Lecture 13

$\begin{cases} \text{Ch.1} & \mathbb{R}^2 \\ \text{Ch.2} & \text{Surface} \end{cases}$

$(\mathbb{R}^2, d_{Euc})$ given $P, Q \in \mathbb{R}^2$, compute $d(P,Q)$

We consider isometries $f \in \underbrace{Iso(\mathbb{R}^2, d)}_{\text{group of isometries}}$:

$d(P,Q) = d(f(P), f(Q))$

How to classify f ?

"Characterization of isometries by images of 3 pts"

需加入条件.

Thm I: Let $A, B, C \in \mathbb{R}^2$ be 3 Non colinear points

Then, $f: \mathbb{R}^2 \to \mathbb{R}^2$ is uniquely determined by the images of $f(A), f(B), f(C)$

FALSE $\Uparrow$

Review Proof: given $D \in \mathbb{R}^2$, where is $f(D) \in \mathbb{R}^2$?

Thm II (Classification of Isometries)

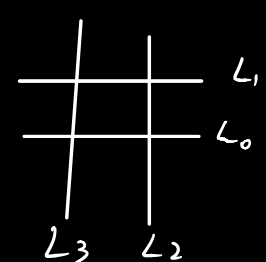
Any $f: \mathbb{R}^2 \to \mathbb{R}^2$, $\exists\, L_1 \dots L_k$ lines s.t.

$$f = \overline{r}_{L_k} \circ \overline{r}_{L_{k-1}} \circ \cdots \circ \overline{r}_{L_2} \circ \overline{r}_{L_1}$$

"$Iso(\mathbb{R}^2)$ is generated by reflection"

In fact, $\boxed{k \leq 3}$ i.e. f is 0, 1, 2, or 3 reflections.

Example: (1)



$$f = \overline{r}_3 \circ \overline{r}_2 \circ \overline{r}_1 \circ \overline{r}_0$$

P. Y 2. R ?

Thm III (Type of Isometries)

Every isometry $f \in Iso(\mathbb{R}^2)$

(1) Reflection $\overline{r}_L$ 1 (3) Translation $t_{\langle\alpha,\beta\rangle}$ 2 refs

(2) Rotation $R_{P,\theta}$ 2 refs (4) Glide Ref. 3 refs.

Part 2 : Geometry in $\mathbb{R}^2/\Gamma$ and $\Gamma \subseteq Iso(\mathbb{R}^2)$

Given $P, Q \in \mathbb{R}^2$, they define points $\Gamma P, \Gamma Q \in \mathbb{R}^2/\Gamma$.

(*) compute $d(\Gamma P, \Gamma Q)$ in $\mathbb{R}^2/\Gamma$

↳ 1st Draw Γ-orbit of P

2nd Min. distance.

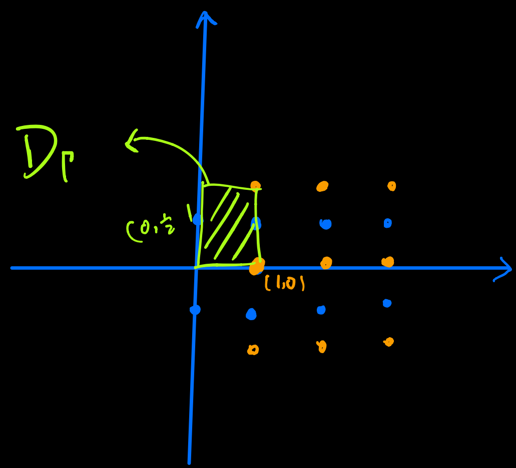
(Def^n) A fundamental domain D_Γ for Γ

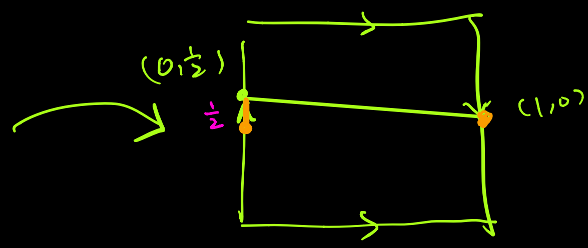
- if Γ is given, find such D_Γ
= Find distances in D_Γ and computing intersection between lines in $\mathbb{R}^2/\Gamma$

→ computationally efficient matter

Example

Consider $\Gamma = \langle t_{(1,0)} \circ \bar{r},\ t_{(0,1)} \rangle \leq \mathrm{Iso}(\mathbb{R}^2)$





• 相同的点

$d[(0,\frac{1}{2}),(1,0)] = \frac{1}{2}$

$(x,y) \sim (x,\ y\pm 1) \sim (x\pm 1,\ -y)$