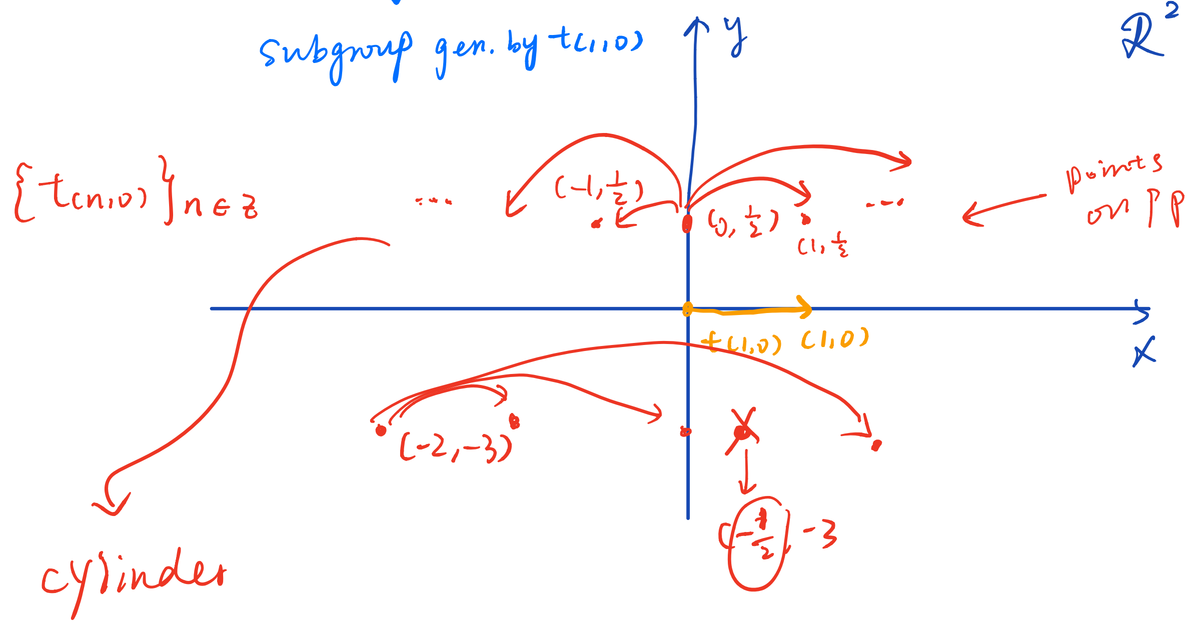
Lecture 8

§1 Let $\Gamma \leq \mathrm{Iso}(\mathbb{R}^2)$ be a subgroup.

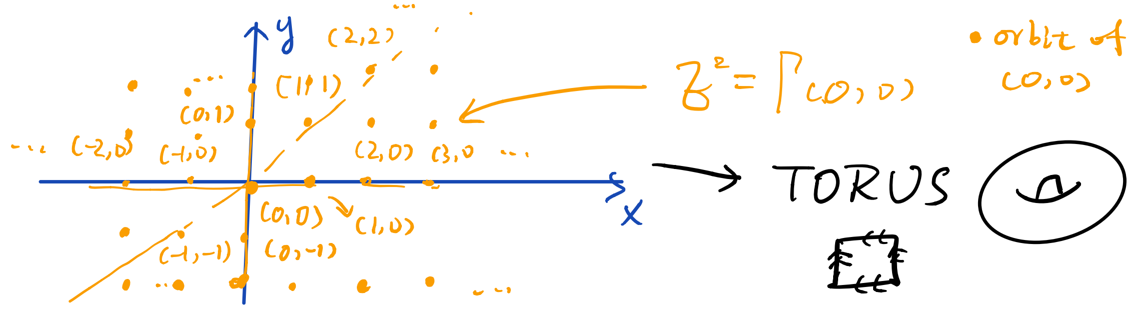
<u>Def^n</u> The Γ-orbit of $p \in \mathbb{R}^2$ is the set

$$\Gamma p := \{ g(p) : g \in \Gamma \}$$

<u>Ex.1</u> $\underbrace{\Gamma = \langle t_{(1,0)} \rangle}_{\text{subgroup gen. by } t_{(1,0)}} = \{ t^n_{(1,0)}, n \in \mathbb{Z} \} \leq \mathrm{Iso}(\mathbb{R}^2)$

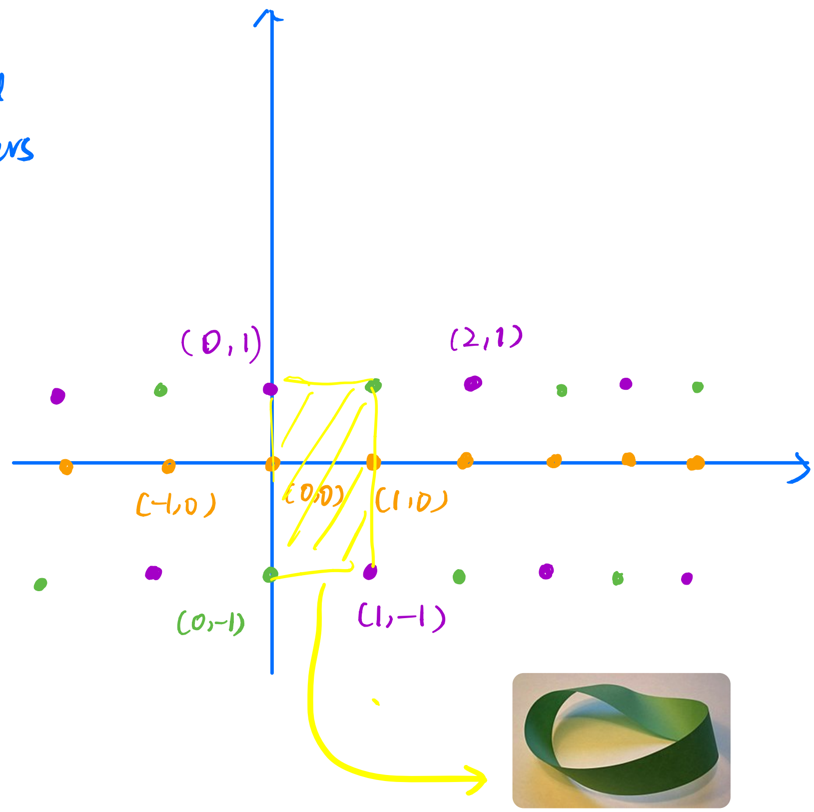


<u>Ex.2</u> $\Gamma = \langle t_{(1,0)}, t_{(0,1)} \rangle = \{ t_{(n,m)}, n,m \in \mathbb{Z} \}$



Ex. 3: $\Gamma = \langle t_{(1,0)} \circ \overline{r_{x\text{-axis}}} \rangle = \{ (t_{(1,0)} \circ \bar{r})^n, n \in \mathbb{Z} \}$

twisted cylinders (aka Mobius band)



● → orbit of (0,1)

● → orbit of (0,-1)

● → orbit of (0,0)

Def^n: The Euclidean surface S_Γ associated to a subgroup $\Gamma \leq \mathrm{Iso}(\mathbb{R}^2)$ is the set of Γ-orbits

i.e. $S_\Gamma := \{ \Gamma p : p \in \mathbb{R}^2 \} \left(= \mathbb{R}^2/\Gamma \right)$ ∎

Remark A point $q \in S_\Gamma$ is a Γ-orbit

§2. The distance in S_Γ: the Euclidean distance $(\mathbb{R}^2, d_{Euc})$ descends to $\mathbb{R}^2/\Gamma$ via

$$d_{S_\Gamma}(\Gamma P, \Gamma Q) := \min\{d_{Euc}(P', Q') \mid P' \in \Gamma P,\ Q' \in \Gamma Q\}$$

$$\underset{\text{Exercise}}{=} \min\{d_{Euc}(P, Q') : Q' \in \Gamma Q\}$$