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Biến đổi thuật toán PCA.

Tập hợp đầu vào

$$X = \{x_1; x_2; x_3; \dots x_n\} \ (x_i \in \mathbb{R}^D)$$

$$X = \begin{bmatrix} x_1^T \\ x_2^T \\ \dots \\ x_n^T \end{bmatrix} \in \mathbb{R}^{N \times D}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_N \end{bmatrix}$$

$$Z = X . B$$

$$\Leftrightarrow \begin{bmatrix} z_1^T \\ z_2^T \\ \dots \\ z_N^T \end{bmatrix} = \begin{bmatrix} x_1^T \\ x_2^T \\ \dots \\ x_N^T \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_N \end{bmatrix} = \begin{bmatrix} x_1^T b_1 & x_1^T b_2 & \dots & x_1^T b_M \\ x_2^T b_1 & x_2^T b_2 & \dots & x_2^T b_M \\ & \dots & & \\ x_N^T b_1 & x_N^T b_2 & \dots & x_N^T b_M \end{bmatrix}$$

Maximize variance :

$$V_1 = \frac{1}{N} \sum_{n=1}^{N} z_{1n}^2 \text{ where } z_{1n}^2 = x_i^T b_1$$

$$V_1 = \frac{1}{N} \sum_{n=1}^{N} (x_i^T b_1)^2 = \frac{1}{N} \sum_{n=1}^{N} b_1^T x_i \, x_i^T b_1 = b_1^T \left(\frac{1}{N} \sum_{n=1}^{N} x_i x_i^T \right) b_1$$

$$= b_1^T S \, b_1$$

$\rightarrow$ maximize $b_1^T S b_1$ subject to $\|b_1\|^2 = 1$

Langrangian

$$L = b_1^T S b_1 + \alpha (1 - b_1^T b_1)$$

$$\frac{\partial L}{\partial b_1} = 0 \Leftrightarrow 2 S b_1 - 2 \alpha b_1 = 0 \Leftrightarrow S b_1 = \alpha b_1$$

We can see that b_1, α is an eigenvector and eigen value of S

$$\frac{\partial L}{\partial \alpha} = 0 \Leftrightarrow b_1^T b_1 = 1$$

$$\rightarrow var = b_1^T S b_1 = b_1^T \alpha b_1 = \alpha b_1^T b_1 = \alpha.$$