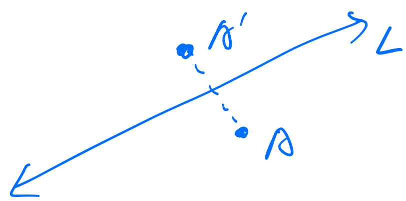
* Reflections
* Groups
* Symmetries

• What data is needed for a reflection?

Just a line L



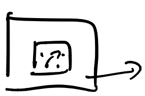
$\bar{r}_L(x,y) = (x,y)$

• Which point is $\boxed{\text{fixed}}$ by $\bar{r}_L$?

Exactly the points on line L

A function $f: A \to A$, "preserve" a subset $X \subseteq A$, $f: f(X) = X$ ($f(x) \in X$, for all $x \in X$)

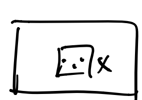
set X

 points move around box

↓ more restricted def.

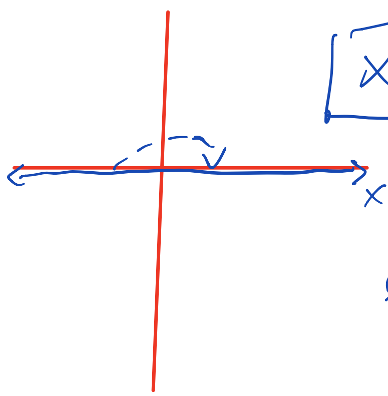
point x

f fixes X, $f: f(x) = x$, for all $x \in X$

$f|_X = id_X \to$ isometry(X) 

↳ points are pointed to one point.

Q: $f(x,y) = (-x,-y)$, is this a reflection? $\boxed{\text{NO}}$



$\boxed{\text{X axis}}$ is preserved.

↳ not fixed.

every pt should be fixed

↓

line fixed

ONLY fixed pt here: origin

f is a rotation $f = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{bmatrix}$

$= R_\pi$.

counter clock

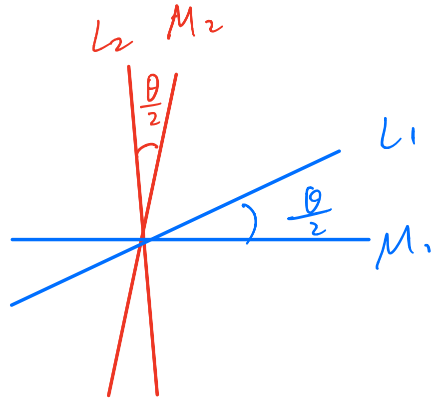
rotation c/w by $180°$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$(x,y) \to (-x,y)$ $\quad$ $(x,y) \to (x,-y)$.

<u>Thm</u> In general, $R_{\theta,p} = \overline{r_L}\,\overline{r_M}$ where L, M are any line

Q.



$$\overline{r_{L_1}}\,\overline{r_{M_1}} = \overline{r_{L_2}}\,\overline{r_{M_2}} = R_{\theta,p}$$

Non-unique factorization of rotation.

Take any line L, M, is $\overline{r_M}\,\overline{r_L} = \overline{r_L}\,\overline{r_M}$ NO

$$(\overline{r_M}\,\overline{r_L})\circ(\overline{r_L}\,\overline{r_M}) = \overline{r_M}(\overline{r_L}\circ\overline{r_L})\overline{r_M} = \overline{r_M}\,\overline{r_M} = id$$

(identity)

$$r_M r_L = (r_L r_M)^{-1} \quad (\overset{?}{=} r_L r_M)$$

maybe

$$r_L r_M = \begin{cases} \text{Rotation} \\ \text{Translation} \end{cases} \Rightarrow r_L r_M \neq (r_L r_M)^{-1} \quad \boxed{\text{except } R_\pi,\ id}.$$

GROUP THEORY

A Group is a pair $(G, *)$, G is a set, and a function (binary op)

$* : G \times G \to G$

- associativity $(g * h) * k = g * (h * k)$ for all g, h, k ex [illegible]
- identity $\exists\, e \in G$, $e * g = g * e = g$ $\forall g = G$ $e = id = f_0$
- Inverse $\forall g \in G$, $\exists$ element in G, g^{-1} s.t. $g * g^{-1} = g^{-1} * g = e$ $(f_{10})^{-1} = f_{-10}$

Ex. Positive isometry of $\mathbb{R}$

$$G = \{ f_a \; \mathbb{R} \to \mathbb{R},\ f_a(x) = x + a \mid a \in \mathbb{R} \}$$

$f_2(x) f_3(x) = x^2 + 5x + 6 \notin G$ $\boxed{\text{not ISO}}$

$* = \circ$, func. composition.

$(f_2 \circ f_3)(x) = (x + 3) + 2 = x + 5 = f_5(x).$

$f_2 \circ f_3 = f_5 \in G$

Note: Never said $g * h = h * g$

$(f_a \circ f_b) = f_{a+b} = f_{b+a} = f_b \circ f_a$, commutes

<u>Def</u> If $g * h = h * g$ $\forall g, h \in G$, G is called $\boxed{\text{abelian}}$

<u>otherwise</u> $\exists g, h \in G$, s.t. $g * h \neq h * g$

$\boxed{\text{non-abelian}}$

Q: do ref. $(\mathbb{R}^2)$ make a group?

<u>NO</u> $r_L r_m$ is not a reflection

They generate $Iso(\mathbb{R}^2)$, a non-abelian group

Q: What isometry preserve a

$\triangle$

if $f \in \mathrm{Isom}(\mathbb{R}^2)$ preserve $\triangle$

it preserves

if f fixes 3 vertices

... 2

say $f(1) = 1$