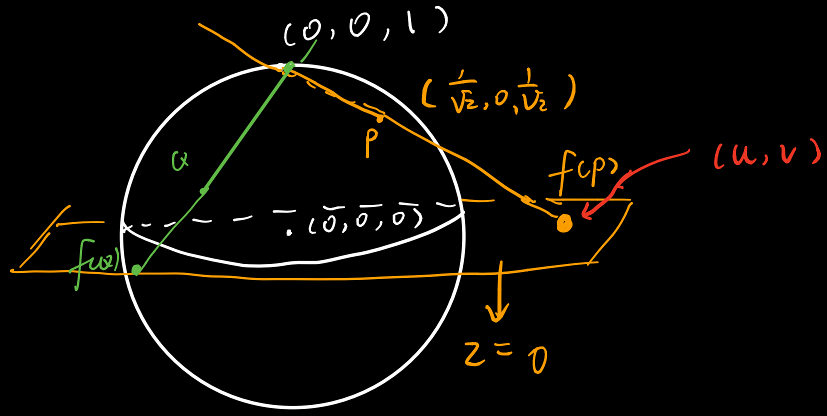


By definition, the stereographic project. is a bijection $\boxed{f: S^2 \setminus \{N\} \to \mathbb{R}^2}$

$(x, y, z) \qquad (u, v)$

Not iso. conformal

§ 1. Formula for Stereographic Proj.

$$f: S^2 \setminus \{N\} \longrightarrow \mathbb{R}^2$$

(x, y, z)

s.t. $x^2 + y^2 + z^2 = 1 \longrightarrow (u, v) \stackrel{?}{=} (u(x,y,z), v(x,y,z))$

North Pole: $z = 1$ being S^2

$z \neq 1$ Not being S^2

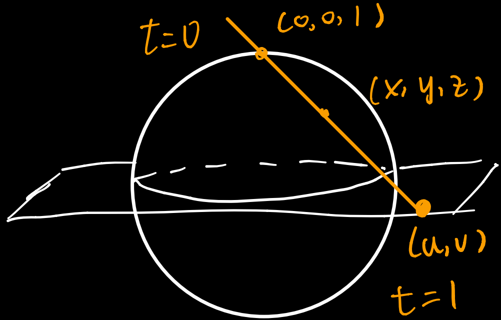
$\Updownarrow$

$t \neq 0$

Since f is bijection, $\exists f^{-1}$ which is $x = x(u, v)$
$y = y(u, v)$ $z = z(u, v)$

The geometric description of f states:

① Consider line through (x, y, z) and $(0,0,1)$ and (u,v) in $\mathbb{R}^2$



$t \in [0,1]$ a parameter.

$t=0 \to (0,0,1)$

$t=1 \to (u,v,0)$

$x = 0 + tu, \quad y = 0 + tv, \quad z = 1 - t.$

② Find $t \in [0,1]$ s.t. $x^2+y^2+z^2=1$, i.e. find the intersection of $S^2 \cap$ (orange segment above)

$$x^2+y^2+z^2=1 \qquad t \neq 1$$

$$\Updownarrow$$

$$(ut)^2+(vt)^2+(1-t)^2=1 \qquad t \neq 0$$

$$\Updownarrow$$

$$(u^2+v^2+1)\,t^2-2t+1=1 \qquad t \neq 0$$

$$\Updownarrow$$

$$(u^2+v^2+1)\,t=2$$

$$\boxed{t = \frac{2}{u^2+v^2+1}}$$

③ Now that $t = \frac{2}{u^2+v^2+1}$ is the intersection point

$$(u,v) \longmapsto (x,y,z) = \left(\frac{2u}{u^2+v^2+1}, \frac{2v}{u^2+v^2+1}, \frac{u^2+v^2-1}{u^2+v^2+1} \right)$$

refer : $x = 0 + tu$

$y = 0 + tv$

$z = 1 - t$

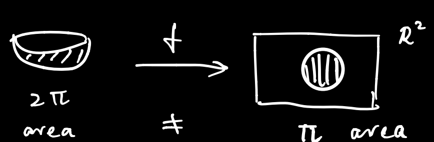
④ The map $f \; S^2 / \{N\} \longrightarrow \mathbb{R}^2$ is

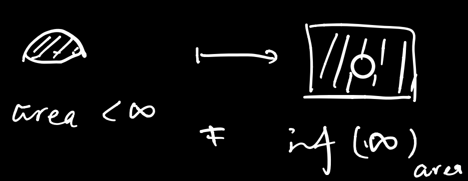
$$(x,y,z) \longmapsto (u,v) = \left(\frac{x}{1-z}, \frac{y}{1-z} \right)$$

refer $\underset{t=1-z}{u = \frac{x}{t}}$, $v = \frac{y}{t}$

Properties of Stereographic Proj.

(1) f does <u>not</u> preserve area. Thus f is <u>not</u> an isometry.





(2) f preserves area