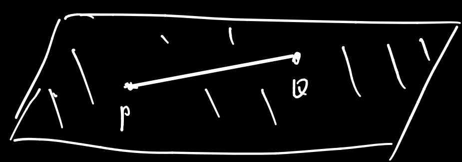
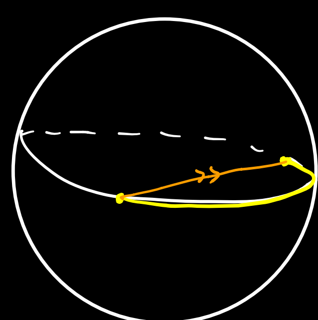
Lecture 14 : Midterm

Lecture 15 (Chap 3. Sect 3.1)

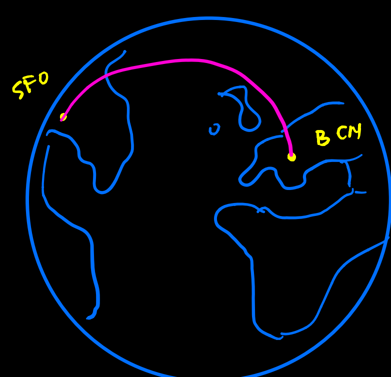
$\mathbb{R}^2$ $\qquad \psi(x,y)$ predict in x



$$S^2 = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2=1\}$$



Ex.



What are the geodesics of the 2-sphere?

"What is the shortest path between 2 points?"

"How do we compute in the 2-sphere S^2?"

§1. Euclidean Space $\mathbb{R}^2$ and the 2-sphere.

<u>Def</u> Euclidean Space $\mathbb{R}^3$ is the set $\mathbb{R}^3 = \{(x, y, z),\ x, y, z \in \mathbb{R}\}$ endowed with the distance

$$d(P, Q) = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

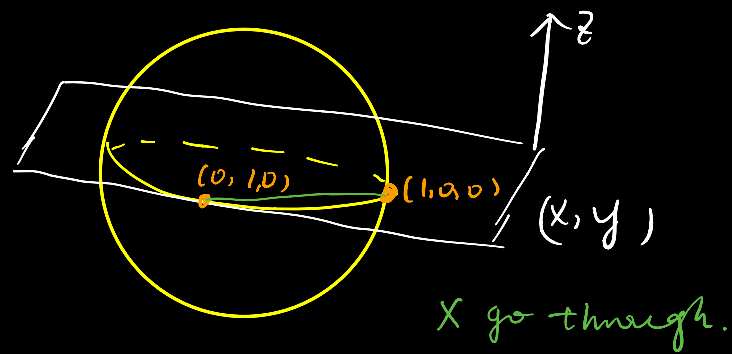
$P = (x, y, z)$ $Q = (x', y', z')$

<u>Def</u> The 2-sphere $S^2 \subseteq \mathbb{R}^3$ is the set of points $P \in \mathbb{R}^3$ s.t. $d(P, O) = 1$

With $O=(0,0,0)$ the origin.

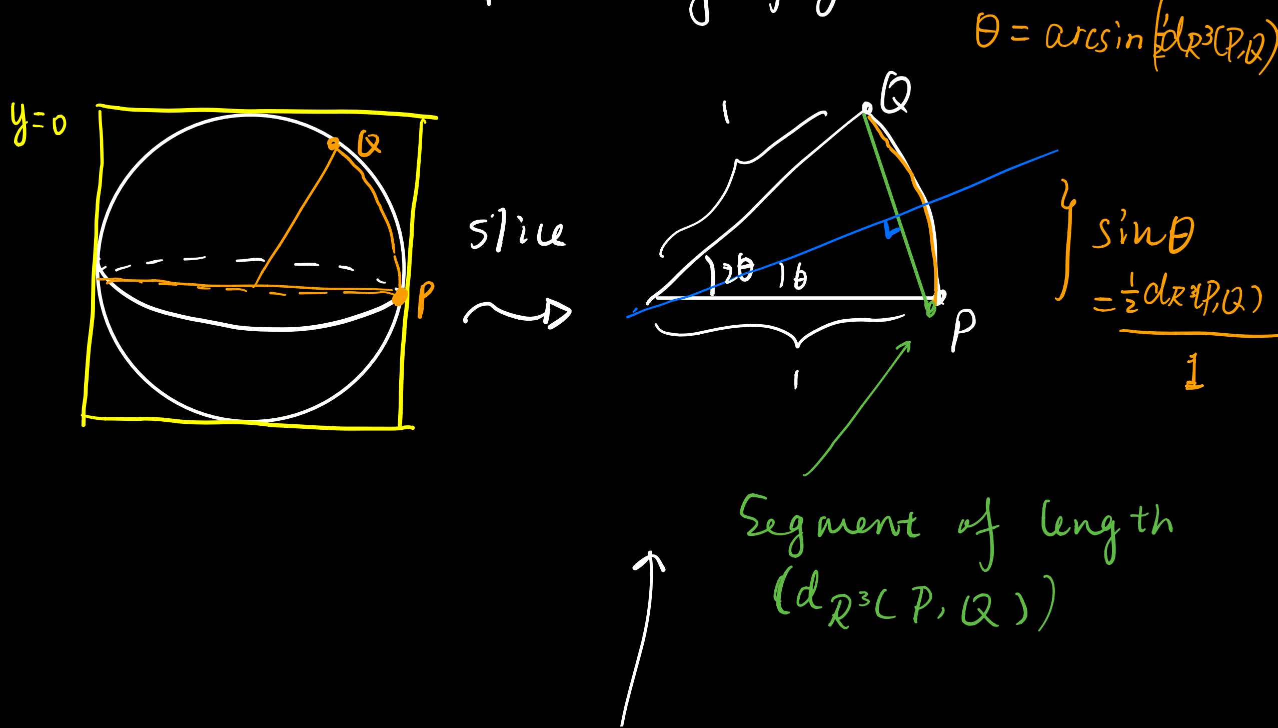
Equiv.
$$S^2=\{(x,y,z)\in\mathbb{R}^3 = \sqrt{x^2+y^2+z^2}=1\}$$

Pictorially



§2. Distance in S^2

Consider the following figure



Def (from the figure)

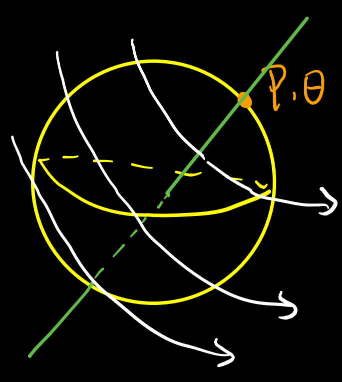
$$d_{S^2}(P, Q) := 2\theta = 2 \cdot \underbrace{\sin^{-1}}_{\text{arcsin}}\left(\frac{1}{2} d_{\mathbb{R}^3}(P, Q)\right)$$

$\Rightarrow$ lesson " we learn to compute distance in S^2 by computing in $\mathbb{R}^3$ <u>and</u> using sin "

§3. Isometry of (S^2, d_{S^2})

First, we observe that there exists two classes.

(1) Rotations : $R_{P,\theta}$



also $R_{A,\theta}$

$\downarrow$

A axis defined by $\vec{OP}$ in $\mathbb{R}^3$

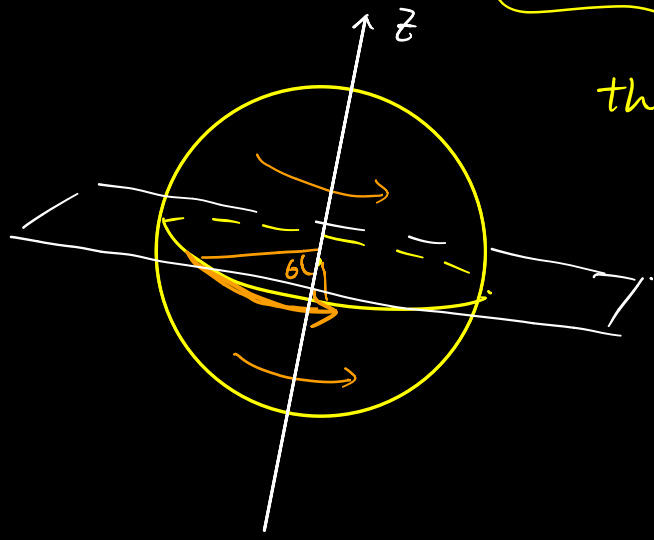
__Example:__ $R_{x,\theta}$ = rotation of angle θ along x-axis

[Same $R_{y,\theta}$ and $R_{z,\theta}$.]

In formula,

$$R_{z,\theta}(x,y,z) = (x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta,\ z)$$

this in $\mathbb{R}^2$ in $R_{(0,0,1),\theta}$



(2) Reflections along lines in S^2