Lecture 10

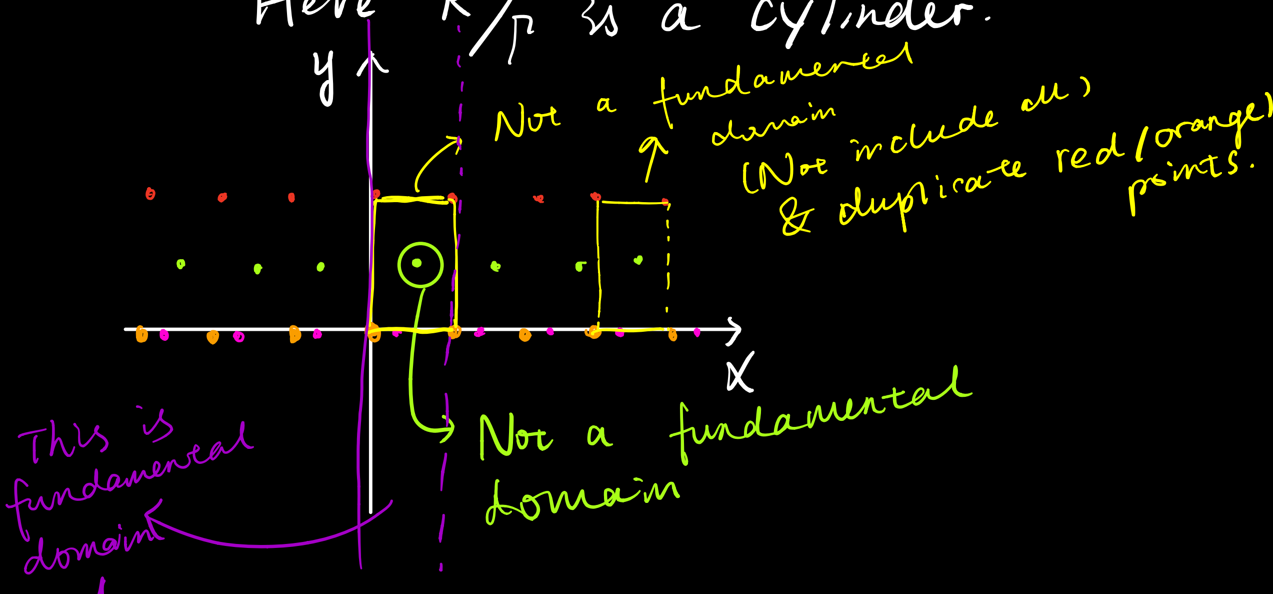
Let $\Gamma \in \mathrm{Iso}(\mathbb{R}^2)$ be a discontinuous fixed point free subgroup.

Def A fundamental domain $D_\Gamma \subseteq \mathbb{R}^2$ for Γ is any subset D_Γ s.t. for any p in $\mathbb{R}^2$, $|\Gamma p \cap D_\Gamma| = 1$ ☐

§1. Γ generated by translations

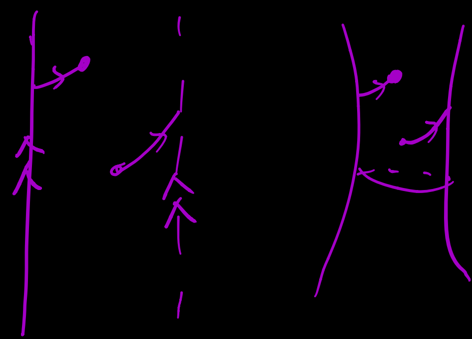
(a) $\Gamma = \langle t_{(1,0)} \rangle \leq \mathrm{Iso}(\mathbb{R}^2)$.

Here $\mathbb{R}^2/\Gamma$ is a cylinder.



→ AND not unique.

passage to D_Γ

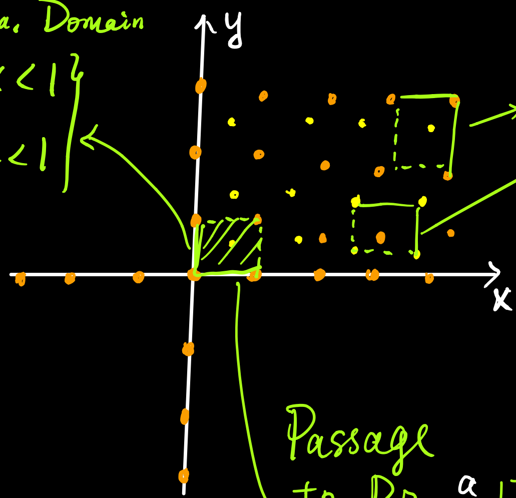


(b) $\Gamma = \langle t_{(1,0)}, t_{(0,1)} \rangle \subseteq Iso(\mathbb{R}^2)$, then

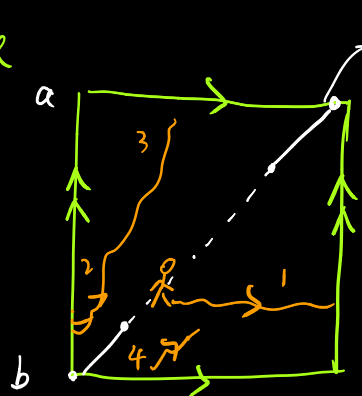
$\mathbb{R}^2/\Gamma$ is a 2-torus.

funda. Domain $\begin{cases} 0 \le x < 1 \\ 0 \le y < 1 \end{cases}$

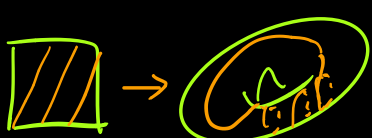
Also a funda. domain.



Passage to D_Γ



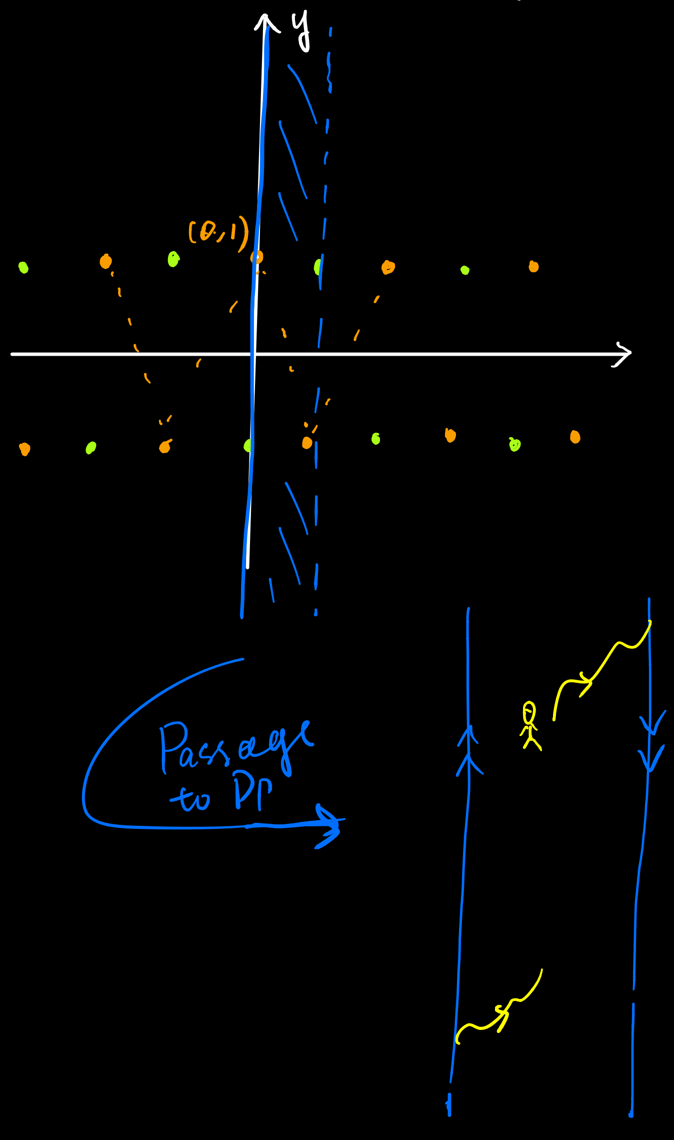
实线 = shortest path intersect a/b at one point.



$(x, y+1) \sim (x, y) \sim (x+1, y)$

§2. Γ generated with glide reflections.

(c) $\Gamma = \langle t_{(1,0)} \cdot T \rangle \leq \mathrm{Iso}(\mathbb{R}^2)$. Here $\mathbb{R}^2/\Gamma$ is a twisted cylinder.



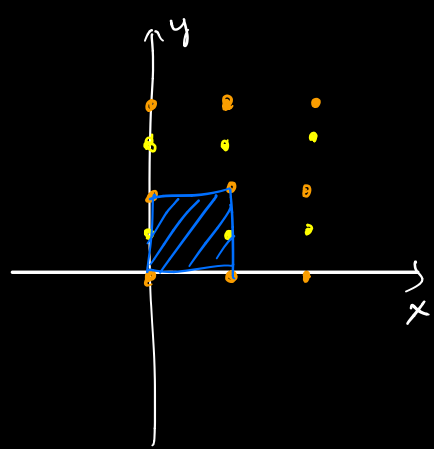
$(x+2, y)$
$\wr$
(x, y).

$(x, y) \sim (x+1, -y)$

(d) Klein bottle

can glide ref.

$$\Gamma = \langle t_{(1,0)} \circ \bar{T} , t_{(0,1)} \rangle$$



Passage to D_Γ