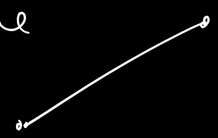
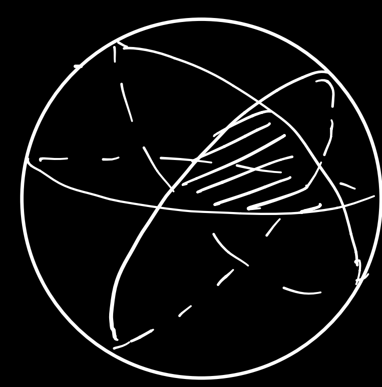
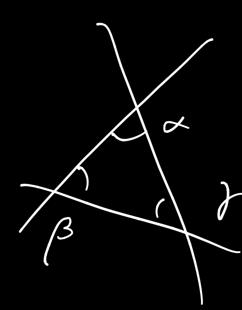
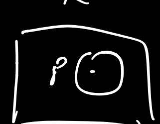
$\mathbb{R}^2$ Eucl. plane	S^2 2-sphere
line 	

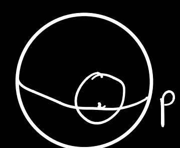
<u>Thm</u> Let T be a triangle $(\mathbb{R}^2, d_{Eucl})$ if α, β, γ are the interior angles of T. The $\underbrace{\alpha + \beta + \gamma = \pi}_{\text{angles in } \mathbb{R}^2}$



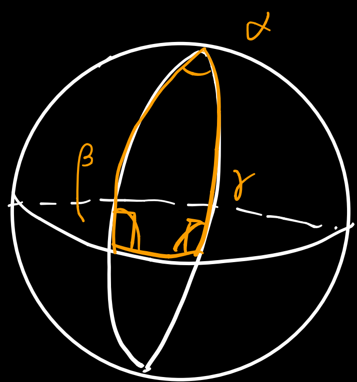
$\mathbb{R}^2$



S^2



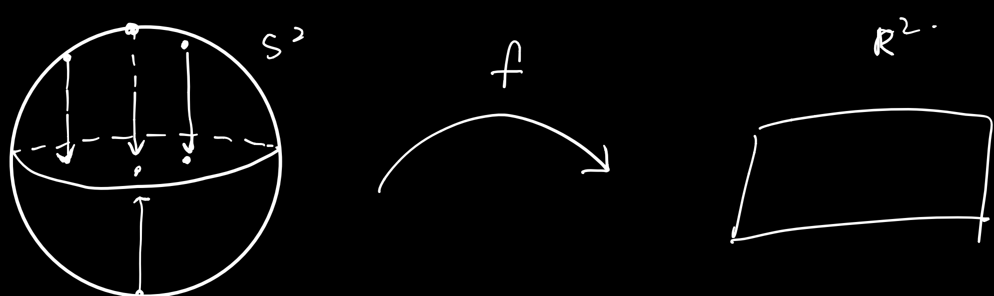
<u>Thm</u> (triang in S^2) Let $T \subseteq S^2$ be a triangle with interior angles α, β, γ. Then $\alpha+\beta+\gamma = \pi + \text{Area}(\Delta)$



$$\frac{\pi}{2} + \frac{\pi}{2} + \alpha$$

<u>Q1</u>: Does $\exists$ a bijection $f: S^2 \to \mathbb{R}^2$ s.t. f continuous & preserve distances?

No
today

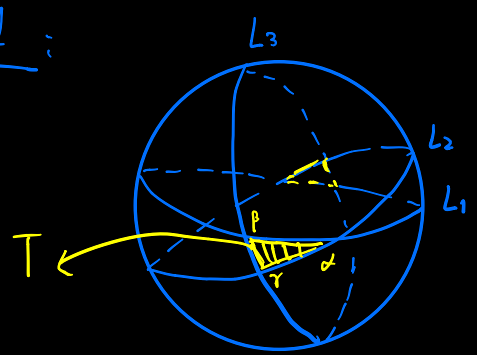


$(x, y, z) \longrightarrow (x, y)$

Thm: Let $T \in S^2$ be a triangle, with interior angles α, β, γ. Then,

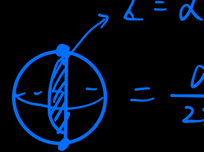
$$\alpha+\beta+\gamma = \pi + \text{Area}(T)$$

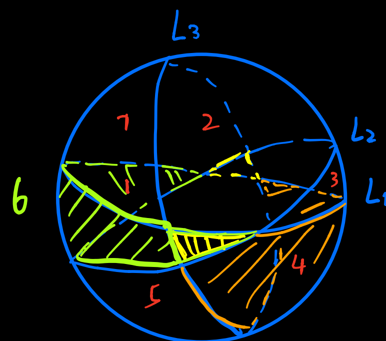
Proof:



Area(T): the yellow region

Area(S^2) = 4π.

We know the area of  $= \dfrac{\alpha}{2\pi} \cdot 4\pi = 2\alpha$



$1 \cup 6$ sector of area 2α

$1 \cup 4 \rightarrow 2\gamma$

$1 \cup 2 \rightarrow 2\beta$

Sector $8 \cup 7$ sector of area 2γ

$8 \cup 3 \rightarrow 2\beta$

$8 \cup 5 \rightarrow 2\alpha$

The union of all sectors covers the 2-sphere, the area$(1 \cup 2) + A(1 \cup 4)$

$$+ A(1 \cup 6) + A(8 \cup 7) + A(8 \cup 3)$$

$$+ A(8 \cup 5)$$

$$= 4\pi + 2\,\mathrm{Area}(T) \;\; \underbrace{+2\,\mathrm{Area}(T)}_{\text{over counting}}$$

$$\Downarrow$$

$$4(\alpha+\beta+\gamma) = 4\pi + 4\,\mathrm{Area}(T)$$

$$\boxed{\mathrm{Area}(T) = \alpha+\beta+\gamma-\pi.}$$

§2. Stereographic Projection.

"Rotates geometry in $\mathbb{R}^2$" to geometry in $\mathbb{R}^2$