Functional Dependency

Learning Maps

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2	Relational Databases
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Intro > Overview

☐ A : Voice and PPT Overview
☐ B : Text-based Overview
■ C : Video and PPT Overview

Opening Message	→ In this lesson, we will study the functional dependency concepts, Armstrong axioms and their' secondary rules, closure of a set of functional dependencies, closure of a set of attributes under a set of functional dependencies, algorithm to find a minimal key, equivalence of sets of functional dependencies, minimal set of a set of functional dependencies
Lesson topic	1. Functional Dependency 2. Armstrong 's Axioms and secondary rules 3. closure of a FD set, closure of a set of attributes 4. A minimal key 5. equivalence of sets of functional dependencies 6. Minimal Sets of FDs
Learning Goals	Upon completion of this lesson, students will be able to: 1. Recall the concepts of functional dependency, Armstrong 's axioms and secondary rulesl 2. Identify closure of a FD set, closure of a set of attributes 3. Find a minimal key of a relation under a set of FDs 4. Identify the equivalence of sets of FDs and find the minimal cover of a set of FDs

Intro > Keywords

Keyword	Description
Functional dependency	the values of the X component of a tuple uniquely (or **functionally**) determine the values of the Y component
Armstrong 's axioms	Axioms about FDs
closure of a FD set	the set of all dependencies that include F as well as all dependencie s that can be inferred from F
Closure of a set of attributes	All attributes that are functionally determined by them
A minimal key	A minimal set of attributes that determines R
Equivalence of S ets of FDs	Two sets of FDs, in which, this set covers the other
A minimal cover of a set of FDs	A set of dependencies in a standard or canonical form and with no re dundancies

Lesson > Topic 1: Functional Dependency

- 1.1. Introduction
- 1.2. Definition

1.1. Introduction

- We have to deal with the problem of database design: anomalies, redundancies
- The single most important concept in relational schema design theory

1.2. Definition

- Suppose that $R = \{A_1, A_2, \dots, A_n\}$, X and Y are non-empty subsets of R.
- A functional dependency (FD), denoted by $X \rightarrow Y$, specifies a constraint on the possible tuples that can form a relation state r of R. The constraint is that, for any two tuples t_1 and t_2 in r that have $t_1[X] = t_2[X]$, they must also have $t_1[Y] = t_2[Y]$.
  - X: the left-hand side of the FD
  - Y: the right-hand side of the FD

1.2. Definition (cont.)

- This means that the values of the X component of a tuple uniquely (or **functionally**) determine the values of the Y component.
- A FD $X \rightarrow Y$ is **trivial** if $X \supseteq Y$
- If X is a candidate key of R, then $X \rightarrow R$

1.2. Definition (cont.)

- Examples
  - AB → C

A	B	C	D
a1	b1	c1	d1
a1	b1	c1	d2
a1	b2	c2	d1
a2	b1	c3	d1

subject_id	name	credit	percentage_final_exam
IT3090	Databases	3	0.7
IT4843	Data integration	3	0.7
IT4868	Web mining	2	0.6
IT2000	Introduction to ICT	2	0.5
IT3020	Discrete Mathematics	2	0.7
IT3030	Computer Architectures	3	0.7

  - subject_id → name,
  - subject_id → credit,
  - subject_id → percentage_final_exam,
  - subject_id → {name, credit}

Lesson > Topic 2: Armstrong 's axioms

- 2.1. Armstrong 's axioms
- 2.2. Secondary rules
- 2.3. An example

2.1. Armstrong axioms

- $R = \{A_1, A_2, \ldots, A_n\}$, X, Y, Z, W are subsets of R.
- XY denoted for $X \cup Y$

- Reflexivity
  - If $Y \subseteq X$ then $X \rightarrow Y$
- Augmentation
  - If $X \rightarrow Y$ then $XZ \rightarrow YZ$
- Transitivity
  - If $X \rightarrow Y$, $Y \rightarrow Z$ then $X \rightarrow Z$

2.2. Secondary rules

- Union
  - If $X \rightarrow Y$, $X \rightarrow Z$ then $X \rightarrow YZ$.
- Pseudo-transitivity
  - If $X \rightarrow Y$, $WY \rightarrow Z$ then $XW \rightarrow Z$.
- Decomposition
  - If $X \rightarrow Y$, $Z \subseteq Y$ then $X \rightarrow Z$

2.3. An example

- Given a set of FDs: $F = \{AB \rightarrow C, C \rightarrow A\}$
- Prove: $BC \rightarrow ABC$
  - From $C \rightarrow A$, we have $BC \rightarrow AB$ (Augmentation)
  - From $AB \rightarrow C$, we have $AB \rightarrow ABC$ (Augmentation)
  - And we can conclude $BC \rightarrow ABC$ (Transitivity)

Lesson > Topic 3: closure of a FD set, closure of a set of attributes

- 3.1. Closure of a FD set
- 3.2. Closure of a set of attributes
- 3.3. A problem

3.1. Closure of a FD set

- Suppose that $F = \{A \to B, B \to C\}$ on $R(A, B, C, \ldots)$. We can infer many FD such as: $A \to C$, $AC \to BC, \ldots$
- Definition
  - Formally, the set of all dependencies that include F as well as all dependencies that can be inferred from F is called the **closure** of F, denoted by F^+.
- $F \vDash X \to Y$ to denote that the FD $X \to Y$ is inferred from the set of FDs F.

3.2. Closure of a set of attributes

- Problem
  - We have F, and $X \rightarrow Y$, we have to check if $F \models X \rightarrow Y$ or not
- Should we calculate F^+ ?

  $\Rightarrow$closure of a set of attributes
- Definition
  - For each such set of attributes X, we determine the set X^+ of attributes that are functionally determined by X based on F; X^+ is called the **closure of X under F**.

3.2. Closure of a set of attributes (cont.)

- To find the closure of an attribute set X^+ under *F*

Input: A set F of FDs on a relation schema R, and a set of attributes X, which is a subset of R.

$X^0 := X$;
repeat
    for each functional dependency $Y \rightarrow Z$ in F do
        if $X^{i-1} \supseteq Y$ then $X^i := X^{i-1} \cup Z$;
        else $X^i := X^{i-1}$
until (X^i unchanged);
$X^+ := X^i$

3.2. Closure of a set of attributes (cont.)

- An example
  - Given R(A, B, C, D, E, F) and F = {$AB \rightarrow C$, $BC \rightarrow AD$, $D \rightarrow E$, $CF \rightarrow B$}. Calculate $(AB)^+_F$
    - $X^0 = AB$
    - $X^1 = ABC$ (from $AB \rightarrow C$)
    - $X^2 = ABCD$ (from $BC \rightarrow AD$)
    - $X^3 = ABCDE$ (from $D \rightarrow E$)
    - $X^4 = ABCDE$
    - $(AB)^+_F = ABCDE$

3.3. A problem

- $X \rightarrow Y$ can be inferred from F if and only if $Y \subseteq X^+_F$
- $F \vDash X \rightarrow Y \Leftrightarrow Y \subseteq X^+_F$
- An example
  - Let R(A, B, C, D, E), $F = \{A \rightarrow B, B \rightarrow CD, AB \rightarrow CE\}$. Consider whether or not $F \vDash A \rightarrow C$
  - $(A)^+_F = ABCDE \supseteq \{C\}$

Lesson > Topic 4: A minimal key

- 4.1. Definition
- 4.2. An algorithm to find a minimal key
- 4.3. An example

4.1. Definition

- Minimal key
  - Given R(U), U = $\{A_1, A_2, \ldots, A_n\}$, a set of FDs F
  - K is considered as a minimal key of R if:
    - $K \subseteq U$
    - $K \rightarrow U \in F^+$
    - Với $\forall K' \subset K$, thì $K' \rightarrow U \notin F^+$
  - $K^+ = U$ and $K \backslash \{A_i\} \rightarrow U \notin F^+$

4.2. An algorithm to find a minimal key

- To find a minimal key

Input: $R(U)$, $U = \{A_1, A_2, \dots, A_n\}$, a set of FDs F

- $Step^0$ $K^0 = U$
- $Step^i$ If $(K^{i-1} \setminus \{A_i\}) \rightarrow U$ then $K^i = K^{i-1} \setminus \{A_i\}$
  else $K^i = K^{i-1}$
- $Step^{n+1}$ $K = K^n$

4.3. An example

Given R(U), U= {A, B, C, D, E}, F = {AB $\rightarrow$ C, AC $\rightarrow$ B, BC $\rightarrow$ DE}.
Find a minimal key
Step^0: K^0= U = ABCDE
Step^1: Check if or not (K^0\{A}) $\rightarrow$ U (i.e, BCDE $\rightarrow$ U).
(BCDE)$^+$= BCDE $\neq$ U. Vậy K^1 = K^0 = ABCDE
Step^2: Check if or not (K^1\{B}) $\rightarrow$ U (i.e, ACDE $\rightarrow$ U).
(ACDE)$^+$ = ABCDE = U. So, K^2 = K^1\{B} = ACDE
Step^3: K^3 = ACDE
Step^4: K^4 = ACE
Step^5: K^5 = AC
We infer that AC is a minimal key

Lesson > Topic 5: Equivalence of Sets of FDs

- 5.1. Definition
- 5.2. An example

5.1. Definition

- Definition.
  - A set of FDs F is said to cover another set of FDs G if every FD in G is also in F^+ (every dependency in G can be inferred from F).
- Definition.
  - Two sets of FDs F and G are equivalent if $F^+ = G^+$. Therefore, equivalence means that every FD in G can be inferred from F, and every FD in F can be inferred from G; that is, G is equivalent to F if both the conditions - G covers F and F covers G - hold.

5.2. An example

- Prove that $F = \{A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H\}$ and $G = \{A \rightarrow CD, E \rightarrow AH\}$ are equivalent
  - For each FD of F, prove that it is in G^+
    - $A \rightarrow C$: $(A)^+_G = ACD \supseteq C$, so $A \rightarrow C \in G^+$
    - $AC \rightarrow D$: $(AC)^+_G = ACD \supseteq D$, so $AC \rightarrow D \in G^+$
    - $E \rightarrow AD$: $(E)^+_G = EAHCD \supseteq AD$, so $E \rightarrow AD \in G^+$
    - $E \rightarrow H$: $(E)^+_G = EAHCD \supseteq H$, so $E \rightarrow H \in G^+$
    - $\Rightarrow F^+ \subseteq G^+$
  - For each FD of G, prove that it is in F^+ (the same)
    - $\Rightarrow G^+ \subseteq F^+$
  - $\Rightarrow F^+ = G^+$

Lesson > Topic 6: A minimal cover of a set of FDs

- 6.1. Definition
- 6.2. An algorithm to find a minimal cover of a set of FDs
- 6.3. An example

6.1. Definition

- Minimal Sets of FDs
  - A set of FDs F to be minimal if it satisfies:
    - Every dependency in F has a single attribute for its right-hand side.
    - We cannot replace any dependency $X \rightarrow A$ in F with a dependency $Y \rightarrow A$, where Y is a proper subset of X, and still have a set of dependencies that is equivalent to F.
    - We cannot remove any dependency from F and still have a set of dependencies that is equivalent to F.
  - a set of dependencies in a standard or canonical form and with no redundancies

6.2. An algorithm to find a minimal cover of a set of FDs

- Finding a Minimal Cover F for a Set of FDs G

Input: A set of FDs G.
1. Set F := G.
2. Replace each functional dependency $X \rightarrow \{A_1, A_2, ..., A_n\}$ in F by the n FDs $X \rightarrow A_1, X \rightarrow A_2, ..., X \rightarrow A_n$.
3. For each FD $X \rightarrow A$ in F
   for each attribute B that is an element of X
   if $\{\{F - \{X \rightarrow A\}\} \cup \{(X - \{B\}) \rightarrow A\}\}$ is equivalent to F
   then replace $X \rightarrow A$ with $(X - \{B\}) \rightarrow A$ in F.
4. For each remaining functional dependency $X \rightarrow A$ in F
if $\{F - \{X \rightarrow A\}\}$ is equivalent to F, then remove $X \rightarrow A$ from F.

6.3. An example

- $G = \{B \rightarrow A, D \rightarrow A, AB \rightarrow D\}$. We have to find the minimal cover of G.

- All above dependencies are in canonical form
- In step 2, we need to determine if $AB \rightarrow D$ has any redundant attribute on the left-hand side; that is, can it be replaced by $B \rightarrow D$ or $A \rightarrow D$?
Since $B \rightarrow A$ then $AB \rightarrow D$ may be replaced by $B \rightarrow D$.
We now have a set equivalent to original G, say G_1: $\{B \rightarrow A, D \rightarrow A, B \rightarrow D\}$.
- In step 3, we look for a redundant FD in G_1. Using the transitive rule on $B \rightarrow D$ and $D \rightarrow A$, we conclude $B \rightarrow A$ is redundant.
Therefore, the minimal cover of G is $\{B \rightarrow D, D \rightarrow A\}$

Remarks

- Functional dependencies
- Armstrong axioms and their’ secondary rules
- Closure of a set of FDs,
- Closure of a set of attributes under a set of FDs
- An algorithm to find a minimal key
- Equivalence of sets of FDs
- Finding a minimal set of a set of FDs

Quiz

No	Question (Multiple Choice)	Answer (1,2,3,4)	Commentary
1	Let R(A, B, C, D, E), F = {A →B, B→CD, AB→CE}. Which of the followings can be inferred by F? 1. A→C 2. C→A 3. C→E 4. B→AC	1	Since B→CD, we have B →C. Together with A → B, we can conclude A → C
2	Let F = {A→D, AB→DE, CE→G, E→H} Calculate $(AB)^{+}_{F}$	ABDEH	Reaction rate: $\frac{1}{6}\frac{d[CO_2]}{dt} = -\frac{1}{2}\frac{d[C_3H_6]}{dt}$ Formation rate of CO_2: $R_{CO_2} = \frac{d[CO_2]}{dt}$ Consumption rate of C_3H_6 $R_{C_3H_6} = -\frac{d[C_3H_6]}{dt}$
3	R = {A,B,C}, F = {A→BC, B→C, A→B, AB→C}. Tìm p hủ tối thiểu của F?	Fc = {A→B, B→C}	

→You have just learnt the following topics:
Functional dependencies
Armstrong axioms and their' secondary rules
Closure of a set of FDs,
Closure of a set of attributes under a set of FDs
An algorithm to find a minimal key
Equivalence of sets of FDs
Finding a minimal cover of a set of FDs

Next lesson:

Normalization