

HOMework WEEK 1

EX1:

a)

Marginal dist:

$$p(X = x_1) = p(x_1, y_1) + p(x_1, y_2) + p(x_1, y_3) = 0.01 + 0.05 + 0.1 = 0.16$$

Similarly:

$$p(X = x_2) = 0.17$$

$$p(X = x_3) = 0.11$$

$$p(X = x_4) = 0.22$$

$$p(X = x_5) = 0.34$$

It's the same with

$$\begin{aligned} p(Y = y_1) &= p(x_1, y_1) + p(x_2, y_1) + p(x_3, y_1) + p(x_4, y_1) + p(x_5, y_1) \\ &= 0.01 + 0.02 + 0.03 + 0.1 + 0.1 = 0.26 \end{aligned}$$

$$p(Y = y_2) = 0.47$$

$$p(Y = y_3) = 0.27$$

b)

To compute conditional dist we have the formula:

$$p(x|Y = y_i) = \frac{p(x, y_i)}{p(Y = y_i)}$$

So conditional dist of $p(x|Y = y_1)$:

$$p(x = x_1|Y = y_1) \approx 0.038$$

$$p(x = x_2|Y = y_1) \approx 0.077$$

$$p(x = x_3|Y = y_1) \approx 0.115$$

$$p(x = x_4|Y = y_1) \approx 0.385$$

$$p(x = x_5|Y = y_1) \approx 0.385$$

Similar with $p(x|Y = y_3)$:

$$p(x = x_1|Y = y_3) \approx 0.370$$

$$p(x = x_2|Y = y_3) \approx 0.185$$

$$p(x = x_3|Y = y_3) \approx 0.111$$

$$p(x = x_4|Y = y_3) \approx 0.185$$

$$p(x = x_5|Y = y_3) \approx 0.148$$

EX2:

a)

Xác suất để người được hỏi sử dụng 2 sản phẩm là:

$$p(x, y) = p(x|y) * p(y) = 0.365 * 0.5 = 0.1825$$

Vậy sẽ có 18,25% người được hỏi sẽ sử dụng cả 2 sản phẩm X và Y

b)

Áp dụng Bayes ta có:

$$\begin{aligned} p(Y|\bar{X}) &= \frac{p(\bar{X}|Y) * p(Y)}{p(\bar{X})} \\ &= \frac{(1 - p(X|Y)) * p(Y)}{1 - p(X)} \approx 0.4 \end{aligned}$$

EX4:

$$\begin{aligned} Var_X &= E_X[(X - E_X[X])^2] \\ &= E_X[X^2 - 2XE_X[X] + (E_X[X])^2] \\ &= E_X[X^2 - 2E_X[X * E_X[X]] + (E_X[X])^2] \\ &= E_X[X^2] - 2E_X[X] * E_X[X] + (E_X[X])^2 \\ &= E_X[X^2] - 2(E_X[X])^2 + (E_X[X])^2 \\ &= E_X[X^2] - (E_X[X])^2 \end{aligned}$$

