1. (a) $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

令 pumping length 为 $p=n$, $w \in L = uvxyz$. 设 $|w| > n$

对于 $|vy| > 0$, $|vxy| \leq n$, 有

1° $vxy = 0^* 1^*$ 则 $uv^2xy^2z = 0^{n+i}1^{n+j}0^n1^n / 0^n1^n0^{n+i}1^{n+j} \notin L$

2° $vxy = 1^* 0^*$ 则 $uv^2xy^2z = 0^n1^{n+i}0^{n+j}1^n \notin L$

∴ 矛盾, L 非 CFG.

(d) $\{t_1 \# t_2 \# \cdots \# t_k \mid k \geq 2, t_i \in \{a,b\}^*, t_i = t_j \text{ for some } i = j\}$

令 pumping length 为 $p=n$, 有 $w \in L = a^nb^n\#a^nb^n$

对于 $|vy| > 0$, $|vxy| \leq p$, 有 $(a^nb^n\#a^{n+i+j}b^n)$

1° $vxy = a^i$ 则 $uv^2xy^2z = a^{n+i+j}b^n\#a^nb^n \notin L$ (∵ $t_i \neq t_j$)

2° $vxy = b^i$ 则 同上

3° $vxy = a^ib^j$ 则 $uv^2xy^2z = a^{n+i}b^{n+j}\#a^nb^n / a^nb^n\#a^{n+i}b^{n+j} \notin L$

4° $vxy = b^i\#a^j$ 则 $uv^2xy^2z =$

① $x = \# \Rightarrow a^n b^{n+i} \# a^{n+j} b^n \notin L$

② otherwise $\Rightarrow a^n b^{n+i} \#^l a^{n+j} b^n \notin L$

综上，矛盾，得证.

2. 令 pumping length 为 $p = n$，有 $w \in L = 0^n 1^n 1^n 0^n$

对于 $|vy| = 0$，$|vxy| \le n$，有

① $vxy = 0^k$，则 $uv^2xy^2z = 0^{n+i}1^n1^n0^n / 0^n1^n1^n0^{n+i} \notin L$

② $vxy = 1^k$，则 $uv^2xy^2z = 0^n 1^{2n+i} 0^n \notin L$

③ $vxy = 0^i1^j$，则 $uv^2xy^2z = 0^{n+i}1^{n+j}1^n0^n \notin L$

④ $vxy = 1^i0^j$，则 $uv^2xy^2z = 0^n1^n1^{n+i}0^{n+j} \notin L$

∴ 矛盾.

3. a. $S \to 0A1 \mid 0A01 \mid 1A1 \mid 1A0$

$$P\begin{cases} A \to CAD \mid 1 \\ C \to 0 \mid 1 \mid 01 \mid 00 \mid 11 \mid 10 \\ D \to 0 \mid 1 \mid 01 \mid 00 \mid 11 \mid 10 \end{cases}$$

$G = (\{S, A, C, D\}, \{0, 1\}, P) = G_1$

显然，由G生成的string有长度为 $|\Sigma^i||\Sigma^j| = i+j+1$ 且极端情况下

$i = 2j \Rightarrow \frac{3j+1}{3} = j + \frac{1}{3} \Rightarrow j + \frac{1}{3} \sim 2j + \frac{2}{3} \Rightarrow 1$ 在此范围中.

得证.

(b) 令 pumping length 为 $p=n$，$w=0^{n+2}10^n10^{n+2}\in C_2=uvxyz$

对于 $|yz|>0$ 且 $|xyz|\le n$，有

① $vxy=0^i$，$|v^kxy^k|>4n+8$ 则 $uv^kxy^kz=0^m10^n10^{n+2}/0^{n+2}10^m10^{n+2}$

$0^{n+2}10^n10^m\notin L$

（$\because |uv^kxy^kz|>6n+12$，所有情况有1均不在middle third）

② $v=0^i, y=0^j, x=1$ 则 $uv^2xy^2z=0^{n+i+2}10^{n+j}10^{n+2}/$

$0^{n+2}10^{n+i}10^{n+2+j}$

$\notin L$

因此，矛盾，$C_2\notin CFL$.

4. INT(L)即为L中任一字符串的前缀

给定识别L的PDA：P_1，将P_1中的所有非终态S_i与终态F连边，

边上为$\varepsilon, \varepsilon \to \varepsilon$，则得$P_2$，有$P_2$识别INT(L)

5. $S \to AB \mid BC$

$A \to BA \mid a$

$B \to CC \mid b$

$C \to AB \mid a$

S A C				
B	B			
B	S, C			
S, C	A, S	S	A, S	
A, C	B	A, C	B	A, C

a b a b a

$\therefore ababa \in G$