9513,1.1246,1.2845)
(1,2,0.6)	(0.8522,1.7811,0.7523)	(1.0675,2.0494,0.5387)
(2,0,2)	(1.8548,0.1629,1.8711)	(2.1027,0.0786,1.9397)
(2,1,1.2)	(2.1465,0.8611,1.3789)	(2.0682,0.9548,1.1376)
(2,2,0.6)	(2.1850,2.2083,0.8144)	(1.9365,1.9546,0.7114)
(3,0,2)	(3.2047,0.1202,2.1816)	(2.9486,0.0593,2.0872)
(3,1,1.2)	(3.1800,1.2543,1.0475)	(3.1148,1.0763,1.2794)
(3,2,0.6)	(3.1599,2.1848,0.7865)	(3.0596,1.9372,0.5289)

IV. CONCLUSION

The proposed 3D TOA positioning algorithm in this paper breaks through the limitations of considering the two-dimensional in the previous positioning algorithm. The modified 3D TOA positioning algorithm improves the positioning accuracy, and joint Kalman filtering to further eliminates the error interference in the transmission process. The simulation results show that the positioning accuracy can reach about 5~10/cm, which verifying the feasibility of the joint the modified 3D TOA positioning algorithm and Kalman filtering algorithm. Based on the results of this paper, future research will focus on visualization tag trajectories.

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