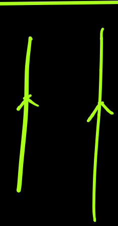
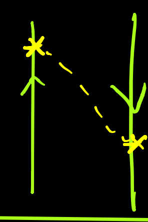
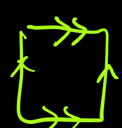
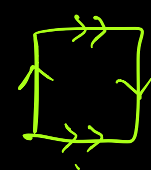
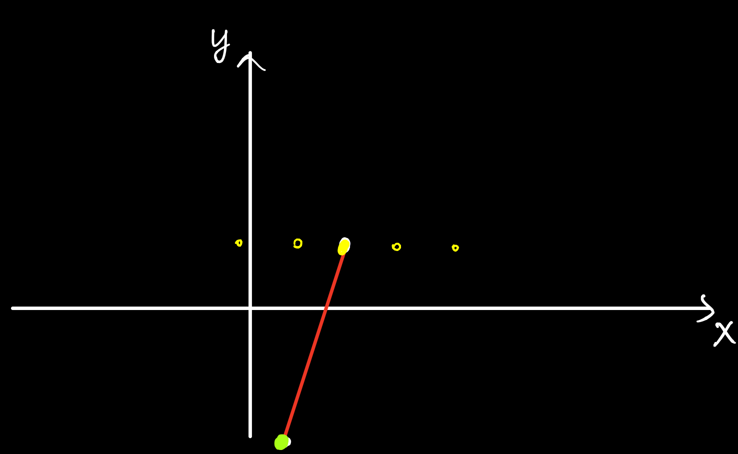
* Γ is a subgroup of $I(\mathbb{R}^2)$

$$\mathbb{R}^2/\Gamma = \{\Gamma(p), p \in \mathbb{R}^2\}$$

$$\Gamma(p) = \{g(p) : g \in \Gamma\}$$

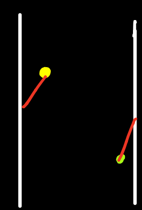
cylinder $\underline{C}$ 	$\underline{T}$wisted cylinder 
Torus $\underline{M}$ 	$\underline{K}$lein bottle 

* $(2.1, 1.25) \sim (0.8, -5) \in \mathbb{R}^2/\Gamma$



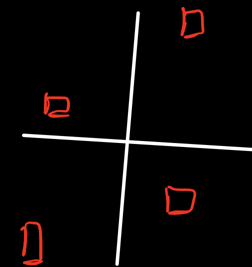
$d_c = \sqrt{(0.3)^2 + (6-2.5)^2}$

$d_M = \sqrt{(0.3)^2 + (3.75)^2}$



$d_T = \sqrt{(0.3)^2 + (0.25)^2}$

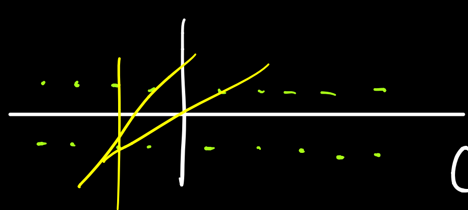
$d_K =$



* $\exists$ a line through any 2 points? $\boxed{\text{YES}}$

C: ✓ T: ✓

* That line is unique? $\boxed{\text{FALSE}}$



inf. Lines

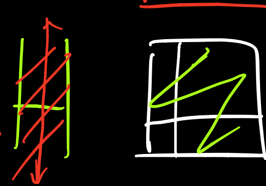
C: X T: X

* 2 lines meet in at most 1 point? $\boxed{\text{FALSE}}$

* All Lines has ∞-length? $\boxed{\text{FALSE}}$

C: X T: X

except horizontal line



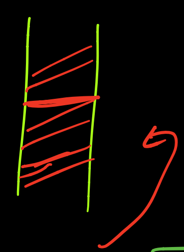
* Lines give shortest distance between 2 points? $\boxed{\text{TRUE}}$

C: √ T: √

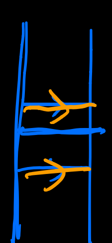
$d_C(\Gamma(p),\Gamma(Q)) = d_{\mathbb{R}^2}(p', Q')$ take the line in $\mathbb{R}^2$.

* Lines dont across themselves? $\boxed{\text{TRUE}}$

C: √ T: √

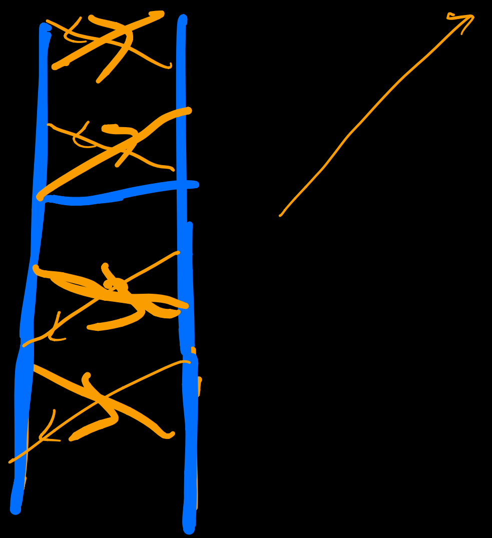


* ∃ Parallel lines? $\boxed{\text{TRUE}}$

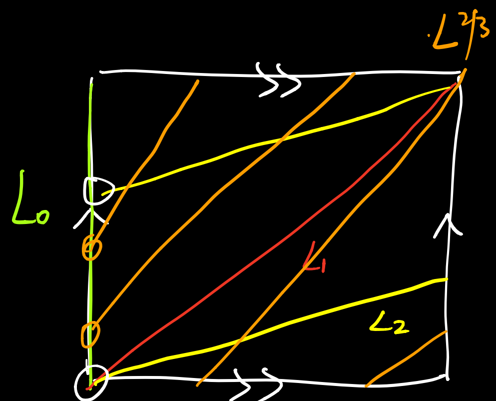


⇒ orange is one line

Mobius band



$$L_\alpha = \{x = \alpha y\} \subseteq T \qquad L_\alpha \cap L_0$$



$$|L_1 \cap L_0| = 1$$

$$|L_2 \cap L_0| = 2$$

$$|L_{2/3} \cap L_0| = 2.$$

$$\{x = 0\}$$

$$\begin{cases} \pi(L_0) = \bigcup\limits_{n \in \mathbb{Z}} \{x - n = 0\} \\[2ex] \pi(L_\alpha) = \bigcup\limits_{k,n \in \mathbb{Z}} \{x - k = \alpha(y - m)\} \end{cases}$$

$\pi(L_0) \cap \pi(L_\alpha)$ $\qquad x = n$

$\|$ $\qquad x - k = \alpha(y - m)$.

$\left\{ \left(n, \frac{n-k}{\alpha} + m\right) : n, k, m \in \mathbb{Z} \right\}$.

$= \left\{ \left(0, \frac{n-k}{\alpha}\right) : \quad \cdots \right\}$

$r = n - k$

$= \left\{ \left(0, \frac{r}{\alpha}\right) : r \in \mathbb{Z} \right\}$

- if α is rational, $\alpha = \frac{p}{q}$ in lowest.

  $p > 0$

  $\{ \ldots p \text{ elements} \ldots \}$

  $(0,0), \left(0, \frac{q}{p}\right), \left(0, \frac{q}{p} \cdot 2\right) \cdots \left(0, \frac{q}{p}(p-1)\right)$

- if α is irrational

  $\frac{r}{\alpha} \notin \mathbb{Z}$

then $\{(0, \frac{r}{\alpha}) \; r \in \mathbb{Z}\}$ is simplified.