$$E[f(X)g(Y)] = \int_{\mathbb{R}^2} f(x)g(y)\rho_{X,Y}(x,y)\,dx\,dy. = \int_{\mathbb{R}^2} f(x)\rho_X(x)\,dx \cdot \int_{\mathbb{R}^2} g(y)\rho_Y(y)\,dy = \mathbb{E}[f(X)]\,\mathbb{E}[g(Y)].$$

$$\rho_X(x_1, \cdots, x_n) = \frac{1}{(2\pi)^{n/2}\,|\det \Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right) = \prod_{i=1}^{n} \rho_{X_i}(x_i).$$

$$\rho_{W_{t_2}|W_{t_1}}(x_2|x_1) = \frac{\rho_{W_{t_1},W_{t_2}}(x_1,x_2)}{\rho_{W_{t_1}}(x_1)}$$

$$\rho_{W_{t_1},W_{t_2}}(x_1,x_2) = \frac{1}{\sqrt{2\pi t_1}} e^{-\frac{(x_1-x_0)^2}{2t_1}} \frac{1}{\sqrt{2\pi(t_2-t_1)}} e^{-\frac{(x_2-x_1)^2}{2(t_2-t_1)}}.$$

$$\star\ \nu_{t_1,\cdots,t_n}(\Gamma_1 \times \cdots \times \Gamma_n) = \mathbb{P}((w_1,\cdots,w_n) \in \Gamma_1 \times \cdots \times \Gamma_n)$$
$$= \int_{\Gamma_1\times\cdots\times\Gamma_n} p(t_1, x_0, x_1)\, p(t_2-t_1, x_1, x_2) \cdots p(t_n-t_{n-1}, x_{n-1}, x_n)\, dx_1 \cdots dx_n.$$

$$p(t_1, x_0, x_1) \cdots p(t_n-t_{n-1}, x_{n-1}, x_n) = (2^n \pi^n \delta t_1 \cdots \delta t_n)^{-m/2} e^{-\sum_{i=1}^{n} \frac{1}{2\delta t_i}|x_i - x_{i-1}|^2}.$$

joint probability density function of $(W_{t_1}, \cdots, W_{t_n})$:

$$\frac{1}{(2\pi)^{mn/2}[(t_1-t_0)(t_2-t_1)\cdots(t_n-t_{n-1})]^{m/2}} \exp\left(-\frac{|x_1-x_0|^2}{2(t_1-t_0)} - \frac{|x_2-x_1|^2}{2(t_2-t_1)} - \cdots - \frac{|x_n-x_{n-1}|^2}{2(t_n-t_{n-1})}\right).$$