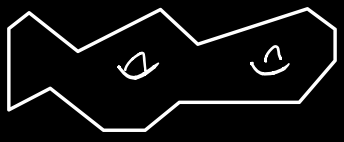
Lecture 22

$(\mathbb{R}^2, d_{Eu})$	(S^2, ds^2)
* Isometry $Iso(\mathbb{R}^2)$	* Isometry $Iso(S^2)$ (no transl
* lines & distances	* Lines & distances
$(\mathbb{R}^2/\Gamma, d_{Eu})$	* Triangles & Area
* Triangles & Regions	(new!)

?? (H^2, d_{hyp})

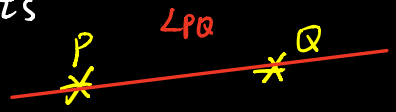
* Isometries Iso



§1. Euclid's Elements (300 BC, Alexandria)
"The elements"

Euclid introduced 5 Axioms for geometry:

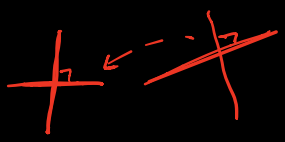
A1. ∃ unique line through 2 pts



A2. Any segment between P & Q can be extended to line.

A3. Given P a point and a radius, ∃ circle centered at P.

A4. Right angles are congruent.



A5. (contversial) Given a line L and apt P & L
∃ unique parallel line L through P

Q: is A5 { really needed?
follow from A1 - A4?

Negate A5 $\longrightarrow$ $\exists$ at least 2 parallel lines.

$\searrow$ $\exists$ parallel line $\rightarrow$ failed in (S^2, ds^2)

Thm (Balyai – Lobocherstic – Gauss)

$\exists$ a geometry (H^2, d_{H^2}) which satisfies Axioms A_1、A_2、A_3、A_4, but $\exists$ at least 2 parallel lines

$\hookrightarrow$ A5 fails

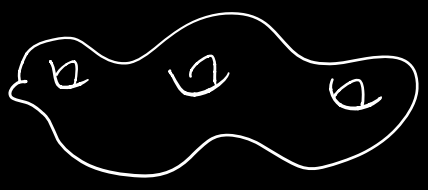
Cor: $A_1 - A_4$ does NOT imply Axiom 5

Thm (Classfi of Surfaces)

Every surfaces has one of the 3 geometries (R^2、S^2、H^2)

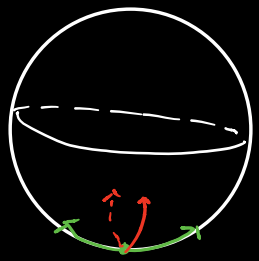
R^2、S^2 见前页.

(H^2, d_{H^2}) where $H^2 := \{(x, y) \in R^2, y > 0\}$

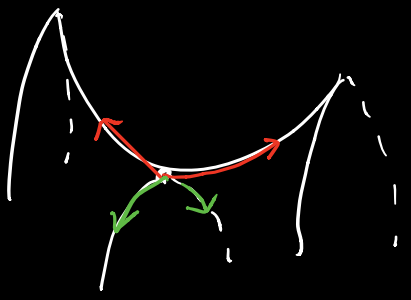


§2. What is hyperbolic geometry?

flat plane
R^2



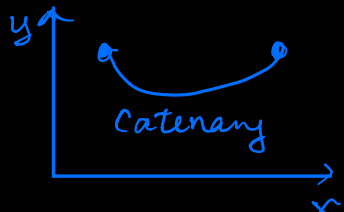
S^2 sphere



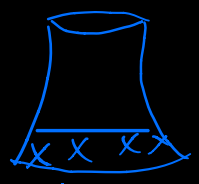
Saddle
(pringle)

Examples (Nature)

① Architecture



$y = \cosh(x)$



nuclear plant

② Nature

(leaves of plant) & butterflies' wings & Mantis shrimp. kick & coral reefs

③ Mathematically

Completes classif of all higher genus surfaces
≥ 2 holes