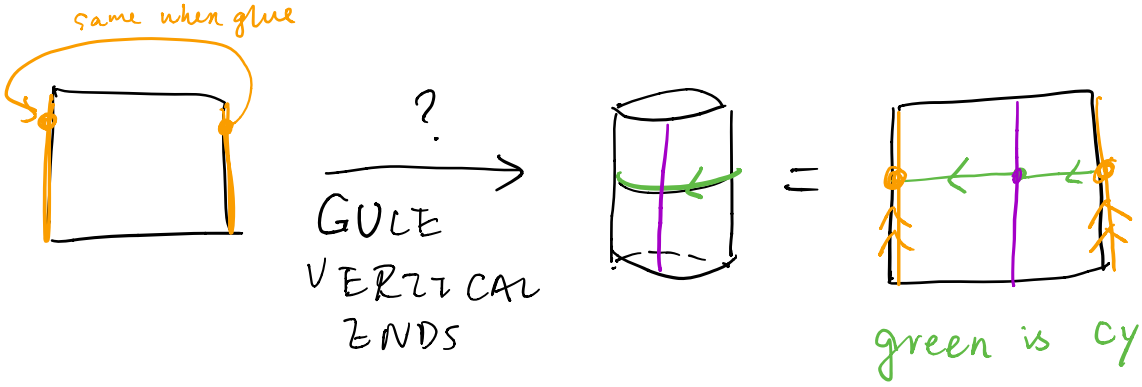
Section 2.1 & 2.2

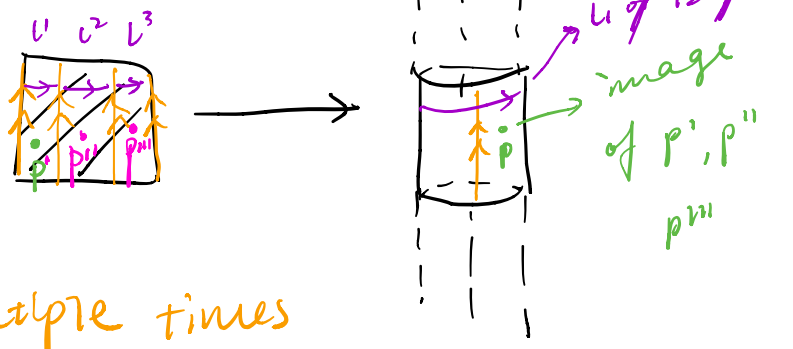
Euclidean Surface

klein bottle.

§1. The quotients construction:



Now, give $\mathbb{R}^2$



Roll multiple times

(∞ rolling)

What are the main goals for cylinder?

① Rigor. def. as a set, then add distance.

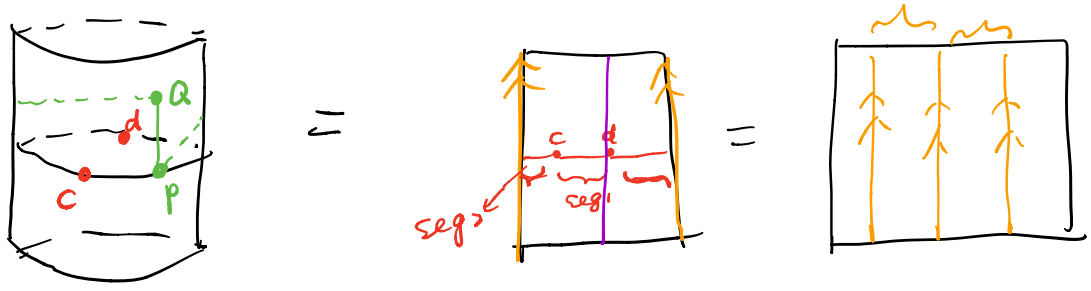
② What is distance (P, Q)? (# of segments?)

③ What are lines?

④ What are triangle?

⑤ What is the isometry group of the cylinder?

Example



§2. Rigorous Definition : Euclidean cylinder

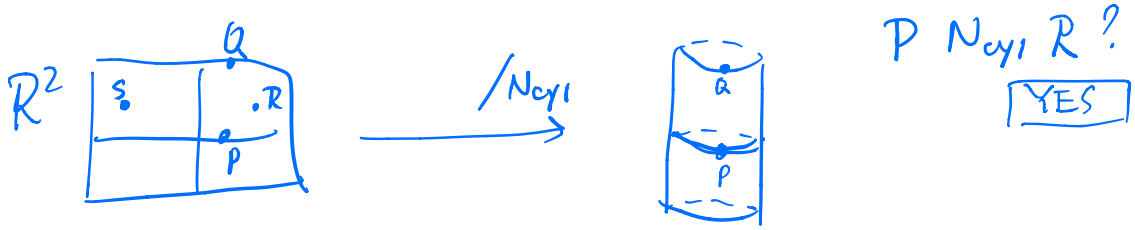
Def : Let $t_{(1,0)} \in \mathcal{Z}_{so}(\mathbb{R}^2)$, then

$C := \mathbb{R}^2 / N_{cyl}$ is the cylinder,

where $P \underset{cyl}{\sim} Q \iff \exists n \in \mathbb{Z}$ such that

$t^n_{(1,0)}(P) = Q$ ▨

Ex $P=(3,4)$ $Q=(3,6)$, $R=(5,4)$, $S=(-7,4)$



$P \; N_{cyl} \; R$?

YES

Equivalently, $C := \{(x,y) \in \mathbb{R}^2\} / (x \sim x+1)$

Ex $(3,4) \sim (5,4)$, $(3,4) \sim (-7,4)$

$(3,4) \not\sim (3.2,4)$, $(3,4) \not\sim (3,5)$ ▨

The distance in C will $(\mathbb{R}^2, d_{EUCL})$ descended to $\mathbb{R}^2 / N_{cyl}$

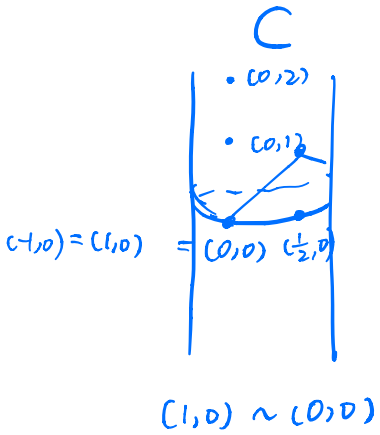
Def The distance d_{cyl} for cylinder C is defined by

$$d_{cyl}(P,Q) := \min\{\text{dist}(P',Q')\} \equiv \min\{d(P,Q')\}$$

(one Q' of P)

where $P, Q \in C$, and P', Q' run over all possible lifts of P, Q respectively

Ex.



$\text{dist}((0,0),(\frac{1}{2},1))$?

Note: Also refer to Disc 3