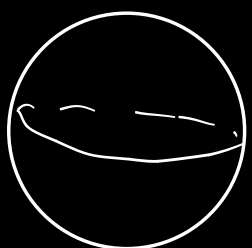
Disc 7

General procedures for rotating about $P = (p_x, p_y, p_z) \in S^2$ w/ angle θ (reflecting through a plane)

Want to rotate P to $(0,0,1)$



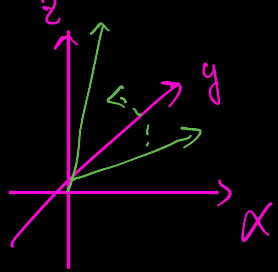
$$R_{p,\theta} = T^{-1} \cdot S \cdot T$$

$$R_{z,\theta} = \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

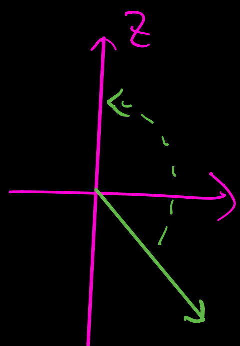
To find T, given $p = (p_x, p_y, p_z)$

1^{st} rotate into yz-plane ($P_x^* = 0$)

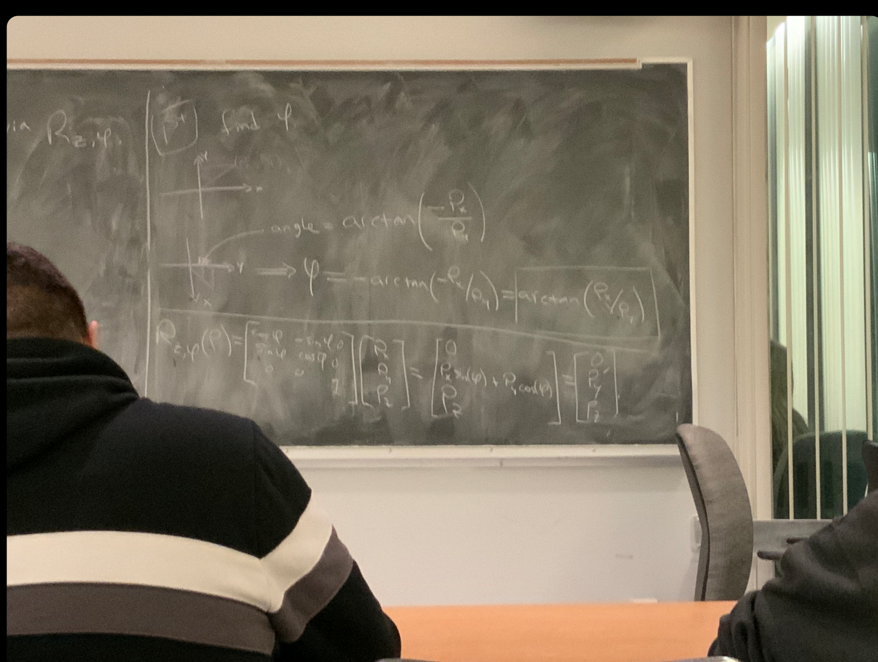
via $R_{z,\varphi}$



2^{nd} rotate to $(0,0,1)$ via $R_{x,\psi}$



$$T = R_{x,\psi} \circ R_{z,\varphi}$$

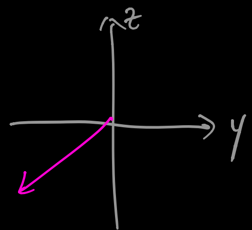


$T^{-1} \circ R_{z,\theta} \circ T$

Ex. Rotate about π about $p = (0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$.

1^{st} $\quad \varphi = 0$ $\qquad (\xi, -$

2^{nd}



$$\varphi = \frac{5\pi}{4}$$

$$\left(\varphi = \arctan\left(\frac{p_y}{p_z}\right) + \pi\right).$$

$$\left(= \arctan(1) + \pi\right)$$

$$T = R_{x,\frac{5\pi}{4}} \circ R_{z,0} = R_{x,\frac{5\pi}{4}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\frac{5\pi}{4} & -S\frac{5\pi}{4} \\ 0 & S\frac{5\pi}{4} & C\frac{5\pi}{4} \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 1 & & \\ & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \qquad = \begin{bmatrix} 1 & & \\ & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 \\ -1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

$$R_{p,\pi} = T^{-1} R_{z\pi} \circ T = T^{-1} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} T$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Check: If $R_{p,\pi}$ fix the axis

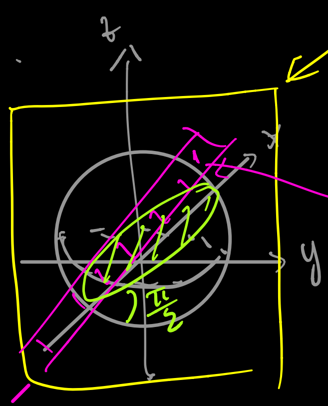
$\langle p \rangle = \{(0,t,t) \in \mathbb{R}^3 : t \in \mathbb{R}\}$.

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ t \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ t \end{bmatrix} \checkmark$$

Det $= 1$ ✓

Orthogonal matrix (colums form orthonormal basis) ✓

Q: Decompose $R_{p,\pi}$ into 2 refs. $\forall \xi_1 =$



$L_1 = \{x=0\}$ $\hat{n} = (1,0,0)$

ortho $\updownarrow$ $\xrightarrow{\text{ortho}}$ so angle $\frac{\pi}{2}$

90°

$L_2 = \{y=z\}$ $\hat{n} = (0,1,-1)$

$$R_{p\pi} = \vec{r}_{L_2} \circ \vec{r}_{L_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\Downarrow$

Same as above.